Spin Hall Effect in a 2DEG in the presence of magnetic couplings

Roberto Raimondi

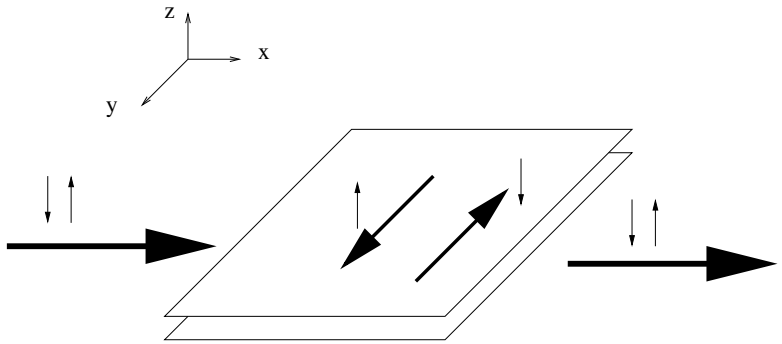
Coworkers:
Peter Schwab (Augsburg)
Cosimo Gorini (Augsburg)
Michael Dzierzawa (Augsburg)
Mirco Milletarì (Lancaster)

- Phys. Rev. B **74**, 035340 (2006)
- Physica E **40** , 1078 (2008)
- arXiv:0801.1786

Outline

1. SHE in a 2DEG: why vanishes
2. Finite SHE: in-plane magnetic field
3. Finite SHE: Magnetic impurities
4. Sketch of the derivation
5. Conclusions

SHE in a 2DEG: why is zero



Definition

$$j^z_{s,y} = \sigma_{sH}\mathcal{E}_x$$

$$H_{so} = \alpha \mathbf{p} \times \hat{\mathbf{e}}_z \cdot \boldsymbol{\sigma}, \ \sigma^0_{sH} = \frac{|e|}{8\pi}$$

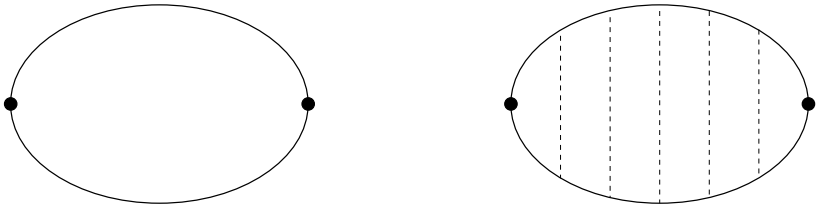
- J. Sinova et al., Phys. Rev. Lett. **92**, 126603 (2004).

Continuity-like equation

$$\frac{\partial s_y}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{j}^y_s = -2m\alpha j^z_{s,y}$$

- O. V. Dimitrova, Phys. Rev. B **71**, 245327 (2005).
- O. Chalaev and D. Loss, Phys. Rev. B **71**, 245318 (2005).
- E. I. Rashba, Phys. Rev. B **70**, 201309(R) (2004).

SHE in a 2DEG: how it happens



Vertex corrections cancel bubble

- J. I. Inoue, et al. Phys. Rev. B **70**, 041303(R) (2004).
- E. G. Mishchenko, et al., Phys. Rev. Lett. **93**, 226602 (2004).
- R. Raimondi and P. Schwab, Phys. Rev. B **71**, 033311 (2005).
- A. Khaetskii, Phys. Rev. Lett. **96**, 056602 (2006).

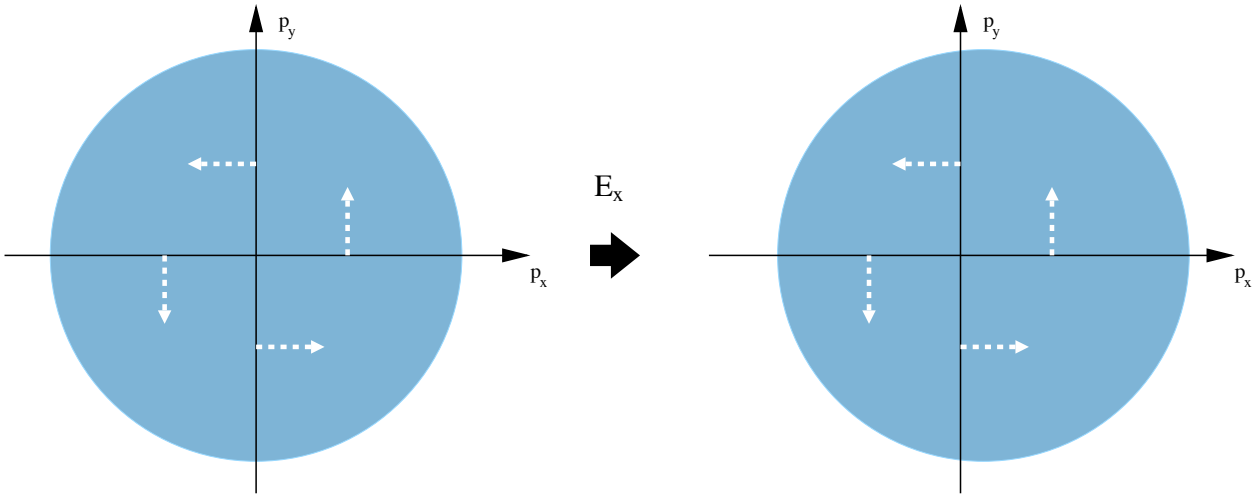
Connection with in-plane polarization

Physical insight from the quasiclassical Green function approach

$$j^z_{s,y} \propto \sigma^0_{sH}\mathcal{E}_x + \frac{1}{8\pi\alpha N_0\tau} s^0_y, \quad \sigma^0_{sH} = \frac{|e|}{8\pi}$$

- R. Raimondi, et al., Phys. Rev. B **74**, 035340 (2006).

Electrically-induced in-plane spin polarization



Effective spin-orbit field

$$s_y = -N_0 \alpha \delta p$$

$$\delta p = |e| \mathcal{E}_x \tau$$

Edelstein's result

$$s_0 = -N_0 |e| \alpha \tau \mathcal{E}_x$$

- V. M. Edelstein, Solid State Commun. **73**, 233 (1990).

Spin relaxation

Bloch's equations

$$\begin{aligned}\partial_t s_x &= -\frac{1}{\tau_s} s_x \\ \partial_t s_y &= -\frac{1}{\tau_s}(s_y + s_0) \\ \partial_t s_z &= -\frac{2}{\tau_s} s_z\end{aligned}$$

$$\tau_s^{-1} = 2\alpha p_F(\alpha p_F \tau)$$

Dyakonov-Perel relaxation

The static solution allows in-plane spin polarization only

$$s_y = -s_0, \quad s_z = 0$$

Question

What changes in order to have a finite SHE?

In-plane magnetic field (Cf. Milletarì's talk)

$$H_{mf} = \frac{1}{2} g_L \mu_B \mathcal{B}_x \hat{\mathbf{e}}_x \cdot \boldsymbol{\sigma}$$

Zeeman energy $2\omega_s \equiv g_L \mu_B \mathcal{B}_x$

Continuity-like equation with extra term

$$\frac{\partial s_y}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{j}_s^y = -2m\alpha j_{s,y}^z + 2\omega_s s_z$$

Related work:

- Q. Lin, et al., Appl. Phys. Lett. **88**, 122105 (2006).
- H. A. Engel, et al., Phys. Rev. Lett. **98**, 036602 (2007).

$$\partial_t s_x = -\frac{1}{\tau_s}(s_x - s_x^{eq})$$

$s_x^{eq} = N_0 \omega_s$ depends on magnetic field

$$\partial_t s_y = -\frac{1}{\tau_s}[s_y + s_1] + 2\omega_s s_z$$

s_1 and s_2 depend on electric field

$$\partial_t s_z = -\frac{2}{\tau_s} s_z - 2\omega_s [s_y + s_2]$$

If $s_1 = s_2$ no out-of-plane polarization!

With short-range disorder $s_1 = s_2 = s_0 = -|e| N_0 \alpha \tau \mathcal{E}_x$

Magnetic field with long-range disorder

Scattering kernel

$$W(\theta = \phi - \phi') = V_0 + 2V_1 \cos(\phi - \phi') + 2V_2 \cos(2\phi - 2\phi') + \dots .$$

Naively, one would use the transport time:

$$\tau \to \tau_{tr} \quad \tau_{tr}^{-1} = N_0 \int_0^{2\pi} \mathrm{d}\theta\, W(\theta)(1 - \cos(\theta)) = \frac{1}{\tau} \frac{V_0 - V_1}{V_0}$$

However:

- It would give no effect, since $s_1 = s_2 = -s_0 \tau_{tr}/\tau$
- But it is also wrong!

The correct generalization: more scattering times

$$s_1 = -s_0 \frac{\tau_E}{\tau}, \quad s_2 = -s_0 \frac{\tau_{tr}}{\tau} \qquad \frac{1}{\tau_E} = N_0 \int_0^{2\pi} \mathrm{d}\theta\, W(\theta)(1-\cos(2\theta)) = \frac{V_0 - V_2}{\tau V_0}$$

Consequence from Bloch's equations: magnetic field dependence

$$\begin{aligned} s_y &= -\frac{V_0\, s_0}{V_0 - V_2} \frac{1 + 2\omega_s^2 \tau_s^2 (V_0 - V_2)(V_0 - V_1)^{-1}}{1 + 2\omega_s^2 \tau_s^2} \\ s_z &= -\frac{V_0\, s_0}{V_0 - V_2} \frac{V_1 - V_2}{V_0 - V_1} \frac{\omega_s \tau_s}{1 + 2\omega_s^2 \tau_s^2} \end{aligned}$$

For SHE see Milletarì's talk

Note also the Dyakonov-Perel relaxation time renormalization

$$\tau_s^{-1} = 2\alpha p_F (\alpha p_F \tau_{tr})$$

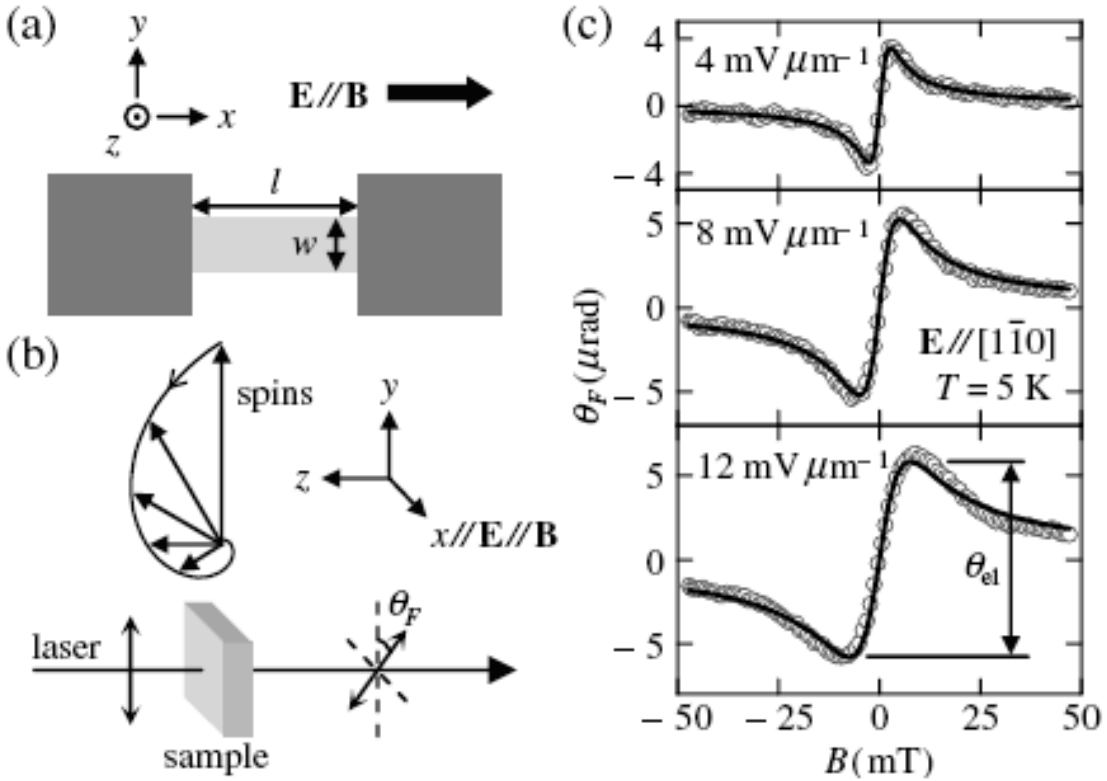
Physical interpretation

$$s_y \sim \delta n_+ - \delta n_- \quad \delta n_\pm = \frac{\partial \epsilon_{F\pm}}{\partial p_x} \delta p_{\pm x} N_\pm \;\; \delta p_{\pm x} = -|e| \mathcal{E}_x \tau_{tr,\pm}$$

The new time τ_E means a modification of the Edelstein result due to different DOS and transport times in the spin-orbit splitted bands

$$s_y = \frac{v_F}{4}(N_+ \tau_{tr,+} - N_- \tau_{tr,-})|e|\mathcal{E}_x = -s_0 \frac{V_0}{V_0 - V_2}$$

Experimental relevance



InGaAs

Faraday rotation spectroscopy $\theta_F \propto s_z$

$w = 60\mu m$ and $l = 200\mu m$

Some estimates:

- $\tau_s \sim 10ns$ $\tau_{tr} \sim 1ps$
- $\alpha p_F \sim 10^{-5} eV$
- $\omega_s \sim 10^{-6} eV$

- Y.K. Kato, R.C. Myers, A.C. Gossard, and D.D. Awshalom, Phys. Rev. Lett. **93**, 176601 (2004).

Magnetic impurities with short-range correlations

$$H_{mi} = V_m(\mathbf{x})\mathbf{S}\cdot\boldsymbol{\sigma}$$

Related work

- J. Inoue, et al., Phys. Rev. Lett. **97**, 046604 (2006).
- P. Wang, et al., Phys. Rev. B **75**, 075326 (2007).

Spin-flip time

$$\tau_{sf}^{-1} = 2\pi N_0 V_m^2 S(S+1)$$

Continuity-like equation with extra source terms

$$\frac{\partial s_y}{\partial t} + \frac{\partial}{\partial \mathbf{x}}\cdot \mathbf{j}_s^y = -2m\alpha j_{s,y}^z - \frac{4}{3\tau_{sf}} s_y$$

For weak magnetic scattering

$\tau \ll \tau_{sf}$

$$\sigma_{sH} = \frac{|e|}{3\pi}\frac{\tau}{\tau_{sf}}$$

However, one can do better!
What about $\tau \sim \tau_{sf}$?

Let us go back to Bloch's equations

$$\partial_t s_x = -\left(\frac{1}{\tau_s} + \frac{4}{3\tau_{sf}}\right) s_x$$

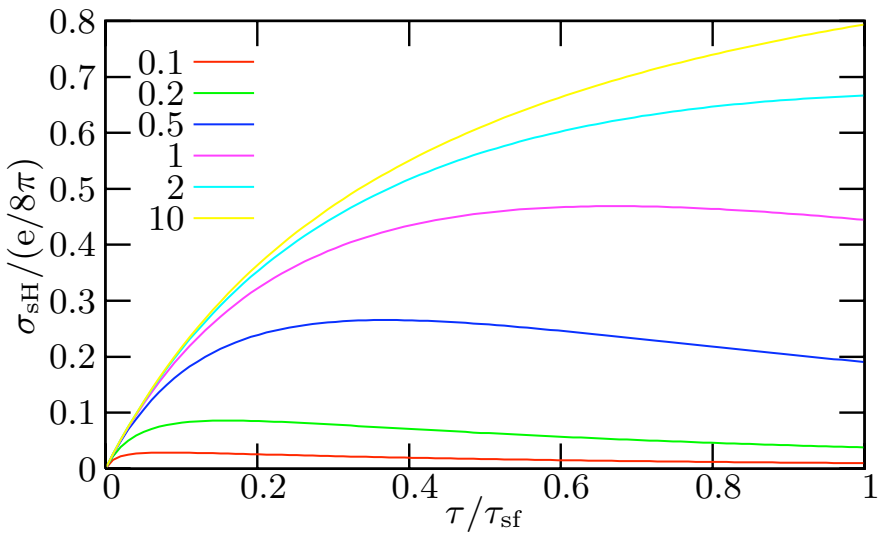
$$\partial_t s_y = -\left(\frac{1}{\tau_s} + \frac{4}{3\tau_{sf}}\right) [s_y + \tilde{s}_0]$$

$$\partial_t s_z = -\left(\frac{2}{\tau_s} + \frac{4}{3\tau_{sf}}\right) s_z$$

$$\tilde{s}_0 = -s_0 \frac{\tilde{\tau}_E}{\tau} \quad \tau_s^{-1} = 2\alpha p_F (\alpha p_F \tau_{tr})$$

$$\frac{1}{\tilde{\tau}_E} = \left(\frac{1}{\tau_E} + \frac{1}{\tau_{sf}}\right)\left(1 + \frac{4\tau_s}{3\tau_{sf}}\right) + \frac{4}{3\tau_{sf}}$$

$$\tau_{tr}^{-1} = \frac{1}{\tau}\frac{V_0 - V_1}{V_0} + \frac{1}{\tau_{sf}}$$



Different colors for different value of $\alpha p_F \tau$ and $V_1 = V_2 = 0$

SHE

$$\sigma_{sH} = \frac{|e|}{3\pi}\frac{\tilde{\tau}_E}{\tau_{sf}}$$

when $\tau_{sf} \sim \tau$, $\sigma_{sH} \longrightarrow$ the universal value

Relevant for mercury telluride-cadmium telluride quantum wells

- Konig et al. Science **318**, 766 (2007)
- Wen Yang, Kai Chang and Shou-Cheng Zhang, PRL **100**, 056602 (2008)
- Bernevig, Hughes, Shou-Cheng Zhang, Science **314**, 1757 (2006)

Sketch of calculation

Quasiclassical Green function

$$\check{g} \equiv \check{g}_{t_1 t_2}(\hat{\mathbf{p}}; \mathbf{x}) = \frac{\mathrm{i}}{\pi} \int \mathrm{d}\xi \, \check{G}_{t_1 t_2}(\mathbf{p}, \mathbf{x}), \;\; \check{G} = \begin{pmatrix} G^R & G \\ 0 & G^A \end{pmatrix}$$

Eilenberger-like equation (Phys. Rev. B **74**, 035340 (2006))

$$\partial_t \check{g} = -\frac{1}{2} \sum_{\mu=\pm} \left\{ \frac{\mathbf{p}_\mu}{m} + \frac{\partial}{\partial \mathbf{p}} (\mathbf{b}_\mu \cdot \boldsymbol{\sigma}), \frac{\partial}{\partial \mathbf{x}} \check{g}_\mu \right\} - \mathrm{i} \sum_{\mu=\pm} [\mathbf{b}_\mu \cdot \boldsymbol{\sigma}, \check{g}_\mu] - \mathrm{i} \left[\check{\Sigma}, \check{g} \right]$$

$$\mathbf{b}(\mathbf{p}) = \alpha \mathbf{p} \times \hat{\mathbf{e}}_z - \omega_s \hat{\mathbf{e}}_x \;\; \check{\Sigma} = -\mathrm{i} \left(\frac{1}{2\tau} \langle K \check{g} \rangle + \frac{1}{6\tau_{sf}} \sum_l \sigma_l \langle \check{g} \rangle \sigma_l \right)$$

Disorder parameter

$$\frac{1}{\epsilon_F \tau} \ll 1$$

Leading terms

SO parameter

$$\frac{\alpha p_F}{\epsilon_F} = \frac{\alpha}{v_F} \ll 1$$

Leading terms

Additional parameter

$\alpha p_F \tau \ll 1$	diffusive
$\alpha p_F \tau \gg 1$	clean

Conclusions

- Magnetic couplings yield finite SHE in a 2DEG
- Effective Bloch's equation for spin dynamics
- Different time scales with long-range disorder
- Future: inclusion of quantum corrections and extrinsic effects