# 表 <br> UNIVERSITÀ DEGLI STUDI 

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# Measurement of $\eta$-meson production in $\gamma \gamma$ interactions and $\Gamma(\eta \rightarrow \gamma \gamma)$ with the KLOE detector 

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## Cecilia Taccini

Three quarks for Muster Mark!
Sure he has not got much of a bark
And sure any he has it's all beside the mark.

## - James Joyce, Finnegans Wake -

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## Introduction

Photon-photon interactions are forbidden in classical electrodynamics. According to Maxwell's linear equations electromagnetic waves cross each other without any disturbance. In quantum electrodynamics (QED), however, the uncertainty principle allows a photon of energy $E_{\gamma}$ to fluctuate into states of charged particle pairs with mass $m_{\text {pair }}$ and lifetime $\Delta t \approx 2 E_{\gamma} / m_{\text {pair }}^{2}$, and photon-photon scattering becomes possible due to the interaction of the intermediate particles. The cross section for photon-photon interactions is of the order $\alpha^{4}$, but increases with the beam energy $E$ like $\left(\log E / m_{e}\right)^{2}$, and therefore already dominates over the $O\left(\alpha^{2}\right) e^{+} e^{-}$annihilation process for beam energies of a few GeV . Experimentally it is difficult to collide high energy photon beams. A simple way of avoiding this problem is to use virtual particles, e.g., the quantum fluctuations of an electron into an electron-photon state. This is done at electron and positron colliders. In the basic diagram of photon-photon reactions both the incoming $e^{+}$ and $e^{-}$radiate a photon, predominantly at small angles and with small energies, and the two photons produce the final state $X$. Photon-photon production of neutral mesons provides basic information on their internal structure. The strength of the coupling, measured by the partial decay width $\Gamma(X \rightarrow \gamma \gamma)$, is related to the quark content of the meson and gives information on the relations between the hadronic state and its $q \bar{q}$ representation. The measurement of the radiative width of pseudoscalar mesons is indeed a crucial input for the determination of the pseudoscalar mixing angle and for testing the valence gluon content in the $\eta^{\prime}$ wavefunction. Moreover, a precise study of the form factors of the transition $\gamma \gamma^{*} \rightarrow X$, where one photon is off-shell and the other is real, is of particular interest in evaluating the light-by-light contribution to the anomalous magnetic moment of the muon.
Photon-photon interactions in electron-positron colliders were pioneered in the 1970s at ADONE in Frascati and since then have been used to study the production of hadrons in almost all $e^{+} e^{-}$ colliders in a variety of conditions in low- and high $-\eta^{2}$ processes. In particular, measurements of the $\gamma \gamma$ partial width of $\eta$ and $\eta^{\prime}$ mesons have been done measuring the $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta\left(\eta^{\prime}\right)$ cross section.
This thesis is focused on the measurement of the cross section $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$ and the extraction of the partial width $\Gamma(\eta \rightarrow \gamma \gamma)$ with the KLOE detector at the $\phi$-factory DAФNE. DA $\Phi$ NE is an $e^{+} e^{-}$collider that operates at the mass of the $\phi$ resonance, 1020 MeV . The measurement is done with off-peak data, at $\sqrt{s}=1 \mathrm{GeV}$, to reduce the large background from $\phi$ decays, and with an integrated luminosity of about $240 \mathrm{pb}^{-1}$. The final state $e^{+}$and $e^{-}$are not detected, being emitted with high probability in the forward directions, outside the acceptance of the
detector. The production of the $\eta$ meson is identified in two decay modes, $\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}$ and $\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}$, that exploit in a complementary way the tracking and calorimeter of the detector. The most relevant background for both measurements is the radiative process $e^{+} e^{-} \rightarrow \eta \gamma$ when the monochromatic photon is emitted at small polar angles and escapes detection. The cross section for $e^{+} e^{-} \rightarrow \eta \gamma$ is measured in the same data sample with a dedicated analysis. The cross section of the process $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$ is a convolution of the differential $\gamma \gamma$ luminosity and the $\gamma \gamma \rightarrow \eta$ cross section. The $\eta$ partial decay width $\Gamma(\eta \rightarrow \gamma \gamma)$ is obtained by extrapolating the value of $\sigma(\gamma \gamma \rightarrow \eta)$ for real photons, using a parametrization for the $\eta$ form factor based on recent measurements.
The value obtained for the $\eta$ partial decay width $\Gamma(\eta \rightarrow \gamma \gamma)$ is in agreement with the world average and is the most precise measurement to date.

## Chapter 1

## Mesons in the quark model

### 1.1 The quark model

In the 1960s and 1970s the number of observed hadronic resonances rapidly grew. Several attempts were made to build a new classification scheme for summarizing the regularities of the quantum numbers of all the particles. In 1964 M. Gell-Mann and G. Zweig independently proposed the "quark model" [1, 2, 3], which was the follow-up to a classification system known as the Eightfold Way, or $\operatorname{SU}(3)$ flavour symmetry. According to the quark model, the hadrons are composed of three fundamental particles: the "up", "down" and "strange" quarks, denoted as $u, d$ and $s$. For every quark flavour there is a corresponding antiparticle, known as an antiquark, that differs from the quark in that some of its quantum numbers have opposite sign. The antiquarks corresponding to the $u, d$ and $s$ quarks are denoted as $\bar{u}, \bar{d}, \bar{s}$. Both quarks and antiquarks are strongly interacting fermions with spin $1 / 2$. Quarks have, by convention, positive parity, while antiquarks have negative parity. In the quark model, all known hadrons are composed of quarks and antiquarks according to the following simple rules:

- each meson is a quark-antiquark pair;
- each baryon consists of three quarks, and each antibaryon of three antiquarks.

This simple model accounts to perfection for the properties of all the hadrons known in the 1960s. The electric charge, baryon number and isospin of the particles equal the sum of the corresponding quantum numbers of the composing quarks. One of the outstanding features of quarks is their electric charge. Contrary to all the previously discovered particles, they have non-integer charges (in units of the proton charge): $2 / 3$ for $u$ and $-1 / 3$ for $d$ and $s$. This is a consequence of the fact that the baryon number of each quark is $1 / 3$ (the resulting baryon number for baryons is 1 ); as for strangeness, it is 0 for $u$ and $d$, and -1 for $s$. The flavour quantum numbers of the quarks are related to the charge $Q$ through the Gell-Mann-Nishijima formula $Q=I_{3}+(B+S) / 2=I_{3}+Y / 2$, where $I_{3}$ is the third component of the isospin, $B$ is the baryon number, $S$ is the strangeness and $Y$ is the hypercharge. The $u$ and $d$ quarks form an isospin doublet ( $I=1 / 2$ ) with $S=0$, where the $u$ quark has $I_{3}=+1 / 2$ and the $d$ quark has $I_{3}=-1 / 2$. The $s$ quark is an isospin singlet $(I=0)$ with $S=-1$. The flavor of a quark
$\left(I_{z}, S, B\right)$ has, by convention, the same sign as its charge $Q$. Therefore any flavor carried by a charged meson has the same sign as its charge. Antiquarks have the opposite flavor signs. The properties of quarks and antiquarks are summarized in Tab. 1.1. Mesons, consisting of a quark

|  | $Q$ | $I_{3}$ | $B$ | $S$ | $Y$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $u$ | $+2 / 3$ | $+1 / 2$ | $1 / 3$ | 0 | $1 / 3$ |
| $d$ | $-1 / 3$ | $-1 / 2$ | $1 / 3$ | 0 | $1 / 3$ |
| $s$ | $-1 / 3$ | 0 | $1 / 3$ | -1 | $-2 / 3$ |
| $\bar{u}$ | $-2 / 3$ | $-1 / 2$ | $-1 / 3$ | 0 | $-1 / 3$ |
| $\bar{d}$ | $+1 / 3$ | $+1 / 2$ | $-1 / 3$ | 0 | $-1 / 3$ |
| $\bar{s}$ | $+1 / 3$ | 0 | $-1 / 3$ | +1 | $+2 / 3$ |

Table 1.1: Properties of quarks. $Q$ is the electric charge, $I_{3}$ the third component of the isospin, $B$ the baryon number, $S$ the strangeness, $Y$ the hypercharge.
and an antiquark, have baryon number $B=0$. If the orbital angular momentum of the $q \bar{q}$ state is $l$, then the parity is $P=(-1)^{l+1}$, where the factor $(-1)^{l}$ comes from the orbital motion and the factor -1 is due to the opposite intrinsic parities of quark and antiquark. The meson spin $J$ is given by the usual relation $|l-s| \leq J \leq|l+s|$, where $s$ is 0 (antiparallel quark spins) or 1 (parallel quark spins). The charge conjugation, or C-parity, is $C=(-1)^{l+s}$ and is defined only for the $q \bar{q}$ states made of quarks and their own antiquarks. The C-parity can be generalized to the G-parity $G=(-1)^{I+l+s}$, where $I$ is the isospin. Mesons are classified in $J^{P C}$ multiplets. The $l=0$ states are the pseudoscalars $0^{-+}$and the vectors $1^{--}$. The orbital excitations $l=1$ are the scalars $0^{++}$, the axial vectors $1^{++}$and $1^{+-}$, and the tensors $2^{++}$. According to the $\mathrm{SU}(3)$ symmetry, the nine possible $q \bar{q}$ combinations containing the light $u, d$, and $s$ quarks are grouped into an octet and a singlet of light quark mesons: $\mathbf{3} \otimes \overline{\mathbf{3}}=\mathbf{8} \oplus \mathbf{1}$, as shown in Fig. 1.1 for the pseudoscalar mesons. The singlet with $Y=0$ and $I=0$ contains all the quarks on an equal


Figure 1.1: Pseudoscalar mesons arranged in SU(3) octet and singlet.
footing. The normalized singlet state is

$$
\begin{equation*}
\left|\mathbf{1}_{S U(3)} ;|\vec{I}|=0\right\rangle \equiv \psi_{1}=1 / \sqrt{3}(u \bar{u}+d \bar{d}+s \bar{s}), \tag{1.1}
\end{equation*}
$$

symmetric in flavour, where $n$ is the dimensionality of the representation. Of the two states at the centre of the octet, one belongs to an isospin triplet (isovector) and the other is an isospin singlet (isoscalar), both have $I_{3}=0$. The quark wavefunction of the $I_{3}=0$ triplet state is

$$
\begin{equation*}
\left|8_{S U(3)} ;|\vec{I}|=1\right\rangle \equiv \psi_{V}=1 / \sqrt{2}(u \bar{u}-d \bar{d}) . \tag{1.2}
\end{equation*}
$$

Since the $s$ and $\bar{s}$ quarks are isospin singlets, they cannot couple to give an $I=1$ state. However, they can couple to give an $I=0$ state so that the $I=0$ state at the center of the octet is a linear combination of $u \bar{u}, d \bar{d}$ and $s \bar{s}$. The properly normalized state, orthogonal to (1.1) and (1.2), is

$$
\begin{equation*}
\left|8_{S U(3)} ;|\vec{I}|=0\right\rangle \equiv \psi_{8}=1 / \sqrt{6}(u \bar{u}+d \bar{d}-2 s \bar{s}) \tag{1.3}
\end{equation*}
$$

Pseudoscalar mesons have angular momentum $J=0$ : quark and antiquark are in the minimum energy state, with relative angular momentum $l=0$, and have opposite spin. The correspondence between pseudoscalar mesons and $q \bar{q}$ states is: $K^{+}=u \bar{s}, K^{0}=d \bar{s}, \pi^{-}=\bar{u} d$, $K^{-}=\bar{u} s, \overline{K^{0}}=\bar{d} s, \pi^{+}=u \bar{d}$. The pseudoscalar mesons $\pi^{0}, \eta, \eta^{\prime}$, that have quantum numbers $Q=0, I_{3}=0, Y=0$, are represented as orthogonal combinations $A_{u} u \bar{u}+A_{d} d \bar{d}+A_{s} s \bar{s}$, with normalized amplitudes $\left|A_{u}\right|^{2}+\left|A_{d}\right|^{2}+\left|A_{s}\right|^{2}=1$.

### 1.2 Mesons mixing

The mass splitting in the multiplets show that although flavour $\mathrm{SU}(3)$ describes the hadron spectrum very well, it is not an exact symmetry. If it were, indeed, the states in a given multiplet would be degenerate. In the pseudoscalar sector, assuming that the $\pi^{0}$ meson has no strangeness component $\left(m_{\pi^{0}}<m_{s}\right)$ and is the $(u \bar{u} d \bar{d}) / \sqrt{2}$ state, $\mathrm{SU}(3)$-breaking causes the physical $\eta^{0}$ and $\eta^{\prime}$ mesons to be mixtures of the $\mathrm{SU}(3)$ octet and singlet states:

$$
\begin{align*}
& \eta=\eta_{8} \cos \theta_{P}-\eta_{1} \sin \theta_{P} \\
& \eta^{\prime}=\eta_{8} \sin \theta_{P}+\eta_{1} \cos \theta_{P} \tag{1.4}
\end{align*}
$$

where $\eta^{\prime}$ and $\eta$ are the physical states, $\eta_{1}$ and $\eta_{8}$ are the singlet and octet state respectively and $\theta_{P}$ is the mixing angle in the pseudoscalar nonet. The physical states $\eta^{\prime}$ and $\eta$ are related to the $\operatorname{SU}(3)$ singlet and octet states by a rotation of the angle $\theta_{P}$. For small values of $\theta_{P}$ the parametrization (1.4) implies that $\eta^{\prime}$ is mainly a singlet state and $\eta$ mainly an octet state. Assuming the mass matrix elements to be quadratic rather than linear (according to chiral perturbation theory),

$$
H\binom{\eta_{1}}{\eta_{8}}=\left(\begin{array}{ll}
M_{11}^{2} & M_{18}^{2}  \tag{1.5}\\
M_{18}^{2} & M_{88}^{2}
\end{array}\right)\binom{\eta_{1}}{\eta_{8}} .
$$

After diagonalization of the mass matrix one derives [2]:

$$
\begin{equation*}
\tan ^{2} \theta_{P}=\frac{M_{88}^{2}-m_{\eta}^{2}}{m_{\eta^{\prime}}^{2}-M_{88}^{2}} \tag{1.6}
\end{equation*}
$$

with $M_{88}^{2}=1 / 3\left(m_{K}^{2}-m_{\pi}^{2}\right)$. Similar expressions exist for the vector and tensor meson nonets in which there are $\phi-\omega$ and $f_{2}^{\prime}-f_{2}$ mixing respectively. The sign of the mixing angle is negative (positive) according to whether the mass of the mainly octet member is smaller than (greater than) that of the mainly singlet member. Important predictions about the dominant decay modes of the isoscalar states come from the observation that the $1^{-}$and $2^{+}$nonets are almost "ideally mixed". The singlet and octet wavefunctions for the isoscalar states are defined in (1.1) and (1.3), and the octet-singlet mixing is, for the general case, parametrized by

$$
\begin{align*}
& m_{8}=\psi_{8} \cos \theta-\psi_{1} \sin \theta \\
& m_{1}=\psi_{8} \sin \theta+\psi_{1} \cos \theta, \tag{1.7}
\end{align*}
$$

where $m_{1}$ and $m_{8}$ are the physical, mainly singlet meson and the physical, mainly octet meson respectively. If $\sin \theta=1 / \sqrt{3}$, where $\theta$ is the mixing angle, one has $m_{1} \approx u \bar{u}+d \bar{d}$ and $m_{8} \approx s \bar{s}$. In this case the nonet is said to be ideally mixed because the singlet state consists only of $u \bar{u}$ and $d \bar{d}$ quarks and the octet state only of $s \bar{s}$ quarks. Ideal mixing happens for $\theta \approx 35^{\circ}$, which is approximately the case for the $1^{-}$and $2^{+}$nonets but not for the pseudoscalar nonet. Therefore, for the members of these nonets, the mainly singlet states decay predominantly to pseudoscalar mesons consisting of $u$ and $d$ quarks (pions) and the mainly octet states to strange pseudoscalar mesons (kaons): $B R(\phi \rightarrow K \bar{K}) \approx 83 \%, B R\left(\omega \rightarrow \pi^{+} \pi^{-} \pi^{0}\right) \approx 89 \%, B R\left(f_{2}^{\prime} \rightarrow K \bar{K}\right) \approx 89 \%$, $B R\left(f_{2} \rightarrow \pi \pi\right) \approx 85 \%$.

## $1.3 \gamma \gamma$ coupling of mesons

The nonet mixing angles can be measured in $\gamma \gamma$ collisions. The $\gamma \gamma$ couplings of mesons can be expressed in terms of coupling constants $g_{M \gamma \gamma}$. For pseudoscalar and scalar resonances one can define [4]:

$$
\begin{gather*}
\Gamma_{P \gamma \gamma}=\frac{m_{P}^{3}}{64 \pi} g_{P \gamma \gamma}^{2}, \\
\Gamma_{S \gamma \gamma}=\frac{m_{S}^{3}}{16 \pi} g_{S \gamma \gamma}^{2} . \tag{1.8}
\end{gather*}
$$

In the quark model mesons are represented as

$$
\begin{equation*}
|M\rangle=\Sigma_{q} c_{q}|q \bar{q}\rangle . \tag{1.9}
\end{equation*}
$$

The coupling of two photons (with a given $\gamma \gamma$ helicity) to a quark-antiquark pair is proportional to the square of the quark charge:

$$
\begin{align*}
& \langle q \bar{q} \mid \gamma \gamma\rangle \sim e_{q}^{2} \psi_{q}(0) \text { (S wave) }, \\
& \langle q \bar{q} \mid \gamma \gamma\rangle \sim e_{q}^{2} \psi_{q}^{\prime}(0) \text { (P wave) } \tag{1.10}
\end{align*}
$$

where $\psi_{q}(0)$ is the radial quark wave function at the origin and $\psi_{q}^{\prime}(0)$ is the first derivative of $\psi_{q}(0)$ at zero. If $\psi_{q}(0)$ is independent of the quark flavour the $\gamma \gamma$ coupling constant of a meson $M$ can be related to the quark charge through (1.9) and (1.10):

$$
\begin{equation*}
g_{M \gamma \gamma} \sim\langle M \mid \gamma \gamma\rangle \sim \Sigma_{q} c_{q} e_{q}^{2}=\left\langle e_{q}^{2}\right\rangle . \tag{1.11}
\end{equation*}
$$

The coefficients $c_{q}$ are given by the $\mathrm{SU}(3)$ representations shown in (1.1)-(1.3). The effective squared charges defined in (1.11) are

$$
\begin{align*}
\left\langle e_{q}^{2}\right\rangle_{V}=\left(e_{d}^{2}-e_{u}^{2}\right) / \sqrt{2} & =1 /(3 \sqrt{2}), \\
\left\langle e_{q}^{2}\right\rangle_{8}=\left(e_{u}^{2}+e_{d}^{2}-2 e_{s}^{2}\right) / \sqrt{6} & =1 /(3 \sqrt{6}),  \tag{1.12}\\
\left\langle e_{q}^{2}\right\rangle_{1}=\left(e_{u}^{2}+e_{d}^{2}+e_{s}^{2}\right) / \sqrt{3} & =2 /(3 \sqrt{3}),
\end{align*}
$$

where the indices 8 and 1 denote the flavour octet and flavour singlet isoscalars, while the symbol $V$ denotes the isovectors (e.g. $\pi^{0}, a_{2}$ ). Since the $\mathrm{SU}(3)$ symmetry is broken by the mass of the $s$ quark, the physical states are mixtures of the $\mathrm{SU}(3)$ singlet and octet states, as explained in the previous section. Therefore, neglecting any possible mass dependence, the ratios of the coupling constants depend only on the quark charges and on the mixing angle:

$$
\begin{align*}
g_{\pi \gamma \gamma}: g_{\eta \gamma \gamma}: g_{\eta^{\prime} \gamma \gamma}=g_{a_{2} \gamma \gamma}: g_{f^{\prime} \gamma \gamma}: g_{f \gamma \gamma} & = \\
\left\langle e_{q}^{2}\right\rangle_{V}: \cos \theta\left\langle e_{q}^{2}\right\rangle_{8}-\sin \theta\left\langle e_{q}^{2}\right\rangle_{1}: \sin \theta\left\langle e_{q}^{2}\right\rangle_{8}+\cos \theta\left\langle e_{q}^{2}\right\rangle_{1} & = \\
\sqrt{3}: \cos \theta-2 \sqrt{2} \sin \theta: \sin \theta+2 \sqrt{2} \cos \theta . & \tag{1.13}
\end{align*}
$$

A possible mass dependence for the coupling constant $g_{M_{\gamma \gamma}}$ is strongly model dependent.
The first two-photon experiment proposed for $e^{+} e^{-}$storage rings was the measurement of the $\pi^{0}$ width [5]. The $\pi^{0} \rightarrow \gamma \gamma$ decay played a fundamental role in the determination of the number of color degrees of freedom of the quarks and therefore became a milestone for the development of the color gauge theory, "quantum chromodynamics" (QCD). The $\gamma \gamma$ width of the $\pi^{0}$ is connected to the $\gamma \gamma$ widths of the $\eta$ and the $\eta^{\prime}$ mesons through the relations [6]:

$$
\begin{align*}
& \frac{\Gamma_{\eta \rightarrow \gamma \gamma}}{\Gamma_{\pi^{0} \rightarrow \gamma \gamma}}=\frac{1}{3} \frac{m_{\eta}^{3}}{m_{\pi}^{3}}\left[\frac{f_{\pi} \cos \theta_{P}}{f_{\eta 8}}-\frac{\sqrt{8} f_{\pi} \sin \theta_{P}}{f_{\eta 1}}\right]^{2},  \tag{1.14}\\
& \frac{\Gamma_{\eta^{\prime} \rightarrow \gamma \gamma}}{\Gamma_{\pi^{0} \rightarrow \gamma \gamma}}=\frac{1}{3} \frac{m_{\eta^{\prime}}^{3}}{m_{\pi}^{3}}\left[\frac{f_{\pi} \sin \theta_{P}}{f_{\eta 8}}-\frac{\sqrt{8} f_{\pi} \cos \theta_{P}}{f_{\eta 1}}\right]^{2}, \tag{1.15}
\end{align*}
$$

where $f_{\pi}$ is the pion decay constant, $f_{\pi} \approx 93 \mathrm{MeV}, \theta_{P}$ is the pseudoscalar mixing angle, $f_{\eta 1}$ and $f_{\eta 8}$ are the decay constants for the combinations $\eta_{1}$ and $\eta_{8}$. The ratio $f_{8} / f_{\pi} \approx 1.25$ has been calculated using chiral perturbation theory. Therefore the measurement of the partial $\gamma \gamma$ width $\Gamma(X \rightarrow \gamma \gamma)$ is a crucial input for the determination of the pseudoscalar mixing angle.

## Chapter 1. Mesons in the quark model

### 1.4 Scalar mesons

Scalar mesons belong to the multiplet $J^{P}=0^{+}$. They are grouped in a nonet, like the pseudoscalar mesons, but the mass spectrum is inverted, as shown in Fig. 1.2. This inversion does not have any explanation within the usual description of mesons in terms of $q \bar{q}$ couples. Moreover, the scalar mesons have positive parity, which is not possible in a $q \bar{q}$ combination


Figure 1.2: Mass spectrum of the scalar mesons (left) and pseudoscalar mesons (right).
with angular momentum $l=0$. One of the models used to describe the nature of the scalar mesons is the "tetraquark model", that predicts the existence of four valence quarks: a couple of quarks (diquark) and a couple of antiquarks (antidiquark). This model explains the inverted mass spectrum. In Tab. 1.2 the quantum numbers of the light scalar mesons (with mass $<1$ GeV ) are shown. In contrast to the vector and tensor mesons, the identification of the scalar

|  | $I$ | $I_{3}$ | $S$ | $Y$ | composition |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $a^{+}$ | 1 | +1 | 0 | 0 | $[s u][\bar{s} \bar{d}]$ |
| $a^{0}$ | 1 | 0 | 0 | 0 | $\frac{1}{\sqrt{2}}([s u][\bar{s} \bar{u}]-[s d][\bar{s} \bar{d}])$ |
| $a^{-}$ | 1 | -1 | 0 | 0 | $[s d][\bar{s} \bar{u}]$ |
| $f_{0}$ | 0 | 0 | 0 | 0 | $\frac{1}{2}([s u][\bar{s} \bar{u}]+[s d] \bar{s} \bar{d} \bar{d})$ |
| $\sigma$ | 0 | 0 | 0 | 0 | $[u d][\bar{u} \bar{d}]$ |
| $K^{+}$ | $1 / 2$ | $+1 / 2$ | +1 | +1 | $[u \bar{d}][\bar{s} \bar{d}]$ |
| $K^{0}$ | $1 / 2$ | $-1 / 2$ | +1 | +1 | $[u \bar{d}][\bar{s} \bar{u}]$ |
| $\bar{K}^{0}$ | $1 / 2$ | $+1 / 2$ | -1 | -1 | $[u s][\bar{d} \bar{u}]$ |
| $K^{-}$ | $1 / 2$ | $-1 / 2$ | -1 | -1 | $[d s][\bar{d} \bar{u}]$ |

Table 1.2: Quantum numbers and scalar mesons composition in the diquark-antidiquark model.
mesons is a long-standing puzzle. Scalar resonances are difficult to resolve because some of them have large decay widths which cause a strong overlap between resonances and background. Scalar mesons can be produced in $\pi N$ scattering, $p \bar{p}$ annihilation, $J / \psi, B-, D$ - and $K$-meson decays, $\phi$ radiative decays and $\gamma \gamma$ interactions.

## Chapter 2

## Photon-photon interactions

### 2.1 Physics with two photons

Light by light scattering $[4,7,8,9,10]$ is forbidden in classical electrodynamics because according to Maxwell's classical linear equations electromagnetic waves cross each other without any disturbance. In quantum electrodynamics (QED), however, the situation is different. The uncertainty principle allows a photon of energy $E_{\gamma}$ to fluctuate into states of charged particle pairs with mass $m_{\text {pair }}$. The lifetime of this intermediate state, $\Delta t \approx 2 E_{\gamma} / m_{\text {pair }}^{2}$, can be very large for high values of $E_{\gamma}$, and photon-photon scattering becomes possible due to the interaction of the intermediate particles. In other words, the photons create virtual pairs by quantum fluctuations of the vacuum. The simplest mechanism for elastic $\gamma \gamma$ scattering is given by the box diagram (Fig. 2.1). Very intense sources of photons are electron and positron storage rings,


Figure 2.1: Box diagram for elastic $\gamma \gamma$ scattering.
which were built to investigate the annihilation of electrons and positrons. In the lowest order of the electromagnetic coupling constant $\alpha, O\left(\alpha^{2}\right)$, this process can be seen as the annihilation of $e^{+} e^{-}$into a virtual time-like photon, which then materializes into a final state $X$ of hadrons or leptons, as shown on the left side of Fig. 2.2. In the dominant diagram of the two-photon mechanism (right side of Fig. 2.2), instead, electrons and positrons of both incident beams emit virtual space-like photons that annihilate producing the final state $X$, which is some arbitrary final state allowed by conservation laws. In particular, hadronic states with $J^{P C}=0^{ \pm+} ; 2^{ \pm+}$ are directly produced through the $\gamma \gamma \rightarrow X$ subprocess. The cross section of the process is of the order $\alpha^{4}$ and is very small at low beam energies (up to several hundred MeV ) if compared with the $e^{+} e^{-}$annihilation cross section. However, the two-photon cross section increases with


Figure 2.2: Feynman diagrams for $e^{+} e^{-}$annihilation (left) and $\gamma \gamma$ interactions (right).
the beam energy $E$ like $\left(\log E / m_{e}\right)^{2}$, while the $e^{-} e^{-}$annihilation cross section decreases at least like $1 / E^{2}$. Therefore the two-photon process, despite of the order $\alpha^{4}$, already dominates over the annihilation process (of the order $\alpha^{2}$ ) for beam energies of a few GeV . The structure of the photon propagators in $\gamma \gamma$ reactions causes the photons to be radiated nearly on-mass-shell (almost real photons) and at small angles ( $\sim m_{e} / E$ ) relative to the beam axis. The momentum transferred to the system $X$ is therefore small. Most $\gamma \gamma$ events produce a low invariant mass final system, because of the typical bremsstrahlung spectrum of the emitted photons ( $\sim 1 / E_{\gamma}$ ). The two-photon production of lepton pairs can be used to test quantum electrodynamics (QED) up to the order $\alpha^{4}$, while the production of hadronic final states gives the possibility of probing hadron dynamics and studying the coupling of the photons to quarks. In the regime of large four-momentum transfer $q^{2}$ of the photons and high transverse momenta of the produced hadrons, the elementary nature of the photon is emphasized and the two photons have a pointlike coupling to a quark pair (high- $p_{T}$ jets, structure functions). The investigation of the production of high transverse momentum particles, jets, and scattering of highly virtual photons, allows for tests of QCD. In the regime of low four momentum transfer $q^{2}$ and low transferse momenta of the hadrons, the hadronic nature of the photon is emphasized, and almost real photons are emitted. In the vector meson dominance (VMD) model [11], which works fairly well in most processes involving real or almost real photons, the photons turn into virtual vector mesons (e.g. $\rho, \omega, \phi$ ) which then interact strongly with hadrons (Fig. 2.3).


Figure 2.3: The dual nature of the photon: QED coupling (left) and and VMD coupling (right).
The first theoretical papers related to two-photon physics at $e^{+} e^{-}$storage rings appeared in 1960. At the end of the 1960s and early 1970s the storage rings in Frascati, Novosibirsk, Orsay, Stanford and Hamburg became available. The first experimental results were obtained by ADONE in Frascati [12] and by VEPP-2 in Novosibirsk [13]. Since then, $\gamma \gamma$ interactions have
been used to study the production of hadrons in almost all $e^{+} e^{-}$colliders in a variety of conditions in low- and high- $q^{2}$ processes [10]. Both $\gamma \gamma^{*}$ (one almost real photon and one virtual photon) and $\gamma \gamma$ (two almost real photons) reactions, exclusive and inclusive and for different regimes of photon-photon center of mass energy $W$, give crucial information on hadronic structure. At low to medium $W$, the main goal of exclusive $\gamma \gamma$ studies is to extract the two-photon widths of meson resonances that couple to two photons. The measurement of the radiative width of pseudoscalar mesons $\Gamma(X \rightarrow \gamma \gamma)$ is a crucial input for the determination of the pseudoscalar mixing angle (see sections 1.2-1.3) and for testing the valence gluon content in the $\eta^{\prime}$ wavefunction [14]. On the other hand, a precise study of the form factors of the transition $\gamma \gamma^{*} \rightarrow X, F\left(Q_{\gamma^{*}}^{2}, 0\right)$, where one photon is off-shell and the other one is real, allows one to test phenomenological models used for computing the light-by-light contribution to the $(g-2)_{\mu}$ prediction in the Standard Model [15] and the dynamics of the $\pi^{0} \rightarrow e^{+} e^{-}$transition [16].

### 2.2 Kinematics and cross section of $\gamma \gamma$ reactions

Two-photon scattering at $e^{+} e^{-}$storage rings can be observed through the reaction $e^{+} e^{-} \rightarrow$ $e^{+} e^{-} \gamma^{(*)} \gamma^{(*)} \rightarrow e^{+} e^{-}$X: an electron and a positron radiate photons, and these photons produce the final system $X$ with even C-parity. The kinematics of the reaction is completely determined by the four-momenta of the incoming and of the scattered electron and positron (see Fig. 2.4). The main goal is to find the amplitudes for the $\gamma \gamma \rightarrow X$ process, both for virtual and almost


Figure 2.4: Kinematics of the two photon reaction $e^{+} e^{-} \rightarrow e^{+} e^{-} X$.
real photons. The colliding photons with momenta $q_{1}$ and $q_{2}$ are space-like ( $q^{2}<0$ ) and may have both a transverse ( $T$ ) polarization and a longitudinal $(L)$ polarization. The mass of the produced system is $W^{2}=\left(q_{1}+q_{2}\right)^{2}$. The observed $e^{+} e^{-} \rightarrow e^{+} e^{-} X$ cross section is expressed in terms of the $\gamma \gamma \rightarrow X$ cross sections for the corresponding photons: $\sigma_{T T}, \sigma_{T L}, \sigma_{L T}, \sigma_{L L}$, e.g. $\sigma_{T L}$ is the cross section for the collision of a transverse photon $q_{1}$ with a longitudinal photon $q_{2}$. Moreover, the result has four additional interfering terms: $\tau_{T T}, \tau_{T L}, \tau_{T T}^{a}, \tau_{T L}^{a}$, where $\tau_{T T}$ is the difference between cross sections for scattering transverse photons with parallel $(\|)$ and
orthogonal $(\perp)$ linear polarizations, and $\tau_{T T}^{a}$ is the difference between the cross sections for scattering of transverse photons in states with total helicity $0\left(\sigma_{0}\right)$ and $2\left(\sigma_{2}\right)$ :

$$
\begin{equation*}
\tau_{T T}=\sigma_{\|}-\sigma_{\perp} ; \quad \tau_{T T}^{a}=\sigma_{0}-\sigma_{2} ; \quad \sigma_{T T}=1 / 2\left(\sigma_{\|}+\sigma_{\perp}\right)=1 / 2\left(\sigma_{0}+\sigma_{2}\right) \tag{2.1}
\end{equation*}
$$

The terms $\tau_{T L}$ and $\tau_{T L}^{a}$ are connected with the interference terms of amplitudes for the transition $\gamma \gamma \rightarrow X$ where both transverse and longitudinal photons participate. All these quantities depend on $W^{2}, q_{1}^{2}$ and $q_{2}^{2}$ only. The differential cross section for the two-photon production has the form

$$
\begin{array}{r}
d \sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} X\right)=\frac{\alpha^{2} \sqrt{\left(q_{1} q_{2}\right)^{2}-q_{1}^{2} q_{2}^{2}}}{32 \pi^{4} E^{2} q_{1}^{2} q_{2}^{2}} \times \frac{d^{3} p_{1}^{\prime} d^{3} p_{2}^{\prime}}{E_{1}^{\prime} E_{2}^{\prime}} \times\left[4 \rho_{1}^{++} \rho_{2}^{++} \sigma_{T T}\right. \\
+2 \rho_{1}^{++} \rho_{2}^{00} \sigma_{T L}+2 \rho_{1}^{00} \rho_{2}^{++} \sigma_{L T}+\rho_{1}^{00} \rho_{2}^{00} \sigma_{L L}+2\left|\rho_{1}^{+-} \rho_{2}^{+-}\right| \tau_{T T} \cos 2 \tilde{\phi} \\
\left.-8\left|\rho_{1}^{+0} \rho_{2}^{+0}\right| \tau_{T L} \cos 2 \tilde{\phi}+A \tau_{T T}^{a}+B \tau_{T L}^{a}\right] \tag{2.2}
\end{array}
$$

The mixture of polarization states of the photon is given by a $3 \times 3$ density matrix with elements $\rho_{i}^{\mu v}$ ( $i=1,2$ for the two photons). The quantities $\rho_{1,2}^{a b}$ are the elements of the density matrix of the virtual photons in the $\gamma \gamma$ helicity basis ( $a, b= \pm 1$ for transverse photons, 0 for longitudinal photons), and can be expressed in terms of the momenta $p_{i}$ and $q_{i}$ and the particle form factors. The transformation between the two different bases (linear polarization basis and helicity basis) is derived in [9]. The term $\tilde{\phi}$ is the angle between the lepton scattering planes in the $\gamma \gamma$ center-of-mass system. Symmetry between photons requires $\sigma_{T L}\left(W, q_{1}^{2}, q_{2}^{2}\right)=\sigma_{L T}\left(W, q_{2}^{2}, q_{1}^{2}\right)$, reducing the number of independent functions to be determined. The coefficients of $\tau_{T T}$ and $\tau_{T L}$ both vanish after the integration over $\tilde{\phi}$. The terms $\tau_{T T}^{a}$ and $\tau_{T L}^{a}$ can only be measured with polarized lepton beams, otherwise $A=B=0$. All terms with an index $L$ vanish if the corresponding photon is on-shell. Only $\sigma_{T T}$ and $\tau_{T T}$ survive as both photons become real.

### 2.3 Approximations for the cross section formula

The complicated helicity structure of the cross section in equation (2.2) can in some cases be simplified [7, 8, 9]. Because of the photon propagators in $\gamma \gamma$ processes, the emitted photons are almost real, and one can make the approximation that only transverse photons contribute. The cross section 2.2 contains then only the term $\sigma_{T T}$ (the term $\tau_{T T}$ vanishes after integrating over $\tilde{\phi})$ :

$$
\begin{equation*}
d \sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} X\right)=\frac{\alpha^{2} \sqrt{\left(q_{1} q_{2}\right)^{2}-q_{1}^{2} q_{2}^{2}}}{32 \pi^{4} E^{2} q_{1}^{2} q_{2}^{2}} \times 4 \rho_{1}^{++} \rho_{2}^{++} \sigma_{T T} \times \frac{d^{3} p_{1}^{\prime} d^{3} p_{2}^{\prime}}{E_{1}^{\prime} E_{2}^{\prime}} \tag{2.3}
\end{equation*}
$$

The $e^{+} e^{-} \rightarrow e^{+} e^{-} X$ cross section has been approximated by a product of the transverse photons densities and a cross section for the process $\gamma \gamma \rightarrow X$. Introducing a "two-photon luminosity
function" for transverse photons, $L_{\gamma \gamma}^{T T}$, the cross section can be rewritten as

$$
\begin{equation*}
\frac{d^{5} \sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} X\right)}{d \omega_{1} \times d \omega_{2} \times d \cos \theta_{1} \times d \cos \theta_{2} \times d \phi}=\frac{d^{5} L_{\gamma \gamma}^{T T}}{d \omega_{1} \times d \omega_{2} \times d \cos \theta_{1} \times d \cos \theta_{2} \times d \phi} \times \sigma_{T T} \tag{2.4}
\end{equation*}
$$

where $\omega_{i}=E_{\gamma i} / E$. The differential luminosity function is

$$
\begin{equation*}
\frac{d^{5} L_{\gamma \gamma}^{T T}}{d \omega_{1} \times d \omega_{2} \times d \cos \theta_{1} \times d \cos \theta_{2} \times d \phi}=\frac{\alpha^{2} E_{1}^{\prime} E_{2}^{\prime}}{16 \pi^{3} q_{1}^{2} q_{2}^{2}} \times 4 \rho_{1}^{++} \rho_{2}^{++} \sqrt{X} \tag{2.5}
\end{equation*}
$$

where $X=1 / 4\left(W^{2}-q_{1}^{2}-q_{2}^{2}\right)^{2}-q_{1}^{2} q_{2}^{2}$. The terms $\rho_{i}^{++}$contain in general the variables of both photons. However, for $q_{i}^{2} \rightarrow 0, q_{i}^{2} \ll W^{2}$, it is possible to write the photon luminosity function as a product of two fluxes. Since $q_{i}^{2} \ll W^{2}, W^{2}$ depends only on the energies of the photons: $W^{2}=\left(q_{1}+q_{2}\right)^{2} \approx 4 E_{\gamma 1} E_{\gamma 2}$. After integrating over the angular distribution of the leptons, one obtains the factorized luminosity function:

$$
\begin{equation*}
\frac{d^{2} L_{\gamma \gamma}}{d \omega_{1} \times d \omega_{2}}=\frac{d N_{\gamma}\left(\omega_{1}\right)}{d \omega_{1}} \times \frac{d N_{\gamma}\left(\omega_{2}\right)}{d \omega_{2}} \tag{2.6}
\end{equation*}
$$

The photon spectra, integrated between $q_{\min }^{2}$ and $q_{\max }^{2} \ll W^{2}$, become

$$
\begin{equation*}
\frac{d N_{\gamma}(\omega)}{d \omega}=\frac{\alpha}{2 \pi \omega}\left\{\left[1+(1-\omega)^{2}\right] \ln \frac{q_{\max }^{2}}{q_{\min }^{2}}-(1-\omega)\left(1-\frac{q_{\min }^{2}}{q_{\max }^{2}}\right)\right\} \tag{2.7}
\end{equation*}
$$

Keeping only the leading term in (2.7) the photon spectrum is approximated by

$$
\begin{equation*}
d N_{\gamma}(\omega) / d \omega=(\alpha / \pi)(1 / \omega) \ln \left(E / m_{e}\right)\left[1+(1-\omega)^{2}\right] \tag{2.8}
\end{equation*}
$$

The differential luminosity $d L_{\gamma \gamma} / d z$ (where $z=W / 2 E$ ) can be obtained by integrating (2.6) with the constraint $\omega_{1} \omega_{2}=z^{2}$ and with the approximation (2.8):

$$
\begin{equation*}
\frac{d L_{\gamma \gamma}}{d z}=\left(\frac{2 \alpha}{\pi}\right)^{2}\left(\ln \frac{E}{m_{e}}\right)^{2} \frac{f(z)}{z} \tag{2.9}
\end{equation*}
$$

where the "Low function" $f(z)$ is defined as

$$
\begin{equation*}
f(z)=\left(2+z^{2}\right)^{2} \ln (1 / z)-\left(1-z^{2}\right)\left(3+z^{2}\right) \tag{2.10}
\end{equation*}
$$

For not too large values of $z(z<0.8)$, this formula overestimates the exact luminosity function by about $10 \%$ to $20 \%$, but reproduces quite well the shape of the function. The factorization (2.6) is called "Equivalent Photon Approximation" (EPA) or "Weizsäcker-Williams Approximation". This approximation gives the exact cross section in the case where both scattered $e^{+} e^{-}$are detected within small forward angles. Fig. 2.5 shows the luminosity function multiplied by an integrated $e^{+} e^{-}$luminosity $L_{e e}=1 \mathrm{fb}^{-1}$, as a function of the $\gamma \gamma$ invariant mass for three different center-of-mass energies.


Figure 2.5: Differential $\gamma \gamma$ luminosity as a function of the center- of-mass energy. Accessible final states are also indicated.

### 2.4 Resonance production in $\gamma \gamma$ interactions

The total cross section for the production of a hadronic resonance $R$ by two real photons is given by the (relativistic) Breit-Wigner formula

$$
\begin{equation*}
\sigma(\gamma \gamma \rightarrow R)=8 \pi(2 J+1) \frac{\Gamma \Gamma_{\gamma \gamma}}{\left(W^{2}-M_{R}^{2}\right)^{2}+\Gamma^{2} M_{R}^{2}} \tag{2.11}
\end{equation*}
$$

where $J$ denotes the spin of the resonance, $M_{R}$ its mass, $\Gamma$ and $\Gamma_{\gamma \gamma}$ its total and two-photon decay width, and $W$ the $\gamma \gamma$ center of mass energy. For a narrow resonance with $J=0$ (e.g. pseudoscalar mesons) the cross section is

$$
\begin{equation*}
\sigma(\gamma \gamma \rightarrow R)=8 \pi^{2} \frac{\Gamma_{\gamma \gamma}}{M_{R}} \delta\left(W^{2}-M_{R}^{2}\right) . \tag{2.12}
\end{equation*}
$$

For almost real photons (Equivalent Photon Approximation) the cross section of the process $e^{+} e^{-} \rightarrow e^{+} e^{-} R$ can be estimated from the expression:

$$
\begin{equation*}
\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} R\right)=\int d z \frac{d L_{\gamma \gamma}}{d z} \sigma_{\gamma \gamma \rightarrow R}(z) . \tag{2.13}
\end{equation*}
$$

Implementing the cross section formula for a narrow resonance $R, \sigma(\gamma \gamma \rightarrow R)$, in equation (2.13) one obtains the resulting cross section:

$$
\begin{equation*}
\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} R\right)=\frac{16 \alpha^{2} \Gamma_{\gamma \gamma}}{M_{R}^{3}}\left(\ln \frac{E}{m_{e}}\right)^{2}\left[\left(z^{2}+2\right)^{2} \ln \frac{1}{z}-\left(1-z^{2}\right)\left(3+z^{2}\right)\right] \tag{2.14}
\end{equation*}
$$

where the Low function has been used. This formula can be used to study the processes $e^{+} e^{-} \rightarrow$ $e^{+} e^{-} \pi^{0}, \eta, \eta^{\prime}$. Tab. 2.1 shows the cross section values for pseudoscalar mesons production in $\gamma \gamma$ interactions for different values of $\sqrt{s}$.

| R | $\sqrt{s}=1 \mathrm{GeV}$ | $\sqrt{s}=1.02 \mathrm{GeV}$ | $\sqrt{s}=1.2 \mathrm{GeV}$ | $\sqrt{s}=1.4 \mathrm{GeV}$ |
| :--- | :---: | :---: | :---: | :---: |
| $\pi^{0}$ | 266 | 271 | 317 | 364 |
| $\eta$ | 43 | 45 | 66 | 90 |
| $\eta^{\prime}$ | 3.3 | 4.9 | 20.0 | 39.7 |

Table 2.1: $\sigma_{e^{+} e^{-} \rightarrow e^{+} e^{-} R}[\mathrm{pb}]$ calculated with the Equivalent Photon Approximation for different values of $\sqrt{s}$.

### 2.5 Tagging of the photons

In experiments with two-photon interactions it is possible to "tag" the interacting photons by detecting the scattered leptons. There are three different kinematical conditions: both scattered leptons are detected (double-tag), only one scattered lepton is detected (single-tag), neither of the leptons is detected (no-tag). In principle the double-tag condition is the best one for measuring $\gamma \gamma$ processes because it gives complete information on the $\gamma \gamma$ system. However, most of the photons are emitted at small angles with respect to the beam axis, and the rate of events drops steeply when the leptons scatter away from the very forward direction. Tagging at very small angles is in most cases not easy because of background problems (small angle Bhabha scattering, beam-gas interaction). Furthermore, the energy loss of the scattered leptons is measured less accurately at higher energies, and the resolution of the $\gamma \gamma$ center of mass energy becomes worse. Most experimental results have been obtained with the single-tag or no-tag conditions. Single-tag is required when the background from one-photon annihilation events is not small or when one wants to determine the $q^{2}$ dependence of resonance couplings or of the total cross section (see section 2.6). Experimental experience has shown that if one wants to study exclusive final states with almost real photons, tagging is often not necessary. For the rejection of the background it is possible to take advantage of the fact that photons are mainly radiated along the beam direction, and the transverse momentum of the $\gamma \gamma$ system is small.

### 2.6 Form factors in meson-photon-photon transitions

The study of $\gamma \gamma$ interactions is useful to learn the properties of the strong interactions. Despite the probe and the target are both photons that carry electromagnetic force, they can produce a pair of quarks that interact strongly and are observed as hadrons, e.g. pseudoscalar mesons. However, the transition between a meson and two photons cannot be calculated from QCD directly, because at low energies the strong coupling constant $\alpha_{S}$ is too large for a perturbative approach to work, and approximations are needed. Therefore, the process is calculated without
including the effects of quarks and gluons, and at the end these effects are taken into account in an extra factor, known as "transition form-factor" (see Fig. 2.6). The form-factor connects three


Figure 2.6: General $P \gamma^{(*)} \gamma^{(*)}$ vertex described by a transition form factor, where $P$ is a pseudoscalar meson.
particles and therefore depends on three variables, the squared momenta $q_{i}^{2}$ of the photons and the $q_{P S}^{2}$ of the pseudoscalar. However, under the approximation that the quark mass is zero, the pseudoscalar becomes mass-less, $q_{P S}^{2}=0$ (unlike the photons, which can be virtual). Therefore $F\left(q_{1}^{2}, q_{2}^{2}, q_{P S}^{2}\right)=F\left(q_{1}^{2}, q_{2}^{2}, 0\right)=F\left(q_{1}^{2}, q_{2}^{2}\right)$. The shape of the form-factor is not known exactly, but there exist some constraints on it [19]. Three of them come from QCD calculations:

- when the two photons are real, $q_{i}^{2}=0$, the transition can be seen as a point-like process and the form-factor must fulfill the relation $F(0,0)=1$;
- when both photons are are highly virtual and with equal value of $q^{2}, q_{1}^{2}=q_{2}^{2}=q^{2} \ll 0$, the form factor is $F\left(q_{1}^{2}, q_{2}^{2}\right)=-8\left(\pi f_{\pi}\right)^{2} / N_{c} q^{2}$, where $N_{c}$ is the number of colors in the Standard Model, $N_{c}=3$, and $f_{\pi}$ is the pion decay constant;
- when one of the photons is real and the other one highly virtual, $q_{i}^{2}=0$ and $q_{j}^{2}=q^{2} \ll 0$, one must have $F\left(0, q^{2}\right)=-8\left(\pi f_{\pi}\right)^{2} C / N_{c} q^{2}$, where $C$ is a constant.

A fourth constraint derives from the fact that one believes that quark and gluon effects (not included in the pointlike description) correspond to intermediate states with other mesons, like the $\rho$ meson. Therefore it should be possible to explain the shape of the form factor within the VMD model [20]. It is very difficult to find a form factor that satisfies all of these four constraints. The form factor

$$
\begin{equation*}
F\left(q_{1}^{2}, q_{2}^{2}\right)=1 \tag{2.15}
\end{equation*}
$$

satisfies only the first constraint. In this case there is no form factor, and the reaction is seen as pointlike. The form factor

$$
\begin{equation*}
F\left(q_{1}^{2}, q_{2}^{2}\right)=\frac{m_{\rho}^{4}}{\left(m_{\rho}^{2}-q_{1}^{2}\right)\left(m_{\rho}^{2}-q_{2}^{2}\right)} \tag{2.16}
\end{equation*}
$$

satisfies the first and the fourth constraint, and with $f_{\pi}=92.4 \mathrm{MeV}$ and $m_{\rho}=770 \mathrm{MeV}$ it satisfies the third constraint within the uncertainty on the constant parameter. The form factor

$$
\begin{equation*}
F\left(q_{1}^{2}, q_{2}^{2}\right)=\frac{m_{\rho}^{2}}{\left(m_{\rho}^{2}-q_{1}^{2}-q_{2}^{2}\right)} \tag{2.17}
\end{equation*}
$$

satisfies the first three constraints but not the fourth one. The last form factor,

$$
\begin{equation*}
F\left(q_{1}^{2}, q_{2}^{2}\right)=\frac{m_{\rho}^{4}-\frac{4 \pi^{2} F_{\pi}^{2}}{N_{C}}\left(q_{1}^{2}+q_{2}^{2}\right)}{\left(m_{\rho}^{2}-q_{1}^{2}\right)\left(m_{\rho}^{2}-q_{2}^{2}\right)} \tag{2.18}
\end{equation*}
$$

satisfies constraint one, two and four.
The cross section for the process $\gamma \gamma \rightarrow R$, where $R$ is a narrow resonance, can be written as:

$$
\begin{equation*}
\sigma_{\gamma \gamma \rightarrow R}\left(q_{1}, q_{2}\right)=\Gamma_{R \rightarrow \gamma \gamma} \frac{8 \pi^{2}}{M_{R}} \delta\left(\left(q_{1}+q_{2}\right)^{2}-M_{R}^{2}\right)\left|F\left(q_{1}^{2}, q_{2}^{2}\right)\right|^{2} \tag{2.19}
\end{equation*}
$$

If the photons are real the form factor dependence disappears. The transition form factors associated to the meson-photon-photon vertex can be accessed in the space-like region $\left(q^{2}<0\right)$ by single-tag two photon experiments, when the momentum $q_{i}^{2}$ of one photon is varied and the $q_{j}^{2}$ of the other photon is kept small (single-tag condition). Available data on $\left|F_{\pi^{0}}\left(q^{2}, 0\right)\right|$ and $\left|F_{\eta}\left(q^{2}, 0\right)\right|$ for low $\left|q^{2}\right|$ values are presented in Figs. 2.7 and 2.8 [21]. Both processes $\pi^{0} \rightarrow \gamma \gamma *$ and $\eta \rightarrow \gamma \gamma *$ can be described by the VMD model.


Figure 2.7: Single off-shell $\pi^{0}$ meson transition form factor in the low $\left|q^{2}\right|$ region from SND [22] and CMD-2 [23] data on the reaction $e^{+} e^{-} \rightarrow \pi^{0} \gamma$ and CELLO [24] data on the reaction $e^{+} e^{-} \rightarrow$ $e^{+} e^{-} \gamma^{*} \gamma^{*} \rightarrow e^{+} e^{-} \pi^{0}$.


Figure 2.8: Single off-shell $\eta$ meson transition form factor from NA60 [25] data on $\eta \rightarrow \gamma \mu^{+} \mu^{-}$ decay; from SND [22] and CMD-2 [23] data on the reaction $e^{+} e^{-} \rightarrow \eta \gamma$, and from CELLO [24] data on the reaction $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma^{*} \gamma^{*} \rightarrow e^{+} e^{-} \eta$.

### 2.7 Measurements of radiative widths of mesons

In this section some measurements of partial widths $\Gamma_{R \rightarrow \gamma \gamma}$ obtained with experiments at $e^{+} e^{-}$ storage rings are reported. With the MD-1 detector [17] at the VEPP-4 storage ring the following processes have been studied:

- $e^{+} e^{-} \rightarrow e^{+} e^{-} a_{2}$, with $a_{2} \rightarrow \pi^{+} \pi^{-} \gamma \gamma$,
- $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta^{\prime}$, with $\eta^{\prime} \rightarrow \pi^{+} \pi^{-} \gamma$,
- $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$, with $\eta \rightarrow \gamma \gamma$.

The center of mass energy is in the range [7.2-10.4] GeV, and the integrated luminosity is 20.8 $\mathrm{pb}^{-1}$. The results for the $\gamma \gamma$ widths are:

- $\Gamma\left(a_{2} \rightarrow \gamma \gamma\right)=(1.26 \pm 0.26 \pm 0.18) \mathrm{keV}$,
- $\Gamma\left(\eta^{\prime} \rightarrow \gamma \gamma\right)=(4.6 \pm 1.1 \pm 0.6) \mathrm{keV}$,
- $\Gamma(\eta \rightarrow \gamma \gamma)=(0.51 \pm 0.12 \pm 0.05) \mathrm{keV}$.

The Crystal Ball detector [6] at DORIS II (DESY) has been used to study the process $e^{+} e^{-} \rightarrow$ $e^{+} e^{-} R$, with $R \rightarrow \gamma \gamma$, where $R$ is a generic narrow resonance with mass between 100 and 3000 MeV . With an integrated luminosity of $114 \mathrm{pb}^{-1}$ and a center of mass energy between 9.4 and 10.6 GeV , three peaks are observed in the invariant $\gamma \gamma$ mass spectrum, corresponding to the pseudoscalar mesons $\pi^{0}, \eta$ and $\eta^{\prime}$ (see Fig. 2.9). The results obtained for the $\gamma \gamma$ widths are:

- $\Gamma\left(\pi^{0} \rightarrow \gamma \gamma\right)=(7.7 \pm 0.5 \pm 0.5) \mathrm{keV}$,
- $\Gamma(\eta \rightarrow \gamma \gamma)=(0.514 \pm 0.017 \pm 0.035) \mathrm{keV}$,
- $\Gamma\left(\eta^{\prime} \rightarrow \gamma \gamma\right)=(4.7 \pm 0.5 \pm 0.5) \mathrm{keV}$.

The production of the $\eta$ and $\eta^{\prime}$ mesons in $\gamma \gamma$ interactions has also been observed with the detector ASP [18] at the PEP storage ring (SLAC), with a data sample of $108 \mathrm{pb}^{-1}$ and a center of mass energy $\sqrt{s}=29 \mathrm{GeV}$. The process studied is $e^{+} e^{-} \rightarrow e^{+} e^{-} R$, with $R \rightarrow \gamma \gamma$. After a selection of $2287 \eta$ events and $547 \eta^{\prime}$ events, the following $\gamma \gamma$ partial widths are obtained:

- $\Gamma(\eta \rightarrow \gamma \gamma)=(0.490 \pm 0.010 \pm 0.048) \mathrm{keV}$,
- $\Gamma\left(\eta^{\prime} \rightarrow \gamma \gamma\right)=(4.96 \pm 0.23 \pm 0.72) \mathrm{keV}$.


Figure 2.9: Distribution of the invariant $\gamma \gamma$ mass in the region of the $\pi^{0}$ mass (a), of the $\eta$ mass (b) and of the $\eta^{\prime}$ mass (c) obtained with the Crystal Ball experiment at DORIS II.

## Chapter 3

## The KLOE experiment at DA $\Phi$ NE

The K LOng Experiment, KLOE, operates at the Frascati $\phi$-factory DAФNE. The main goal of the experiment is to measure direct CP violation in the neutral kaon system, analyzing $K \bar{K}$ couples produced in the $\phi$ meson decays. However, DAФNE also produces a huge statistics of $\rho, \omega, \eta, \eta^{\prime}, f_{0}, a_{0}$ mesons. The KLOE physics program goes thus beyond the study of symmetry violations in kaons, and covers many other topics, among which high precision studies on light hadron spectroscopy.

### 3.1 The collider DAФNE

DAФNE [26] (Double Annular $\Phi$-factory for Nice Experiments) is an electron- positron collider designed to work at a center of mass energy corresponding to the mass of the $\phi$ resonance, $M_{\phi}=(1019.456 \pm 0.020) \mathrm{MeV}$ [1]. The accelerator complex consists of a LINAC, an accumulator and a two-ring collider, as shown in Fig. 3.1. Electrons and positrons are accelerated up to 510 MeV in the LINAC and are then stored in the accumulator, where they are prepared for injection in the main rings. In order to reduce beam-beam interactions and to achieve high


Figure 3.1: Layout of the $D A \Phi N E$ facility.
values of luminosity ( $\approx 10^{32} \mathrm{~cm}^{-1} \mathrm{~s}^{-2}$ ), in DAФNE electrons and positrons are stored in two
different rings and cross at the interaction point, IP, in two interaction regions with an angle in the horizontal plane ( $\mathrm{x}-\mathrm{z}$ ) of 25 mrad , as shown in Fig. 3.2. This angle results in a small average $e^{+} e^{-}$-momentum along the $x$-axis: $\left\langle p_{x, e^{+} e^{-}}\right\rangle \approx-12.7 \mathrm{MeV}$. Electrons and positrons circulate


Figure 3.2: Layout of the DAФNE main rings. The boxes indicate the two interaction regions. The KLOE detector is located in the lower one.
in the rings grouped in bunches. If $L_{0}$ is the single bunch luminosity, the total luminosity can be expressed as:

$$
\begin{equation*}
L=n \times L_{0}=n \times \frac{v N_{+} N_{-}}{4 \pi \sigma_{x} \sigma_{y}}, \tag{3.1}
\end{equation*}
$$

where $n$ is the number of bunches, $N_{+}$and $N_{-}$the number of positrons and electrons per bunch, $v$ the collision frequency, $\sigma_{x}$ and $\sigma_{y}$ the transverse (horizontal and vertical respectively) dimensions of the bunch at the IP. The bunch dimensions are kept small at the IP by using a triplet of quadrupoles, which focus the beam in the vertical direction. The bunch sizes are $\sigma_{x}=0.2 \mathrm{~cm}, \sigma_{y}=20 \mu \mathrm{~m}, \sigma_{z}=3 \mathrm{~cm}$. The beams collide with a frequency up to 370 MHz , corresponding to a minimum bunch crossing period of $T_{\text {bunch }}=2.7 \mathrm{~ns}$ and a maximum number of 120 bunches in each ring.

### 3.2 The KLOE detector

The KLOE detector was designed to collect the largest amount of neutral kaons from $\phi$ decays. The size of the apparatus is driven by the decay length of the $K_{L}$, which at DA $\Phi$ NE is about 340 cm . The detectable decay products of neutral kaons are pions, electrons, muons and photons, the latter coming mainly from neutral pion decays. The momentum spectra, limited by the kaons low energy, range between 50 and $300 \mathrm{MeV} / \mathrm{c}$ for charged particles and between 20 and $300 \mathrm{MeV} / \mathrm{c}$ for photons [27]. The detector has to be efficient for these energy ranges, minimizing the losses due to geometrical acceptance. Fig. 3.3 shows a section of the detector. The main components are:

- a large, highly efficient drift chamber which measures trajectories and momenta of charged particles;


Figure 3.3: Vertical section of the KLOE detector.

- an electromagnetic calorimeter (barrel and endcaps) with excellent timing capabilities, to measure the energy deposits and the impact points of photons;
- a second electromagnetic calorimeter located in the narrow space between the drift chamber and the beam focusing quadrupoles, to improve acceptance and hermeticity;
- a superconducting coil which surrounds all the detectors and produces an axial magnetic field $B=0.52 \mathrm{~T}$.

The beam-pipe at the interaction point is made of a beryllium-aluminum alloy, 0.5 mm thick, to reduce multiple scattering, kaon regeneration, energy loss of particles and photon conversion, and encloses an interaction region made of a 10 cm radius sphere.

### 3.2.1 The drift chamber

The design of the KLOE drift chamber (DC) [28] was guided by the event topology of the $K_{L}$ decays. Five main physics requirements have to be fulfilled:

- high and uniform reconstruction efficiency over a large volume;
- good momentum resolution ( $\delta p_{T} / p_{T} \approx 0.4 \%$ ) for low momentum tracks ( $50<p<300$ $\mathrm{MeV})$. The dominant contribution to the momentum resolution is multiple scattering:

$$
\begin{equation*}
\frac{\delta p_{T}}{p_{T}}=\frac{0.053}{|B| L \beta} \sqrt{\frac{L}{X_{0}}} \tag{3.2}
\end{equation*}
$$

where $p_{T}$ is the transverse momentum in $\mathrm{GeV}, \beta$ is the velocity of the particle, $L$ is the track length in $\mathrm{m}, B$ is the magnetic field in T and $X_{0}$ is the radiation length;

- transparency to low energy photons (down to 20 MeV ), and minimization of $K_{L}$ regeneration;
- a track resolution in the transverse plane $\sigma_{R \Phi} \approx 200 \mu \mathrm{~m}$, a vertex resolution $\sigma_{v t x} \approx 1 \mathrm{~mm}$, and a z resolution $\sigma_{z} \approx 2 \mathrm{~mm}$ over the whole sensitive volume;
- fast trigger for neutral and charged particles.

The solution that meets the above requirements is a large cylindrical drift chamber, 3.3 m in length and 2 m in radius, around the IP. The uniform filling of the chamber has been achieved through a structure of drift cells almost square shaped, arranged in coaxial layers with alternating stereo angles which increase in magnitude with the radius from $\pm 60$ to $\pm 150 \mathrm{mrad}$ (Fig. 3.4). The stereo angle is defined as the angle between the wire and a line parallel to the


Figure 3.4: The KLOE drift chamber without the external wall.
z-axis passing through the point on the plate of the DC, where the wire is connected (Fig. 3.5). Uniformity of response is obtained with a ratio of field to sense wires of 3:1, which is a satisfac-


Figure 3.5: Scheme of the stereo angle geometry of the cells.
tory solution that ensures good electrostatic properties of the drift cell while still maintaining an acceptable track sampling frequency. Gold-plated tungsten wires are used as anodes (diam. $=25 \mu \mathrm{~m})$. For the field wires, silver-plated aluminum wires have been chosen (diam. $=80 \mu \mathrm{~m}$ ). There are 12 inner and 46 outer layers, the corresponding cell areas are $(2 \times 2) \mathrm{cm}^{2}$ and $(3 \times 3)$ $\mathrm{cm}^{2}$, respectively, for a total of 12582 single-sense-wire cells and 52140 wires. The gas used is a $90 \%$ helium, $10 \%$ isobutane mixture. The helium is the active component, its low atomic mass reduces multiple scattering and regeneration. The isobutane acts as quencher, absorbing the photons produced in recombination processes and avoiding the production of discharges in the DC . The mixture has a radiation length $X_{0} \approx 1300 \mathrm{~m}$. Taking into account also the presence of the wires, the average radiation length in the whole DC volume is $X_{0} \approx 900 \mathrm{~m}$. The signals coming from sense wire are amplified, discriminated and transmitted to read-out system: ADC for $\mathrm{dE} / \mathrm{dx}$ measurement and TDC for time meausurement. Samples of Bhabha-scattering events allow evaluation of the momentum resolution for $510 \mathrm{MeV} e^{ \pm}$(Fig. 3.6), as well as the beam energy at the IP, and the position and the shape of the luminous region. In the interval


Figure 3.6: Momentum resolution for Bhabha tracks as a function of the polar angle.
$50^{\circ}<\theta<130^{\circ}$ the momentum resolution is $\sigma_{p} \approx 1.3 \mathrm{MeV}, \sigma_{p} / p=2.5 \times 10^{-3}$.

### 3.2.2 The electromagnetic calorimeter

The KLOE electromagnetic calorimeter (EMC) [29] was designed to fulfill four main requirements:

- good time resolution ( $\approx 100 \mathrm{ps}$ ) and good spatial determination of the photon conversion point ( $\approx 1 \mathrm{~cm}$ );
- hermeticity ( $98 \%$ of the solid angle), good energy resolution $(\approx 5 \% / \sqrt{E[\mathrm{GeV}]})$ and high efficiency over the range $20-300 \mathrm{MeV}$;
- particle identification power for electrons, muons and charged pions;
- fast first level trigger.

The above considerations have led to the choice of a lead-scintillating fiber sampling calorimeter. Scintillating fibers offer many advantages, in particular they provide good light transmission over the required distances, up to about 4.3 m . It is easy to adapt the calorimeter shape (Fig. 3.7) to geometrical requirements, obtaining good hermeticity. The cylindrical barrel con-


Figure 3.7: The KLOE electromagnetic calorimeter.
sists of 24 modules 4.3 m long, 23 cm thick and trapezoidal in cross-section, with fibers running parallel to the beam line. Each of the two endcaps consists of 32 vertical C-shaped modules 0.7 to 3.9 m long and 23 cm thick, with fibers running perpendicular to the beam line. The whole structure has a $98 \%$ solid angle coverage. All modules are stacks of 0.55 mm thick lead foils (passive material) alternating with layers of 1 mm diameter scintillating fibers (active material) (Fig. 3.8). The average density is $5 \mathrm{~g} / \mathrm{cm}^{3}$, the radiation length is about 1.5 cm and the over-


Figure 3.8: Schematic view of the fiber-lead structure of the electromagnetic calorimeter for a barrel module.
all thickness of the calorimeter is about 15 radiation lengths. Light is collected at both ends of the fibers through light pipes, which match almost square portions of the module to 4880
photo-tubes (PMs). The read-out subdivides the calorimeter into five planes in depth, each 4.4 cm thick. In the transverse direction each plane is subdivided into cells 4.4 cm wide. The resulting read-out granularity is about $4.4 \times 4.4 \mathrm{~cm}^{2}$. Signals from the PMs are split and sent to ADC's for energy measurements and trigger, and to the TDC's for time measurements. The time difference of the signal at both ends allows to reconstruct the coordinate along the fiber with a resolution $\sigma_{\|} \approx 1.4 \mathrm{~cm} / \sqrt{E[\mathrm{GeV}]}$. The resolution in the orthogonal direction is $\sigma_{\perp} \approx 1.3$ cm . Energy resolution and linearity have been measured using photons from radiative Bhabha events. Event reconstruction from tracking informations determines the photon direction and energy, $E_{\gamma}$, with good accuracy. The photon energy is then compared with the measured cluster energy $E_{c l u}$. The resolution $\sigma_{E} / E_{\gamma}$ and the deviation from linearity $\left(E_{\gamma}-E_{c l u}\right) / E_{\gamma}$ are shown in Fig. 3.9 as a function of the photon energy. Linearity is better than $1 \%$ for $E_{\gamma}>75 \mathrm{MeV}$.


Figure 3.9: Top: linearity of the calorimeter energy response as a function of the photon energy. Bottom: energy resolution of the calorimeter as a function of the photon energy.

By fitting the energy resolution with a function $a / \sqrt{E[\mathrm{GeV}]}+b$, one obtains a stochastic term $a=5.7 \%$ and a negligible constant term, showing that the resolution is dominated by sampling fluctuations:

$$
\begin{equation*}
\frac{\sigma_{E}}{E_{\gamma}}=\frac{0.057}{\sqrt{E_{\gamma}[\mathrm{GeV}]}} . \tag{3.3}
\end{equation*}
$$

The time resolution has been obtained from the analysis of Bhabha events and radiative $\phi$ decays, and is shown in Fig. 3.10 as a function of the energy of the photon:

$$
\begin{equation*}
\sigma_{t}=\frac{57 \mathrm{ps}}{\sqrt{E_{\gamma}[\mathrm{GeV}]}} \oplus 100 \mathrm{ps} \tag{3.4}
\end{equation*}
$$

where the sampling fluctuation term is in agreement with test beam data. The second term is a constant to be added in quadrature and is given by two contributions: the intrinsic time spread due to the finite length of the luminous point in the beam direction, and the resolution of the synchronization with the DA $\Phi$ NE radiofrequency.


Figure 3.10: Time resolution of the calorimeter as a function of the photon energy.

### 3.2.3 The quadrupole calorimeters

The quadrupole tile calorimeters of KLOE (QCAL) [30] are two compact detectors, made of lead plates and scintillator tiles, that surround the focusing quadrupoles (Fig. 3.11). Their aim is to complete the hermeticity of the KLOE calorimeter for photons coming from the $K_{L} \rightarrow \pi^{0} \pi^{0} \pi^{0}$ decays. Each sector contains 16 absorber plates made of 1.9 mm thick lead, alternating with


Figure 3.11: The KLOE QCAL.

16 scintillator layers 1 mm thick. The scintillator layers are divided into three equal tiles. In the cylindrical section the tyles have rectangular shape, while in the conical section the shape is trapezoidal. In each layer, four 190 cm long wavelength shifting (WLS) fibers run along the sides of the tiles. Light is collected by PMs. The overall radial thickness is $5.5 X_{0}$.

### 3.2.4 The trigger system

The KLOE trigger system [31] was designed to:

- produce a trigger signal for all $\phi$ decays;
- recognize Bhabha and cosmic-ray events;
- reject machine background.

The trigger is based on local energy deposit in the calorimeter and multiplicity information from the DC. It is composed of two levels in order to both produce an early trigger with good timing to start the acquisition operations and to use as much information as possible from the DC. After the arrival of a first level trigger, additional information is collected from the DC, which is used, together with the calorimetric information, to confirm the trigger and start the data acquisition system.

## The EMC trigger

For trigger purposes the fine granularity of the calorimeter is not needed, therefore adjacent calorimeter columns are grouped together to form a "trigger sector" and their signals are summed. In order to guarantee that each "particle" is fully contained in at least one sum, the calorimeter signals form a set of totally overlapping sectors: "normal" and "overlap". In the barrel, each trigger sector is made of 5 cells $\times 6$ columns, being the columns of each series placed on top of the other by half sector width (see Fig. 3.12). Since the particle multiplicity is


Figure 3.12: Trigger sectors in the barrel.
higher in the forward region, mostly for background events, the geometry of the end caps is more complex: they are segmented in groups of four columns in the zone close to the beam axis and of five/six columns elsewhere. The outer layer of the entire calorimeter is used as a cosmic ray detector. The calorimeter triggers on local energy deposits larger than a programmable threshold. Two thresholds are used, one for the barrel $(\approx 50 \mathrm{MeV})$ and one for the endcaps $(\approx 150 \mathrm{MeV})$. In practice it is not easy to apply a threshold which corresponds to a constant energy deposit, because the signal amplitude depends on the position along the fibers of the
incident particle. This is due to the attenuation of the scintillator light in the optical fibers. In order to reduce the effect, the analog signals from both sides (A and B) of each calorimeter sector are compared to two thresholds $\left(T_{\text {low, }}, T_{\text {high }}\right)$. The threshold settings and comparator output are shown in Fig. 3.13. This scheme allows to apply a large variety of effective thresholds.



Figure 3.13: Effective trigger threshold as a function of the $z$-coordinate (along the fiber) of the incident particle.

## The DC trigger

DC information can be used to produce a trigger for the $\pi^{ \pm, \mp}$ and $\pi^{ \pm, 00}$ channels, for which the calorimeter trigger is less efficient. The DC trigger is based on the multiplicity of hit wires, i.e. on the sum of all signals from the 12582 DC sense wires. The sense wire signals, after preamplification, are brought to the ADS (Amplifier/Discriminator/Shaper) boards. On the ADS, each signal is discriminated, buffered, and split into two different paths. The first path is directed to the DC readout front-end; the second path is used for the trigger. In the trigger path, signals are formed with a width of 250 ns . The signals coming from the DC wires are organized in nine concentric ring sections called "superlayers", which represent the multiplicity of hit wires in eight, six or four (from the innermost to the outermost) contiguous planes. The superlayers are defined in order to reduce the effect of low-momentum electrons spiralling inside the DC volume, which produce a large number of hit wires in the inner region of the detector.

## The two-level trigger logic

The KLOE trigger is composed of two levels. The first level trigger (T1) is activated if there are two calorimeter fired sectors with barrel-barrel, barrel-endcap or endcap- endcap (not the same endcap) topology OR 15 DC hits within 250 ns . The T1 trigger sets $\mathrm{a} \approx 2 \mu$ long acknowledge
signal, which vetoes other T1 triggers and allows signals formation from the DC cells. The second level trigger (T2) validates the T1 trigger and starts the data acquisition. Events with two fired sectors in the external planes of the calorimeter (see Fig. 3.12) with barrel-barrel or barrel-endcap topology are passed to a third level hardware algorithm, based on DC hits, which recognizes cosmic-ray events and rejects them.

### 3.2.5 Data acquisition

The KLOE data acquisition (DAQ) [32] has been designed to cope with a rate of $10^{4}$ events per second, due to $\phi$ decays, downscaled Bhabha events, non vetoed cosmic rays and DAФNE machine background. An average event size of 5 kbytes is estimated, corresponding to a total bandwidth requirement of $50 \mathrm{Mbytes} / \mathrm{s}$. The DAQ readout system involves some 23000 channels of front end electronics (FEE) from EMC, DC and trigger system. For each event, relevant data coming from the whole FEE system have to be concentrated in a single CPU where a dedicated process builds the complete event. A three level scheme has been implemented. The first level reads data from single FEE crates. The second level combines information from different crates. The last level, responsible for final event building, relies on standard network media and protocols (TCP/IP).

### 3.3 Data reconstruction

Data reconstruction starts immediately after the completion of the calibration jobs. The reconstruction program, DATAREC [33], provides additional data-quality and monitoring information, and consists of several modules, among which EMC reconstruction, DC reconstruction, and track-to-cluster association.

### 3.3.1 Cluster reconstruction

The calorimeter is segmented into 2440 cells, which are read out by PMs at both ends (A, B). This segmentation provides the determination of the position of energy deposits in $r-\phi$ for the barrel and in $x-z$ for the endcaps. Both charges $Q_{A D C}^{A, B}$ and times $t_{T D C}^{A, B}$ are recorded. For each cell, the particle arrival time $t$ and its coordinate $s$ along the fiber direction (the zero being taken at the fiber center) are obtained using the times at the two ends as

$$
\begin{array}{r}
t(\mathrm{~ns})=\frac{1}{2}\left(t^{A}+t^{B}-t_{0}^{A}-t_{0}^{B}\right)-\frac{L}{2 v}, \\
s(\mathrm{~cm})=\frac{v}{2}\left(t^{A}-t^{B}-t_{0}^{A}+t_{0}^{B}\right), \tag{3.6}
\end{array}
$$

with $t^{A, B}=c^{A, B} \times t_{T D C}^{A, B}$, where $c^{A, B}$ are the TDC calibration constants, $t_{0}^{A, B}$ are the overall time offsets, $L$ and $v$ are the cell length and the light velocity in the fibers, respectively. The energy

## Chapter 3. The KLOE experiment at DAФNE

on each side of a cell $i$ is obtained as

$$
\begin{equation*}
E_{i}^{A, B}(\mathrm{MeV})=k_{E} \times g_{i}(s) \times \frac{S_{i}^{A, B}}{S_{M I P, i}^{A, B}}, \tag{3.7}
\end{equation*}
$$

where $S=Q_{A D C}-Q_{0, A D C}$ is the charge collected after subtraction of the zero-offsets (ADC "pedestals"), and $S_{\text {MIP }}$ is the response to a minimum ionizing particle crossing the calorimeter center. The correction factor $g(s)$ accounts for light attenuation as a function of the impact position $s$ along the fiber, while $k_{E}$ is the energy scale factor, obtained from showers of particles of known energy (for more information about global offsets and calibration constants see refs. [29, 33]). The cell energy $E_{i}$ is taken as the mean of the energies at each end:

$$
\begin{equation*}
E_{i}(\mathrm{MeV})=\frac{E_{i}^{A}+E_{i}^{B}}{2} \tag{3.8}
\end{equation*}
$$

Calorimeter reconstruction starts by applying the calibration constants to transform the measured quantities $Q_{A D C}$ and $t_{T D C}$ into the physical quantities $S$ and $t$. Position reconstruction and energy/time corrections are applied to each fired cell. Then a clustering algorithm looks for groups of cells contiguous in $r-\phi$ or $x-z$ and groups them into pre-clusters. In a second step, the longitudinal coordinates and arrival times of the pre-clusters are used for further merging and/or splitting. The cluster energy, $E_{c l u}$, is the sum of the energies of all cells assigned to a cluster. The cluster position, $\{x, y, z\}_{c l u}$, and time, $t_{c l u}$, are evaluated as energy-weighted averages over the contributing cells. Cells are included in the cluster search only if times and amplitudes are available on both sides; otherwise, they are recorded as "incomplete" cells. The available information from most of the incomplete cells is added to the existing clusters at a later stage, by comparing the positions of such cells with the cluster centroid.

### 3.3.2 Track reconstruction

Track reconstruction is performed in three steps: pattern recognition, track fit, and vertex fit. Each step is managed separately and produces the information for the following step.

## Pattern recognition

The pattern recognition algorithm searches for track candidates. It begins by associating hits, working inward from the outermost layer, and then obtains track segments and approximate trajectories parameters. The DC wires form alternating positive and negative stereo angles with respect to the $z$ direction. When the hits are projected on the $x-y$ plane, they are seen in the stereo views as two distinct images. The pattern recognition procedure first associates separately the hits of each projection, using only two dimensional information, and in a second step combines the track candidates of the two views, according to their curvature values and geometrical compatibility.

## Track fit

The track-fit procedure minimizes a $\chi_{t r k}^{2}$ function based on the comparison between the measured and the expected drift distance for each hit: $\chi_{\text {trk }}^{2}=\sum_{i=1}^{n}\left(d_{i}-d_{i}^{f i t}\right)^{2} / \sigma_{i}^{2}$, where $n$ is the number of hits, $d_{i}\left(t_{\text {drift }}\right)$ is the drift distance, obtained via the space-time (s-t) relation from the measured drift time, $d_{i}^{\text {fit }}$ is the result of the fit and $\sigma_{i}^{2}$ is the estimate of the hit resolution. The procedure is iterative because the s-t relation depends on the track parameters. At each tracking step, the effects due to energy loss and multiple scattering are estimated.

## Vertex fit

The track parameters are used to look for primary and secondary vertices. For each track pair, a $\chi_{\nu t x}^{2}$ function is computed from the distances of closest approach between tracks; the covariance matrices from the track-fit stage are used to evaluate the errors. The vertex position is obtained minimizing the $\chi_{v t x}^{2}$.

### 3.3.3 Track-to-cluster association

The track-to-cluster association module makes correspondences between tracks in the DC and clusters in the EMC. The procedure starts by assembling the reconstructed tracks and vertices into decay chains and by isolating the tracks at the end of these chains. For each of these tracks, the measured momentum and the position of the last hit in the DC are used to extrapolate the track to the EMC. The extrapolation gives the track length $L_{e x}$ from the last hit in the chamber to the calorimeter surface, and the momentum $\mathbf{p}_{e x}$ and position $\boldsymbol{x}_{e x}$ of the particle at the surface. The resulting impact point is then compared with the positions $\mathbf{x}_{c l}$ of the reconstructed cluster centroids. A track is associated to a cluster if the distance to the centroid in the plane orthogonal to the direction of incidence of the particle on the calorimeter, $D=\left|\left(\mathbf{x}_{c l}-\mathbf{x}_{e x}\right) \times \mathbf{p}_{e x} /\left|\mathbf{p}_{e x}\right|\right|$, is less than 60 cm .

## Chapter 4

## Data and background sample

### 4.1 Data sample and preselection filter

The data used in this analysis were collected by the KLOE detector at DAФNE in 2006 at $\sqrt{s}=1$ GeV . The average data taking conditions are summarized in Tab.4.1. The analysis is performed

| $\sqrt{s}$ | 1000.1 MeV |
| :---: | :---: |
| $e^{+} e^{-}$transverse momentum | 12.7 MeV |
| $e^{+}$current | 0.7 A |
| $e^{-}$current | 1.1 A |
| luminosity | $7 \times 10^{31} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$ |
| trigger rate | 1.7 kHz |

Table 4.1: Average values during the run.
on data taken at $\sqrt{s}=1 \mathrm{GeV}$ to reduce background due to $\phi$ decays, and is based on an integrated luminosity of $242.5 \mathrm{pb}^{-1}$, measured with a precision of about $0.3 \%$ recording large angle Bhabha scattering events [34]. The KLOE trigger uses both calorimeter and DC information. For this analysis the events are selected by the calorimeter trigger, requiring two energy deposits with $E>50 \mathrm{MeV}$ in the barrel or $E>150 \mathrm{MeV}$ in the endcaps. The data were filtered with the background rejection filter FILFO [33] before event reconstruction. FILFO is an offline filter used to recognize and reject cosmic rays, machine background events and Bhabha scattering events with electrons (positrons) emitted with polar angles $\theta<20^{\circ}$ that interact with the low-beta focusing quadrupoles. To reject background events, cuts are applied on the number of clusters, the number of DC hits, the total energy deposited in the calorimeter, the position of the most energetic clusters, and the ratio of the number of hits in the internal DC layers to the total number of hits. A 1/20th sample of unfiltered data is used to control the filter efficiency. Usual analysis filters applied to the reconstructed data are dedicated to the analysis of kaons and radiative $\phi$-decays, and require a large amount of energy deposited in the final state. In $\gamma \gamma$ interactions, on the contrary, most of the energy deposited in the final state is carried away by positrons and electrons that go undetected in the beam direction. In this analysis a filter that

## Chapter 4. Data and background sample

optimizes the selection of $\gamma \gamma$ events has been used, which requires:

- at least two energy clusters, neutral (not associated to any track) and prompt (with $\mid t-$ $r / c \mid<5 \sigma_{t}$ );
- all prompt neutral clusters are required to have energy $E_{\gamma}>15 \mathrm{MeV}$ and polar angle $20^{\circ}<\theta_{\gamma}<160^{\circ}$;
- at least one prompt neutral cluster with energy greater than 50 MeV ;
- a ratio of the energy of the two highest energy prompt clusters over the total calorimeter energy $R=\sum_{\gamma} E_{\gamma} / E_{\text {tot }}>0.3 ;$
- $100 \mathrm{MeV}<E_{\text {tot }}<900 \mathrm{MeV}$, to reject low energy background events and the high rate processes $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma, e^{+} e^{-} \rightarrow \gamma \gamma(\gamma)$.


### 4.2 Simulation of signal and background

The detector response for signal and background events is fully simulated with the program GEANFI [33], based on the package GEANT3. For a given process, the momenta of the particles in the final state are generated according to the data taking conditions, and GEANFI simulates the detector response. Moreover, it allows one to simulate the machine background on a run-by-run basis, i.e. simulation of accidental clusters and tracks follows the real data taking conditions, and the accidental activity is monitored analyzing $e^{+} e^{-} \rightarrow \gamma \gamma$ collinear events. The beam-induced background events are added to simulated events in the Monte Carlo, MC. The calorimeter clusters are simulated for all the particles, and also the DC hits for the charged particles. For the reconstruction of the events the same procedure applied to data (and explained in section 3.3) is used. The trigger response is simulated as well. While GEANFI contains the event generator for all background processes, a new generator for $e^{+} e^{-} \rightarrow e^{+} e^{-} X$ events is developed and interfaced to the detector simulation. While the Equivalent Photon Approximation involves almost real photons, the simulation used in this analysis generates events with exact matrix element according to full 3-body phase space distributions [35] (see Appendix A). This results in the production of $\eta$ mesons with non negligible transverse momentum. The relative error due to high-order radiative corrections is estimated to be $1 \%$ [36]. The $\eta$ transition form factor is parametrized as

$$
\begin{equation*}
F_{\eta}\left(q_{1}^{2}, q_{2}^{2}\right)=\left(\frac{1}{1-b_{\eta} q_{1}^{2}}\right)\left(\frac{1}{1-b_{\eta} q_{2}^{2}}\right) \tag{4.1}
\end{equation*}
$$

where $q_{i}^{2}$ are the 4-momenta of the virtual photons and $b_{\eta}$ is the slope parameter. The parameter $b_{\eta}$ for the $\eta$ meson has been measured at high $q^{2}$ values in $\gamma \gamma$ experiments with single-tagging [37, 38, 24] and with measurements of the $\eta$ leptonic radiative decays $\eta \rightarrow$ $\ell^{+} \ell^{-} \gamma[39,25,40]$ at low $q^{2}$ values, closer to those of this measurement (see Tab. 4.2). The results do not show appreciable dependence on $q^{2}$, as shown in Fig. 4.1, and the average value
assumed in this analysis is the weighted average out of the last three measurements in Tab. 4.2: $b_{\eta}=(1.94 \pm 0.15) \mathrm{GeV}^{-2}$, with $\chi^{2} / n_{d o f}=0.12 / 1$. The kinematics shows that there is a

| $b_{\eta}$ | Experiment | $q^{2}$ range $\left(\mathrm{GeV}^{2}\right)$ |
| :--- | :---: | :---: |
| $1.67 \pm 0.13$ | CLEO [37] | $1.5-20$ |
| $2.04 \pm 0.46$ | TPC [38] | $0.1-7$ |
| $1.42 \pm 0.21$ | CELLO [24] | $0.3-3.4$ |
| $1.9 \pm 0.4$ | Lepton-G [39] | $0.05-0.25$ |
| $1.95 \pm 0.17 \pm 0.05$ | NA60 [25] | $0.04-0.25$ |
| $1.92 \pm 0.35 \pm 0.13$ | CB/TAPS [40] | $0.025-0.25$ |

Table 4.2: Measurements of $b_{\eta}$ performed by the quoted experiments.


Figure 4.1: Measurements of $b_{\eta}$ performed by the quoted experiments as a function of the average $q^{2}$ value of the photons.
strong correlation between the $\eta$ longitudinal momentum in the $e^{+} e^{-}$center of mass, $p_{L}$, and the squared missing mass $M_{\text {miss }}^{2}$

$$
\begin{equation*}
M_{m i s s}^{2}=s+M_{\eta}^{2}-2 \sqrt{s} \sqrt{M_{\eta}^{2}+p_{T}^{2}+p_{L}^{2}}=s+M_{\eta}^{2}-2 \sqrt{s} E_{T} \sqrt{1+p_{L}^{2} / E_{T}^{2}} \tag{4.2}
\end{equation*}
$$

and, for small values of $p_{T}\left(E_{T} \simeq M_{\eta}\right)$,

$$
\begin{equation*}
M_{m i s s}^{2} \simeq s+M_{\eta}^{2}-2 M_{\eta} \sqrt{s}-\sqrt{s} \frac{p_{L}^{2}}{M_{\eta}} \tag{4.3}
\end{equation*}
$$

This correlation is shown in Fig. 4.2 for events generated according to the Equivalent Photon Approximation, i.e. with negligible transverse momentum of the outgoing $e^{+} e^{-}$, for events generated with exact matrix element and for MC reconstructed events that pass the preselection cuts of the $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta, \eta \rightarrow \pi^{+} \pi^{-} \pi^{0}$ analysis. All background processes are simulated


Figure 4.2: Correlation between the $\eta$ longitudinal momentum and the squared missing mass for events generated according to the EPA approximation (top left), for events generated with exact matrix element (bottom left), and for the reconstructed events that pass the preselection cuts of the $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta, \eta \rightarrow \pi^{+} \pi^{-} \pi^{0}$ analysis (right).
in GEANFI and have been extensively studied in other analyses. A source of irreducible background is originated by $e^{+} e^{-} \rightarrow \eta \gamma$ when the monochromatic photon is emitted at small angles and is not detected. The cross section for this process is measured in the same data sample and is used for normalization of the $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$ cross section.
The MC energy scale has been corrected taking into account data-MC comparisons for the processes $e^{+} e^{-} \rightarrow K_{S} K_{L}$ and $e^{+} e^{-} \rightarrow \eta \gamma$. The details of the procedure are described in Appendix B. The effect of tracking efficiency has been studied on a clean sample of $\phi \rightarrow \rho \pi \rightarrow \pi^{+} \pi^{-} \pi^{0}$ events (purity $>99 \%$ ), using 2002 and 2005 data [41]. The correction to the tracking efficiency depends on the longitudinal $\left(P_{z}\right)$ and transverse $\left(P_{T}\right)$ momentum of the track, and is given as a function of $P_{T}$ in slices of $P_{z}$. A gaussian smearing [42] is applied as well to the momenta of the reconstructed tracks.

## Chapter 5

## Cross section for $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$ with <br> $\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}$

### 5.1 Event selection

In addition to the pre-selection described in section 4.1, candidate events $\gamma^{(*)} \gamma^{(*)} \rightarrow \eta \rightarrow$ $\pi^{+} \pi^{-} \pi^{0}$ must fulfill the following requirements:

1. two and only two neutral prompt clusters with $|t-r / c|<3 \sigma_{t}$ and polar angle $23^{\circ}<\theta<$ $157^{\circ}$;
2. at least two tracks with opposite curvature extrapolated inside a cylinder with $\rho=\sqrt{x^{2}+y^{2}}<$ 8 cm and $|z|<8 \mathrm{~cm}$ centered on the average beam collision point;
3. distance of the first DC hit to the average beam collision point $<50 \mathrm{~cm}$ for both tracks (in case of two or more tracks with the same curvature, the track with best quality parameters is chosen);
4. sum of the momenta $\left|\vec{p}_{1}\right|+\left|\vec{p}_{2}\right|<700 \mathrm{MeV}$.

To minimize any selection bias and to optimize the selection efficiency, there is no requirement for the tracks to be associated to calorimeter clusters nor that they form a vertex. The number of selected events is $3.9 \times 10^{6}$. Events with fully neutral final states survive the tracks requirements, because of $\gamma N \rightarrow e^{+} e^{-} N$ conversions or $\pi^{0}$ Dalitz decays.

### 5.2 Background rejection

Many background contributions have been considered, of which the most important are $e^{+} e^{-} \rightarrow$ $\eta \gamma, e^{+} e^{-} \rightarrow \omega \pi^{0}, e^{+} e^{-} \rightarrow K_{S} K_{L}, e^{+} e^{-} \rightarrow K^{+} K^{-}$and $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$.

- The $e^{+} e^{-} \rightarrow \eta \gamma$ process is a source of irreducible background when $\eta$ decays to $\pi^{+} \pi^{-} \pi^{0}$ and the monochromatic photon, $E_{\gamma}=350 \mathrm{MeV}$, is emitted at small polar angles and is not
detected. However, the correlation of $M_{\text {miss }}^{2}$ vs. $p_{L}$ is rather different from the signal since $p_{L}=E_{\gamma} \cos \theta \simeq 350 \mathrm{MeV}$ and $M_{\text {miss }}^{2} \simeq 0$.
- The $e^{+} e^{-} \rightarrow \omega \pi^{0}$ process has four photons in the final state and therefore gives the same final state of the signal only when two photons are lost. The cross section has been measured by KLOE at $\sqrt{s}=1 \mathrm{GeV}$ with data of the same run: $\sigma\left(e^{+} e^{-} \rightarrow \omega \pi^{0} \rightarrow\right.$ $\left.\pi^{+} \pi^{-} \pi^{0} \pi^{0}\right)=(5.72 \pm 0.05) \mathrm{nb}$ [43].
- $e^{+} e^{-} \rightarrow K_{S} K_{L}$ events can mimic the signal either when the $K_{L}$ decays to $\pi^{ \pm} l^{\mp} v$ close to the collision point and $K_{S} \rightarrow \pi^{0} \pi^{0}$ with one neutral pion not detected, or when the $K_{S} \rightarrow \pi^{0} \pi^{0}$ decay gives rise to one $e^{+} e^{-}$pair from conversion or $\pi^{0}$ Dalitz decay and the $K_{L}$ escapes detection.
- $e^{+} e^{-} \rightarrow K^{+} K^{-}$events can mimic the signal when both kaons decay close to the collision point: either $K^{ \pm} \rightarrow \pi^{ \pm} \pi^{0}$ or semileptonic $K^{ \pm}$decays in coincidence with $K^{\mp} \rightarrow \mu^{\mp} v$ decays.
- $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ events have a very large cross section at $\sqrt{s}=1 \mathrm{GeV}$, about 400 nb , and can be an important background source in case of accidental or split clusters.


### 5.2.1 Photon pairing

The difference between the $\gamma \gamma$ invariant mass and the $\pi^{0}$ mass can be used to check if the two photons come from the $\pi^{0}$. This is performed using a pseudo- $\chi^{2}$ variable

$$
\begin{equation*}
\chi_{\gamma \gamma}^{2}=\frac{\left(m_{\gamma \gamma}-m_{\pi^{0}}\right)^{2}}{\sigma_{m \gamma \gamma}^{2}} \quad \text { with } \quad \frac{\sigma_{m \gamma \gamma}}{m_{\gamma \gamma}}=\frac{1}{2}\left(\frac{\sigma_{E \gamma i}}{E_{\gamma i}} \oplus \frac{\sigma_{E \gamma j}}{E_{\gamma j}}\right) \tag{5.1}
\end{equation*}
$$

The energy resolution function is given in Tab. 5.1; the $\gamma \gamma$ invariant mass is dominated by the calorimeter energy resolution while the angle measurement gives a negligible contribution. Fig. 5.1 shows the distribution of the $\chi_{\gamma \gamma}^{2}$ variable for MC signal events and data. Candidate

| $\sigma_{E} / E$ | $0.057 / \sqrt{E(\mathrm{GeV})}$ |
| :--- | :---: |
| $\sigma_{t}$ | $57 \mathrm{ps} / \sqrt{E(\mathrm{GeV})} \oplus 100 \mathrm{ps}$ |
| $\sigma_{x y}$ (barrel), $\sigma_{x z}$ (endcap) | 1.3 cm |
| $\sigma_{z}$ (barrel), $\sigma_{y}$ (endcap) | $1.4 \mathrm{~cm} / \sqrt{E(\mathrm{GeV})}$ |

Table 5.1: Resolution function for the cluster measurements.
events are selected asking for $\chi_{\gamma \gamma}^{2}<8$.

### 5.2.2 Kinematic fit

The two tracks momenta are combined with the $\pi^{0}$ to identify $\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}$ decay candidates, assigning the charged pion mass to the tracks. A kinematic fit is then performed, using Lagrange Multipliers (see Appendix C). The fit requires that the $\pi^{+} \pi^{-} \gamma \gamma$ invariant mass is equal


Figure 5.1: $\chi_{\gamma \gamma}^{2}$ distribution for data (top) and MC signal events (bottom).
to the $\eta$ mass. The function to be minimized is the following:

$$
\begin{equation*}
\chi_{\eta}^{2}=\sum_{i=1}^{N} \frac{\left(V_{i}-V_{i}^{\text {meas }}\right)^{2}}{\sigma_{i}^{2}}+\sum_{j=1}^{M} \lambda_{j}^{k} C_{j}\left(V_{1}^{k}, \ldots V_{N}^{k}\right), \tag{5.2}
\end{equation*}
$$

where $V_{i}^{\text {meas }}$ are the measured values of the $V_{i}$ variable, $C_{j}\left(V_{i}\right)$ are $M$ constraints, $\lambda_{j}$ are the Lagrange Multipliers and $k$ is the iteration index. The number of degrees of freedom is given by the number of constraints. To search for the minimum of equation (5.2) the following 10 quantities are used:

- the energy, $E_{i}$;
- the time, $t_{i}$;
- the cluster centroid position, $x_{i}, y_{i}, z_{i}$.
for the two photons. The track momenta are not varied in the minimization since these are measured with much better precision than the cluster energies. There are four constraints:
- promptness of the two photons assumed to originate at the IP, $t_{i}-r_{i} / c=0$;
- $\eta$ and $\pi^{0}$ masses, $M_{\pi^{+} \pi^{-} \gamma \gamma}=m_{\eta}$ and $M_{\gamma \gamma}=m_{\pi^{0}}$.

The resolution functions used in the fit are given in Tab. 5.1. Fig. 5.2 shows the distribution of the $\chi^{2}$ of the kinematic fit, $\chi_{\eta}^{2}$, for data and MC signal events. The cut $\chi_{\eta}^{2}<20$ is applied. This

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cut reduces the $\eta\left(\rightarrow \pi^{+} \pi^{-} \pi^{0}\right) \gamma$ background, which has has a long tail for $\chi_{\eta}^{2}>20$ values, due to events with the monochromatic photon in the detector acceptance and one undetected photon coming from the $\pi^{0}$ that are not rejected by the $\chi_{\gamma \gamma}^{2}<8$ requirement. Fig. 5.3 shows the


Figure 5.2: Distribution of the $\chi^{2}$ of the kinematic fit for data (top) and MC signal events (bottom).
correlation between the energy of the most energetic photon, $E_{\gamma 1}$, and the $\chi_{\eta}^{2}$ variable for MC $\eta\left(\rightarrow \pi^{+} \pi^{-} \pi^{0}\right) \gamma$ events before and after the $\chi_{\gamma \gamma}^{2}<8$ requirement.

### 5.2.3 Track identification and rejection of the QED background

At this stage of the selection, radiative Bhabha scattering, $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$, and $e^{+} e^{-} \rightarrow \gamma \gamma$ annihilation followed by photon conversion are still a source of background. Separation of charged pion from electron/positron tracks is done using a $\pi-e$ likelihood method [42]. This method is applicable when a cluster is associated to the track, i.e. the distance between the centroid and the extrapolation of the track to the calorimeter wall is less than 60 cm . The likelihood is based on the following variables:

- the time of flight difference $t-\ell / c$, where $t$ is the time assigned to the cluster, and $\ell$ is the track length;
- the cluster energy deposited in the calorimeter;
- the fraction of energy deposited in the first and the fifth calorimeter layers.


Figure 5.3: Correlation between $E_{\gamma 1}$ and $\chi_{\eta}^{2}$ for MC $\eta\left(\rightarrow \pi^{+} \pi^{-} \pi^{0}\right) \gamma$ events before (top) and after (bottom) the $\chi_{\gamma \gamma}^{2}<8$ selection.

In this analysis events with a cluster associated to each track and a value of the likelihood estimator $\log \mathcal{L}_{\pi} / \mathcal{L}_{e}<0$ for both clusters are rejected. Bhabha radiative events, $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$, are characterized by a small acollinearity, with small values of $\theta_{+}$and large values of $\theta_{-}$, where $\theta_{+}$and $\theta_{-}$are the polar angle of the positive and the negative track, respectively. In the case of $\gamma N \rightarrow e^{+} e^{-} N$ conversions, instead, both tracks have either small or large values of $\theta$. Fig. 5.4 shows the correlation between the polar angle of the positive and the negative track before and after the cuts on the fit $\chi^{2}$ and on the likelihood estimator. Events along the diagonal, $\theta_{+}+\theta_{-} \simeq 180^{\circ}$, survive the cut on the likelihood estimator, because they are not the result of QED reactions. These events are due to a peculiarity of the track reconstruction program that does not assume that a track originates in the beam collision point. In fact, if a single track with positive, or negative, curvature is split in two track segments, and both are extrapolated back close to the collision point, it may happen that they are reconstructed as two opposite curvature tracks with opposite momenta originated close to the point where the original track was split. An example is shown in the event display of Fig 5.5 where a single track is split in two tracks


Figure 5.4: Correlation between positive and negative track polar angles before (top) and after (bottom) the cuts on the fit $\chi^{2}$ and on the likelihood estimator.
with origin far away from the collision point, thus with opposite curvature. These events are characterized by

- large values of the angle $\alpha$ between the two candidate tracks, i.e. $\theta_{+} \simeq 180-\theta_{-}$;
- opposite values of the momenta, i.e. $\vec{p}+\simeq-\vec{p}-$;
- small distance between the first DC hits of the two tracks.

Fig. 5.6 shows the correlation between the opening angle of the two candidate tracks $\alpha$ (close to $180^{\circ}$ ) and the momentum difference $\Delta p$ (centered around zero) for data and for signal simulated events. Indeed also the simulation shows the same split-track pathology, though much less abundant. The characteristics of these events suggest a way to reject a large part of them by cutting on the tracks opening angle $\alpha$ and the distance between the first hit of the positive track and the first hit of the negative track, as shown in Fig. 5.7. Also the cut $\alpha_{\pi^{+} \pi^{-}}<176^{\circ}$ has been applied. Additional cuts to suppress specifically $e^{+} e^{-} \rightarrow \gamma \gamma(\gamma)$ events and to reduce


Figure 5.5: Display of an event characterized by a large angle between the two tracks and a small difference between the absolute values of the momenta.
$\eta\left(\rightarrow \pi^{+} \pi^{-} \gamma\right) \gamma$ events where the monochromatic photon is mis-identified as coming from the neutral pion are

- a cut on the energy of the most energetic photon, $E_{\gamma 1}<230 \mathrm{MeV}$,
- a cut on the polar angle of the most energetic photon, $27.5^{\circ}<\theta_{\gamma 1}<152.5^{\circ}$,
- a cut on the angle between the tracks, $\alpha_{\pi^{+} \pi^{-}}>50^{\circ}$, which is useful in reducing $\gamma$ conversions into an $e^{+} e^{-}$pair and background from kaons.

Fig. 5.8 shows the correlation between the energy and the polar angle of the most energetic photon. Fig. 5.9 shows the distribution of the angle between the tracks, $\alpha_{\pi^{+} \pi^{-}}$, for data, MC $K^{+} K^{-}$events, MC signal events and $\mathrm{MC} K_{S} K_{L}$ events.

### 5.2.4 Time and energy quality cuts

Improved time and energy measurements, derived using the kinematic fit described in section 5.2.2, are compared with the direct measurements from the calorimeter so that the following pull statistics are defined

$$
\begin{align*}
\chi_{\mathrm{t}}^{2} & =\sum_{2 \gamma}\left(\frac{t_{\text {meas }}-t_{i m p r}}{\sigma_{t}}\right)^{2}  \tag{5.3}\\
\chi_{\mathrm{E}}^{2} & =\sum_{2 \gamma}\left(\frac{E_{\text {meas }}-E_{i m p r}}{\sigma_{E}}\right)^{2} \tag{5.4}
\end{align*}
$$



Figure 5.6: Correlation between the momentum difference $\Delta p$ and the two tracks opening angle $\alpha$ for data (top) and for signal simulated events (bottom).
where the superscript meas and impr indicate the values measured and returned by the fit, respectively. These variables are used to further suppress background events from $K_{S} K_{L}$ and $K^{+} K^{-}$. In fact, deviations from time intervals expected for photons coming from the collision point and from energy deposits expected for the $\eta$ decay products are a signature of kaon decays. The cuts applied are $\chi_{\mathrm{t}}^{2}<7, \chi_{\mathrm{E}}^{2}<8$. Figs. 5.10 and 5.11 show the time and energy pulls of the two photons for data, $\mathrm{MC} \mathrm{K}^{+} \mathrm{K}^{-}$events, MC signal events and $\mathrm{MC} K_{S} K_{L}$ events.


Figure 5.7: Correlation between the tracks opening angle $\alpha$ and the distance between the first hit of the positive track and the first hit of the negative track for data (top) and for signal simulated events (bottom). The cut to reject pathological events is indicated by the straight line.


Figure 5.8: Correlation between the energy and polar angle of the most energetic photon for data (top) and MC signal events (bottom). Selected events are included in the black rectangle.


Figure 5.9: Distribution of the angle between the tracks, $\alpha_{\pi^{+} \pi^{-}}$, for data (top left), MC $K^{+} K^{-}$ events (top right), MC signal events (bottom left) and MC $K_{S} K_{L}$ events (bottom right).


Figure 5.10: Time pulls of the two photons for data (top left), $\mathrm{MC} \mathrm{K}{ }^{+} K^{-}$events (top right), MC signal events (bottom left) and MC $K_{S} K_{L}$ events (bottom right). Selected events ( $\chi_{\mathrm{t}}^{2}<7$ ) are included in the black circle.


Figure 5.11: Energy pulls of the two photons for data (top left), MC $K^{+} K^{-}$events (top right), MC signal events (bottom left) and MC $K_{S} K_{L}$ events (bottom right). Selected events ( $\chi_{\mathrm{E}}^{2}<8$ ) are included in the black circle.

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### 5.3 Selection efficiency for signal and backgrounds

The selection efficiencies are evaluated with the MC simulation described in section 4.2. The number of MC events after each analysis cut is shown in Tabs. 5.2-5.4 for the signal and the backgrounds. The first row lists the number of generated events; the second row, selection, includes FILFO, trigger, and the filters described in sections 4.1 and 5.1. The trigger efficiency is controlled by comparison of the calorimeter trigger with a complementary trigger based on the DC hit patterns. The $e^{+} e^{-} \rightarrow \eta^{\prime} \gamma$ and $e^{+} e^{-} \rightarrow a_{0}(980) \gamma$ background processes do not survive the analysis cuts. Despite the number of $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ generated events is very large, the number of events surviving the cut on the $\pi-e$ likelihood is not sufficient for the analysis. Then, in this case, this cut is not applied since there is no correlation between the likelihood estimator and the variables used in the fit. Tab. 5.5 lists the number of data events after the same analysis cuts of Tabs. 5.2-5.4. The signal is simulated with different values of the $b_{\eta}$

|  | MC signal | $\eta\left(\rightarrow \pi^{+} \pi^{-} \pi^{0}\right) \gamma$ | $\omega \pi^{0}$ | $e^{+} e^{-} \gamma$ |
| :--- | :---: | :---: | :---: | :---: |
| generated | 19150 | 4907485 | 9208930 | 95580900 |
| selection | 6587 | 654119 | 283859 | 427533 |
| $\chi_{\gamma \gamma}^{2}<8$ | 6487 | 254487 | 156846 | 133280 |
| $\chi_{\eta}^{2}<20$ | 5528 | 135791 | 5162 | 1366 |
| likelihood | 4760 | 122670 | 4286 | - |
| split tracks | 4524 | 119755 | 3866 | 1037 |
| $E_{\gamma 1}<230 \mathrm{MeV}$ | 4492 | 115009 | 3691 | 692 |
| $27.5^{\circ}<\theta_{\gamma 1}<152.5^{\circ}$ | 4306 | 106341 | 3595 | 599 |
| $\chi_{\mathrm{t}}^{2}<7, \chi_{\mathrm{E}}^{2}<8$ | 4077 | 101090 | 2404 | 379 |
| $\alpha_{\pi^{+} \pi^{-}}>50^{\circ}$ | 3974 | 94704 | 2083 | 356 |

Table 5.2: Number of MC events after each analysis cut.

|  | $K^{+} K^{-}$ | $K_{S} K_{L}$ | $\eta\left(\rightarrow \pi^{+} \pi^{-} \gamma\right) \gamma$ | $\eta(\rightarrow$ neutrals $) \gamma$ |
| :--- | :---: | :---: | :---: | :---: |
| generated | 29487300 | 19872200 | 1024527 | 9822290 |
| selection | 124604 | 33632 | 449625 | 18170 |
| $\chi_{\gamma \gamma}^{2}<8$ | 118713 | 21083 | 68784 | 1615 |
| $\chi_{\eta}^{2}<20$ | 7813 | 5977 | 548 | 292 |
| likelihood | 6913 | 4712 | 463 | 208 |
| split tracks | 5032 | 4080 | 344 | 184 |
| $E_{\gamma 1}<230 \mathrm{MeV}$ | 4664 | 3768 | 170 | 70 |
| $27.5^{\circ}<\theta_{\gamma 1}<152.5^{\circ}$ | 4500 | 3625 | 148 | 60 |
| $\chi_{\mathrm{t}}^{2}<7, \chi_{\mathrm{E}}^{2}<8$ | 2741 | 2448 | 93 | 47 |
| $\alpha_{\pi^{+} \pi^{-}}>50^{\circ}$ | 2200 | 1170 | 92 | 29 |

Table 5.3: Number of MC events after each analysis cut.

|  | $\pi^{+} \pi^{-} \gamma$ | $\gamma \gamma(\gamma)$ | $\pi^{+} \pi^{-} \pi^{0}$ | $\pi^{+} \pi^{-} \pi^{0} \gamma$ |
| :--- | :---: | :---: | :---: | :---: |
| generated | 71939100 | 142775000 | 6057553 | 2964527 |
| selection | 204772 | 54278 | 2356549 | 984584 |
| $\chi_{\gamma \gamma}^{2}<8$ | 107532 | 18851 | 2318201 | 938952 |
| $\chi_{\eta}^{2}<20$ | 382 | 1462 | 169 | 1294 |
| likelihood | 372 | 444 | 155 | 1138 |
| split tracks | 334 | 362 | 120 | 503 |
| $E_{\gamma 1}<230 \mathrm{MeV}$ | 322 | 45 | 81 | 450 |
| $27.5^{\circ}<\theta_{\gamma 1}<152.5^{\circ}$ | 313 | 40 | 75 | 391 |
| $\chi_{\mathrm{t}}^{2}<7, \chi_{\mathrm{E}}^{2}<8$ | 207 | 20 | 57 | 258 |
| $\alpha_{\pi^{+} \pi^{-}}>50^{\circ}$ | 111 | 1 | 42 | 213 |

Table 5.4: Number of MC events after each analysis cut.

|  | data sample |
| :--- | :---: |
| filtered | $1.4 \times 10^{8}$ |
| selection | 3886001 |
| $\chi_{\gamma \gamma}^{2}<8$ | 2555196 |
| $\chi_{\eta}^{2}<20$ | 16539 |
| likelihood | 10210 |
| split tracks | 6788 |
| $E_{\gamma 1}<230 \mathrm{MeV}$ | 5687 |
| $27.5^{\circ}<\theta_{\gamma 1}<152.5^{\circ}$ | 5057 |
| $\chi_{\mathrm{t}}^{2}<7, \chi_{\mathrm{E}}^{2}<8$ | 3516 |
| $\alpha_{\pi^{+} \pi^{-}}>50^{\circ}$ | 2977 |

Table 5.5: Number of data events after each analysis cut.
parameter of the form factor, varying in the range $(0.7-2.24) \mathrm{GeV}^{-2}$ and the fit to derive the signal yield is repeated for each value. The values of the efficiencies shown in Tabs. 5.2-5.4 correspond to $b_{\eta}=1.94 \mathrm{GeV}^{-2}$.

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### 5.4 Cross section evaluation

To evaluate the number of signal events and the cross section, a 2-dimensional fit (see Appendix D ) of the data is performed. The distributions fitted are the $\pi^{+} \pi^{-} \gamma \gamma$ transverse momentum, $p_{T}$, and the squared missing mass, $M_{\text {miss }}^{2}$, in the window $p_{T}<300 \mathrm{MeV}$ and -0.15 $\mathrm{GeV}^{2}<M_{\text {miss }}^{2}<0.25 \mathrm{GeV}^{2}$. The 2-dimensional distributions used in the fit are shown in Fig. 5.12. The $p_{T}$ variable is preferred with respect to the longitudinal $\pi^{+} \pi^{-} \gamma \gamma$ momentum, $p_{L}$, because it allows a better separation between kaons and signal events, as can be deduced by comparing Figs. 5.12-5.13. The number of data events in this window is 2720, and the number of MC signal events is 3970 . The signal efficiency is $(20.73 \pm 0.29) \%$. The weights are left free for the signal and the backgrounds, except for the $\eta\left(\rightarrow \pi^{+} \pi^{-} \pi^{0}\right) \gamma$ background, whose cross section, measured in the same data sample (see chapter 9 ), is implemented by adding the following $\chi^{2}$-like term in the 2-dimensional fit:

$$
\begin{equation*}
-2 \ln \mathcal{L} \rightarrow-2 \ln \mathcal{L}+\left(\frac{f_{\eta \gamma} N_{d a t a} /\left(\epsilon_{\eta \gamma} L\right)-\sigma_{\eta \gamma}}{\delta \sigma_{\eta \gamma}}\right)^{2} \tag{5.5}
\end{equation*}
$$

where $\sigma_{\eta \gamma}=\sigma_{e^{+} e^{-} \rightarrow \eta \gamma \rightarrow \pi^{+} \pi^{-} \pi^{0} \gamma}=(194.7 \pm 3.3) \mathrm{pb}$ is the measured $\eta \gamma$ cross section, and the other quantities refer to $\eta \gamma$ events as selected in the present measurement. The fit returns the fraction of data events $f_{i}=n_{i} / n_{\text {tot }}$ with the constraint $\sum_{i} f_{i}=1$. Tab. 5.6 lists the fraction of events returned by the fit. The error on $f_{\eta \gamma}$ is constrained by the $\chi^{2}$-like term of equation

| $f_{\text {signal }}$ | $(14.49 \pm 1.06) \%$ |
| :--- | ---: |
| $f_{\eta \gamma}$ | $(32.02 \pm 0.54) \%$ |
| $f_{\omega \pi^{0}}$ | $(20.48 \pm 1.81) \%$ |
| $f_{K^{+} K^{-}}$ | $(15.13 \pm 1.81) \%$ |
| $f_{K_{S} K_{L}}$ | $(11.36 \pm 1.70) \%$ |
| $f_{e^{+} e^{-} \gamma}$ | $(7.54 \pm 0.87) \%$ |
| $\chi^{2} / n_{\text {dof }}=2670 / 2637$ |  |
| $n_{\text {data }}=2720$ |  |

Table 5.6: Fit results
(5.5). Other contributions either do not survive the analysis cuts or result in fractions by far negligible compared to sensitivities in Tab. 5.6. The projections of the fit are shown in Fig. 5.14 for data and backgrounds weighted by their fractions $f_{i}$, and the distribution of $p_{L}$ is shown in Figure 5.15, The most relevant background is $e^{+} e^{-} \rightarrow \eta \gamma$, characterized by $M_{m i s s}^{2} \simeq 0$ and $p_{L} \simeq \pm 350 \mathrm{MeV}$. The fit returns $394 \pm 28$ signal events, that correspond to a cross section $\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta \rightarrow e^{+} e^{-} \pi^{+} \pi^{-} \pi^{0}\right)=\left(7.84 \pm 0.57_{\text {stat }}\right) \mathrm{pb}$. Fig. 5.16 shows the distributions of $\chi_{\gamma \gamma}^{2}$ and $\chi_{\eta}^{2}$ with signal and backgrounds weighted by their fractions $f_{i}$.


Figure 5.12: Top: 2-dimensional distributions of $M_{m i s s}^{2}$ versus $p_{T}$ for MC signal events (top left), MC $\eta \gamma$ events (top right), MC $e^{+} e^{-} \gamma$ events (center left), MC $\omega \pi^{0}$ events (center right), MC $K^{+} K^{-}$events (bottom left), and MC $K_{S} K_{L}$ events (bottom right). Bottom: 2-dimensional distribution of $M_{\text {miss }}^{2}$ versus $p_{T}$ for data.


Figure 5.13: Top: 2-dimensional distributions of $M_{m i s s}^{2}$ versus $p_{L}$ for MC signal events (top left), MC $\eta \gamma$ events (top right), MC $e^{+} e^{-} \gamma$ events (center left), MC $\omega \pi^{0}$ events (center right), MC $K^{+} K^{-}$events (bottom left), and MC $K_{S} K_{L}$ events (bottom right). Bottom: 2-dimensional distribution of $M_{m i s s}^{2}$ versus $p_{L}$ for data.


Figure 5.14: Projections of the 2-dimensional fit. Top: distribution of the transverse momentum of the $\pi^{+} \pi^{-} \gamma \gamma$ system. Bottom: distribution of the squared missing mass. The contribution of the signal is blue, $e^{+} e^{-} \rightarrow \eta \gamma$ is red, $e^{+} e^{-} \rightarrow \omega \pi^{0}$ is black, $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ is green, $e^{+} e^{-} \rightarrow$ $K^{+} K^{-}$is light blue and $e^{+} e^{-} \rightarrow K_{S} K_{L}$ is purple.


Figure 5.15: Distribution of the longitudinal $\pi^{+} \pi^{-} \gamma \gamma$ momentum. The contribution of the signal is blue, $e^{+} e^{-} \rightarrow \eta \gamma$ is red, $e^{+} e^{-} \rightarrow \omega \pi^{0}$ is black, $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ is green, $e^{+} e^{-} \rightarrow K^{+} K^{-}$ is light blue and $e^{+} e^{-} \rightarrow K_{S} K_{L}$ is purple.


Figure 5.16: Distributions of the $\chi_{\gamma \gamma}^{2}$ variable (left) and the $\chi^{2}$ of the kinematic fit (right). The contribution of the signal is blue, $e^{+} e^{-} \rightarrow \eta \gamma$ is red, $e^{+} e^{-} \rightarrow \omega \pi^{0}$ is black, $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ is green, $e^{+} e^{-} \rightarrow K^{+} K^{-}$is light blue and $e^{+} e^{-} \rightarrow K_{S} K_{L}$ is purple.

Fig. 5.17 shows the distributions of the sum of the energies of the two photons, $E_{\gamma 1}+E_{\gamma 2}$, the $\pi^{+} \pi^{-}$invariant mass, $M_{\pi^{+} \pi^{-}}$, the angle between the two photons, $\alpha_{\gamma 1 \gamma 2}$, and the polar angle of the two photons system, $\theta_{\pi^{0}}$. In all the distributions there is good agreement between data and MC.


Figure 5.17: Distributions of $E_{\gamma 1}+E_{\gamma 2}$ (top left), $M_{\pi^{+} \pi^{-}}$(top right), $\alpha_{\gamma 1 \gamma 2}$ (bottom left) and $\theta_{\pi^{0}}$ (bottom right). The contribution of the signal is blue, $e^{+} e^{-} \rightarrow \eta \gamma$ is red, $e^{+} e^{-} \rightarrow \omega \pi^{0}$ is black, $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ is green, $e^{+} e^{-} \rightarrow K^{+} K^{-}$is light blue and $e^{+} e^{-} \rightarrow K_{S} K_{L}$ is purple.

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### 5.4.1 Estimates of background yields from a control region

The background weights, defined as $f_{X} N_{\text {data }} / N_{X, \mathrm{MC}}^{c}$, for the $X$ background with $N_{X, \mathrm{MC}}^{\text {cuts }}$ events after analysis cuts, are left free in the 2-dimensional fit. To check these weight values, a second 2-dimensional fit is performed in a sideband region. This independent control region is selected by the requirements:

- $20<\chi_{\eta}^{2}<60$,
- no cuts on the pulls variables,
- $p_{T}>150 \mathrm{MeV}$.

The fit is performed using the $p_{T}$ and $M_{m i s s}^{2}$ variables in the window $150 \mathrm{MeV}<p_{T}<320 \mathrm{MeV}$ and - $0.05 \mathrm{GeV}^{2}<M_{m i s s}^{2}<0.25 \mathrm{GeV}^{2}$. The 2-dimensional distributions used in the fit are shown in Fig. 5.18. The $\eta\left(\rightarrow \pi^{+} \pi^{-} \pi^{0}\right) \gamma$ cross section is constrained by the $\chi^{2}$-like term described in equation (5.5). Tab. 5.7 shows the fit results. The projections of the fit are shown in Fig. 5.19 for data and backgrounds weighted by their fractions $f_{i}$. There is good agreement between the weights obtained with the "standard" fit and those obtained with the fit in the sideband region, as shown in Tab. 5.8.

| $f_{\omega \pi^{0}}$ | $(21.11 \pm 4.31) \%$ |
| :---: | :---: |
| $f_{K^{+} K^{-}}$ | $(57.90 \pm 5.63) \%$ |
| $f_{K_{S} K_{L}}$ | $(20.16 \pm 4.06) \%$ |
| $\chi^{2} / n_{\text {dof }}=787 / 774$ |  |
| $n_{\text {data }}=582$ |  |

Table 5.7: Fit results.

|  | standard fit weights (\%) | sideband weights (\%) |
| :--- | :---: | :---: |
| $e^{+} e^{-} \rightarrow \omega \pi^{0}$ | $28.2 \pm 2.5$ | $23.9 \pm 4.9$ |
| $e^{+} e^{-} \rightarrow K^{+} K^{-}$ | $21.6 \pm 2.6$ | $23.9 \pm 2.3$ |
| $e^{+} e^{-} \rightarrow K_{S} K_{L}$ | $26.8 \pm 4.0$ | $31.0 \pm 6.2$ |

Table 5.8: Background weights. Comparison between standard fit and control region.


Figure 5.18: 2-dimensional distributions of $M_{\text {miss }}^{2}$ versus $p_{T}$ in the control region for MC $\eta \gamma$ events (top left), MC $\omega \pi^{0}$ events (top right), $\mathrm{MC} K^{+} K^{-}$events (center left), MC $K_{S} K_{L}$ events (center right), and data (bottom).


Figure 5.19: Projections of the 2-dimensional fit in the control region. Top: distribution of the transverse momentum of the $\pi^{+} \pi^{-} \gamma \gamma$ system. Bottom: distribution of the squared missing mass. The contribution of $e^{+} e^{-} \rightarrow \omega \pi^{0}$ is black, $e^{+} e^{-} \rightarrow K^{+} K^{-}$is light blue and $e^{+} e^{-} \rightarrow K_{S} K_{L}$ is purple.

### 5.5 Evaluation of the systematic uncertainties

The possible contributions to the systematic error are investigated by varying the analysis cuts by the r.m.s. width of the distributions of each variable. The results are shown in Tab. 5.9, where $\delta \sigma / \sigma(\%)$ is the percentage variation of the signal cross section. The total positive and negative variations are $+2.6 \%$ and $-2.4 \%$, and $2.6 \%$ is taken as final systematic fractional error. To this the MC simulation statistical error of $1.4 \%$ is added in quadrature, and the

|  | $f_{\text {signal }} N_{\text {data }}$ | $\epsilon_{\text {signal }}(\%)$ | $\delta \sigma / \sigma(\%)$ |
| :--- | :---: | :---: | :---: |
| $\chi_{\gamma \gamma}^{2}<6.6$ | 396 | 20.70 | +0.67 |
| $\chi_{\gamma \gamma}^{2}<10.8$ | 391 | 20.73 | -0.73 |
| $\chi_{\eta}^{2}<18.5$ | 393 | 20.72 | +0.06 |
| $\chi_{\eta}^{2}<23.5$ | 393 | 20.82 | -0.68 |
| $E_{\gamma 1}<210 \mathrm{MeV}$ | 385 | 20.54 | -1.17 |
| $E_{\gamma 1}<250 \mathrm{MeV}$ | 393 | 20.78 | -0.33 |
| $26.5^{\circ}<\theta_{\gamma 1}<153.5^{\circ}$ | 403 | 20.96 | +1.21 |
| $28.5^{\circ}<\theta_{\gamma 1}<151.5^{\circ}$ | 390 | 20.46 | +0.46 |
| $\chi_{\mathrm{t}}^{2}<6$ | 393 | 20.46 | +1.10 |
| $\chi_{\mathrm{t}}^{2}<8$ | 391 | 20.87 | -1.22 |
| $\chi_{\mathrm{E}}^{2}<7$ | 395 | 20.42 | +1.89 |
| $\chi_{\mathrm{E}}^{2}<9$ | 395 | 21.10 | -1.39 |
| $\alpha_{\pi^{+} \pi^{-}}>48^{\circ}$ | 394 | 20.79 | -0.21 |
| $\alpha_{\pi^{+} \pi^{-}}>52^{\circ}$ | 394 | 20.69 | +0.20 |

Table 5.9: Systematics for the $\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta \rightarrow e^{+} e^{-} \pi^{+} \pi^{-} \pi^{0}\right)$ measurement.
errors due to knowledge of the form factor and to the branching ratio are kept separate to account for correlations between the two $\eta$ decay modes. The changes of the result due to the variation of $b_{\eta}$ in the transition form factor formula are shown in Tab. 5.10. A $\pm 1 \sigma$ variation of $b_{\eta}$ leads to a $2.0 \%$ fractional error. The final result for the cross section is $\sigma\left(e^{+} e^{-} \rightarrow\right.$ $\left.e^{+} e^{-} \eta \rightarrow e^{+} e^{-} \pi^{+} \pi^{-} \pi^{0}\right)=\left(7.84 \pm 0.57_{\mathrm{stat}} \pm 0.23_{\mathrm{syst}} \pm 0.16_{\mathrm{FF}}\right) \mathrm{pb}$ at $\sqrt{s}=1 \mathrm{GeV}$. Using for the branching fraction the value $\operatorname{BR}\left(\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}\right)=0.2274 \pm 0.0028$ [1], one derives $\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta\right)=\left[34.5 \pm 2.5_{\text {stat }} \pm 1.0_{\text {syst }} \pm 0.7_{\mathrm{FF}} \pm 0.4_{\mathrm{BR}\left(\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}\right)}\right] \mathrm{pb}$.

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| $b_{\eta}$ | $N_{M C}$ | signal efficiency (\%) | signal yield (\%) | cross section $(\mathrm{pb})$ |
| ---: | :---: | :---: | :---: | :---: |
| 0.7 | 18807 | 20.16 | 14.161 | 34.65 |
| 1.5 | 18441 | 20.63 | 14.467 | 34.59 |
| 1.64 | 18994 | 20.53 | 14.058 | 33.77 |
| 1.8 | 18749 | 20.68 | 14.743 | 35.17 |
| 1.94 | 19150 | 20.73 | 14.487 | 34.47 |
| 2 | 18997 | 20.75 | 14.208 | 33.78 |
| 2.2 | 18909 | 20.28 | 14.396 | 35.01 |
| 2.24 | 18662 | 20.50 | 14.548 | 35.01 |

Table 5.10: Signal efficiency, event yield and cross section as a function of the $b_{\eta}$ parameter.

## Chapter 6

## Cross section for $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$ with $\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}$

### 6.1 Event selection

In addition to the pre-selection described in section 4.1, candidate decays $\gamma^{(*)} \gamma^{(*)} \rightarrow \eta \rightarrow$ $\pi^{0} \pi^{0} \pi^{0}$ must fulfill the following requirements:

- six and only six neutral prompt clusters with $|t-r / c|<3 \sigma_{t}$ and polar angle $23^{\circ}<\theta<$ $157^{\circ}$;
- no tracks in the DC.

The number of selected events is 9857 .

### 6.2 Background rejection

Many background contributions have been considered, of which the most important are $e^{+} e^{-} \rightarrow$ $\eta \gamma, e^{+} e^{-} \rightarrow \omega \pi^{0}, e^{+} e^{-} \rightarrow K_{S} K_{L}$ and $e^{+} e^{-} \rightarrow a_{0}(980) \gamma$.

- the $e^{+} e^{-} \rightarrow \eta \gamma$ process, as for the charged $\eta$ decay, is a source of irreducible background when $\eta$ decays to $\pi^{0} \pi^{0} \pi^{0}$ and the monochromatic photon, $E_{\gamma}=350 \mathrm{MeV}$, is emitted at small polar angles and is not detected. Anyway, also in this case the correlation of $M_{\text {miss }}^{2}$ vs. $p_{L}$ is rather different from the signal.
- The $e^{+} e^{-} \rightarrow \omega\left(\rightarrow \pi^{0} \gamma\right) \pi^{0}$ process has 5 photons in the final state and therefore is important only in case of accidental or split photons. The cross section has been measured by KLOE at $\sqrt{s}=1 \mathrm{GeV}$ with the same data set: $\sigma\left(e^{+} e^{-} \rightarrow \omega \pi^{0} \rightarrow \pi^{0} \pi^{0} \gamma\right)=(0.550 \pm 0.005)$ nb [43].
- $e^{+} e^{-} \rightarrow K_{S} K_{L}$ events mimic the signal when $K_{S} \rightarrow \pi^{0} \pi^{0}$, the $K_{L}$ is not detected and there are two additional split or accidental clusters.

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- The $e^{+} e^{-} \rightarrow a_{0}(980) \gamma \rightarrow \eta \pi^{0} \gamma$ process can mimic the signal only when $\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}$, with undetected photons, or $\eta \rightarrow 2 \gamma$, in presence of split or accidental clusters. Similar arguments apply to the $e^{+} e^{-} \rightarrow \eta^{\prime} \gamma$ background with $\eta^{\prime} \rightarrow$ neutrals topologies, more suppressed by products of branching fractions: $\operatorname{BR}\left(\eta^{\prime} \rightarrow \eta \pi^{0} \pi^{0}\right) \times \operatorname{BR}(\eta \rightarrow$ neutrals $)$ or $\operatorname{BR}\left(\eta^{\prime} \rightarrow \omega \gamma\right) \times \operatorname{BR}\left(\omega \rightarrow \pi^{0} \gamma\right)$ as the major components.


### 6.2.1 Photon pairing

The six photons are paired chosing the combination that minimizes the difference between the $\gamma \gamma$ invariant masses and the masses of the three neutral pions. This is performed using a pseudo- $\chi^{2}$ variable

$$
\begin{equation*}
\chi_{\gamma \gamma}^{2}=\sum_{\gamma \gamma p a i r} \frac{\left(m_{\gamma \gamma}-m_{\pi^{0}}\right)^{2}}{\sigma_{m \gamma \gamma}^{2}} \quad \text { with } \quad \frac{\sigma_{m \gamma \gamma}}{m_{\gamma \gamma}}=\frac{1}{2}\left(\frac{\sigma_{E \gamma i}}{E_{\gamma i}} \oplus \frac{\sigma_{E \gamma j}}{E_{\gamma j}}\right) . \tag{6.1}
\end{equation*}
$$

The energy resolution function is given in Tab. 5.1. Fig. 6.1 shows the distribution of the $\chi_{\gamma \gamma}^{2}$ variable for MC signal events and data. In the following analysis events with $\chi_{\gamma \gamma}^{2}<14$ are selected.


Figure 6.1: Distribution of $\chi_{\gamma \gamma}^{2}$ for data (top) and MC signal events (bottom).

### 6.2.2 Kinematic fit

A kinematic fit is then performed, requiring the $6 \gamma$ invariant mass to be equal to the $\eta$ mass.
The function to be minimized is the same used for the charged channel (equation (5.2)). To search for the minimum of equation (5.2) 30 variables are used:

- the energy, $E_{i} ;$
- the time, $t_{i}$;
- the cluster centroid position, $x_{i}, y_{i}, z_{i}$.
for the six photons. There are seven constraints:
- $t_{i}-r_{i} / c=0$ for the six photons;
- $M_{6 \gamma}^{2}=M_{\eta}^{2}$, where $M_{6 \gamma}^{2}$ is the invariant mass of the six photons and $M_{\eta}^{2}$ is the invariant mass of the $\eta$ meson.

The resolution functions used in the fit are given in Tab. 5.1. Fig. 6.2 shows the distribution of the $\chi^{2}$ of the kinematic fit for data and MC signal events. The cut applied is $\chi_{\eta}^{2}<20$. Fig. 6.3 shows the distribution of the $\chi^{2}$ of the kinematic fit for the backgrounds $e^{+} e^{-} \rightarrow \eta \gamma$, $e^{+} e^{-} \rightarrow \omega \pi^{0}, e^{+} e^{-} \rightarrow K_{S} K_{L}$ and $e^{+} e^{-} \rightarrow a_{0}(980) \gamma$.


Figure 6.2: Distribution of the $\chi^{2}$ of the kinematic fit for data (top) and MC signal events (bottom).


Figure 6.3: Distribution of the $\chi^{2}$ of the kinematic fit for $e^{+} e^{-} \rightarrow \eta \gamma$ (top left), $e^{+} e^{-} \rightarrow \omega \pi^{0}$ (top right), $e^{+} e^{-} \rightarrow K_{S} K_{L}$ (bottom left) and $e^{+} e^{-} \rightarrow a_{0}(980) \gamma$ (bottom right).

### 6.2.3 Cut on the energy of the most energetic photon

Also in this analysis the $e^{+} e^{-} \rightarrow \eta \gamma \rightarrow \pi^{0} \pi^{0} \pi^{0} \gamma$ background has a very big tail for bad $\chi_{\eta}^{2}$ values (i.e. greater than 20). This is due to events with the monochromatic photon in the detector acceptance. Fig. 6.4 (left) shows the distribution of the energy of the most energetic photon, $E_{\gamma 1}$, for data, MC $\eta \gamma$ events and MC signal events; the peak of the monochromatic photon, which is visible in the data and in the MC $\eta \gamma$ events, is shifted due to the preselection requirement $E_{\text {tot }}<900 \mathrm{MeV}$. On the right side of Fig. 6.4 the correlation between $E_{\gamma 1}$ and $\chi_{\eta}^{2}$ for $\eta \gamma$ events is shown. It is possible to see that the $\eta \gamma$ events with the monochromatic photon in the detector acceptance (e.g. $E_{\gamma}>300 \mathrm{MeV}$ ) have a bad $\chi_{\eta}^{2}$ value. To eliminate these events, in addition to the $\chi_{\eta}^{2}$ requirement, the cut $E_{\gamma 1}<260 \mathrm{MeV}$ is performed. Fig. 6.5 shows a good data-MC agreement also in the sideband region, selected requiring $\chi_{\eta}^{2}>20$ and $E_{\gamma 1}>260$ MeV .


Figure 6.4: Left: distribution of the energy of the most energetic photon, $E_{\gamma 1}$, for data (top), MC $\eta \gamma$ (center) and MC signal events (bottom). The peak of the monochromatic photon, which is visible in the data and in the MC $\eta \gamma$ distribution, is shifted due to the preselection requirement $E_{t o t}<900 \mathrm{MeV}$. Right: Correlation between $E_{\gamma 1}$ and $\chi_{\eta}^{2}$ for MC $\eta \gamma$ events.


Figure 6.5: Distribution of $E_{\gamma 1}$ in the sideband region ( $\chi_{\eta}^{2}>20, E_{\gamma 1}>260 \mathrm{MeV}$ ). Data are black, MC $\eta \gamma$ events are red.

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### 6.2.4 Cut on the $6 \gamma$ invariant mass

Fig. 6.6 shows the invariant mass distribution of the $6 \gamma, M_{6 \gamma}$, for data, MC signal events and MC $\eta \gamma$ events. A long tail at high values of $M_{6 \gamma}$ is visible in the data. This tail is not well reproduced by the signal and neither by the $\eta \gamma$ background. The cut $M_{6 \gamma}<630 \mathrm{MeV}$ is therefore applied. The systematics due to this cut are small (see section 6.5).


Figure 6.6: Invariant mass distribution of the $6 \gamma$ for data (top), MC signal events (center) and MC $\eta \gamma$ events (bottom).

### 6.3 Selection efficiency for signal and backgrounds

The selection efficiencies are evaluated with the MC simulation described in section 4.2. The number of MC events after each analysis cut is shown in Tabs. 6.1-6.2 for the signal and the backgrounds. The first row lists the number of generated events; the second row, selection, includes FILFO, trigger, and the filters described in sections 4.1 and 6.1. The $e^{+} e^{-} \rightarrow \gamma \gamma(\gamma)$ events do not survive the selection cuts. Tab. 6.3 lists the number of data events after the same analysis cuts of Tabs. 6.1-6.2.

|  | MC signal | $\eta\left(\rightarrow \pi^{0} \pi^{0} \pi^{0}\right) \gamma$ | $\omega\left(\rightarrow \pi^{0} \gamma\right) \pi^{0}$ |
| :--- | :---: | :---: | :---: |
| generated | 27159 | 6293520 | 914472 |
| selection | 8386 | 689018 | 1328 |
| $\chi_{\gamma \gamma}^{2}<14$ | 8231 | 585297 | 906 |
| $\chi_{\eta}^{2}<20$ | 8184 | 228699 | 274 |
| $E_{\gamma 1}<260 \mathrm{MeV}$ | 8024 | 142578 | 103 |
| $M_{6 \gamma}<630 \mathrm{MeV}$ | 7768 | 134788 | 70 |

Table 6.1: Number of MC events after each analysis cut.

|  | $K_{S} K_{L}$ | $a_{0}(980) \gamma$ | $f_{0}(980) \gamma$ | $\eta^{\prime} \gamma$ |
| :--- | :---: | :---: | :---: | :---: |
| generated | 19571400 | 53340 | 129115 | 43865 |
| selection | 2468 | 1440 | 190 | 934 |
| $\chi_{\gamma \gamma}^{2}<14$ | 2143 | 1243 | 134 | 815 |
| $\chi_{\eta}^{2}<20$ | 1566 | 900 | 42 | 337 |
| $E_{\gamma 1}<260 \mathrm{MeV}$ | 1552 | 802 | 15 | 120 |
| $M_{6 \gamma}<630 \mathrm{MeV}$ | 1437 | 453 | 9 | 93 |

Table 6.2: Number of MC events after each analysis cut.

|  | data sample |
| :--- | :---: |
| filtered | $3.77 \times 10^{8}$ |
| selection | 9857 |
| $\chi_{\gamma \gamma}^{2}<14$ | 6794 |
| $\chi_{\eta}^{2}<20$ | 3297 |
| $E_{\gamma 1}<260 \mathrm{MeV}$ | 2405 |
| $M_{6 \gamma}>630 \mathrm{MeV}$ | 2166 |

Table 6.3: Number of data events after each analysis cut.

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### 6.4 Cross section evaluation

To evaluate the number of signal events and the cross section, a 2-dimensional fit of the data is performed. The variables used to discriminate the signal from the background are the $6 \gamma$ longitudinal momentum, $p_{L}$, and the squared missing mass, $M_{\text {miss }}^{2}$, in the interval $-0.15 \mathrm{GeV}^{2}$ $<M_{\text {miss }}^{2}<0.35 \mathrm{GeV}^{2}$ and $-450 \mathrm{MeV}<p_{L}<450 \mathrm{MeV}$, that contains 2166 events. The signal efficiency is $(28.60 \pm 0.27) \%$. The fit to the data is done using the simulated shapes for the signal and backgrounds and the fit returns the fraction of data events $f_{i}=n_{i} / n_{\text {tot }}$ with the constraint $\sum_{i} f_{i}=1$. The weights are left free for the signal and the backgrounds. Background yields from $\omega\left(\rightarrow \pi^{0} \gamma\right) \pi^{0}, a_{0}(980) \gamma$ and $\eta^{\prime} \gamma$ result in the fractions estimated using the MC efficiencies from Tabs. 6.1-6.2:

$$
\begin{align*}
\mathcal{L} \sigma_{\omega \pi^{0}}(\sqrt{s}=1 \mathrm{GeV}) \epsilon_{\omega \pi^{0}} & \rightarrow f_{\omega \pi^{0}}=0.47 \%  \tag{6.2}\\
\sigma_{a_{0}(980) \gamma}(\sqrt{s}=1 \mathrm{GeV})<\sigma_{\eta \gamma}(\sqrt{s}=1 \mathrm{GeV}) \frac{\mathrm{BR}_{\phi \rightarrow a_{0} \gamma}}{\mathrm{BR}_{\phi \rightarrow \eta \gamma}} & \rightarrow f_{a_{0}(980) \gamma}<0.34 \%  \tag{6.3}\\
\sigma_{\eta^{\prime} \gamma}(\sqrt{s}=1 \mathrm{GeV}) \leq \sigma_{\eta \gamma}(\sqrt{s}=1 \mathrm{GeV}) \frac{\mathrm{BR}_{\phi \rightarrow \eta^{\prime} \gamma}}{\mathrm{BR}_{\phi \rightarrow \eta \gamma}} \quad & \rightarrow f_{\eta^{\prime} \gamma} \leq 0.02 \% \tag{6.4}
\end{align*}
$$

where the estimate of equation (6.2) comes from the knowledge of the $\omega \pi^{0}$ cross section value [43], and the estimates of equations $(6.3,6.4)$ are based on phase space arguments when moving from $\sqrt{s}=M_{\phi}$ to $\sqrt{s}=1 \mathrm{GeV}$, together with the knowledge of the $\eta \gamma$ cross section value (see chapter 9). These background contributions are by far below the expected statistics sensitivity of about $4-5 \%$ and are neglected. Fits with the $\omega \pi^{0}$ contribution left free are performed resulting in a negligible $f_{\omega \pi^{0}}$ value. The 2-dimensional distributions used in the fit are shown in Fig. 6.7. The contribution of the $K_{S} K_{L}$ background turns out completely negligible. Only the signal and the $\eta\left(\rightarrow \pi^{0} \pi^{0} \pi^{0}\right) \gamma$ background are left. Tab. 6.4 shows the fit results. The projections of the

| $f_{\text {signal }}$ | $(33.4 \pm 1.5) \%$ |
| :--- | :--- |
| $f_{\eta \gamma}$ | $(66.6 \pm 1.9) \%$ |
| $\chi^{2} / n_{\text {dof }}=1140 / 1536$ |  |
| $n_{\text {data }}=2166$ |  |

Table 6.4: Fit results
$M_{\text {miss }}^{2} \times p_{L}$ distribution are shown in Fig. 6.8 for the data and the backgrounds weighted by their relative factors $f_{i}$, and the $p_{T}$ distribution is shown in Fig. 6.9. From 2166 data events, $723 \pm 33$ signal events are obtained, and a cross section $\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta \rightarrow e^{+} e^{-} \pi^{0} \pi^{0} \pi^{0}\right)=$ $\left(10.43 \pm 0.48_{\text {stat }}\right) \mathrm{pb}$. The fit also returns the fraction of $\eta \gamma$ events, resulting in $1442 \pm 41$ events and a cross section $\sigma\left(e^{+} e^{-} \rightarrow \eta \gamma \rightarrow \pi^{0} \pi^{0} \pi^{0} \gamma\right)=\left(278.0 \pm 8.1_{\text {stat }}\right)$ pb. Fig. 6.10 shows the distributions of the variables $\chi_{\gamma \gamma}^{2}$ and $\chi_{\eta}^{2}$, where the MC distributions have the weights obtained from the fit. In all the distributions there is good agreement between data and MC.


Figure 6.7: 2-dimensional distributions of $M_{\text {miss }}^{2}$ versus $p_{L}$ for data (top left), MC signal events (bottom left), MC $\eta \gamma$ events (top right), and MC $K_{S} K_{L}$ events (bottom right).


Figure 6.8: Projections of the 2-dimensional fit. Top: distribution of the longitudinal monentum of the $6 \gamma$. Bottom: distribution of the squared missing mass. The contribution of the signal is blue and $e^{+} e^{-} \rightarrow \eta \gamma$ is red.


Figure 6.9: Distribution of the transverse momentum. The signal is blue and $e^{+} e^{-} \rightarrow \eta \gamma$ is red.


Figure 6.10: Distributions of the variables $\chi_{\gamma \gamma}^{2}$ (left) and $\chi_{\eta}^{2}$ (right). The contribution of the signal is blue and $e^{+} e^{-} \rightarrow \eta \gamma$ is red.

Chapter 6. Cross section for $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$ with $\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}$

### 6.5 Evaluation of the systematic uncertainties

The contributions to the systematic error are evaluated by varying the analysis cuts by the r.m.s. width of the distributions of the variables $\chi_{\gamma \gamma}^{2}, \chi_{\eta}^{2}$ and $M_{6 \gamma}$. The results are shown in Tab. 6.5, where $\delta \sigma / \sigma(\%)$ is the percentage variation of the signal cross section. In subsection 6.2.3 it was shown that in the $E_{\gamma 1}$ sideband region there is a very good data-MC agreement (see Fig. 6.5), both in shape and normalizazion, therefore a negligible systematic error is expected from the cut on this variable. The total positive and negative variations are $+2.6 \%$ and $-1.5 \%$, and $2.6 \%$ is taken as final systematic fractional error. To this the MC simulation statistical error of $1.0 \%$ is

|  | $f_{\text {signal }} N_{\text {data }}$ | $\epsilon_{\text {signal }}(\%)$ | $\delta \sigma / \sigma(\%)$ |
| :--- | :---: | :---: | :---: |
| central value | 723 | 28.60 | - |
| $\chi_{\gamma \gamma}^{2}<12$ | 716 | 28.46 | -0.51 |
| $\chi_{\gamma \gamma}^{2}<16$ | 732 | 28.69 | +0.83 |
| $\chi_{\eta}^{2}<17$ | 716 | 28.41 | -0.26 |
| $\chi_{\eta}^{2}<23$ | 730 | 28.66 | +0.68 |
| $M_{6 \gamma}<610$ | 686 | 27.49 | -1.33 |
| $M_{6 \gamma}<650$ | 754 | 29.13 | +2.38 |

Table 6.5: Systematics for the $\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta \rightarrow e^{+} e^{-} \pi^{0} \pi^{0} \pi^{0}\right)$ measurement.
added in quadrature, and the errors due to knowledge of the form factor and to the branching ratio are kept separate. The changes of the result due to the variation of $b_{\eta}$ in the transition form factor formula are shown in Tab. 6.6. The changes of the result due to the variation of $b_{\eta}$ in the

| $b_{\eta}$ | $N_{M C}$ | signal efficiency (\%) | signal yield (\%) | cross section $(\mathrm{pb})$ |
| ---: | :---: | :---: | :---: | :---: |
| 0.7 | 27413 | 27.51 | 32.974 | 32.87 |
| 1.5 | 27425 | 28.42 | 33.332 | 32.16 |
| 1.64 | 27011 | 28.93 | 32.882 | 31.18 |
| 1.8 | 27416 | 28.70 | 33.290 | 31.81 |
| 1.94 | 27159 | 28.60 | 33.409 | 32.03 |
| 2 | 27239 | 28.40 | 32.969 | 31.84 |
| 2.2 | 27051 | 28.22 | 33.169 | 32.24 |
| 2.24 | 27291 | 28.67 | 32.986 | 31.56 |

Table 6.6: Signal efficiency, event yield and cross section as a function of the $b_{\eta}$ parameter.
transition form factor formula lead to a $0.7 \%$ fractional error. The final result for the cross section is $\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta \rightarrow e^{+} e^{-} \pi^{0} \pi^{0} \pi^{0}\right)=\left(10.43 \pm 0.48_{\text {stat }} \pm 0.29_{\mathrm{syst}} \pm 0.07_{\mathrm{FF}}\right) \mathrm{pb}$ at $\sqrt{s}=1$ GeV . Using the value $\operatorname{BR}\left(\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}\right)=0.3257 \pm 0.0023$, one obtains $\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta\right)=$ $\left[32.0 \pm 1.5_{\mathrm{stat}} \pm 0.9_{\mathrm{syst}} \pm 0.2_{\mathrm{FF}} \pm 0.2_{\mathrm{BR}\left(\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}\right)}\right] \mathrm{pb}$. Similar systematics measurements are performed for the $\sigma\left(e^{+} e^{-} \rightarrow \eta \gamma\right)$ cross section, as shown in Tab. 6.7. The total positive and nega-

|  | $f_{\eta \gamma} N_{\text {data }}$ | $\epsilon_{\eta \gamma}(\%)$ | $\delta \sigma / \sigma(\%)$ |
| :--- | :---: | :---: | :---: |
| central value | 1442 | 2.14 | - |
| $\chi_{\gamma \gamma}^{2}<12$ | 1421 | 2.12 | -0.36 |
| $\chi_{\gamma \gamma}^{2}<16$ | 1451 | 2.16 | -0.20 |
| $\chi_{\eta}^{2}<17$ | 1429 | 2.11 | +0.43 |
| $\chi_{\eta}^{2}<23$ | 1446 | 2.15 | -0.44 |
| $M_{6 \gamma}<610$ | 1390 | 2.07 | -0.17 |
| $M_{6 \gamma}<650$ | 1470 | 2.18 | +0.13 |

Table 6.7: Systematics for the $\sigma\left(e^{+} e^{-} \rightarrow \eta \gamma \rightarrow \pi^{0} \pi^{0} \pi^{0} \gamma\right)$ measurement.
tive variations are $+0.5 \%$ and $-0.6 \%$, and $0.6 \%$ is taken as final systematic fractional error. The result for the cross section is $\sigma\left(e^{+} e^{-} \rightarrow \eta \gamma \rightarrow \pi^{0} \pi^{0} \pi^{0} \gamma\right)=\left(278.0 \pm 8.1_{\text {stat }} \pm 1.7_{\text {syst }}\right) \mathrm{pb}$ at $\sqrt{s}=1 \mathrm{GeV}$. With the same $\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}$ branching fraction value, one obtains $\sigma\left(e^{+} e^{-} \rightarrow\right.$ $\eta \gamma)=\left[853 \pm 25_{\text {stat }} \pm 5_{\text {syst }} \pm 6_{\mathrm{BR}\left(\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}\right)}\right] \mathrm{pb}$. This $e^{+} e^{-} \rightarrow \eta \gamma$ cross section is in agreement with the value measured in the dedicated analysis (see chapter 9 ).

Chapter 6. Cross section for $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$ with $\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}$

## Chapter 7

## Combination of the two cross section values

The cross section values from the $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta \rightarrow e^{+} e^{-} 3 \pi^{0}$ and $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta \rightarrow e^{+} e^{-} \pi^{+} \pi^{-} \pi^{0}$ processes are combined accounting for the following sources of correlation:
a) systematic uncertainties are correlated due to the photon requirement, i.e. the features of photons reconstructed in the calorimeter, essentially the cluster energy scale and so the derived efficiency, energy resolution and time resolution that are in common to the results of the two selections;
b) the common systematics due to the transition form factor, i.e. the $b_{\eta}$ parameter;
c) each exclusive cross section is divided by the appropriate branching fraction from PDG [1], and their correlation coefficients are accounted for.

The correlation coefficient for the type a) is evaluated varying the data/MC energy scale correction to the MC cluster from $0.5 \%$ to $1.1 \%$ in steps of $0.1 \%$, in such a way to have seven signal efficiencies for each analysis (see Tab. 7.1) and to perform statistical regression, resulting in $\rho_{\text {syst }}=-0.37$, obtained with the formula:

$$
\begin{equation*}
\rho=\frac{\sum_{i}\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)}{\sqrt{\sum_{j}\left(x_{j}-\bar{x}\right)^{2} \sum_{k}\left(y_{k}-\bar{y}\right)^{2}}} . \tag{7.1}
\end{equation*}
$$

The same formula is used to estimate the correlation coefficient for the type b), due to the form factor, evaluated by varying $b_{\eta}$ as in Tabs. 5.10-6.6. The correlation coefficient obtained is $\rho_{\mathrm{FF}}=0.34$. The correlation due to the branching fraction, $\rho_{\mathrm{BR}}=-0.73$, is taken from PDG [1]. The covariance matrices have the form

$$
\mathrm{V}_{\text {covariance }}=\left(\begin{array}{ll}
\delta_{3 \pi^{0}}^{2} & \rho \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}}  \tag{7.2}\\
\rho \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}} & \delta_{\pi^{+} \pi^{-} \pi^{0}}^{2}
\end{array}\right)
$$

| $\epsilon_{\text {data }} / \mathrm{MC}$ | $\epsilon_{\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}}(\%)$ | $\epsilon_{\eta \rightarrow 3 \pi^{0}}(\%)$ |
| :--- | :---: | :---: |
| $0.5 \%$ | 20.73107 | 28.65349 |
| $0.6 \%$ | 20.74151 | 28.64244 |
| $0.7 \%$ | 20.72585 | 28.62771 |
| $0.8 \%$ | 20.73107 | 28.60194 |
| $0.9 \%$ | 20.73107 | 28.57616 |
| $1.0 \%$ | 20.72585 | 28.56143 |
| $1.1 \%$ | 20.75196 | 28.52830 |

Table 7.1: Signal efficiencies for different values of the data/MC energy scale correction.

The following covariance matrices, whose elements are the cross section errors, are used:

$$
\begin{gather*}
\mathrm{V}_{\text {stat }}=\left(\begin{array}{ll}
\delta_{3 \pi^{0}}^{2} & 0 \\
0 & \delta_{\pi^{+} \pi^{-} \pi^{0}}^{2}
\end{array}\right), \mathrm{V}_{\mathrm{syst}}=\left(\begin{array}{ll}
\delta_{3 \pi^{0}}^{2} & -0.37 \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}} \\
-0.37 \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}} & \delta_{\pi^{+} \pi^{-} \pi^{0}}^{2}
\end{array}\right),  \tag{7.3}\\
\mathrm{V}_{\mathrm{FF}}=\left(\begin{array}{ll}
\delta_{3 \pi^{0}}^{2} & 0.34 \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}} \\
0.34 \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}} & \delta_{\pi^{+} \pi^{-} \pi^{0}}^{2}
\end{array}\right), \mathrm{V}_{\mathrm{BR}}=\left(\begin{array}{ll}
\delta_{3 \pi^{0}}^{2} & -0.73 \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}} \\
-0.73 \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}} & \delta_{\pi^{+} \pi^{-} \pi^{0}}^{2}
\end{array}\right) . \tag{7.4}
\end{gather*}
$$

The resulting combination covariance matrix, $\mathrm{V}_{\text {comb }}=\mathrm{V}_{\text {stat }}+\mathrm{V}_{\text {syst }}+\mathrm{V}_{\mathrm{FF}}+\mathrm{V}_{\mathrm{BR}}$,

$$
\mathrm{V}_{\mathrm{comb}}=\left(\begin{array}{ll}
\delta_{3 \pi^{0}}^{2} & \rho \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}}  \tag{7.5}\\
\rho \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}} & \delta_{\pi^{+} \pi^{-} \pi^{0}}^{2}
\end{array}\right)
$$

displays all the elements for determining the average cross section value

$$
\begin{equation*}
\sigma_{\mathrm{comb}}\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta\right)=w \sigma_{3 \pi^{0}}+(1-w) \sigma_{\pi^{+} \pi^{-} \pi^{0}} \tag{7.6}
\end{equation*}
$$

where the weight $w$ is given by

$$
\begin{equation*}
w=\frac{\delta_{\pi^{+} \pi^{-} \pi^{0}}^{2}-\rho \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}}}{\delta_{3 \pi^{0}}^{2}+\delta_{\pi^{+} \pi^{-} \pi^{0}}^{2}-2 \rho \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}}} \tag{7.7}
\end{equation*}
$$

and $\rho$ is the total correlation coefficient. The uncertainty is

$$
\begin{equation*}
\delta_{\mathrm{comb}}^{2}=\frac{\left(1-\rho^{2}\right) \delta_{3 \pi^{0}}^{2} \delta_{\pi^{+} \pi^{-} \pi^{0}}^{2}}{\delta_{3 \pi^{0}}^{2}+\delta_{\pi^{+} \pi^{-} \pi^{0}}^{2}-2 \rho \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}}} \tag{7.8}
\end{equation*}
$$

The values computed for $w$ and $\delta_{\text {comb }}^{2}$ are $w=0.7$ and $\delta_{\text {comb }}^{2}=2 \mathrm{pb}^{2}$, resulting in $\sigma_{\text {comb }}\left(e^{+} e^{-} \rightarrow\right.$ $\left.e^{+} e^{-} \eta\right)=(32.7 \pm 1.4) \mathrm{pb}=\left(32.7 \pm 1.3_{\text {stat }} \pm 0.7_{\text {syst }}\right) \mathrm{pb}$. The total correlation between the two measurements is found to be small, and the result obtained for the combined cross section is comparable to the average value obtained assuming independent uncertainties.

## Chapter 8

## Extraction of the partial width <br> $\Gamma(\eta \rightarrow \gamma \gamma)$

In order to extract the partial width $\Gamma(\eta \rightarrow \gamma \gamma) \equiv \Gamma_{\eta \gamma \gamma}$ each $\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta\right) \equiv \sigma_{\mathrm{X}}^{\text {obs }}$ measurement, done in the charged (CD) or neutral (ND) $\eta$ decay channel, is divided for the theoretical value $\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta \mid b_{\eta} ; \Gamma_{\eta \gamma \gamma}\right) \equiv \sigma^{\text {th }}$. The theoretical cross section value is obtained from a MC code with exact QED matrix element [36] (see Appendix A), using the form factor

$$
\begin{equation*}
F \eta\left(q_{1}^{2}, q_{2}^{2}\right)=\left(\frac{1}{1-b_{\eta} q_{1}^{2}}\right)\left(\frac{1}{1-b_{\eta} q_{2}^{2}}\right), \tag{8.1}
\end{equation*}
$$

and $\Gamma_{\eta \gamma \gamma}=1 \mathrm{keV}$. The width is obtained from

$$
\begin{equation*}
\Gamma_{\eta \gamma \gamma}=\frac{\sigma_{X}^{\mathrm{obs}}}{\sigma^{\mathrm{th}}}=\frac{N_{\mathrm{X}}}{\epsilon \mathrm{BR} \mathrm{R}_{\mathrm{X}} L \sigma^{\text {th }}} \tag{8.2}
\end{equation*}
$$

where $N_{X}$ is the number of $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta \rightarrow e^{+} e^{-} \pi^{+} \pi^{-} \pi^{0} / e^{+} e^{-} \rightarrow e^{+} e^{-} \eta \rightarrow e^{+} e^{-} \pi^{0} \pi^{0} \pi^{0}$ events observed, $\epsilon$ is the total efficiency, $L$ the integrated luminosity and $B R_{X}$ the branching fraction for each decay channel. In this formula, the form factor enters both in the $\sigma_{X}^{\text {obs }}$ measurement, i.e. in the signal efficiency and in the shape used for fitting data (events are generated using the same form factor of equation (8.1)), and in the theoretical cross section $\sigma^{\text {th }}\left(b_{\eta}\right)$. In order to exploit possible compensations, the width $\Gamma_{\eta \gamma \gamma}$ is extracted conditional to the $b_{\eta}$ value, consistently used in both $\sigma_{\mathrm{X}}^{\mathrm{obs}}$ and $\sigma^{\text {th }}$. Since the values of the 4-momenta $q_{1}$ and $q_{2}$ sampled in the two decay modes analyzed in chapters 5 and 6 can be slightly different, the partial width is determined separately for the two decays. Tab. 8.1 shows the cross sections and the partial width values as a function of the $b_{\eta}$ parameter. Using the reference $b_{\eta}=1.94 \mathrm{GeV}^{-2}$ value, one obtains:

$$
\begin{align*}
\Gamma_{\eta \gamma \gamma}^{\mathrm{CD}}\left[\text { from } \pi^{+} \pi^{-} \pi^{0}\right] & =\left(548 \pm 40_{\mathrm{stat}} \pm 16_{\mathrm{syst}} \pm 7_{\mathrm{BR}} \pm 14_{\mathrm{FF}}\right) \mathrm{eV}  \tag{8.3}\\
\Gamma_{\eta \gamma \gamma}^{\mathrm{ND}}\left[\text { from } \pi^{0} \pi^{0} \pi^{0}\right] & =\left(509 \pm 23_{\mathrm{stat}} \pm 14_{\mathrm{syst}} \pm 4_{\mathrm{BR}} \pm 8_{\mathrm{FF}}\right) \mathrm{eV} . \tag{8.4}
\end{align*}
$$

Chapter 8. Extraction of the partial width $\Gamma(\eta \rightarrow \gamma \gamma)$

| $b_{\eta}\left(\mathrm{GeV}^{-2}\right)$ | $\sigma_{\mathrm{CD}}^{\mathrm{obs}}(\mathrm{pb})$ | $\sigma_{\mathrm{ND}}^{\mathrm{obs}}(\mathrm{pb})$ | $\sigma^{\text {th }}(\mathrm{pb} / 1 \mathrm{keV})$ | $\Gamma_{\eta \gamma \gamma}^{\mathrm{CD}}(\mathrm{eV})$ | $\Gamma_{\eta \gamma \gamma}^{\mathrm{ND}}(\mathrm{eV})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.70 | 34.65 | 32.87 | 66.49 | 521.20 | 494.37 |
| 1.50 | 34.59 | 32.16 | 64.04 | 540.23 | 502.29 |
| 1.64 | 33.77 | 31.18 | 63.67 | 530.36 | 489.64 |
| 1.80 | 35.17 | 31.81 | 62.27 | 555.84 | 502.72 |
| 1.94 | 34.47 | 32.03 | 62.93 | 547.69 | 509.02 |
| 2.00 | 33.78 | 31.84 | 62.73 | 537.97 | 507.05 |
| 2.20 | 35.01 | 32.24 | 62.34 | 561.62 | 517.11 |
| 2.24 | 35.01 | 31.56 | 62.25 | 562.37 | 506.92 |

Table 8.1: Observed cross section, theoretical cross section and partial width as a function of the $b_{\eta}$ parameter.

Using the criterion of taking the largest fractional difference between the partial width evaluated at the $b_{\eta}=1.94 \mathrm{GeV}^{-2}$ reference and the ones evaluated at $b_{\eta}=(1.8-2.2) \mathrm{GeV}^{-2}$, the systematic error due to the form factor gets slightly modified with respect to the one assigned to the cross sections. It is relevant to notice that upon combining the two partial widths, also the form factor covariance matrix gets modified:
$\mathrm{V}_{\mathrm{FF}}=\left(\begin{array}{ll}\delta_{3 \pi^{0}}^{2} & 0.34 \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}} \\ 0.34 \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}} & \delta_{\pi^{+} \pi^{-} \pi^{0}}^{2}\end{array}\right) \rightarrow \mathrm{V}_{\mathrm{FF}}=\left(\begin{array}{ll}\delta_{3 \pi \pi^{0}}^{2} & 0.77 \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}} \\ 0.77 \delta_{3 \pi^{0}} \delta_{\pi^{+} \pi^{-} \pi^{0}} & \delta_{\pi^{+} \pi^{-} \pi^{0}}^{2}\end{array}\right)$.
The other correlation coefficients remain the same. Using the formulas given in the previous chapter, one finds $w=0.7$ and

$$
\begin{equation*}
\Gamma_{\mathrm{comb}}(\eta \rightarrow \gamma \gamma)=w \Gamma^{\mathrm{ND}}+(1-w) \Gamma^{\mathrm{CD}}=\left(520 \pm 20_{\text {stat }} \pm 13_{\text {syst }}\right) \mathrm{eV} \tag{8.5}
\end{equation*}
$$

resulting at present as the best determination of the $\Gamma_{\eta \gamma \gamma}$ partial width. Tab. 8 and Fig. 8.1 show the experimental results for the $\Gamma_{\eta \gamma \gamma}$ partial width measurement obtained studying the process $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$.

| $\Gamma_{\eta \rightarrow \gamma \gamma}(\mathrm{keV})$ | Experiment | Year |  |
| :--- | :---: | :---: | :---: |
| $0.56 \pm 0.16$ | Crystal Ball | 1983 | $[44]$ |
| $0.64 \pm 0.14 \pm 0.13$ | TPC-2 | 1986 | $[46]$ |
| $0.53 \pm 0.04 \pm 0.04$ | JADE | 1985 | $[45]$ |
| $0.514 \pm 0.017 \pm 0.035$ | Crystal Ball | 1988 | $[6]$ |
| $0.490 \pm 0.010 \pm 0.048$ | ASP | 1990 | $[18]$ |
| $0.51 \pm 0.12 \pm 0.05$ | MD-1 | 1990 | $[17]$ |
| $0.510 \pm 0.026$ | PDG average ${ }^{1}$ | $[1]$ |  |
| $0.520 \pm 0.020 \pm 0.013$ | KLOE | 2012 |  |

${ }^{1}$ The first two measurements are not taken into account for the calculation of the PDG average.

Table 8.2: Measurements of the $\Gamma_{\eta \gamma \gamma}$ partial width performed by different experiments studying the decay $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$.


Figure 8.1: Experimental points for the measurement of the $\Gamma_{\eta \gamma \gamma}$ partial width. The statistical error is black, while the total error is green. For the Crystal Ball 1983 measurement and the PDG average only the total error is quoted. The main contribution to the total KLOE error comes from the statistical error.

## Chapter 9

## Measurement of the cross section for $e^{+} e^{-} \rightarrow \eta \gamma$

The most relevant background in the measurement of the $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$ cross section is due to the radiative process $e^{+} e^{-} \rightarrow \eta \gamma$. The value of the cross section at $\sqrt{s}=1 \mathrm{GeV}$ has been used as a constraint in the fit in case of the $\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}$ decay while it has been derived as a by-product of the analysis of the $\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}$ decay. The cross section has been measured by the SND experiment [47] at VEPP-2M in the range $\sqrt{s}=(0.6-1.38) \mathrm{GeV}$, but with less precision than needed for this analysis.

### 9.1 Event selection

The cross section for $e^{+} e^{-} \rightarrow \eta \gamma$ is measured exploiting the $\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}$ decay using the same data sample and the same pre-selection and selection procedures described in sections 4.1,5.1 and 6.1 with the only difference that in this case events with three and only three neutral prompt clusters are selected. The event selection aims at finding two tracks of opposite curvature, compatible with being due to $\pi^{ \pm}$, two neutral prompt clusters compatible with being originated by a $\pi^{0}$ decay, and a third neutral prompt cluster compatible with the photon recoiling against the $\pi^{+} \pi^{-} \pi^{0}$ system.

### 9.2 Background rejection

Many background contributions have been considered:

- $e^{+} e^{-} \rightarrow \omega \pi^{0}$ events followed by the decay $\omega \rightarrow \pi^{+} \pi^{-} \pi^{0}$ are characterized by two tracks and four photons and can simulate the signal if one photon is not detected. The contribution from the decay $\omega \rightarrow \pi^{0} \gamma$ is negligible since there are no tracks in the final state.
- $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0} \gamma$ events (including $e^{+} e^{-} \rightarrow \omega \gamma$ events, with $\omega \rightarrow \pi^{+} \pi^{-} \pi^{0}$ ) have the same final state of the signal, with two photons and two tracks. The $e^{+} e^{-} \rightarrow \omega \gamma$ cross
section at $\sqrt{s}=1 \mathrm{GeV}$ has not been measured. According to the MC code Phokhara [48] $\sigma\left(e^{+} e^{-} \rightarrow \omega \gamma \rightarrow \pi^{+} \pi^{-} \pi^{0} \gamma, \sqrt{s}=1 \mathrm{GeV}\right)=1.4 \mathrm{nb}$.
- $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ events have a very large cross section at $\sqrt{s}=1 \mathrm{GeV}$, about 400 nb , and can be an important background source when an electron/positron track is recognized as a pion track. The two extra photons in the final state may come from split or accidental clusters.
- $e^{+} e^{-} \rightarrow K_{S} K_{L}$ events can mimic the signal when the $K_{L}$ decays to $\pi^{ \pm} l^{\mp} v$ close to the collision point and $K_{S} \rightarrow \pi^{0} \pi^{0}$ but one photon is not detected.
- $e^{+} e^{-} \rightarrow K^{+} K^{-}$events can mimic the signal when both kaons decay close to the collision point: $K^{ \pm} \rightarrow \pi^{ \pm} \pi^{0}$ and $K^{\mp} \rightarrow \pi^{\mp} \pi^{0}$, with four photons in the final state and one photon undetected, or $K^{ \pm} \rightarrow \pi^{ \pm} \pi^{0}$ and $K^{\mp} \rightarrow \mu^{\mp} v$, with two photons in the final state plus one accidental or split cluster.
- $e^{+} e^{-} \rightarrow \eta \gamma$ with $\eta \rightarrow \pi^{+} \pi^{-} \gamma$ events $(\mathrm{BR}=0.046)$ have only two photons in the final state, but one extra photon may come from split or accidental clusters. The presence of the monochromatic photon, $E_{\gamma}=350 \mathrm{MeV}$, and of the $\eta$ meson makes this contribution not easily distinguishable from the signal.
- $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ events can mimic the final state of the signal in case of accidental or split clusters.
- $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \gamma$ events can mimic the final state of the signal in case of at least two accidental or split clusters.
- $e^{+} e^{-} \rightarrow \gamma \gamma$ may become important in case of conversions in the material close to the beam line and when there are split or accidental clusters.
- $e^{+} e^{-} \rightarrow \eta \gamma$ with $\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}$ events have seven photons and no tracks in the final state. This can become an important background only in case of conversions; in most cases however there are some undetected photons.


### 9.2.1 Photon pairing

In this analysis the same pseudo- $\chi^{2}$ variable introduced in chapter 5 is used to identify the $\pi^{0}$ meson by selecting two out of the three total photons detected. Photons are paired choosing the combination that minimizes the difference between the $\gamma \gamma$ invariant mass and the mass of the neutral pion. No cut is applied to the value of $\chi_{\gamma \gamma}^{2}$.

### 9.2.2 Kinematic fit

A kinematic fit is then performed, using Lagrange Multipliers. The function to be minimized is the same used for the $\gamma^{(*)} \gamma^{(*)} \rightarrow \eta$ analyses (equation (5.2)), but in this case the leading
requirement is to satisfy energy and momentum conservation. To search for the minimum of equation (5.2) 15 variables are used:

- the energy, $E_{i} ;$
- the time, $t_{i}$;
- the cluster centroid position, $x_{i}, y_{i}, z_{i}$.
for the three photons. The track momenta are not varied in the minimization since these are measured with much better precision than the cluster energies. There are seven constraints:
- $t_{i}-r_{i} / c=0$ for the three photons;
- $\sqrt{s}=\sum_{3 \gamma} E_{\gamma}+E_{\pi^{+}}+E_{\pi^{-}} ;$
- $\sum_{3 \gamma} \vec{p}_{\gamma}+\vec{p}_{\pi^{+}}+\vec{p}_{\pi^{-}}=\vec{p}_{e^{+} e^{-}}$.

The resolution functions used in the fit are given in Tab. 5.1. Fig. 9.1 shows the distribution of the $\chi^{2}$ of the kinematic fit for data, MC signal events, MC $e^{+} e^{-} \rightarrow K^{+} K^{-}$events and MC $e^{+} e^{-} \rightarrow K_{S} K_{L}$ events. The cut $\chi^{2}<50$ is applied. The improved variables are used to fit the distribution of the recoil photon energy. Fig. 9.2 shows the pull statistics

$$
\begin{array}{ll}
\frac{E_{i m p r}-E_{\text {meas }}}{\sigma_{E}}, & \frac{t_{i m p r}-t_{\text {meas }}}{\sigma_{t}}, \\
\frac{x_{i m p r}-x_{\text {meas }}}{\sigma_{x}}, & \frac{z_{i m p r}-z_{\text {meas }}}{\sigma_{z}}, \tag{9.2}
\end{array}
$$

for the energy and the $z, x, t$ coordinates of the unpaired photon. The pairing procedure is repeated after the fit, to minimize the events in which the selected unpaired photon is one of the photons coming from the $\pi^{0}$. This is shown in Fig. 9.3, where the distributions of the "improved" (after the fit) energy of the unpaired photon, before, in black, and after, in red, the second pairing procedure are compared. Figs. $9.4-9.5$ show the distributions of the energy of the unpaired photon and of the $\pi^{+} \pi^{-} \pi^{0}$ invariant mass before and after the kinematic fit for data and MC signal events.

### 9.2.3 Track identification and rejection of the QED background

The background of $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ and $e^{+} e^{-} \rightarrow \gamma \gamma$ is reduced using the $\pi-e$ likelihood estimator as described in subsection 5.2.3. Fig. 9.6 shows the correlation between the polar angles of the positive and negative tracks before and after the cuts on the fit $\chi^{2}$ and on the likelihood estimator. Split-track events are completely removed at this stage of the analysis. Additional cuts to reduce $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ events are a cut on the angle between the tracks, $\alpha_{\pi^{+} \pi^{-}}<160^{\circ}$, and a cut on the angles between any photon pair, $\alpha_{\gamma_{i} \gamma_{j}}>20^{\circ}$. The last cut aims to reduce $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ events with one or more split photons. Fig. 9.7 shows the distribution of the angle between the two tracks for $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ events and for the signal, while Figs. 9.8 and 9.9 show some $\alpha_{\gamma_{i} \gamma_{j}}$ distributions for data, $\mathrm{MC} e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ events and MC signal events.


Figure 9.1: Distribution of the $\chi^{2}$ of the kinematic fit for data (top left), MC signal events (top right), $\mathrm{MC} e^{+} e^{-} \rightarrow K^{+} K^{-}$events (bottom left), $\mathrm{MC} e^{+} e^{-} \rightarrow K_{S} K_{L}$ events (bottom right).

### 9.2.4 Cut on the sum of the photon energies

A cut on the sum of the photon energies requires $\sum E_{\gamma}<660 \mathrm{MeV}$. This cut is performed to suppress the background $e^{+} e^{-} \rightarrow \eta \gamma \rightarrow \pi^{0} \pi^{0} \pi^{0} \gamma$ with conversions. Fig. 9.10 shows the distribution of the sum of the photon energies for MC signal events and MC $e^{+} e^{-} \rightarrow \eta \gamma \rightarrow$ $\pi^{0} \pi^{0} \pi^{0} \gamma$ events with conversions.

### 9.2.5 Cut on the sum of the track momenta

A cut on the sum of the momenta of the tracks requires $\left|\vec{p}\left(\pi^{+}\right)\right|+\left|\vec{p}\left(\pi^{-}\right)\right|<440 \mathrm{MeV}$. This cut is performed basically to suppress the backgrounds $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ and $e^{+} e^{-} \rightarrow \eta(\rightarrow$ $\left.\pi^{+} \pi^{-} \gamma\right) \gamma$, but it also reduces the contribution of $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0} \gamma$ and $e^{+} e^{-} \rightarrow \omega \pi^{0}$. Fig. 9.11


Figure 9.2: Pull statistics for the energy (top left) and the $z$ (top right), $x$ (bottom left), $t$ (bottom right) coordinates of the unpaired photon.
shows the distribution of the sum of the tracks momenta for MC signal events and MC $e^{+} e^{-} \rightarrow$ $\pi^{+} \pi^{-} \pi^{0}$ events, and the distribution of the energy of the unpaired photon for data before and after the cut on the sum of the tracks momenta. The $e^{+} e^{-} \rightarrow \omega \gamma$ peak is visible, with $E_{\gamma}=194$ MeV.


Figure 9.3: Distribution of the improved energy of the unpaired photon, before, in black, and after, in red, the second pairing procedure.


Figure 9.4: Distribution of the energy of the unpaired photon before (left) and after (right) the kinematic fit for data (top) and MC signal events (bottom). The peaks of the monochromatic photons $\gamma_{\eta}$ and $\gamma_{\omega}$ (from the processes $e^{+} e^{-} \rightarrow \eta \gamma_{\eta}$ and $e^{+} e^{-} \rightarrow \omega \gamma_{\omega}$ respectively) are clearly visible in the data distribution after the fit.


Figure 9.5: Distribution of the $\pi^{+} \pi^{-} \pi^{0}$ invariant mass before (left) and after (right) the kinematic fit for data (top) and MC signal events (bottom). The $\eta$ and $\omega$ peaks are clearly visible in the data distributions.


Figure 9.6: Correlation between positive and negative track polar angles in data before (top) and after (bottom) the cuts on the fit $\chi^{2}$ and on the likelihood estimator. Split-track events are completely removed at this stage of the analysis.


Figure 9.7: Distribution of the angle between the two tracks for MC $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ events (top) and MC signal events (bottom).


Figure 9.8: Distribution of the angle between the two paired photons (top), the angle between the most energetic paired photon and the unpaired photon (center) and the angle between the least energetic paired photon and the unpaired photon (bottom) for data.


Figure 9.9: Left: distribution of the angle between the two paired photons (top) and the angle between the most energetic paired photon and the unpaired photon (bottom) for MC signal events. Right: distribution of the angle between the least energetic paired photon and the unpaired photon for MC signal events (top) and MC $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ events (bottom).


Figure 9.10: Distribution of the sum of the photon energies for MC signal events (top) and for the MC of the process $e^{+} e^{-} \rightarrow \eta \gamma \rightarrow \pi^{0} \pi^{0} \pi^{0} \gamma$ with conversions (bottom).


Figure 9.11: Distribution of the sum of the tracks momenta for MC signal events (top) and MC $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ events (center). The plot at the bottom shows the distribution of the energy of the unpaired photon for data before (black) and after (red) the cut on the sum of the tracks momenta. The $e^{+} e^{-} \rightarrow \omega \gamma$ peak is clearly visible, with $E_{\gamma}=194 \mathrm{MeV}$.

### 9.3 Selection efficiency for signal and backgrounds

The selection efficiencies are evaluated with the MC simulation described in section 4.2. Tabs.9.1 - 9.2 list the number of MC events after each analysis cut for the signal and the backgrounds. The first row lists the number of generated events. The second row, selection, includes FILFO, trigger, and the filters described in sections 4.1 and 9.1. Table 9.3 lists the number of data events after the same analysis cuts of Tabs. 9.1-9.2.

|  | $\eta\left(\rightarrow \pi^{+} \pi^{-} \pi^{0}\right) \gamma$ | $\pi^{+} \pi^{-} \pi^{0} \gamma$ | $\omega \pi^{0}$ | $e^{+} e^{-} \gamma$ | $K^{+} K^{-}$ | $K_{S} K_{L}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| generated | 4907485 | 2964527 | 9208930 | 95580900 | 29487300 | 19872200 |
| selection | 1801423 | 180205 | 1823023 | 28605 | 11113 | 76347 |
| $\chi^{2}<50$ | 1609974 | 131916 | 226126 | 7125 | 128 | 1663 |
| likelihood | 1498948 | 129999 | 220610 | 2109 | 108 | 1378 |
| $\theta_{\pi^{+} \pi^{-}}<160^{\circ}$ | 1492246 | 122018 | 203795 | 910 | 105 | 715 |
| $\theta_{\gamma \gamma}>20^{\circ}$ | 1467958 | 116472 | 199770 | 706 | 103 | 714 |
| $\sum E_{\gamma}<660 \mathrm{MeV}$ | 1408102 | 116221 | 199464 | 513 | 103 | 714 |
| $\sum p_{\pi}<440 \mathrm{MeV}$ | 1407643 | 35356 | 98116 | 353 | 84 | 714 |

Table 9.1: Number of MC events after each analysis cut.

|  | $\eta\left(\rightarrow \pi^{+} \pi^{-} \gamma\right) \gamma$ | $\eta(\rightarrow$ neutrals $) \gamma$ | $\pi^{+} \pi^{-} \gamma$ | $\gamma \gamma(\gamma)$ | $\pi^{+} \pi^{-} \pi^{0}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| generated | 1024527 | 9822290 | 71939100 | 142775000 | 6057553 |
| selection | 7406 | 10919 | 7757 | 2710 | 34042 |
| $\chi^{2}<50$ | 3333 | 6641 | 657 | 395 | 8087 |
| likelihood | 3193 | 6054 | 655 | 258 | 8067 |
| $\theta_{\pi^{+} \pi^{-}}<160^{\circ}$ | 3144 | 5744 | 539 | 211 | 8021 |
| $\theta_{\gamma \gamma}>20^{\circ}$ | 1172 | 5715 | 79 | 194 | 500 |
| $\sum E_{\gamma}<660 \mathrm{MeV}$ | 1149 | 215 | 77 | 94 | 499 |
| $\sum p_{\pi}<440 \mathrm{MeV}$ | 712 | 214 | 9 | 93 | 15 |

Table 9.2: Number of MC events after each analysis cut.

### 9.4 Cross section evaluation

The number of signal events is derived with a 2-dimensional fit to the data. The distributions used to discriminate the signal from the background are the energy of the unpaired photon, $E_{\gamma}$, and the invariant mass of the two charged pions, $M_{\pi \pi}$, in the interval $50 \mathrm{MeV}<E_{\gamma}<$ 400 MeV and $280 \mathrm{MeV}<M_{\pi^{+} \pi^{-}}<520 \mathrm{MeV}$ that contains 55150 events. The signal efficiency is $(28.68 \pm 0.02) \%$. The fit to the data is done using the simulated shapes for the signal and backgrounds and the weights are left free. The fit returns the fraction of data events $f_{i}=$ $n_{i} / n_{\text {tot }}$ with the constraint $\sum_{i} f_{i}=1$. The 2-dimensional histograms used in the fit are shown in Fig. 9.12. The projections of the $E_{\gamma} \times M_{\pi^{+} \pi^{-}}$distribution are shown in Fig. 9.13 for the data

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|  | data sample |
| :--- | :---: |
| filtered | $1.4 \times 10^{8}$ |
| selection | 811043 |
| $\chi^{2}<50$ | 177618 |
| likelihood | 166365 |
| $\theta_{\pi^{+} \pi^{-}}<160^{\circ}$ | 154989 |
| $\theta_{\gamma \gamma}>20^{\circ}$ | 142449 |
| $\sum E_{\gamma}<660 \mathrm{MeV}$ | 141271 |
| $\sum p_{\pi}<440 \mathrm{MeV}$ | 55408 |

Table 9.3: Number of data events after each analysis cut.
and the backgrounds weighted by their relative factors $f_{i}$. The result of the fit gives $13536 \pm 121$ signal events resulting in a cross section $\sigma\left(e^{+} e^{-} \rightarrow \eta \gamma \rightarrow \pi^{+} \pi^{-} \pi^{0} \gamma\right)=\left(194.7 \pm 1.8_{\text {stat }}\right) \mathrm{pb}$. The only relevant backgrounds are from $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0} \gamma$ and $e^{+} e^{-} \rightarrow \omega \pi^{0}$. Figs. 9.14-9.17 show some data-MC comparisons. Fig. 9.14 shows the distribution of the cosine of the polar angle of the monochromatic photon, $\cos \theta$, where in the right plot the region around the signal peak, $320 \mathrm{MeV}<E_{\gamma}<380 \mathrm{MeV}$, is selected. Fig. 9.15 shows the distributions of the $\chi^{2}$ of the kinematic fit. The invariant mass of the $\pi^{+} \pi^{-} \pi^{0}$ system, reconstructed with the improved energies, is shown in Fig. 9.16. Fig. 9.17 shows the distributions of the energy of the two paired photons, where $\gamma_{1}$ stands for the most energetic one and $\gamma_{2}$ for the least energetic one. In all these distributions there is good agreement between data and MC.

### 9.5 Evaluation of the systematic uncertainties

The distributions of $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0} \gamma$ and $e^{+} e^{-} \rightarrow \eta \gamma$ are well reproduced by simulation both in shape and relative normalization, while the fraction of $\omega \pi^{0}$ events results slightly higher than expected. The $e^{+} e^{-} \rightarrow \omega \pi^{0}$ cross section can be introduced as a constraint by adding the following $\chi^{2}$-like term in the 2-dimensional fit:

$$
\begin{equation*}
-2 \ln \mathcal{L} \rightarrow-2 \ln \mathcal{L}+\left(\frac{f_{\omega \pi^{0}} N_{d a t a} /\left(\epsilon_{\omega \pi^{0}} \mathcal{L}\right)-\sigma_{\omega \pi^{0}}}{\delta \sigma_{\omega \pi^{0}}}\right)^{2} \tag{9.3}
\end{equation*}
$$

where $\sigma_{\omega \pi^{0}} \pm \delta \sigma_{\omega \pi^{0}}=(5.72 \pm 0.05) \mathrm{nb}$ is the measured value [43], while the other quantities refer to $\omega\left(\rightarrow \pi^{+} \pi^{-} \pi^{0}\right) \pi^{0}$ events as selected in the present measurement. After including the term of eq. (9.3), a result higher by $1.36 \%$ is obtained for the $e^{+} e^{-} \rightarrow \eta \gamma$ cross section. This difference is accounted for in the systematic error. Other possible contributions to the systematic error are investigated by varying the analysis cuts by the r.m.s. width of the distributions of $\chi^{2}, \theta_{\pi^{+} \pi^{-}}, \theta_{\gamma \gamma}, \sum E_{\gamma}$, and $\left|\vec{p}\left(\pi^{-}\right)\right|+\left|\vec{p}\left(\pi^{+}\right)\right|$, as shown in Tab. 9.4. This results in a relative error of $\pm 1.45 \%$ and $\sigma\left(e^{+} e^{-} \rightarrow \eta \gamma \rightarrow \pi^{+} \pi^{-} \pi^{0} \gamma\right)=\left(194.7 \pm 1.8_{\text {stat }} \pm 2.8_{\text {syst }}\right) \mathrm{pb}$. Using for the branching fraction the value $\operatorname{BR}\left(\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}\right)=0.2274 \pm 0.0028$ [1], one obtains $\sigma\left(e^{+} e^{-} \rightarrow \eta \gamma\right)=\left[856 \pm 8_{\text {stat }} \pm 12_{\text {syst }} \pm 11_{\operatorname{BR}\left(\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}\right)}\right] \mathrm{pb}$. This value, obtained from a direct


Figure 9.12: Distribution of the energy of the unpaired photon, $E_{\gamma}$, versus the invariant $\pi^{+} \pi^{-}$ mass, $M_{\pi \pi}$, for data (top left), MC signal events (top right), MC $\omega \pi^{0}$ events (bottom left), MC $\pi^{+} \pi^{-} \pi^{0} \gamma$ events (bottom right).
measurement, agrees well with the value obtained from the analysis of $\gamma \gamma \rightarrow \eta \rightarrow \pi^{0} \pi^{0} \pi^{0}$. The result interpolates well with the measurements of the SND experiment [47] and has a better precision. Fig. 9.18 shows the KLOE value for $\sigma\left(e^{+} e^{-} \rightarrow \eta \gamma\right)$ at $\sqrt{s}=1 \mathrm{GeV}$ compared to the SND results [47] for several values of $\sqrt{s}$.


Figure 9.13: Projections of the 2-dimensional fit. Top: distribution of the energy of the unpaired photon. Bottom: distribution of the invariant mass $M_{\pi^{+} \pi^{-}}$. The contribution of the signal $e^{+} e^{-} \rightarrow \eta \gamma$ is blue, $e^{+} e^{-} \rightarrow \omega \pi^{0}$ is green and $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0} \gamma$ is purple.


Figure 9.14: Distribution of the cosine of the polar angle of the monochromatic photon, $\cos \theta$. The contribution of the signal $e^{+} e^{-} \rightarrow \eta \gamma$ is blue, $e^{+} e^{-} \rightarrow \omega \pi^{0}$ is green and $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0} \gamma$ is purple. The MC distributions have the weights obtained from the fit. In the right plot the region around the signal peak, $320 \mathrm{MeV}<E_{\gamma}<380 \mathrm{MeV}$, is selected.


Figure 9.15: Distribution of the $\chi^{2}$ of the kinematic fit. The contribution of the signal $e^{+} e^{-} \rightarrow \eta \gamma$ is blue, $e^{+} e^{-} \rightarrow \omega \pi^{0}$ is green and $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0} \gamma$ is purple.


Figure 9.16: Distribution of the invariant mass of the $\pi^{+} \pi^{-} \pi^{0}$ system, obtained using the improved energies after the kinematic fit. The contribution of the signal $e^{+} e^{-} \rightarrow \eta \gamma$ is blue, $e^{+} e^{-} \rightarrow \omega \pi^{0}$ is green and $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0} \gamma$ is purple.


Figure 9.17: Left: distribution of the energy of the most energetic paired photon, $E_{\gamma 1}$. Right: distribution of the energy of the least energetic paired photon, $E_{\gamma 2}$. The contribution of the signal $e^{+} e^{-} \rightarrow \eta \gamma$ is blue, $e^{+} e^{-} \rightarrow \omega \pi^{0}$ is green and $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0} \gamma$ is purple.

|  | $n_{\text {signal }}$ | $\epsilon_{\text {signal }}(\%)$ | $\delta \sigma / \sigma(\%)$ |
| :--- | :---: | :---: | :---: |
| $\omega \pi^{0}$ fixed | 13720 | 28.68 | +1.36 |
| $\chi^{2}<45$ | 13489 | 28.54 | +0.12 |
| $\chi^{2}<55$ | 13583 | 28.79 | -0.04 |
| $\theta_{\pi^{+} \pi^{-}}<157^{\circ}$ | 13518 | 28.64 | -0.01 |
| $\theta_{\pi^{+} \pi^{-}}<163^{\circ}$ | 13548 | 28.70 | -0.01 |
| $\theta_{\gamma \gamma}>18^{\circ}$ | 13601 | 28.79 | +0.09 |
| $\theta_{\gamma \gamma}>22^{\circ}$ | 13482 | 28.55 | +0.03 |
| $\sum E_{\gamma}<610$ | 12232 | 25.80 | +0.44 |
| $\sum E_{\gamma}<710$ | 13927 | 29.62 | -0.38 |
| $\sum p_{\pi}<435$ | 13551 | 28.65 | +0.21 |
| $\sum p_{\pi}<445$ | 13522 | 28.68 | -0.11 |

Table 9.4: Systematics for the $\sigma\left(e^{+} e^{-} \rightarrow \eta \gamma \rightarrow \pi^{+} \pi^{-} \pi^{0} \gamma\right)$ measurement.


Figure 9.18: Cross section result for the KLOE $\sigma\left(e^{+} e^{-} \rightarrow \eta \gamma\right)$ measurement (red point) compared to SND [47] for several values of $\sqrt{s}$. On this scale, errors of the present measurement are not visible.

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## Conclusions

The cross section $\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta\right)$ is measured at $\sqrt{s}=1 \mathrm{GeV}$ with the KLOE detector, using an integrated luminosity of about $240 \mathrm{pb}^{-1}$. The $\eta$ mesons are selected through the two decays $\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}$ (charged decay, CD) and $\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}$ (neutral decay, ND) that exploit in a complementary way the tracking and the calorimeter measurements. Many background processes are considered, the most relevant being $e^{+} e^{-} \rightarrow \eta \gamma$ when the photon is emitted at small polar angles and escapes detection.
The cross section for $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$ is obtained independently for the two $\eta$ decay modes with a 2 -dimensional fit to the squared missing mass and the $\eta$ momentum projections. The results are $\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta\right)^{\mathrm{CD}}=\left(34.5 \pm 2.5_{\text {stat }} \pm 1.3_{\text {syst }}\right) \mathrm{pb}$ and $\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta\right)^{\mathrm{ND}}=$ $\left(32.0 \pm 1.5_{\text {stat }} \pm 0.9_{\text {syst }}\right) \mathrm{pb}$. Combining the two measurements, the value $\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta\right)=$ ( $32.7 \pm 1.3_{\text {stat }} \pm 0.7_{\text {syst }}$ ) pb is obtained.
The cross section of the irreducible background $e^{+} e^{-} \rightarrow \eta \gamma$ is measured in the same data sample with a dedicated analysis, and yields $\sigma\left(e^{+} e^{-} \rightarrow \eta \gamma\right)=\left(856 \pm 8_{\text {stat }} \pm 16_{\text {syst }}\right) \mathrm{pb}$. This value interpolates well the previous measurements by the SND experiment and is more precise. The $e^{+} e^{-} \rightarrow \eta \gamma$ cross section is used as a constraint in the fit for the $\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}$ case. In the case of $\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}$ decay, instead, it is measured as a by-product of the main analysis, and yields $\sigma\left(e^{+} e^{-} \rightarrow \eta \gamma\right)=\left(853 \pm 25_{\text {stat }} \pm 8_{\text {syst }}\right) \mathrm{pb}$, in agreement with the value obtained in the dedicated analysis.
Since the values of the four-momenta $q_{1}$ and $q_{2}$ sampled in the two decay modes can be slightly different, the partial width is extracted separately for the two decays, and then combined. The results obtained separately for the two decay channels are $\Gamma_{\eta \gamma \gamma}^{C D}=\left(548 \pm 40_{\text {stat }} \pm 22_{\text {syst }}\right) \mathrm{eV}$ and $\Gamma_{\eta \gamma \gamma}^{\mathrm{ND}}=\left(509 \pm 23_{\text {stat }} \pm 17_{\text {syst }}\right) \mathrm{eV}$. The final result for the $\gamma \gamma$ width of the $\eta$ meson is

$$
\begin{equation*}
\Gamma(\eta \rightarrow \gamma \gamma)=\left(520 \pm 20_{\text {stat }} \pm 13_{\text {syst }}\right) \mathrm{eV} \tag{9.4}
\end{equation*}
$$

This value is in agreement with the world average of $(510 \pm 26) \mathrm{eV}$ and is the most precise measurement to date.
These results are encouraging in view of the forthcoming data taking campaign of the KLOE-2 project [49], when analyses of the data collected at the $\phi$-peak will be possible with the information coming from two dedicated $e^{ \pm}$tagging detectors: the low energy taggers (LET), located in the region between the two low-beta focusing quadrupoles inside KLOE, and the high energy taggers (HET), located at the exit of the first bending magnet.

## Appendix A

## A Monte Carlo generator for $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$

Several MC generators have been developed for $\gamma \gamma$ physics (see ref. [36] for a detailed list). The simulation used in this analysis generates events with exact matrix element according to full 3-body phase space distributions, includes both s- and t-channel amplitudes and their interference, and allows user-defined form factors. The cross section for the process $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$ can be written as

$$
\begin{equation*}
\sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-} \eta\right) \propto \int \frac{1}{4} \frac{1}{2 s} \sum|M|^{2} \mathrm{dL}_{3} \tag{A.1}
\end{equation*}
$$

where $\frac{1}{4}$ is the spin averaging factor, $2 s$ is the flux factor, $M$ is the matrix element, and $\mathrm{dL}_{3}$ is the 3-body Lorentz invariant phase space. The Feynman diagrams for s- and t-channels are shown in Fig. A.1.


Figure A.1: Feynman diagrams for the process $e^{+} e^{-} \rightarrow e^{+} e^{-} \eta$. Left: t-channel. Right: schannel.

The kinematic invariants used are

$$
\begin{align*}
s & =\left(p_{1}+p_{2}\right)^{2},  \tag{A.2}\\
t_{1} & =\left(p_{1}-q_{1}\right)^{2},  \tag{A.3}\\
t_{2} & =\left(p_{2}-q_{2}\right)^{2}, \tag{A.4}
\end{align*}
$$

where $t_{1}$ and $t_{2}$ are the four-momenta of the photons in the t -channel. Photons in s-channel have four-momenta $s_{1}=s=\left(p_{1}+p_{2}\right)^{2}$ and $s_{2}=\left(q_{1}+q_{2}\right)^{2}$. The matrix element for the t -channel can be written in the form

$$
\begin{equation*}
M_{t} \propto g_{\eta \gamma \gamma} \times g_{e \gamma} \times \frac{i}{t_{1} t_{2}} \times F\left(t_{1}, t_{2}\right) \times \epsilon_{\mu v \alpha \beta}\left(q_{1}-p_{1}\right)^{\alpha}\left(q_{2}-p_{2}\right)^{\beta}\left(\bar{v}\left(p_{1}\right) \gamma^{\mu} v\left(q_{1}\right)\right)\left(\bar{u}\left(q_{2}\right) \gamma^{v} u\left(p_{2}\right)\right), \tag{A.5}
\end{equation*}
$$

where $g_{\eta \gamma \gamma}$ refers to the $\eta \gamma \gamma$ coupling, $g_{e \gamma}$ to the $e \gamma$ coupling, $F\left(t_{1}, t_{2}\right)$ is the form factor, and $1 / t_{1}, 1 / t_{2}$ are the photon propagators. The last part of the matrix element contains the currents product. $\epsilon^{\mu \nu \alpha \beta}$ is the Levi-Civita completely antisymmetric tensor. The couplings are defined as

$$
\begin{align*}
g_{\eta \gamma \gamma}^{2} & =\frac{4 \pi \Gamma_{\eta \gamma \gamma}}{m_{\eta}^{3}},  \tag{A.6}\\
g_{e \gamma}^{2} & =(4 \pi \alpha)^{2}, \tag{A.7}
\end{align*}
$$

where $\Gamma_{\eta \gamma \gamma}$ is the $\gamma \gamma$ partial width of the $\eta$ meson, and $\alpha \approx 1 / 137$ is the fine structure constant. The $\eta$ transition form factor is parametrized as

$$
\begin{equation*}
F_{\eta}\left(t_{1}, t_{2}\right)=\left(\frac{1}{1-b_{\eta} t_{1}}\right)\left(\frac{1}{1-b_{\eta} t_{2}}\right), \tag{A.9}
\end{equation*}
$$

where $b_{\eta}$ is the slope parameter. The matrix element for the s-channel has the form

$$
\begin{equation*}
M_{s} \propto g_{\eta \gamma \gamma} \times g_{e \gamma} \times \frac{i}{s_{1} s_{2}} \times F\left(s_{1}, s_{2}\right) \times \epsilon_{\mu v \alpha \beta}\left(p_{1}+p_{2}\right)^{\alpha}\left(q_{1}+q_{2}\right)^{\beta}\left(\bar{v}\left(p_{1}\right) \gamma^{\mu} u\left(p_{2}\right)\right)\left(\bar{u}\left(q_{2}\right) \gamma^{v} v\left(q_{1}\right)\right), \tag{A.10}
\end{equation*}
$$

where the form factor $F\left(s_{1}, s_{2}\right)$ has the same analytical expression as in the t -channel (equation (A.9)). The s-channel amplitude has a peak at small invariant masses of the final $e^{+} e^{-}$pair, and is much smaller than the t-channel amplitude. With this MC code it is possible to calculate the total cross section with the only contribution of the $t$-channel, or with both s- and $t$ - channels. For the analyses described in this thesis only the $t$-channel has been considered, as at a center of mass energy of 1 GeV the s-channel contribution is less than $0.5 \%$.

## Appendix B

## Energy scale correction

## B. $1 e^{+} e^{-} \rightarrow K_{S} K_{L}$

MC $e^{+} e^{-} \rightarrow K_{S} K_{L}$ events with $K_{S} \rightarrow \pi^{0} \pi^{0}$ and with the $K_{L}$ interacting in the calorimeter are selected by requiring:

- four prompt neutral clusters with polar angle $23^{\circ}<\theta<157^{\circ}$;
- one neutral delayed cluster with $0.06<\beta^{*}<0.13$, where $\beta^{*}$ is the $K_{L}$ velocity in the $K_{L} K_{S}$ system (expected value $\approx 0.1$ );
- no tracks in the DC.

A pseudo- $\chi^{2}$ variable is then defined, to pair the four photons:

$$
\begin{equation*}
\chi_{\pi \pi}^{2}=\frac{\left(m_{\pi^{0}}-m_{i j}\right)^{2}}{\sigma_{i j}^{2}}+\frac{\left(m_{\pi^{0}}-m_{k l}\right)^{2}}{\sigma_{k l}^{2}} \quad \text { with } \quad \frac{\sigma_{i j}}{m_{i j}}=\frac{1}{2}\left(\frac{\sigma_{E \gamma i}}{E_{\gamma i}} \oplus \frac{\sigma_{E \gamma j}}{E_{\gamma j}}\right), \tag{B.1}
\end{equation*}
$$

where $m_{i j}$ is the invariant mass of the two photons, $m_{i j}=\sqrt{2 E_{i} E_{j}\left(1-\cos \theta_{i j}\right)}$, and the energy resolution is $\sigma_{E} / E=0.057 / \sqrt{E(G e V)}$. The cut $\chi_{\pi \pi}^{2}<4$ is applied. Fig. B.1, left, shows the four photons invariant mass distribution for data and for MC $e^{+} e^{-} \rightarrow K_{S} K_{L}$ events. By superimposing the two distributions (bottom left part of Fig. B.1) it is possible to see that there is a small shift between data and MC events. A correction of $0.8 \%$ has been therefore applied to the MC clusters energy, Eclu $=1.008 \times$ Eclu. The corrected MC spectrum and its superimposition with data are shown on the right side of Fig. B.1.


Figure B.1: Left: Distribution of the four photons invariant mass for data (top), and for MC $e^{+} e^{-} \rightarrow K_{S} K_{L}$ events (center). In the bottom plot the two distribution are superimposed (black $=$ data, red $=\mathrm{MC} e^{+} e^{-} \rightarrow K_{S} K_{L}$ events). Right: same distributions, but after the energy correction on MC clusters energy.
B. $2 e^{+} e^{-} \rightarrow \eta \gamma \rightarrow 3 \pi^{0} \gamma$
$\mathrm{MC} e^{+} e^{-} \rightarrow \eta \gamma \rightarrow 3 \pi^{0} \gamma$ events are selected by requiring:

- at least five prompt neutral clusters with polar angle $23^{\circ}<\theta<157^{\circ}$;
- no tracks in the DC;
- $\chi_{\pi \pi}^{2}<4$. This is the same variable as defined for the $e^{+} e^{-} \rightarrow K_{S} K_{L}$ process described in section B.1, and is used in this case to pair the four "best" photons, i.e. the photons that better reconstruct two out of the three neutral pions;
- $p_{T}>200 \mathrm{MeV}$, where $p_{T}$ is the transverse momentum of the four "paired" photons.

The $\gamma \gamma$-filter described in Section 4.1 cuts out $e^{+} e^{-} \rightarrow \omega \pi^{0}$ events with five photons in the final state. Fig. B.2 shows the $2 \gamma$ invariant mass distribution (inclusive on the four photons) for data and for MC $e^{+} e^{-} \rightarrow \eta \gamma$ events, where the MC cluster energies are corrected by $0.8 \%$. The bottom part of Fig. B.2 shows the data and MC distributions superimposed after the correction.


Figure B.2: Top: distribution of the $2 \gamma$ invariant mass for data. Center: distribution of the $2 \gamma$ invariant mass for MC $e^{+} e^{-} \rightarrow \eta \gamma$ events after energy correction. Bottom: the two distributions are superimposed (black = data, red $=\mathrm{MC} e^{+} e^{-} \rightarrow \eta \gamma$ events).

Appendix B. Energy scale correction

## Appendix C

## The Lagrange Multipliers method

## C. 1 The Least-Squares method

A set of N independent, experimental values $y_{i}\left(x_{i}\right)$ with variances $\sigma_{i}^{2}$ are given. The true values $\eta_{1}, \eta_{2}, \ldots, \eta_{N}$ of the observables are not known, but it can be assumed that some theoretical model exists, which associates the true values $\eta_{i}$ with each $x_{i}$ through a functional dependence,

$$
\begin{equation*}
f_{i}=f_{i}\left(x_{i} \mid \theta_{1}, \theta_{2}, \ldots, \theta_{L}\right) \tag{C.1}
\end{equation*}
$$

where $x_{i}$ are independent variables, and $\left(\theta_{1}, \theta_{2}, \ldots, \theta_{L}\right)$ is a set of parameters, $\mathrm{L} \leq \mathrm{N}$. According to the Least-Squares (LS) Principle [53] the best values of the unknown parameters are those which minimize the quantity

$$
\begin{equation*}
\chi^{2}=\sum_{i=1}^{N} w_{i}\left(y_{i}-f_{i}\right)^{2} \tag{C.2}
\end{equation*}
$$

where $w_{i}=1 / \sigma_{i}^{2}$ is the weight of the i-th observation, and describes the accuracy of the measurement $y_{i}$. If the observations are corrrelated and a covariance matrix $V(\bar{y})$ is given, equation (C.2) can be written as

$$
\begin{equation*}
\chi^{2}=\sum_{i=1}^{N} \sum_{j=1}^{N}\left(y_{i}-f_{i}\right) V_{i j}^{-1}\left(y_{j}-f_{j}\right) \tag{C.3}
\end{equation*}
$$

or, in terms of matrix notation,

$$
\begin{equation*}
\chi^{2}=(\bar{y}-\bar{f})^{T} V^{-1}(\bar{y}-\bar{f}) \tag{C.4}
\end{equation*}
$$

If there is a linear dependence on the parameters $\bar{f}=A \bar{\theta}$, equation (C.4) becomes

$$
\begin{equation*}
\chi^{2}=(\bar{y}-A \bar{\theta})^{T} V^{-1}(\bar{y}-A \bar{\theta}) \tag{C.5}
\end{equation*}
$$

In many situations the quantities to be estimated are not the $\bar{\theta}$ parameters, but the true observables $\bar{\eta}$. The observations $\bar{y}$ with covariance matrix $V(\bar{y})$ are taken as initial estimates of the true, but unknown, $\bar{\eta}$ values. The best estimates of $\bar{\eta}$ are the values which minimize the
quantity

$$
\begin{equation*}
\chi^{2}=\bar{\epsilon}^{T} V^{-1} \bar{\epsilon} \tag{C.6}
\end{equation*}
$$

where $\bar{\epsilon}$ is the difference between the measured and true values, $\bar{\epsilon}=\bar{y}-\bar{\eta}$. When the LS minimization has been performed, the final estimates $\widehat{\bar{\eta}}$ of the true $\bar{\eta}$ are called "improved measurements", or "fitted variables". The "residuals" $\bar{\epsilon}$ of the LS estimate are defined as the differences between the original measurements and the "improved measurements", $\widehat{\bar{\epsilon}}=\bar{y}-\widehat{\bar{\eta}}$. If the N measurements $y_{i}$ are uncorrelated, independent and normally distributed with variance $\sigma_{i}^{2}$ around the true values $\eta_{i}$, and in the hypothesis that the observables $\eta_{i}$ were known, the quantity

$$
\begin{equation*}
\chi^{2}=\sum_{i=1}^{N}\left(\frac{\epsilon_{i}}{\sigma_{i}}\right)^{2}=\sum_{i=1}^{N}\left(\frac{y_{i}-\eta_{i}}{\sigma_{i}}\right)^{2} \tag{C.7}
\end{equation*}
$$

would be a sum of N independent squared normal variables, distributed as a $\chi^{2}$ variable with N degrees of freedom. However, in practice, the true, unknown $\eta_{i}$ values should be replaced by their estimated values $\widehat{\eta}_{i}$, thus obtaining a weighted sum of squared residuals

$$
\begin{equation*}
\chi_{\min }^{2}=\sum_{i=1}^{N}\left(\frac{\widehat{\epsilon}_{i}}{\sigma_{i}}\right)^{2}=\sum_{i=1}^{N}\left(\frac{y_{i}-\widehat{\eta}_{i}}{\sigma_{i}}\right)^{2} \tag{C.8}
\end{equation*}
$$

which, in the case of a linear model with $L$ parameters, is distributed as a $\chi^{2}$ variable with (N-L) degrees of freedom.

## C. 2 Fitting with constraints: Largange Multipliers

In some cases the observables $\bar{\eta}$ can be related through algebraic constraint equations. This happens, for instance, when one measures independently the three angles of a triangle and wants the LS estimates of the true values to satisfy the requirement that their sum be equal to $180^{\circ}$. In this case the method of Lagrange Multipliers [53] may be used. This method increases the number of unknows in the minimization by adding a set of Lagrange Multipliers, one for each constraint equation. Given N observables $y_{i}$, L parameters $\theta_{i}$, and K linear constraints, the constraint equations can be written as

$$
\begin{equation*}
B \bar{\theta}-\bar{b}=0, \tag{C.9}
\end{equation*}
$$

where $B$ is a $K \times L$ matrix and $\bar{b}$ is a K-component vector. If one introduces a Lagrange Multipliers K-component vector $\bar{\lambda}=\left(\lambda_{1}, \lambda_{2}, \ldots, \lambda_{K}\right)$, the quantity to be minimized becomes

$$
\begin{equation*}
\chi^{2}(\bar{\theta}, \bar{\lambda})=(\bar{y}-A \bar{\theta})^{T} V^{-1}(\bar{y}-A \bar{\theta})+2 \bar{\lambda}^{T}(B \bar{\theta}-\bar{b}), \tag{C.10}
\end{equation*}
$$

which is the analogous of equation (C.5) for the unconstrained case. By equating the $\chi^{2}$ derivatives to zero with respect to $\theta_{i}, i=1,2, \ldots, L$ and $\lambda_{i}, i=1,2, \ldots, K$ one obtains

$$
\begin{align*}
\nabla_{\theta} \chi^{2}=-2\left(A^{T} V^{-1} \bar{y}-A^{T} V^{-1} A \bar{\theta}\right)+2 B^{T} \bar{\lambda} & =\overline{0} \\
\nabla \lambda \chi^{2}=2(B \bar{\theta}-\bar{b}) & =\overline{0} \tag{C.11}
\end{align*}
$$

These are $\mathrm{L}+\mathrm{K}$ linear equations for the unknowns. By introducing the abbreviations

$$
\begin{equation*}
C \equiv A^{T} V^{-1} A, \quad \bar{c} \equiv A^{T} V^{-1} \bar{y} \tag{C.12}
\end{equation*}
$$

equations (C.11) become

$$
\begin{align*}
C \bar{\theta}+B^{T} \bar{\lambda} & =\bar{c}, \\
B \bar{\theta} & =\bar{b} . \tag{C.13}
\end{align*}
$$

If the inverse of the $C$ matrix exists, one can multiply the first of these equations by $B C^{-1}$ and substitute $B \bar{\theta}$ from the second equation, obtaining

$$
\begin{equation*}
\bar{b}+B C^{-1} B^{T} \bar{\lambda}=B C^{-1} \bar{c} \tag{С.14}
\end{equation*}
$$

Writing

$$
\begin{equation*}
V_{B} \equiv B C^{-1} B^{T} \tag{C.15}
\end{equation*}
$$

and assuming that the inverse of the $V_{B}$ matrix exists, the solutions for the Lagrange Multipliers, for the parameters $\bar{\theta}$, and for the "improved measurements" are

$$
\begin{array}{r}
\hat{\bar{\lambda}}=V_{B}^{-1}\left(B C^{-1} \bar{c}-\bar{b}\right), \\
\widehat{\bar{\theta}}=C^{-1} \bar{c}-C^{-1} B^{T} V_{B}^{-1}\left(B C^{-1} \bar{c}-\bar{b}\right), \\
\widehat{\bar{\eta}}=A \widehat{\bar{\theta}}=A\left[C^{-1} \bar{c}-C^{-1} B^{T} V_{B}^{-1}\left(B C^{-1} \bar{c}-\bar{b}\right)\right] . \tag{C.16}
\end{array}
$$

The $\chi_{\text {min }}^{2}$ variable is, in this case, distributed as a $\chi^{2}$ variable with ( $\mathrm{N}-\mathrm{L}+\mathrm{K}$ ) degrees of freedom. If the number of observables is equal to the number of parameters, the number of degrees of freedom is equal to the number of constraints $K$.

Appendix C. The Lagrange Multipliers method

## Appendix D

## The 2-dimensional fit

The code used in this analysis fits the MC 2-dimensional distributions to the data using a binned maximum likelihood fit, which takes into account the finite statistics both from data and from MC samples and allows to determine the fraction yield for each component. The distribution is described by 2-dimensional histograms with $n \times m$ bins. This gives a set of numbers $d_{11}, d_{12} \ldots d_{n m}$, where $d_{i j}$ is the number of events in data that fall into the bin $i j . p_{i j}$ is the predicted number of events in the bin, and is given by the predicted fraction of signal and background events in data $f_{s}, f_{b}$, and by the number of MC signal and background events $s_{i j}, b_{i j}$, in bin $i j$ :

$$
\begin{equation*}
p_{i j}=N_{D}\left(f_{s} \frac{s_{i j}}{N_{S}}+f_{b} \frac{b_{i j}}{N_{B}}\right) \equiv w_{s} s_{i j}+w_{b} b_{i j} \tag{D.1}
\end{equation*}
$$

where $N_{D}$ is the total number of data and $N_{S}, N_{B}$ the total number of MC signal events and MC background events, respectively. In D. $1 w_{s}$ and $w_{b}$ are the normalized weights for signal and background. The probability for observing a particular $d_{i j}$ is given by the Poisson distribution:

$$
\begin{equation*}
P\left(d_{i j} \mid p_{i j}\right)=e^{-p_{i j}} \frac{p_{i j}^{d_{i j}}}{d_{i j}!} \tag{D.2}
\end{equation*}
$$

and the estimates of the weights $w_{s}, w_{b}$ are found by maximizing the likelihood $\mathcal{L}=\prod_{i j} P\left(d_{i j} \mid p_{i j}\right)$. For each source, in each bin, there is some (known with limited statistics) expected number of events $S_{i j}, B_{i j}$ such that $p_{i j}=w_{s} S_{i j}+w_{b} B_{i j}$. The likelihood to be maximized is the combined probability for observing $d_{i j}, s_{i j}$ and $b_{i j}$ given $S_{i j}, B_{i j}$ [50]:

$$
\begin{equation*}
\mathcal{L}=\prod_{i=1}^{n} \prod_{j=1}^{m} P\left(d_{i j} \mid w_{s} S_{i j}+w_{b} B_{i j}\right) P\left(s_{i j} \mid S_{i j}\right) P\left(b_{i j} \mid B_{i j}\right) \tag{D.3}
\end{equation*}
$$

Expanding equation (D.3) in terms of three expressions of type in equation (D.2) one obtains the following function:

$$
\begin{equation*}
\ln \mathcal{L}=\sum_{i=1}^{n} \sum_{j=1}^{m}\left(d_{i j}-p_{i j}-d_{i j} \ln \frac{d_{i j}}{p_{i j}}+s_{i j}-S_{i j}-s_{i j} \ln \frac{s_{i j}}{S_{i j}}+b_{i j}-B_{i j}-b_{i j} \ln \frac{b_{i j}}{B_{i j}}\right) \tag{D.4}
\end{equation*}
$$

To find the maximum of equation (D.4) one has to differentiate $\ln \mathcal{L}$ and set the derivatives to zero. The differentials with respect to the $2 \times n \times m$ variables $S_{i j}, B_{i j}$ give $S_{i j}, B_{i j}$ expressed as functions of the observables $s_{i}$ and $b_{i}$ :

$$
\begin{equation*}
B_{i j}=\frac{b_{i j}}{1+w_{b}\left(1-\frac{d_{i j}}{w_{s} s_{i j}+w_{b} b_{i j}}\right)} \quad S_{i j}=\frac{s_{i j}}{1+w_{s}\left(1-\frac{d_{i j}}{w_{s} s_{i j}+w_{b} b_{i j}}\right)} \tag{D.5}
\end{equation*}
$$

The derivative of $\ln \mathcal{L}$ with respect to the free parameters $f_{s}, f_{b}$ is obtained numerically. Customized versions of the routines HMCLNL and HADJUST (included in the HBOOK package [51]) are used, adapted to fit data with the superposition of 2-dimensional MC histograms, to obtain the $-2 \ln \mathcal{L}$ value for a given set of $f_{s}, f_{b}$ fractions.
Also possible constraints are accounted for, such as the knowledge of a cross section value $\sigma_{b}$ for the $b$ component, obtained in similar analyses:

$$
\begin{equation*}
-2 \ln \mathcal{L} \rightarrow-2 \ln \mathcal{L}+\left(\frac{f_{b} N_{\text {data }} /\left(\epsilon_{b} \mathcal{L}\right)-\sigma_{b}}{\delta \sigma_{b}}\right)^{2} \tag{D.6}
\end{equation*}
$$

where $\epsilon_{b}$ is the analysis efficiency from Monte Carlo. The procedure is iterated with the MINUIT routines [52] and determines the best estimate for the parameters $f_{s}$ and $f_{b}$, the statistical error, the likelihood maximum and the minimum value of $\chi^{2}=-2 \ln \mathcal{L}$.

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Acknowledgements

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