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# Commodity Risk Management:
# a Two-Factor Model with Long-Term Dependency


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*Dedicato ai miei genitori e a mio fratello*

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Finally, I wish to thank my parents and my brother for their love and constant support. My family has always been there, helping me achieving my objectives.

# Abstract


The aim of this thesis is to obtain a risk management model that is able to capture the long-term dependencies among different commodities, keeping unaltered the marginal distributions of the single commodities and preserving the analytical flexibility of factor models.

A method to translate the error correction matrix, result of the cointegration analysis, into correlations, and closed form formulae for term correlations are developed to include long-run co-movements in the calibration. Furthermore, since commodity panels are composed also by quarter and year swap contracts, numerical integration based on sparse grids is implemented to reduce the computational complexity. Working in a multivariate framework, also the nearest correlation matrix problem has to be addressed, therefore, a two steps procedure is proposed. In this way, the definition of the starting point is more flexible and it is possible to avoid the implementation of global optimization algorithms.

The data used are the daily quotations from December 2012 to November 2016 of the forward panels of API2, Brent, EU ETS, TTF and German electricity.

Empirical results within a five commodities framework are provided. Four portfolios are taken into consideration: a *Clean Dark Spread*, a *Clean Spark Spread*, an oil-indexed gas contract, a long position on gas and oil. A comparison between a two-factor model and the two-factor model with the long-term dependency is performed.

Risk measurement and risk management activities can be deeply affected by the new approach, since a usual two-factor model could either overestimate and underestimate the risk arising from the dynamics of the commodities.

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# Chapter 1

# Introduction

The dynamics of commodity prices can affect the economics of a company or a country directly and indirectly. Technologies, geopolitics and regulations have a huge impact on the quotations of the commodities. Shale revolution, Kyoto Protocol, COP21 are just some examples. When it comes to forecast their levels to set up long-term strategies, we have to deal with an high level of uncertainty, therefore, risk measurement and risk management become crucial.

Power market sector is among the most impacted by commodity prices. Kiesel et al. (2009) apply a two-factor model to electricity prices and Edoli et al. (2013) extend it in a multi-commodity environment. The aim of these models is to describe the covariance matrices of one or more commodities forward products. As pointed out in Alexander (1999) and in Alexander (2001), correlation is just a short term measure and cannot perform well in long time horizon. We know from Hicks (1939) that *physical things*, having constant relative prices, can be treated as a single one. Cointegration analysis, based on the works of Granger (1981) and Engle and Granger (1987), can be used to test and retrieve the proportions of different assets that make a portfolio stationary. Through cointegration, the *long run equilibrium* (Engle and Granger (1987)) and *long-run economic relations* (Johansen (2000)) among commodities have been intensively investigated in the literature. In this thesis, an analytical framework is proposed to combine the two-factor model with the cointegration analysis. Implementing the Johansen (1995) method, the long run equilibrium among the commodities is captured and then translated in terms of latent factors correlations. In this way, it is possible to calibrate the model taking into account more information, exceptionally precious in the long-term.

Edoli et al. (2013) are able to treat only instantaneous (daily) correlations. In this thesis, closed form formulae for term correlations are developed to include long-term insights, coming from cointegration analysis, and to allow to consistently work in any kind of discrete time framework. Therefore, when it comes to simulate prices in many years, the dimension of the problem is reduced.

Moreover, the two-factor model calibration is enhanced taking advantage of numerical integration technique based on sparse grids (Heiss and Winschel (2008)). Commodities forward panels are composed also by quarter and year swap contracts, that must be coherent with the average of the monthly contracts. With the use of sparse grids, the computational complexity is so reduced, that it is possible to simulate monthly contracts in every step of the optimization in order to properly value the average contracts. This approach could support the calibration of any model that deal with those kind of products.

Edoli et al. (2013) elaborate a specific algorithm based on the Cholesky decomposition to deal with the *nearest correlation matrix* problem (Higham (2002)). As in Rebonato and Jäckel (1999), a two steps procedure is proposed. First, the extended version of the quadratically convergent Newton method by Qi and Sun (2006) is implemented to guarantee the intial correlation matrix to be semi-definite positive, and then the Edoli et al. (2013) algorithm is applied. With this procedure, the starting point of the optimization can be better defined according to long-term information and the use of a global optimization algorithm is not necessary.

A comparative analysis of the model by Edoli et al. (2013) and the two-factor model with long-term dependency is performed. The commodity association becomes more powerful, allowing for higher correlation between electricity, coal, gas and oil. On the other hand, the EU ETS correlations are reduced. EU ETS correlations with electricity, oil and gas become close to zero, whilst they turns negative with coal. This can be explained by specific characteristics of the market. It is noteworthy that this kind of relation is not identified by the standard two-factor model. Depending on the considered portfolio, it is shown that enabling for long-term dependency can both increase or decrease the risk. Therefore, business plans and hedging strategies decisions can be highly affected by the inclusion of cointegration analysis, as very recently analysed by Gatarek and Johansen (2016).

The thesis is structured as follows. In chapter 2, the commodity markets are described, giving a qualitative assessment of the forces that drive their volatilities and trends. In chapter 3, the univariate two-factor model (Kiesel et al. (2009)), the multivariate two-factor model (Edoli et al. (2013)) and the proposed two-factor model with long-term dependency are presented. In chapter 4, the historical series and some details on data management are shown. In chapter 5, the empirical results of models calibrations on four case studies are provided. Finally, conclusion remarks and further areas of analysis are discussed.

# Chapter 2

# An Overview of the Commodity Market

*After the electric light goes into general use, none but the extravagant will burn tallow candles*

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Edison (1880)

The aim of this thesis is to obtain a model able to capture the long-run co-movements of the commodities. A brief overview of the power, oil, coal, gas and emissions markets is presented, in order to understand the context and to be able to give a consistent economic interpretation of the results.

## 2.1 Electricity

Physically, the power markets works in the same way in all the countries. However, markets can have different organizations and characteristics depending on the legislation and on the geography. In subsection 2.1.1, the fundamentals will be given, while subsection 2.1.2 will deal with some specific details of the German electricity market, since it will be the one considered in chapter 5.

### 2.1.1 Fundamentals

Nowadays electricity is necessary for almost every human activity. It is immediately available for all its consumers and can be employed for a broad range of purposes. Lighting, the main usage electricity is commonly associated with, was its first application. Step by step, it has become necessary for primary, secondary and tertiary sectors. It is barely unthinkable to imagine our houses without a constant supply of electricity. Its consumption is clean, since it does not emit greenhouse gases. Moreover, Niu et al. (2013) state that electricity is a requirement to improve the quality of life and to support social development.

Electricity has some characteristics that make it be unique with respect to the other commodities (Pérez-Arriaga (2013)). It cannot be easily and economically stored. In the last years some important technological advancements have been achieved in batteries, but, still, the only way to store it is through hydro reservoir. Generation and demand must be in balance in every moment and a problem occurring in any place could spread throughout the whole system. The transmission of electricity cannot be planned as for the other commodities or goods. Its flows are not regulated by market participants, but by the laws of physics. Keep the system balanced is a very complicated task. Several systems and models have to be implemented considering different time frames. Supply and demand must be in equilibrium in each minute, but estimates are needed also with months in advance.

The power sector can be divided in generation, transmission, distribution and retail.

The generation can be carried out using several types of power plants. Renewable power plants use water, wind, sunlight, Earth internal heat, etc. Nuclear stations use atomic fission of uranium. Finally, we have thermal power stations that use different kind of fuels, such as coal, gas, oil. Each type has its own advantages and disadvantages. Renewable energies have a limited impact on the environment and do not need any kind of fuel to operate. On the other hand, for most of them the production is not reliable, since it depends on the weather. Thermal power plants need fuel to operate, therefore, they are exposed to price dynamics. However, their production is reliable and can be easily managed by generator companies to meet a variable demand. Therefore, for each power system it is very important to develop and maintain a balanced generation mix. Depending on available resources, technologies and according to national and international regulations, each country is characterized by a particular mix. Therefore, every system will have its own level of prices, more or less dependent on fuels dynamics and on weather conditions.

The transmission grids connect generation hubs to demand hubs and have two important components. Power lines transmit electricity in the system at high-voltage, however, electricity is injected by the generators and used by the consumers at low-voltage. Therefore, substations are needed both in generator and demand hubs to properly control the voltage. Transmission capacity is very important both for the resilience of the system and for the level and volatility of the prices. If the transmission system is not developed, the demand will have to rely more on near power plants, since significant constraints in the transmission will make the grid fragmented. On the other hand, an advanced transmission grid will allow all the hubs to be connected, making possible to use any available capacity.

Distribution is the low voltage network of the power system and its aim is to carry electricity to the final consumer.

The power system price is based on the system marginal price (SMP). It is determined by the intersection of the demand and the *merit order curve*. Every hour all the generators bids are put together into the supply curve. In some regulated markets, the generators have the obligation to bid at their marginal costs, while in more advanced markets the producers are free to apply their bidding strategies. Generally, on the left side of the curve, we have must-run and low cost power plants, while on the right side the more expensive ones.

As we have seen, several factors can influence the price of electricity:

- System marginal price, network capacity, prices of the commodities. The supply curve depends on fuel price dynamics and bidding strategies. Moreover, a small movement of the supply curve and in the demand curve, or issues on network capacity, could trigger a spike in the prices due to a change in the marginal technology.

- Macroeconomics. Country production, interest rates affect the power markets in the medium and in the long-term.

- Technology developments. Power plants efficiency plans, advancements in renewable energies technologies, smart grids and batteries can have a huge impact on the market both on the generator and on the consumer side.

- Weather. Temperature affects the consumers needs, but also the efficiency of the power plants. Moreover, its role is becoming more important with the raise of renewable capacity. In countries with high hydroelectric capacity, the prices are highly affect by the rain (e.g. South America).

- Environmental and taxation policies. National and international regulations are one of the main drivers of the power sectors. The level of liberalization of the market, how the prices and tariffs are set vary across the countries depending on the legislations[1]. International agreements, such as Kyoto Protocol and COP21, also play an important role in shaping tax regimes and fostering technological changes.

### 2.1.2 German Power Market

The historical electricity generation by source is reported in figure 2.1. It can be noticed how the generation is changing. The country is reducing nuclear generation, whilst is investing in renewable energies, that in the last year accounted for about one third of the production. In off-peaks periods, such as the week ends, if the weather conditions are favourable, the renewables are able to offset the entire demand. As a consequence, the power prices are declining. However, it is noteworthy that about half of the production still comes from coal power plants (figure 2.2).

[1] See Pérez-Arriaga (2013) for a detailed analysis.

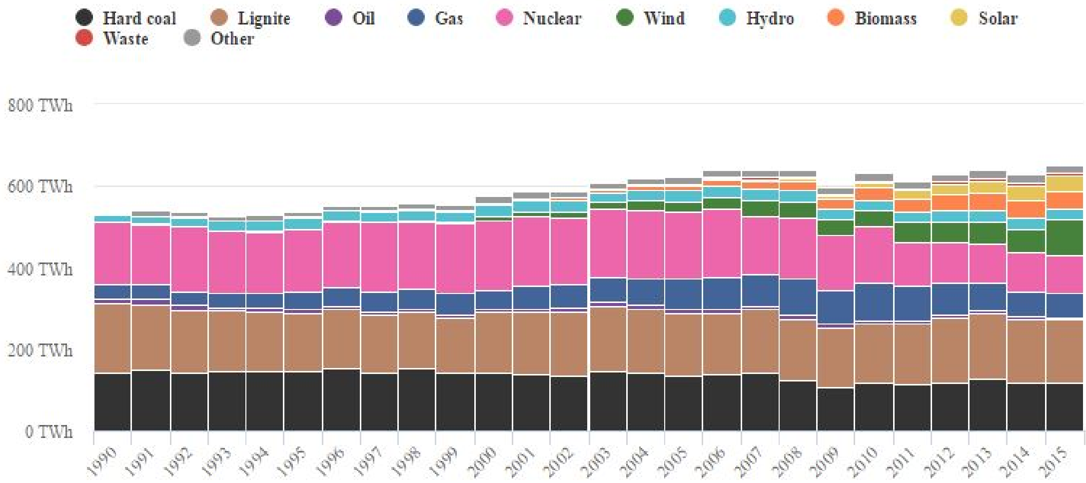


Figure 2.1: Gross electricity generation in Germany (from CarbonBrief (2016a)).

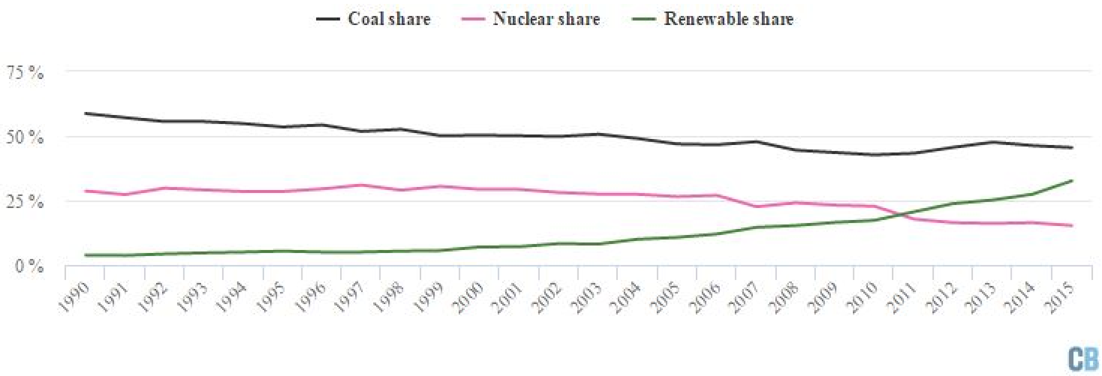


Figure 2.2: German electricity shares of generation by type(from CarbonBrief (2016a)).

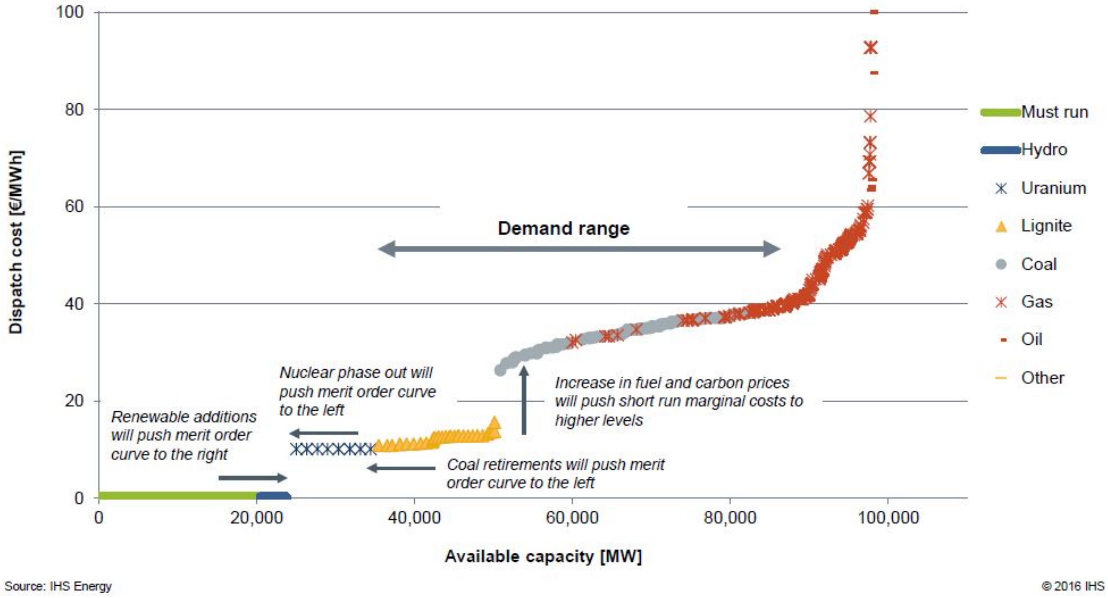


Figure 2.3: German merit order curve (from IHS (2016)).

The *merit order curve* of Germany is reported in figure 2.3. As expected, we can see that the left side of the curve is composed by must-run and hydro power plants. Then, we have nuclear and lignite power plants. Coal and the most economic gas plants compose the central part of the curve. The expensive gas plants are on the right side, but they are rarely used. Therefore, as well known in the markets, the power prices of Germany are linked most of the time to coal. IHS (2016) reports also how the merit order curve could be affected by coal prices, renewables investments and nuclear phaseout.

## 2.2 Oil Market

The oil market is one of the most important in the world. It is huge, international (about two thirds of the production is exported and traded) and affected by several factors, from technological advancements to environmental policies. The oil is also recognized as a leading indicator of the state of the economy, therefore, it is used in forecasting the economic trends (Roncoroni et al. (2015)).

As summarised in Roncoroni et al. (2015), the oil industry can be divided into two processes, the *Upstream* and the *Downstream*. The first one relates to the exploration and the production of crude oil. The second one comprehends the transportation, refining and marketing of the refined oil products. Different firms operate on the supply side, both independent and national oil companies[2], while, as final consumers, we can find utilities, airlines, shipping companies, energy-intensive manufacturers, petrochemical companies, gasoline and diesel retailers.

Roncoroni et al. (2015) make a list of the most important factors that influence this market:

- Macroeconomics. The economic growth is linked to energy consumption, positively correlated with spot and forward prices.

- Technology developments, level of proven reserves, commercial and strategic storage, refining capacity. Exploration and extraction advancements, such as the recent grow of shale oil, the level of the reserves and refinery spare capacity have very important effects on both spot and forward markets.

- Weather. Extreme natural events can damage production sites and logistic structures. Moreover, it has an important effect on the demand side, since both hot and cold periods trigger higher energy consumption.

- Arbitrage among energy commodities, exchange rates and shipping. The relative value of oil products with respect to other fuels, such as gas, is one of the main drivers of spot and forward markets. Being quoted in US dollars, the FX rate can increase or decrease the price of the oil products in local markets. Moreover, shipping rates can influence the prices.

- Geopolitics. Negative events involving oil producers countries can make prices spike. Also OPEC plays an important role in the market.

[2] In Deutsche Bank (2013) can be found a brief profile of the main international oil companies.

- Environmental and taxation policies. Taxes, efficiency plans can change demand behaviours and can also give incentive to innovation processes, such as the growth of the renewable sector.

Given all those sources of uncertainty, risk management is crucial in many sectors, from the aviation industry to the utilities. It is noteworthy that in electricity generation, a firm can have direct exposures to oil prices, if thermal power stations are used, but also indirect exposures, whenever the cost of the fuels, such as the gas, or the price of electricity is indexed to oil.

Briefly analysing the fundamentals of the market, we can see from figure 2.4, that the world demand has increased in the last years. The main regions of the world are following different trends. The US are drastically reducing the import of oil after the rise of shale and fracking technologies. On the other hand, China demand is increasing, although it is also an important producer. Middle East has stable exports, while Russia is increasing its share. Europe and Japan are reducing their consumptions and, consequently, their imports.

## 2.3 Coal Market

According to IEA (2016a), the share of coal in power generation in 2013 was over 41% and it is expected to decline to 36% in 2021.

The cycle of coal from the supply side can be summarised in three phases: mining, preparation, transportation. About two third of the extraction is made by underground mining, while the rest is made by surface mining. In the preparation, the coal is processed in order to be standardized in several typologies. The final product is then used domestically or traded on the market. Industries and power sectors are the main consumers of this commodity. Trade and production of coal were increasing until 2012 (see figure 2.5), while in the last years it has stopped and it is slightly decreasing (CarbonBrief (2016b)). As we can see in figure 2.5, differently from oil, most of the coal is consumed domestically leaving to the export only a marginal part (13% in 2000, 17% in 2014).

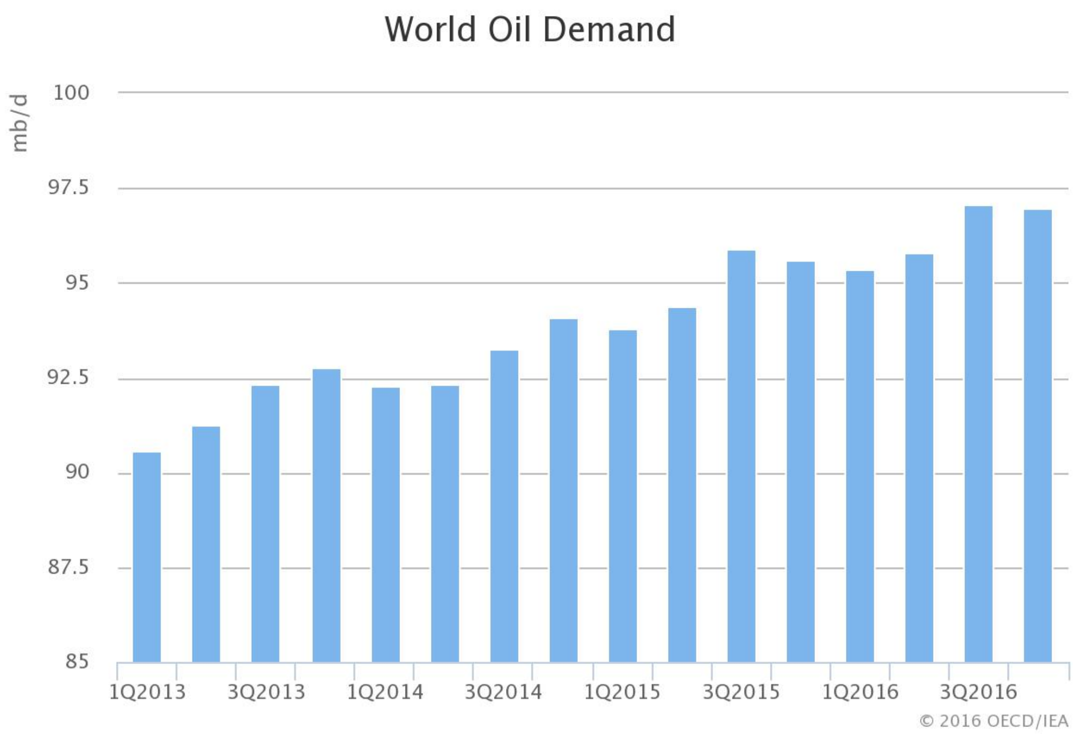


Figure 2.4: World Oil Demand (source IEA (2016b)).

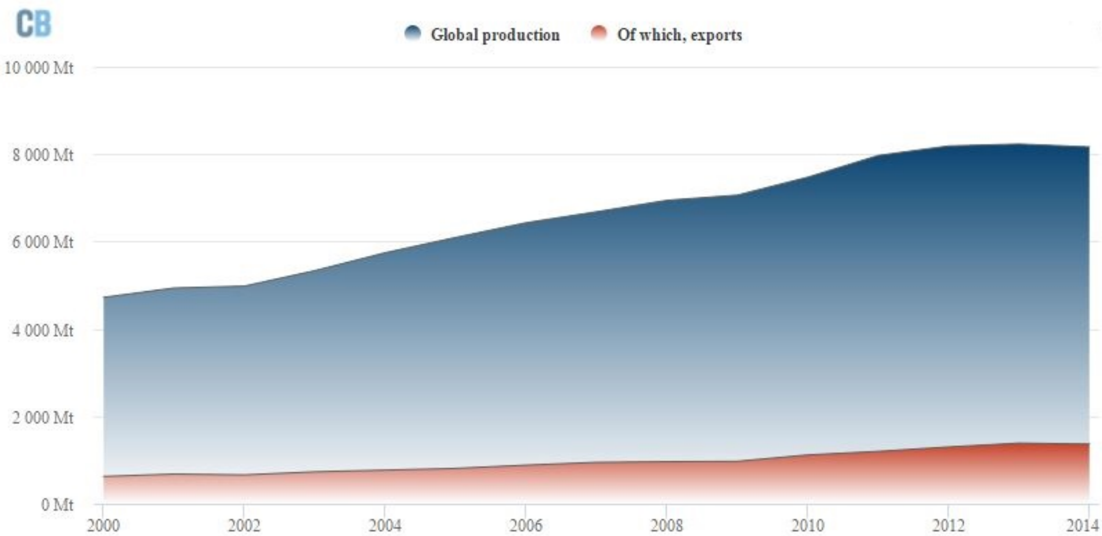


Figure 2.5: World coal production and export (source CarbonBrief (2016b)).

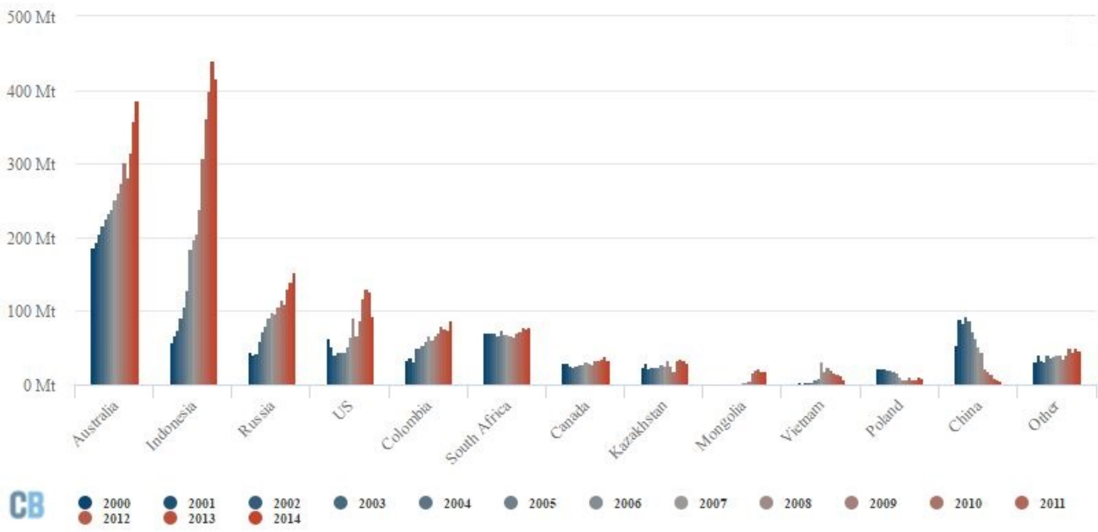


Figure 2.6: World coal exporters (source CarbonBrief (2016b)).

Asia is the highest importer, followed by Europe, while Australia, Indonesia, Russia and US are the most important exporters (figure 2.6).

The drivers of the market are similar to oil but with some differences:

- As we have seen, most of the market is domestic.
- As pointed out in Roncoroni et al. (2015), coal reserves are well distributed in the world, given that they are available in developing countries, but also in USA, Australia and Europe.
- Coal is classified in several typologies, depending on their carbon/energy content (see figure 2.7).
- The transportation costs have an high impact on the price.
- Finally, coal has an high impact on environment, given that it produces the highest quantity of greenhouse gas. For this reason, the last international agreements are trying to reduce the use of coal.

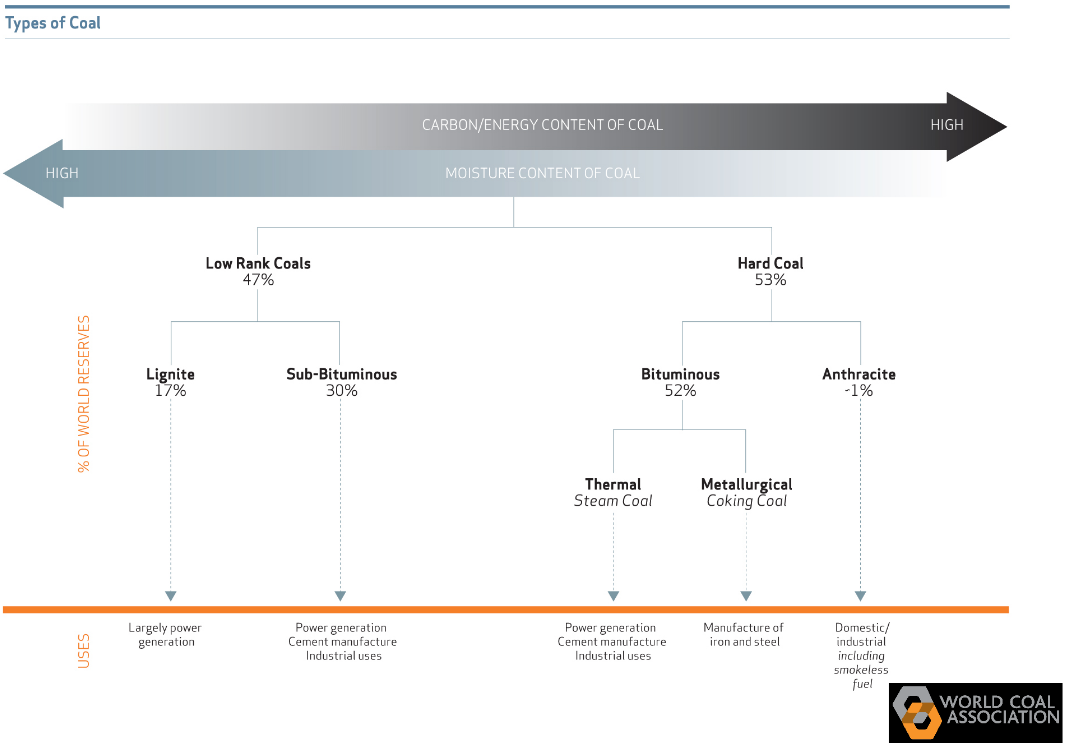


Figure 2.7: Coal classification (source World Coal Association (2016)).

## 2.4 Gas Market

IEA (2016d) data shows that in 2015 the gas production was the highest in history, reaching the level of 3,590 Bcm (see figure 2.8).

As in Roncoroni et al. (2015), the cycle of gas consists in five main activities: exploration and production, processing, transportation, storage. Natural gas can be extracted in oil fields (*associated gas*) or also in specific fields (*non-associated gas*). It is noteworthy that in the last years the shale gas technologies in USA are changing the equilibrium of the market. The processing aim is to remove impurities and water. The transportation can be carried out with pipeline networks or with LNG (liquefied natural gas). We can distinguish between three typologies of networks. Transmission systems deliver the gas from production site to regional systems, that in turn feed local grids, where the end

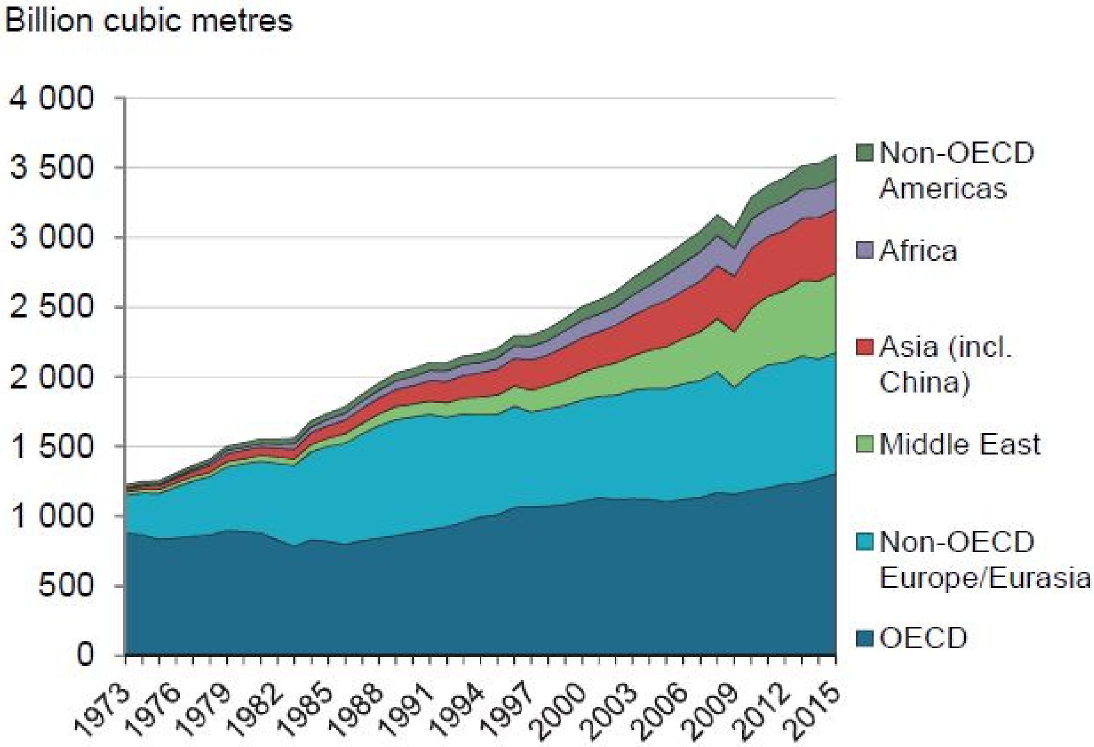


Figure 2.8: World gas production (source IEA (2016d)).

users are connected. LNG can be transported through special freight vessels, since its volume is about 1/600 of natural gas. Special terminals are needed to manage the transportation. Finally, storage is very important in the gas market. Caverns, depleted oil and gas fields, aquifers, and overground steel storage units are used. Large facilities are used to offset seasonal patterns in the demand (*seasonal storage*), injecting gas during the summer, in correspondence of low prices, and withdrawing during the winter, when the prices are higher. Smaller facilities with high injection rates are used to deal with short term variation in the demand (*peak storage*).

The gas market participants can be divided into two levels. In the first, we find producers, power generators, industrial consumers, suppliers and distributors. In the second, we have operators, that buy and sell gas for balancing activities, and traders.

In order to understand the dynamics of gas prices, it is useful to explore the types of price formation mechanism. International Gas Union (2016) finds nine main categories:

- Oil price escalation (OPE). Gas prices are linked to crude oil and oil products. Also coal and electricity price can be used as reference.
- Gas-on-Gas competition (GOG). Supply and demand define the price for different periods and in different hubs. Also bilateral agreements between multiple buyers and sellers are included.
- Bilateral Monopoly (BIM). The price is settled in a bilateral agreement between two large players or between a large player on one side and multiple players on the other.
- Netback from Final Product (NET). The price is linked to the price the buyer is selling its outputs.
- Regulation: Cost of service (RCS). A regulatory authority set a price to cover costs, investments and to guarantee a reasonable rate of return.
- Regulation: Social and Political (RSP). The price is set by a Ministry, or by a similar authority, on political and social basis.
- Regulation: Below Cost (RBC). The price of gas is subsidized in order to be below its cost.
- No Price (NP). The gas is free for population and industries.
- Not Known (NK). No data or evidence.

The historical world price formation by category can be found in figure 2.9, while the breakdown by region for 2015 is in figure 2.10.

As we can see, the Gas-on-Gas competition accounts for about 45% of the total. Oil price escalation share is about 19% and is used in Asia and Europe (in the Mediterranean area the share is over 60%). It is noteworthy that cross border flows account for 27%

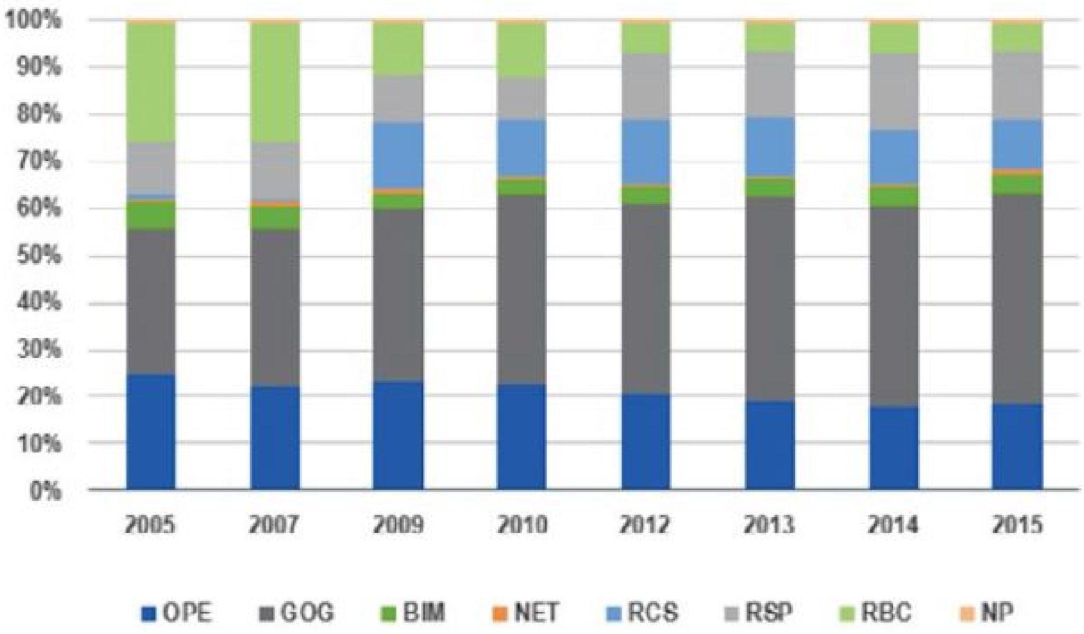


Figure 2.9: World gas price formation 2005 to 2015 - Total Consumption (source International Gas Union (2016)).

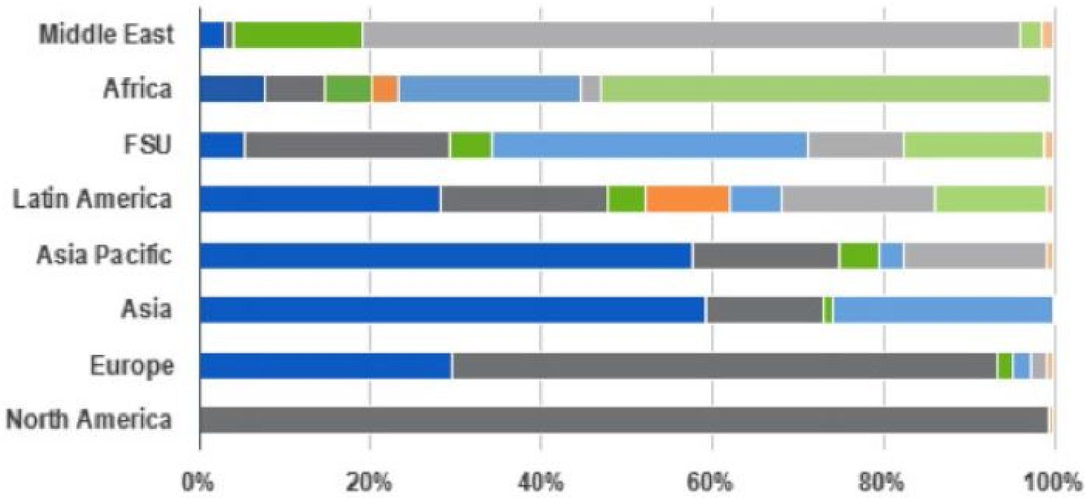


Figure 2.10: World gas price formation 2015 by region - Total Consumption (source International Gas Union (2016)).

of the world total consumption, therefore, we can say that the gas market has a higher degree of internationality than the coal market.

Given the price formation mechanisms and the fundamentals of the market, we can understand that the driving forces of the gas are similar to the commodities presented in the previous sections.

- Macroeconomics and Geopolitics. Gas price is not only sensible to its geopolitics[3], but also to the ones of oil, since OPE establishes a clear link, especially in the long-term.

- Technology developments. New methods for exploration and production can have significant effects on the market. It is noteworthy that the shale revolution in USA involved both oil and gas.

- Weather and Arbitrage among energy commodities. The weather has direct and indirect effects on gas, since it is highly used in the power markets.

- Regulation. The gas is impacted by international regulations directly, because the use of gas produce greenhouse gases, and indirectly, since it is used to substitute the more polluting coal.

## 2.5 The European Union Emissions Trading System

The European Union Emissions Trading System (EU ETS) has been established in 2005 to reduce greenhouse gases. The target is to reduce, with respect to 1990 levels, the emissions by 20% within 2020 ad by 40% within 2030. According to European Commission (2016), the EU ETS is the largest emission trading market, involving 31 countries, 11,000 energy intensive installations, and covering 45% of EU greenhouse emissions. The resources that are collected through the EU ETS are invested to combat the climate changes.

[3] Cordano (2015) makes a focus on the Russian-Ukrainian crisis.

The system is *cap and trade*. For each phase of the program, the European Union sets a limit of the greenhouse gases that can be emitted. It has been designed to be flexible, since the companies can decide to invest in reducing their emissions or they can buy from other players the allowances they need to continue the operations as usual. Therefore, the EU ETS incentives companies to innovate, given that, if they produce less greenhouses gases, they are able to make profits trading their exceeding allowances. On the other hand, companies that are not willing to invest will have to pay extra costs. Each allowance let the owner to emit one ton of CO2[4] and can be used just once.

Four trading periods have been set for EU ETS (source European Commission (2016)):

- 2005-2007 was the first period. It was an experimental phase, where the volume of allowances turned to be excessive and, therefore, the price fell to zero.

- 2008-2012 was the second period. The total of allowances was reduced by 6.5%, however, the recession decreased the demand by a higher rate, leaving a significant amount of unused allowances and credits.

- 2013-2020 is the third and current trading period. An EU-wide cap on emissions has been introduced, reduced by 1.74% each year. Moreover, a transitioning from free allocation of allowances to auctioning based system has started. The aim is to reach that 57% of allowances will be allocated through auctions, in order to increase the transparency of the system and to make polluters pay for their emissions. From 2013, the power generators have to buy all the allowances they need.

- 2021-2030 is the last period. In July 2015, a revision of the system has been presented to the European Commission.

In figure 2.11, we can find the historical trading volumes of emissions since the beginning of the system. We can see that the volume has increased in the second phase. The up-trend lasts until 2013, beginning of the current phase. As pointed out in European Commission (2016), the market *continues to face a challenge in the form of a*

[4] In this thesis, CO2 will be used to identify EU ETS allowances.

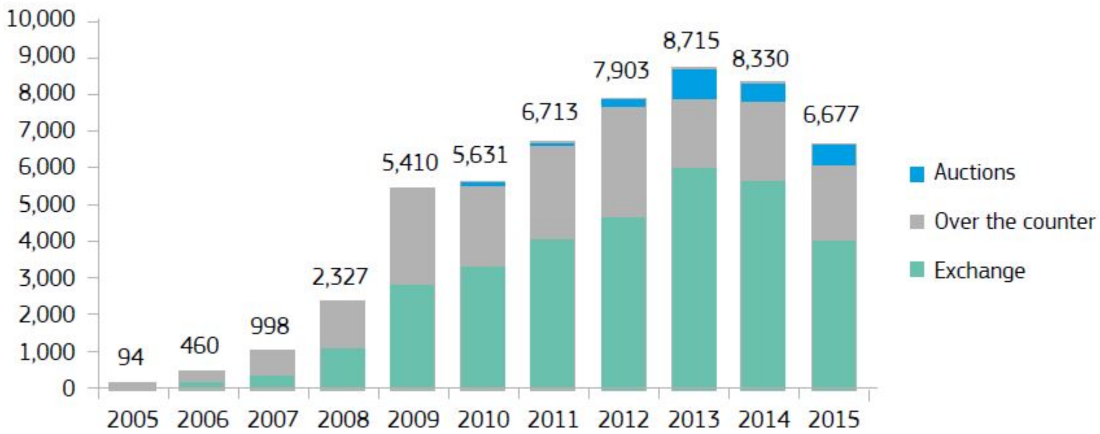


Figure 2.11: Trading volumes in EU emission allowances in mln tons (source European Commission (2016)).

*significant surplus of allowances, largely due to the economic crisis which has substantially depressed emissions. In the short term, this surplus risks undermining the orderly functioning of the carbon market. In the longer term, it could affect the system's ability to meet more demanding emission reduction targets cost-effectively.* An auction of 900 million allowances was planned to take place between 2013 and 2015. However, in order to avoid an increase in the supply, it has been postponed to 2019-2020. In 2015, another decision has been taken to address the carbon market issue. A reserve is going to be established, in which the 900 millions of tons will flow into, in order to be able to manage the supply of allowances and to increase the resilience of the market.

Therefore, it can be said that the EU ETS is linked to the dynamics of the other commodities, but mainly on the demand side. Moreover, shocks on the demand side could affect the daily trading, but in the long-term the main drive of the market is the European regulation. Differently from other commodities, there is no other subject, country or technology that could try to offset a change in the supply of allowances decided by the regulator, as in the case of the 900 millions of tons. This means that, depending on available allowances, also the demand elasticity to price is affected.

## 2.6 Summary

In this chapter, we have briefly reviewed the functioning of the commodity markets object of analysis. We have seen that several common economic forces drive the commodities dynamics, such as macroeconomics, technologies and regulations. However, as discussed in European Commission (2016), the EU ETS market differs significantly from the other markets, since it is not physical, but a "virtual" market driven by the regulators.

# Chapter 3

# The Model

*Most generally, noise makes it very difficult to test either practical or academic theories about the way that financial or economic markets work. We are forced to act largely in the dark.*

---

Black (1986)

## 3.1 Introduction

The model proposed in this thesis is based on factor reduction techniques, where the dynamics of the prices are described through unobservable and latent variables. Kiesel et al. (2009) applied a two-factor model to the electricity market. Edoli et al. (2013) extended Kiesel et al. (2009) by implementing a two-factor model in a multi commodity framework. However, the calibration method proposed by Edoli et al. (2013) uses only the instantaneous correlations among the commodities to model their associations. As stated in Alexander (2001), the *correlation is intrinsically a short-run measure.* A model based only on the daily correlations would not be suited for long-term decisions, such as investments and hedging. Given that the information in the correlation could be incomplete, cointegration analysis could enable the model to be also consistent with long-term dependencies. Previous studies in different commodity markets, in various periods and applying different methodologies prove that most of the commodity markets are cointegrated. Therefore, it is important to extend the two-factor model in order to include long-term information.

It is important to emphasise that non considering long-term dependencies would not only produce a model based on partial information, but would also have negative effects on the hedging strategies: being the correlation only a short term measure, *correlation-based hedging strategies commonly require frequent rebalancing* (Alexander (2001)). It is noteworthy that, as stated in Alexander (1999), *high correlation of returns does not necessarily imply high cointegration in prices*. Even with very high correlation between the returns, prices can diverge. On the other hand, cointegration can be identified also in periods where correlations are low. From a practical perspective, this would mean that whatever the measured correlation is, high or low, the hedging strategy could be radically different when cointegration is taken into account.

There is also another consideration that would lead factor models and cointegration work together in capturing the cross commodity association. Effectively, whenever we take advantage of factor models to reduce the dimension of our problem in a single commodity framework, like in Kiesel et al. (2009), we are basically allowing for cointegration among the products of a commodity. This is straightforward if we use a one-factor model, like in Clewlow and Strickland (1999b), where the correlation among the modelled products is one. Obviously, this is not a simple approximation, but a characteristic already pointed out in the literature. Analysing oil prices, Crowder and Hamed (1993) find that spot and futures markets are cointegrated. Schwarz and Szakmary (1994) and Peroni and McNown (1998) find spot prices to be cointegrated with the future prices in three different oil products (Crude Oil, Heating Oil and Unleaded Gasoline). Moosa and Al-Loughani (1995) studying the speculation effects in the oil market, find spot and futures prices to be cointegrated. Yang et al. (2001) and Adämmer et al. (2016) find cointegration between spot and futures of agricultural commodities. Rittler (2012) proves spot and futures prices of European Union emissions trading scheme (EU-ETS) to be cointegrated using high frequency data.

The model proposed in this thesis extends the models by Kiesel et al. (2009) and by Edoli et al. (2013) in several ways. First, numerical integration techniques based on sparse grids (Heiss and Winschel (2008)) are implemented in order to reduce the computational complexity that arise in dealing with quarter and year swap contracts. Second, a two steps procedure to deal with the nearest correlation matrix problem is applied. A quadratically convergent Newton method (Qi and Sun (2006)) is used to grant flexibility in designing the starting point of the optimization. The algorithm of Edoli et al. (2013), that works directly on the Cholesky decomposition of the matrix, is then applied to fit the historical global correlation matrix. Third, closed form formulae for term correlations are computed to allow to work in any kind of discrete time simulation framework and to link the factor models with cointegration. Finally, long-term dependency is included, translating the error correction matrix, computed through Johansen (1995) cointegration framework, into term correlations.

The chapter is structured as follows. In section 3.2, the literature of commodity models and cointegration analysis is reviewed. In section 3.3, the two-factor model is presented. In section 3.4, the theoretical framework of cointegration will be reported. In section 3.5, the closed form formulae for term correlations will be developed. In section 3.6, it will be shown how the cointegration analysis will be included in the two-factor model to take into consideration the long-term dependency that lies beneath the observed prices of the commodities. Finally, in section 3.7, the calibration methodology of the models is explained.

## 3.2 Literature Review

In literature, a multitude of models have been developed to treat commodity prices both in the spot and in the forward markets. For the special case of electricity, there is a stream of the literature that models it in a structural way. However, as stated by Sims (1980), when a simultaneous equation model in structural form is not identified, a reduced form model can be parameterized instead. Furthermore, cointegration analysis have been intensively applied to commodities to investigate on the long-term relations that drive their joint dynamics.

In subsection 3.2.1, a brief survey of the structural models applied to electricity will be presented. In subsection 3.2.2, the reduced form models will be reviewed. In subsection 3.2.3, cointegration analysis applied to the commodity markets is collected.

### 3.2.1 Structural models for electricity

In structural models, the focus is on modelling the fundamentals of the market, like supply and demand. Once their behaviours and their relation is captured, it will be possible to explain price movements. Electricity has been studied through structural models by different authors. Davison et al. (2002) use power demand and capacity as inputs to model the spot power price as a mixture of two normal distributions. Barlow (2002), assuming that supply is deterministic whilst demand is an Ornstein-Uhlenbeck process, represents the electricity spot price as a nonlinear Ornstein-Uhlenbeck process. Kanamura and Ohashi (2007) approximate the supply curve with a *hockey-stickshaped curve*[1] and define the demand as an Ornstein-Uhlenbeck process. Davison et al. (2002), Barlow (2002) and Kanamura and Ohashi (2007) are able to model electricity price spikes and to reasonable replicate the supply/demand dynamics of this specific market. However, Barlow (2002) in the conclusion states that the model fits well with the spot prices, but it does *not provide a satisfactory explanation of the relation between spot and future prices*. Moreover, also Kanamura and Ohashi (2007) identify in the relationship between spot and future prices an important issue to be further analysed[2]. Pirrong and Jermakyan (2008) let the power price be a function of the demand[3] and of the prices of the fuels. Also in this case, *since one of the state variables is not a traded asset, it is necessary to take account of its market price of risk*. Álvaro Cartea and Villaplana

[1] Kanamura and Ohashi (2007) use two lines, one flat and the other steep, linked by a quadratic curve.

[2] Kanamura and Ohashi (2007) propose to estimate the stochastic discount factor that let its future spot prices to be consistent with the forward curve.

[3] Pirrong and Jermakyan (2008) consider the demand as one of the possible state variables. In their model it is possible to take into consideration also the weather.

(2008) extend the model from Barlow (2002) by introducing the capacity as a random variable. The model is able to analytically express the expected spot prices and the price of forward contracts, thus allowing the forward premium to have a closed form. Lyle and Elliott (2009) develop a supply demand model "simple enough" to provide a closed form solution for European call options.

### 3.2.2 Reduced form models

There are two main families of reduced form models. The first one is based on the definition as stochastic processes of the spot price and the convenience yield. The second one is fully focused on modelling the evolution of the forward curve through time. For this reason, they often leverage on the knowledge developed in the multi-factor framework of interest rate modelling[4].

The convenience yield models find their economical roots throughout history. Keynes (1923) explains the phenomena of *backwardation*[5] stating that *for the sake of certainty, the producer, not unnaturally, is prepared to accept a somewhat lower price in advance than what, on the balance of probability, he thinks the price is likely to be when the time comes*. Kaldor (1939) states that stocks have a yield since they let the producer avoid costs, troubles and delays of ordering frequent deliveries. Working (1949) explores *inter-temporal price relations*[6] through a breakdown of the storage activity. He states that, until a *recognized* level, stocks do imply a convenience yield, since they are necessary to run the main processes of the business. Brennan (1958), analysing the *inverse carrying charges*, proposes the same reasons as Kaldor (1939), since through stocks the wholesaler can be more flexible and resilient to increases in clients demand. Brennan and Schwartz (1985) define the convenience yield as *the flow of services that accrues to an owner of the physical commodity but not to the owner of a contract for future delivery of the commodity*. Wright and Williams (1989) list three main elements that characterize the prices

[4]For a comprehensive review of interest rate models refer to Brigo and Mercurio (2006).

[5]Forward price below the spot price or decreasing forward curve.

[6]Working (1949) excludes from *inter-temporal price relations* the relation between present and past prices.

of storage: *total transformation cost minimization*, *differences among apparently related commodities* and *distortions in commodity markets, such as export subsidies or the strategic petroleum reserve.* There are several models that use the concept of convenience yield to describe the relation between spot and future prices. Gibson and Schwartz (1990) developed a two-factor model[7] where the first one is the spot price and the second is the instantaneous convenience yield. Furthermore, the spot price is supposed to follow a Geometric Brownian Motion like in Black and Scholes (1973), while the instantaneous convenience yield to follow an Ornstein-Uhlenbeck process. After some years, Schwartz (1997) and Miltersen and Schwartz (1998) extended Gibson and Schwartz (1990) considering the instantaneous interest rate to follow a mean reverting process as in Vasicek (1977). Hilliard and Reis (1998) include also jumps in the spot price in order to capture the effect of discrete time events like the one related to the weather. The inclusion of jump diffusion process does not change the pricing of forward contracts, but it significantly affects the valuation of options. All those models are able to describe several commodity forward curve movements, but they have some limitations. As stated by Clewlow and Strickland (1999b), since *the state variables are unobservable - even the spot price is hard to obtain, with the problems exasperated if the convenience yield has to be jointly estimated.*

The second family of models are the forward curve models or factor reduction models. The purpose of this approach is to jointly model the evolution of the forward curve, given the one available at the time of the valuation. The prices are described through unobservable and latent variables, so there is no direct connection or relation with known and explicit external variables. These models are able to let the resulting forward prices have volatilities that decrease with the time to maturity, a very important characteristic in the commodity markets[8]. The most important assumptions of those models are the number of factors to be taken into consideration and their stochastic processes. From an analytical point of view, Gyöngy (1986) was the first to find a one-factor process having

[7]Schwartz (1998) developed a one-factor model that is practically equivalent to the two-factor model for long term time horizon.

[8]Also known as *Samuelson effect* (Samuelson (1965)).

the same marginal distribution as a process with two stochastic parameters. Clewlow and Strickland (1999b) developed a one-factor model with analytical formulae for standard options, caps and floors, collars, swaptions and exotic energy derivatives. Afterwards, Clewlow and Strickland (1999a) extended the previous work to a multi-factor model. Inspired by the calibration of LIBOR market models on swaptions by Brigo and Mercurio (2001), Kiesel et al. (2009) extended Clewlow and Strickland (1999b), in order to work in a two-factor environment applicable to electricity futures market. The model is able to fit swapoption-implied volatilities, even if it has some difficultes with very short term products. Recently, Edoli et al. (2013) developed a calibration methodology that is able to extend the model by Kiesel et al. (2009) in a multi-commodity framework. The calibration is based on the historical forward quotations of WTI, Brent and Gasoil. It is performed in two steps: first, the parameters of each commodity are retrieved and, then, the cross correlations parameters are calibrated to fit the entire covariance matrix.

### 3.2.3 Cointegration and commodity markets

The cointegration concept by Granger (1981) and by Engle and Granger (1987) can find its economic roots in Leontief (1936) and in Hicks (1939). A composite index may be the result of the combination of several goods, so that they can be treated as a single composite good. However, Leontief (1936) states that the choice of the composition is an element of *arbitrariness*. For Hicks (1939) different *physical things* can be treated as a single one as long as we can assume that their relative prices do not change over time. Therefore, one could think of the cointegration as a robust statistical method to overcome the expert based choices of index composition and proportionality. Still, as pointed out in Johansen (2000), the choice of the variables to be analysed depends on the *economic insight* used to formulate the problem, but the cointegration framework can be used to describe and test *the existence of long-run economic relations*.

Cointegration analysis has been extensively applied to the commodity markets.

Girma and Paulson (1999) analyse the relations between daily quotations of two-months futures prices of oil with its end products, gasoline and heating oil, from 1983 to 1994. The spreads taken into consideration are the 3:2:1 *crack spread* (3 contracts of crude oil, 2 contracts of unleaded gasoline and 1 contract of heating oil), the 1:1:0 *gasoline crack spread* (1 contract of crude oil and 1 contract of unleaded gasoline) and the 1:0:1 *heating oil crack spread* (1 contract of crude oil and 1 contract of heating oil). Using the tests by Dickey and Fuller (1981) (ADF, Augmented Dickey-Fuller test) and by Phillips and Perron (1988), they prove that the prices are integrated of order one and that the spreads are stationary. The results are shown to be meaningful both for trading and for hedging purposes.

Simon (1999) conducts a similar study on the future prices of the soybean and its end products, soymeal and soyoil, from 1985 to 1995. The spread taken into consideration is the *crush spread* (soybean meal, soybean oil and soybean). Using the ADF test, the prices are proven to be integrated of order one. Moreover, performing the test by Engle and Granger (1987), it is shown that the prices are cointegrated.

Serletis and Herbert (1999) analyse daily data of Henry Hub gas prices, Transco Zone 6 gas prices, Pennsylvania, New Jersey, Maryland (PJM) electricity prices, and the New York Harbor oil prices from 1996 to 1997. With the implementation of the ADF test, the authors prove the series to be integrated of order one, except for the PJM that result to be integrated of order zero. Using Engle and Granger (1987), gas and oil prices result to be cointegrated. Furthermore, using Granger causality test (Granger (1969)), causality and feedback relations are found among the cointegrated commodities.

Emery and Liu (2002) analyse the California-Oregon Border (COB) and Palo Verde (PV) electricity futures prices with natural gas futures prices from 1996 to 2000[9]. With Dickey and Fuller (1981), they find that the prices are integrated of order one and, using the framework of Engle and Granger (1987), they also find the prices to be cointegrated. The results show also that traders could have generated profits by exploiting the mean reversion of the spark spread[10].

Asche et al. (2006) perform a multivariate cointegration analysis on end of month

[9]Data are daily settlement prices of NYMEX's 1st nearby California-Oregon Border and Palo Verde electricity futures contracts and Henry Hub natural gas futures contract.

[10]The spark spread is the difference between the electricity and the gas prices.

prices of oil, gas and electricity in the UK using the methodology of Johansen (1988). The prices object of analysis result to be cointegrated in the period January 1995 - June 1998, hypothesis that cannot be confirmed in the following period 1998-2002, right after a pipeline connected the UK natural gas grid with Europe.

Wårrel (2006) analyses quarterly data from 1980 to 2000 of European and Japanese coking and steam coal. ADF test shows that the prices are integrated of order one, while using the approach by Engle and Granger (1987) they are also found to be cointegrated, letting the author show the existence of a coal world market.

Panagiotidis and Rutledge (2007) try to find out if the UK gas prices and the Brent oil prices *decoupled* in the period of the UK gas market liberalisation 1996-2003. Using the ADF test, the test by Breitung (2002) and Breitung and Taylor (2003) that takes into account structural breaks, the test by Saikkonen and Lütkepohl (2002) and by Lanne et al. (2002) that considers level shifts, and the test by Phillips and Perron (1988), the authors confirm that prices are integrated of order one. The authors test the presence of cointegration implementing the well known methodology by Johansen (1995) and the non parametric procedure by Breitung (2002). UK gas and Brent prices result to be cointegrated throughout the whole period, regardless of the opening of the gas connection between UK and Europe. This means that, although the market was liberalised, the UK gas market was always linked to the oil price, at least in the long run.

Bunn and Fezzi (2009) build a Vector Error Correction Model, in line with Johansen (1991), to study the relations between gas, electricity and carbon day-ahead prices in Germany and in UK from 2005 to 2006. ADF test and KPSS test (Kwiatkowski et al. (1992)) show that gas and carbon prices are integrated of order one. However, the tests give different results on electricity prices, but the authors decide, however, to treat them as non stationary series following the suggestion by Hendry and Juselius (2000)[11]. Using the methodology by Johansen (1991), both Germany and UK systems result to

[11] Hendry and Juselius (2000), in chapter 6 *Testing for Unit Roots*, state that is better to treat a stationary variable as an integrated one if it has a root close to 1 ($> 0.95$). One of the presented reasons is that the Dickey-Fuller (Dickey and Fuller (1979)) test distribution is skewed with a long left tail, so that is not easy to deal with roots closed to one.

be significantly cointegrated with different cointegrating vectors given that UK has a generation mix with the gas as the marginal technology, whilst Germany has the coal as the marginal technology[12].

De Jong and Schneider (2009) study the dynamics of spot and one month forward contracts of UK, Belgian, Dutch natural gas markets (NBP, ZEE and TTF) and the Dutch power market (APX) from 2004 to 2008. Johansen (1988) is used to test that all the variables are cointegrated. In order to be focused on the relation that lies between spot and forwards and among different markets, the authors develop an innovative multi-market spot-price model by allowing the spot-M1 spreads to be cointegrated. In this way, the correlation between the residuals represent the short term correlation, while the linkages among the spreads of the different markets capture the long-term relationships. Counter-intuitively the spot-M1 spreads of the gas markets are find to be weakly or negatively related with the power market both in the prices and in the returns. The authors explain the results by stating that the relationships have already been captured by the one month contracts. The additional movements of the spot power price with respect to the forwards are independent between power and gas. In the authors opinion, this is mainly due to the gas sourcing practice that is not performed in the spot markets, but usually signing long term contracts or entering in forward transactions. Furthermore, although the relations between the different markets seems to be well captured, the single commodity distributions do not fit well with the history. It is noteworthy that the authors say that an appropriate cointegrated forward price model should be developed in order to capture the co-movements of the gas and electricity one month contracts.

Bosco et al. (2010) analyse the central Europe power markets. They use weekly average of hourly power prices of Netherlands (APX), Germany (EEX), Austria (EXAA), Scandinavia (Nord Pool), Spain (Omel) and France (Powernext) from 2002 to 2007. In order to treat outliers, the non-Gaussian pseudo-likelihood tests by Lucas (1998) is implemented to test if the prices are integrated and cointegrated. All the markets result to be integrated. Moreover, with the exception of Spain and Scandinavia, the prices

[12]For Germany see subsection 2.1.2.

share a common trend. Furthermore, using the data of Belgian gas index (ZEE) and the data of Brent, the authors find that there is an evidence of cointegration between electricity prices and gas, while the same result cannot be confirmed for oil.

Bencivenga et al. (2010) analyse the daily prices of Brent, UK gas (NBP) and EEX electricity from 2001 to 2007. Using the ADF test, the authors conclude that the variables are integrated of order one. Using the Johansen framework (Johansen (1988), Johansen (1991) and Johansen (1995)), the prices result to be cointegrated. Moreover, also the Engle and Granger (1987) method is implemented to study each pair of commodities.

Westgaard et al. (2011) study the relation between Gas oil and Brent Crude oil futures prices. They take into consideration daily prices from 1994 to 2009 of five different maturities for both the commodities: 1 month, 2 months, 3 months, 6 months and 12 months. Using the ADF test, all the variables are found to be integrated of order one. Granger causality test (Granger (1969)) show that *feedback* occurs, meaning that there is not a unidirectional relation. Engle and Granger (1987) and Johansen (1991) frameworks are used to find that the one and two months forwards are cointegrated, whilst the other tenors are not. The authors ascribe those counterintuitive results to the limited liquidity of the longer term forwards and to the volatility of the period 2002-2009.

Joëts and Mignon (2012) use panel cointegration techniques to show that daily forward prices of oil, coal, gas and electricity forward prices on 35 maturities are cointegrated. Oil, gas and coal forward prices are found to be positively linked, whilst electricity and oil forward prices have a negative relation.

Frydenberg et al. (2014) analyse daily future prices of electricity, oil, gas and coal in UK, German and Nordic markets from 2006 to 2012. With ADF test, the prices are proven to be integrated of order one. Using the framework of Johansen, the authors find cointegration among electricity prices, between UK electricity and gas, between UK electricity and coal, between German electricity and coal, and between Nordic electricity and coal. The authors think that maybe there are more cointegrating relations among the commodities. It is noteworthy that they expect to find them using more historical data, but also through an analysis of contracts with longer maturities such as monthly, quarterly and seasonal/yearly forwards.

Madaleno et al. (2015) analyse annual electricity, gas, coal and oil prices in 22 countries[13] from 1996 to 2013. Using panel unit root (Levin et al. (2002)) and panel cointegration tests (Pedroni (2001) and Pedroni (2004)), the authors find the variables to be integrated of order one and to be cointegrated.

## 3.3 Two-Factor Model

In this section we examine the two-factor model framework. The univariate case (subsection 3.3.1) is based on the work of Kiesel et al. (2009), while the multivariate case (subsection 3.3.2) is based on the extension proposed by Edoli et al. (2013).

### 3.3.1 The Univariate Case

In the reduced factor technique, observable forwards prices can be treated directly with Gaussian factors

$$\mathrm{d}F(t,T) = \boldsymbol{\sigma}(t,T)F(t,T)\mathrm{d}\boldsymbol{W}_t \tag{3.1}$$

where

- $F(t,T)$ is the price of the forward with start date $t$ and end date $T$
- $\boldsymbol{\sigma}(t,T)$ are the volatilities of the factors
- $\mathrm{d}\boldsymbol{W}_t$ is the $n$-dimensional Brownian motion
- $n$ is the number of factors.

The solution:

$$F(t,T) = F(0,T)e^{\int_0^t \boldsymbol{\sigma}(s,T)\mathrm{d}\boldsymbol{W}_s - \frac{1}{2}\int_0^t \left\|\boldsymbol{\sigma}(s,T)\right\|^2 \mathrm{d}s} \tag{3.2}$$

[13] Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Netherlands, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, United Kingdom and Switzerland.

As in Kiesel et al. (2009), the two-factor model can be defined as follows

$$\frac{\mathrm{d}F(t,T)}{F(t,T)} = e^{-k(T-t)}\sigma_1 \mathrm{d}W_t^1 + \sigma_2 \mathrm{d}W_t^2 \tag{3.3}$$

where

- $\sigma_1$ is the volatility of the short-term factor
- $\sigma_2$ is the volatility of the long-term factor
- $k$ is the short-term factor mean reversion coefficient
- $\rho$ is the correlation coefficient between the short and the long-term factors

and where $\mathrm{d}W_t^1$ and $\mathrm{d}W_t^2$ are two correlated Wiener processes such that

$$\mathrm{d}W_t^1 \mathrm{d}W_t^2 = \rho \mathrm{d}t \tag{3.4}$$

or in terms of correlation matrix

| | Short | Long |
|---|---|---|
| Short | 1 | $\rho$ |
| Long | $\rho$ | 1 |

Table 3.1: One Commodity Factor Correlation Matrix

Applying Itô's Lemma[14] (Itô (1944)) to the SDE in (3.3), we obtain

$$F(t,T) = F(0,T)e^{\int_0^t \sigma_1 e^{-k(T-s)}\mathrm{d}W_s^1 + \int_0^t \sigma_2 \mathrm{d}W_s^2 - \frac{1}{2}\left(\int_0^t \sigma_1^2 e^{-2k(T-s)}\mathrm{d}s + \int_0^t \sigma_2^2 \mathrm{d}s + 2\int_0^t \sigma_1\sigma_2 e^{-k(T-s)}\rho \mathrm{d}s\right)} \tag{3.5}$$

To simplify the notation, let us have

$$\tilde{\sigma}^2(s,t) = \sigma_1^2 e^{-2k(t-s)} + \sigma_2^2 + 2\sigma_1\sigma_2 e^{-k(t-s)}\rho \tag{3.6}$$

Hence

$$F(t,T) = F(0,T)e^{-\frac{1}{2}\int_0^t \tilde{\sigma}^2(s,T)\mathrm{d}s + \int_0^t \sigma_1 e^{-k(T-s)}\mathrm{d}W_s^1 + \int_0^t \sigma_2 \mathrm{d}W_s^2} \tag{3.7}$$

[14] Shreve (2004) adds Doeblin's name to Itô's formula. In 2000 a sealed envelope, received by the French National Academy of Sciences in February 1940, was opened for the first time. It contained the work by Doeblin (1940) that revealed a construction of the stochastic integral similar to the Itô's one.

From equation (3.7), we can compute the variance of the logarithm of a forward price in time $T_0$

$$\mathrm{Var}(\log F(T_0,T)) = \int_0^{T_0} \sigma_1^2 e^{-2k(T-t)}\mathrm{d}t + \int_0^{T_0} \sigma_2^2 \mathrm{d}t + \int_0^{T_0} \rho\sigma_1\sigma_2 e^{-k(T-t)}\mathrm{d}t \quad (3.8)$$

Solving the integrals, we obtain

$$\mathrm{Var}(\log F(T_0,T)) = \frac{\sigma_1^2}{2k}(e^{-2k(T-T_0)} - e^{-2kT}) + \sigma_2^2 T_0 + 2\frac{\rho\sigma_1\sigma_2}{k}(e^{-k(T-T_0)} - e^{-kT}) \quad (3.9)$$

If we consider two forward contracts with two different maturities ($T_i$ and $T_j$), we can compute the covariance

$$\begin{aligned}\mathrm{Cov}(\log F(T_0,T_i), \log F(T_0,T_j)) = &\int_0^{T_0} \sigma_1^2 e^{-k(T_i+T_j-2t)}\mathrm{d}t + \int_0^{T_0} \sigma_1\sigma_2 e^{-k(T_i-t)}\rho\mathrm{d}t \\ &+ \int_0^{T_0} \sigma_1\sigma_2 e^{-k(T_j-t)}\rho\mathrm{d}t + \int_0^{T_0} \sigma_2^2\mathrm{d}t\end{aligned} \quad (3.10)$$

Hence, the covariance is

$$\begin{aligned}\mathrm{Cov}(\log F(T_0,T_i), \log F(T_0,T_j)) = &e^{-k(T_i+T_j-2T_0)}\frac{\sigma_1^2}{2k}(1-e^{-2kT_0}) + \sigma_2^2 T_0 \\ &+ \frac{\rho\sigma_1\sigma_2}{k}(1-e^{-kT_0})(e^{-k(T_i-T_0)} + e^{-k(T_j-T0)})\end{aligned} \quad (3.11)$$

### 3.3.2 The Multivariate Case

It is possible to generalize equation (3.11) in order to consider a multivariate framework like in Edoli et al. (2013). Let us have two commodities $a$ and $b$, both of them following a two-factor model like in (3.3). In order to have the more generalized framework, let us suppose that the forward $a$ has an expiry $T_a$, while the forward $b$ has an expiry $T_b$.

$$\frac{\mathrm{d}F_a(t,T_a)}{F_a(t,T_a)} = e^{-k_a(T_a-t)}\sigma_{1_a}\mathrm{d}W^1_{t_a} + \sigma_{2_a}\mathrm{d}W^2_{t_a} \quad (3.12a)$$

$$\frac{\mathrm{d}F_b(t,T_b)}{F_b(t,T_b)} = e^{-k_b(T_b-t)}\sigma_{1_b}\mathrm{d}W^1_{t_b} + \sigma_{2_b}\mathrm{d}W^2_{t_b} \quad (3.12b)$$

where the correlations between the short-term factor and the long-term factor of each commodity are

$$\mathrm{d}W^1_{t_a}\mathrm{d}W^2_{t_a} = \rho_{1_a,2_a}\mathrm{d}t \tag{3.13a}$$
$$\mathrm{d}W^1_{t_b}\mathrm{d}W^2_{t_b} = \rho_{1_b,2_b}\mathrm{d}t \tag{3.13b}$$

while the cross correlations are

$$\mathrm{d}W^1_{t_a}\mathrm{d}W^1_{t_b} = \rho_{1_a,1_b}\mathrm{d}t \tag{3.14a}$$
$$\mathrm{d}W^1_{t_a}\mathrm{d}W^2_{t_b} = \rho_{1_a,2_b}\mathrm{d}t \tag{3.14b}$$
$$\mathrm{d}W^2_{t_a}\mathrm{d}W^1_{t_b} = \rho_{2_a,1_b}\mathrm{d}t \tag{3.14c}$$
$$\mathrm{d}W^2_{t_a}\mathrm{d}W^2_{t_b} = \rho_{2_a,2_b}\mathrm{d}t \tag{3.14d}$$

Therefore, instead of a $2\times 2$ correlation matrix like in table 3.1, we have a $4\times 4$ factor correlation matrix (table 3.2).

| | | a | | b | |
|---|---|---|---|---|---|
| | | Short | Long | Short | Long |
| a | Short | 1 | $\rho_{1_a,2_a}$ | $\rho_{1_a,1_b}$ | $\rho_{1_a,2_b}$ |
| | Long | $\rho_{1_a,2_a}$ | 1 | $\rho_{2_a,1_b}$ | $\rho_{2_a,2_b}$ |
| b | Short | $\rho_{1_a,1_b}$ | $\rho_{2_a,1_b}$ | 1 | $\rho_{1_b,2_b}$ |
| | Long | $\rho_{1_a,2_b}$ | $\rho_{2_a,2_b}$ | $\rho_{1_b,2_b}$ | 1 |

Table 3.2: Two Commodities Factor Correlation Matrix

Like in the univariate case, we can obtain the explicit formulae for the logarithm of the prices. Therefore, given (3.12), like in (3.7), we have

$$F_a(t,T_a) = F_a(0,T_a)e^{-\frac{1}{2}\int_0^t \tilde{\sigma_a}^2(s,T_a)\mathrm{d}s+\int_0^t \sigma_{1_a}e^{-k_a(T_a-s)}\mathrm{d}W_s^{1a}+\int_0^t \sigma_{2_a}\mathrm{d}W_s^{2a}} \tag{3.15a}$$
$$F_b(t,T_b) = F_b(0,T_b)e^{-\frac{1}{2}\int_0^t \tilde{\sigma_b}^2(s,T_b)\mathrm{d}s+\int_0^t \sigma_{1_b}e^{-k_b(T_b-s)}\mathrm{d}W_s^{1b}+\int_0^t \sigma_{2_b}\mathrm{d}W_s^{2b}} \tag{3.15b}$$

Now it is possible to compute the closed formula for the covariance at time $T_0$

$$\begin{aligned}\mathrm{Cov}(logF_a(T_0,T_a), logF_b(T_0,T_b)) = &\int_0^{T_0} \sigma_{1_a}\sigma_{1_b}e^{-k_a(T_a-t)-k_b(T_b-t)}\rho_{1_a,1_b}\mathrm{d}t\\ &+\int_0^{T_0} \sigma_{1_a}\sigma_{2_b}e^{-k_a(T_a-t)}\rho_{1_a,2_b}\mathrm{d}t\\ &+\int_0^{T_0} \sigma_{2_a}\sigma_{1_b}e^{-k_b(T_b-t)}\rho_{2_a,1_b}\mathrm{d}t\\ &+\int_0^{T_0} \sigma_{2_a}\sigma_{2_b}\rho_{2_a,2_b}\mathrm{d}t\end{aligned} \tag{3.16}$$

Solving the integrals, we obtain

$$\begin{aligned}\mathrm{Cov}(logF_a(T_0,T_a),logF_b(T_0,T_b)) = &\frac{\rho_{1_a,1_b}\sigma_{1_a}\sigma_{1_b}}{k_a+k_b}e^{-k_a(T_a-T_0)-k_b(T_b-T_0)}(1-e^{-k_aT_0-k_bT_0})\\ &+\frac{\rho_{2_a,1_b}\sigma_{2_a}\sigma_{1_b}}{k_{1_b}}e^{-k_b(T_b-T_0)}(1-e^{-k_bT_0})\\ &+\frac{\rho_{1_a,2_b}\sigma_{1_a}\sigma_{2_b}}{k_{1_a}}e^{-k_a(T_a-T_0)}(1-e^{-k_aT_0})\\ &+\rho_{2_a,2_b}\sigma_{2_a}\sigma_{2_b}T_0\end{aligned} \tag{3.17}$$

Therefore, we can see that formula (3.11) is just (3.17) where

$$\begin{cases}\rho_{1_a,1_b}=1\\ \rho_{2_a,2_b}=1\\ \rho_{1_a,2_b}=\rho_{2_a,1_b}=\rho\\ \sigma_{1_a}=\sigma_{1_b}=\sigma_1\\ \sigma_{2_a}=\sigma_{2_b}=\sigma_2\\ k_a=k_b=k\end{cases} \tag{3.18}$$

Moreover, if we add the condition $T_a = T_b = T$ to (3.18), the (3.17) is equal to the univariate equation of the variance (3.9).

## 3.4 Co-integration Analysis

Cointegration can be applied only if all the series object of analysis are integrated at least of order one. There are several methods to test if a process is stationary or not like

- the Dickey-Fuller test (Dickey and Fuller (1979))
- the Augmented Dickey-Fuller (DF) test (Dickey and Fuller (1981))
- the Phillips-Perron test (Phillips and Perron (1988))
- the KPSS test by Kwiatkowski et al. (1992)
- the test by Breitung (2002) and Breitung and Taylor (2003)

- the test by Saikkonen and Lütkepohl (2002) and by Lanne et al. (2002).

The first two tests are the most used in the literature. Given a process

$$Y_t = \rho Y_{t-1} + u_t \tag{3.19}$$

where $u_t$ have mean zero and finite variance.

According to Dickey and Fuller (1979), if $\rho = 1$, $Y_t$ is not stationary, while, if $|\rho| < 1$, it is stationary. If we express (3.19) in terms of differences, subtracting $Y_{t-1}$ from both sides of the equation, we obtain

$$\Delta Y_t = \delta Y_{t-1} + u_t \tag{3.20}$$

where $\Delta Y_t = Y_t - Y_{t-1}$ and $\delta = \rho - 1$. The null hypothesis of the DF test is that $\delta = 0$ in equation (3.20). However, it is possible that the $u_t$ are correlated. In these cases the Augmented Dickey-Fuller test (Dickey and Fuller (1981)) shall be used. The null hypothesis is the same, but it has to be applied to equation (3.21)

$$\Delta Y_t = b_1 + b_2 t + \delta Y_{t-1} + \sum_{i=1}^{m} a_i \Delta Y_{t-i} + u_t \tag{3.21}$$

where we are basically controlling for $m$ lags in order to get rid of $u_t$ autocorrelation. In case the process results to have a unit root, it is going to be necessary to test if its differences are stationary or not. The order of differences needed to let $Y_t$ be stationary is the order of integration. Generally, the order of integration $k$ is identified by the notation $I(k)$, so that a stationary process is $I(0)$. A process is $I(1)$ if the first order of differences is stationary ($I(0)$).

All the considerations about integration have been made in a univariate framework. However, if we have to manage two processes, it is not correct to treat each process as in the univariate case and then extend the analysis on the bivariate case. The processes could share a common trend and differentiating them before modelling their association, would result in a loss of precious information. From a statistical point of view, this would mean that, even if the processes result to have a unit root, a specific combination of them could result to be stationary. This can be tested by implementing a regression of one process against the other. The regression is often identified as cointegration regression as in (3.22)

$$Y_t = a + \gamma X_{t-1} + u_t \tag{3.22}$$

However, we have to verify that the residuals are $I(0)$. We can express them in autoregressive form (3.23)

$$\Delta u_t = \psi \hat{u}_{t-1} + v_t \text{ with } \psi = \rho - 1 \tag{3.23}$$

As in the ADF test, we have to test the presence of unit root. However, this time we are working on the residuals of the model (3.22) and not directly on the observed data, therefore, the critical values have changed. The test to be performed is the test by Engle and Granger (1987).

In case the cointegration between the processes is confirmed, the bivariate model can be expressed through the Error Correction Model as in (3.24)

$$\Delta Y_t = b_1 \Delta X_t + b_2(Y_{t-1} + \gamma X_{t-1} - a) + u_t \tag{3.24}$$

The term $(Y_{t-1} + \gamma X_{t-1} - a)$ that comes from (3.22) is the mean reversion to the equilibrium and is known as the error correction term, whilst $b_2$ is the speed of the correction toward the equilibrium. Therefore, if due to a shock, the difference between the levels of the two processes increases, the error correction element will become greater and with strength $b_2$ it will pull back the relation to the equilibrium. On the other hand, if in a given $t$, the processes respect the long-term equilibrium, the error correction will be zero. Finally, $b_1$ represents the short term relation among the processes.

Within the Engle and Granger (1987) framework, we can find only one relation among the variables. This is not a problem if we are dealing with only two integrated processes, but, as stated in Alexander (2001), *when there are more than two $I(1)$ series the Engle-Granger method can suffer from a serious bias.* This can happen when more than one spread is stationary. The framework by Johansen (1988), Johansen (1991) and Johansen (1995) is more suited when the number of variables is greater than two. However, as pointed out in Alexander (2001), the two methods have two different aims. Engle and Granger (1987), through OLS estimation, seek for the combination of processes with minimum variance, while Johansen (1995) seeks the more stationary combinations. This framework can be considered a multivariate extension of the DF test, so let us have

$$\boldsymbol{Y}_t = \boldsymbol{a} + \boldsymbol{b}\boldsymbol{Y}_{t-1} + \boldsymbol{\varepsilon}_t \tag{3.25}$$

where $\boldsymbol{\varepsilon}_t$ are independent Gaussian $N_n(0, \boldsymbol{\Sigma})$.

Subtracting $\boldsymbol{Y}_{t-1}$, we have

$$\boldsymbol{\Delta Y}_t = \boldsymbol{a} + \boldsymbol{\Pi Y}_{t-1} + \boldsymbol{\varepsilon}_t \tag{3.26}$$

with $\boldsymbol{\Pi} = \boldsymbol{b} - \boldsymbol{I}$. As for the DF test, the (3.26) can be augmented including lags in order to control for the autocorrelation of the residuals $\boldsymbol{\varepsilon}$

$$\boldsymbol{\Delta Y}_t = \boldsymbol{a} + \boldsymbol{\Pi Y}_{t-1} + \sum_{i=1}^{m} \boldsymbol{\Gamma}_i \boldsymbol{\Delta Y}_{t-i} + \boldsymbol{\varepsilon}_t \tag{3.27}$$

To make an example, if $\boldsymbol{Y}_t$ are $I(1)$, given the differentiation, we will have a stationary process on the left hand side. This implies that also the right hand side has to be stationary. Given our hypothesis, $\boldsymbol{\Delta Y}_{t-i}$ are all stationary by definition, therefore $\boldsymbol{\Pi Y}_{t-1}$ has to be stationary itself[15]. The error correction matrix can be expressed as

$$\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}' \tag{3.28}$$

where $\boldsymbol{\alpha}$ is the error correction or adjustment speed, and $\boldsymbol{\beta}$ is the cointegrating vector. In this framework, the cointegration analysis is nothing else than a test on the rank $r$ of $\boldsymbol{\Pi}$. $r$ is the number of stationary linear combinations of the processes and, therefore, it is also the number of cointegrated relations.

As reviewed in Johansen (2000), a nested sequence of hypothesis has to be formulated in order to find the correct $r$

$$H_0 \subset H_1 \subset ... \subset H_r \subset ... \subset H_n \tag{3.29}$$

where $H_i$, with $i = 0...n$, is the null hypothesis $r \leq i$. $H_0$ will be the test on a vector autoregressive model for $\boldsymbol{Y}_t$ in differences, while $H_1$ will be the unrestricted autoregressive model in the levels. A test on the rank can also be the test to find the $r$ non zero

[15] According to Definition 3 of Johansen (2000), *if $X_t$ is integrated of order 1 but some linear combination, $\beta' X_t$, $\beta \neq 0$ can be made stationary by a suitable choice of $\beta' X_0$, $X_t$ is called cointegrated and $\beta$ is the cointegrated vector. The number of linearly independent cointegrating vectors is called the cointegrating rank, and the space spanned by the cointegrating vectors is the cointegration space.*

eigenvalues. One of the test is

$$\text{Trace Test} = -T \sum_{i=R+1}^{n} \ln(1 - \hat{\lambda}_i) \tag{3.30}$$

where $T$ is the number of the observation and $\hat{\lambda}_i$ are the eigenvalues of $\mathbf{\Pi}$.

## 3.5 Term Correlations

Since the aim of the model presented in this thesis is to consider long-term dependencies among the commodities, we need to extend the analytical framework by Kiesel et al. (2009) and by Edoli et al. (2013). We will need closed form formulae for the term correlations in order to consistently treat products in the future. Controlling the correlations for the integration step will also allow to work in any discrete time simulation framework, increasing the flexibility of the model and reducing the complexity of the problem.

Closed form formulae for term correlations are developed in subsection 3.5.1 for the univariate case and in subsection 3.5.2 for the multivariate case.

### 3.5.1 The Univariate Case

As already pointed out in Kiesel et al. (2009), from (3.9) and (3.11), we can infer that the variance and the covariance obviously depend on the integration step $T_0$. However, Kiesel et al. (2009) do not explicitly treat the closed formula of the correlation $\rho$ between the short and the long-term factors.

In (3.9), we can find the expressions of the variances of the factors and their covariance:

$$\mathrm{Var}(Short(T_0, T)) = \frac{\sigma_1^2}{2k}(e^{-2k(T-T_0)} - e^{-2kT}) \tag{3.31a}$$

$$\mathrm{Var}(Long(T_0, T)) = \sigma_2^2 T_0 \tag{3.31b}$$

$$\mathrm{Cov}(Short(T_0, T), Long(T_0, T)) = \frac{\rho\sigma_1\sigma_2}{k}(e^{-k(T-T_0)} - e^{-kT}) \tag{3.31c}$$

Therefore, the correlation is

$$\mathrm{Corr}(Short(T_0), Long(T_0)) = \rho \frac{\frac{1}{k}(e^{kT_0} - 1)}{\sqrt{\frac{1}{2k}(e^{2kT_0} - 1)T_0}} \tag{3.32}$$

It is noteworthy that (3.32) does not depend on $T$, but just on the mean reversion $k$ and on $T_0$.

### 3.5.2 The Multivariate Case

Differently from Edoli et al. (2013), where an Euler discretization is provided, we are going to set up a not-instantaneous framework for the correlations in the multivariate case.

As done for the univariate case in subsection 3.5.1, let us compute the integrated correlations in the multivariate framework. Obviously, the variances are the same as in (3.31a), whilst the covariances are different.

$$\mathrm{Cov}(Short_i(T_0,T_i), Short_j(T_0,T_j)) = \frac{\rho_{1_i,1_j}\sigma_{1_i}\sigma_{1_j}}{k_i+k_j} e^{-k_i(T_i-T_0)-k_j(T_j-T_0)}(1-e^{-k_iT_0-k_jT_0}) \tag{3.33a}$$

$$\mathrm{Cov}(Short_i(T_0,T_i), Long_j(T_0,T_j)) = \frac{\rho_{1_i,2_j}\sigma_{1_i}\sigma_{2_j}}{k_{1_i}} e^{-k_i(T_i-T_0)}(1-e^{-k_iT_0}) \tag{3.33b}$$

$$\mathrm{Cov}(Long_i(T_0,T_i), Long_j(T_0,T_j)) = \rho_{2_i,2_j}\sigma_{2_i}\sigma_{2_j}T_0 \tag{3.33c}$$

where $i = a, b$ and $j = a, b$ with $i \neq j$.

Hence, the $T_0$ correlations are

$$\mathrm{Corr}(Short_i(T_0), Short_j(T_0)) = \rho_{1_i,1_j} \frac{\frac{1}{k_i+k_j}(e^{(k_i+k_j)T_0}-1)}{\sqrt{\frac{1}{4k_ik_j}(e^{2k_iT_0}-1)(e^{2k_jT_0}-1)}} \tag{3.34a}$$

$$\mathrm{Corr}(Short_i(T_0), Long_j(T_0)) = \rho_{1_i,2_j} \frac{\frac{1}{k_i}(e^{k_iT_0}-1)}{\sqrt{\frac{1}{2k_i}(e^{2k_iT_0}-1)T_0}} \tag{3.34b}$$

$$\mathrm{Corr}(Long_i, Long_j) = \rho_{2_i,2_j} \tag{3.34c}$$

where $i = a, b$ and $j = a, b$ with $i \neq j$.

Therefore, we have shown that the non-instantaneous cross correlations do not depend on the maturity of the forwards, whilst the correlation between the long-term factors equals the instantaneous correlation $\rho_{2_i,2_j}$. Moreover, if we apply the conditions (3.18) to (3.34), we would obtain the same results as in (3.32).

## 3.6 Long-Term Dependency

Edoli et al. (2013) extend in the multivariate case the two-factor model applied in the electricity market by Kiesel et al. (2009). Edoli et al. (2013), as aforementioned in this thesis, deal with the instantaneous correlations. In subsection 3.5.1 and in subsection 3.5.2, closed form formulae of the correlation, taking into consideration the integration steps, have been obtained. However, regardless of the calibration method, this would mean that all the information on the cross commodity correlations is retrieved just on the daily returns of the panels. The question is if something is missing. Is there any additional data that could give some more insight in describing the phenomena that drive the commodity markets? Effectively, we have at least two main reasons to think that the approach until here followed could be incomplete. The first comes from the structural model for electricity literature (see subsection 3.2.1), while the second one comes from the cointegration (see subsection 3.2.3).

We have seen[16] from the studies of Davison et al. (2002), Barlow (2002), Kanamura and Ohashi (2007), Pirrong and Jermakyan (2008), Álvaro Cartea and Villaplana (2008) and Lyle and Elliott (2009), that the electricity is modelled as a direct or an indirect function of the fundamentals of the market, such as demand, supply and cost of the fuels that are used to produce electricity. Part of the literature was concerned in capturing the jumps that characterize the historical spot prices. Even if those models were not suited to fit the observed forward curves, they are based on the concept that the prices of the commodities are linked by some observable factors, that imply a kind of long-term relationship. In the short term, the prices can differ because of jumps, but in the long term they are driven by those forces. Actually, the factor models are based on similar concept, but mainly within the single commodities panel. The long-term correlation among the commodities are mainly described by the correlation of the long term products. However, it is calculated on daily returns, that, by definition, are affected by shocks of different sizes.

[16]Subsection 3.2.1.

The second stream of research is the one related to cointegration. As stated by Johansen (1995), *the reason that cointegration is interesting is that cointegrating relation captures the economic notion of a long-run relation.* The cointegration could be thought as a measure of the long-term dependency, since two variables are cointegrated if they have a stochastic trend in common[17]. As stated in Engle and Granger (1987), even if different economic series *can wander extensively*, some *economic forces* will keep them together. From a statistical point of view, this means that a combination of non stationary time series can be stationary[18]. It is noteworthy that Granger (1981) presents, as the first example of cointegrating series, the *input* and the *output of a black box of limited capacity*. Obviously, this example refers to any kind of physical production process like the electricity generation.

Therefore, the aim of the model is to translate the error correction matrix $\boldsymbol{\Pi}$, which contains *information about the long-run relations in the economy* (Johansen (2000)), in the elements of the multivariate two-factor model of Edoli et al. (2013) that drives the association among the commodities. Since cointegration is related to *long-run economic relations* (Johansen (2000)), to long run equilibrium (Engle and Granger (1987)) and to *long-run co-movements in prices* (Alexander (1999)), the correlations between the long-term factors of the commodities are a reasonable choice. Therefore, the contracts with the longest maturities (i.e. the year swap contracts, commonly known as calendars) will be used in the cointegration analysis. As suggested in Alexander (2001), given that we are dealing with more than two commodities, the Johansen framework (Johansen (1995)) will be implemented. Once the methodology is applied, we will have the estimation of the parameter of the formula (3.26). Expanding the $\boldsymbol{\Delta}$, we have

$$\boldsymbol{Y}_t - \boldsymbol{Y}_{t-1} = \boldsymbol{a} + \boldsymbol{\Pi}\boldsymbol{Y}_{t-1} + \boldsymbol{\varepsilon}_t \tag{3.35}$$

Grouping $\boldsymbol{Y}_{t-1}$

$$\boldsymbol{Y}_t = \boldsymbol{a} + (\boldsymbol{I} + \boldsymbol{\Pi})\boldsymbol{Y}_{t-1} + \boldsymbol{\varepsilon}_t \tag{3.36}$$

[17] Murray (1994) illustrates the concept of cointegration through the humorous example of a drunk and her dog.

[18] See section 3.4.

As in Lütkepohl (1991) and in Lütkepohl and Krätzig (2004), we can compute the forecast at a generic step $t+h$

$$\boldsymbol{Y}_{t+h|t} = \boldsymbol{a} + (\boldsymbol{I} + \boldsymbol{\Pi})\boldsymbol{Y}_{t+h-1|t} \tag{3.37}$$

where the forecast error is

$$\boldsymbol{Y}_{t+h} - \boldsymbol{Y}_{t+h|t} = \boldsymbol{\varepsilon}_{t+h} + (\boldsymbol{I} + \boldsymbol{\Pi})\boldsymbol{\varepsilon}_{t+h-1} + ... + (\boldsymbol{I} + \boldsymbol{\Pi})^{h-1}\boldsymbol{\varepsilon}_{t+1} \tag{3.38}$$

and the covariance matrix of the forecast error at step $t+h$ is

$$\begin{aligned} \boldsymbol{\Sigma}_{\boldsymbol{Y}}(h) &= E\{(\boldsymbol{Y}_{t+h} - \boldsymbol{Y}_{t+h|t})(\boldsymbol{Y}_{t+h} - \boldsymbol{Y}_{t+h|t})'\} \\ &= \sum_{j=0}^{h-1} (\boldsymbol{I} + \boldsymbol{\Pi})^j \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}((\boldsymbol{I} + \boldsymbol{\Pi})^j)' \end{aligned} \tag{3.39}$$

Now, using simple algebra, we are able to extract from the covariance matrix of the forecast of the errors the correlation matrix $\boldsymbol{\rho}(h)$. In formulae

$$\boldsymbol{\rho}(h) = \sqrt{diag(\boldsymbol{\Sigma}_{\boldsymbol{Y}}(h))}^{-1} \boldsymbol{\Sigma}_{\boldsymbol{Y}}(h) \sqrt{diag(\boldsymbol{\Sigma}_{\boldsymbol{Y}}(h))}^{-1} \tag{3.40}$$

## 3.7 Calibration Steps

The calibration is based on a three steps procedure:

1. The first step is the historical calibration on a standalone basis of each commodity. Therefore, we will obtain short and long term volatilities, the mean reversion and the correlation between the short and long-term factor.

2. The second step is based on a cointegration analysis using the framework of Johansen (1995). From the cointegrating relations, we will retrieve the long term implied correlations.

3. The third step is the computation of the cross correlation parameters among the short and long-term factors of the commodities, in order to be able to fit both the daily correlations, as in Edoli et al. (2013), but also the long term implied correlations. In this way, the calibration method by Edoli et al. (2013) is improved by considering cointegrating relations.

### 3.7.1 The Univariate Calibration

As in Kiesel et al. (2009), one of the aim is to find the set of parameters that let the model volatilities of the logarithms of the forward prices to be like the observed volatilities[19]. Moreover, like in Edoli et al. (2013), we are going to calibrate also on the covariances of the panel of each commodity. Let us define the standalone parameters as

$$\phi = (\sigma_1, \sigma_2, k, \rho) \tag{3.41}$$

Hence, in order to find the best parameters, we are going to minimize the square differences between the theoretical variances and covariances and the historical variances and covariances.

$$\sum_{i=1}^{np} (\mathrm{Var}_{historical}(log(F(t,T_i))) - \mathrm{Var}^{\phi}_{model}(log(F(t,T_i))))^2 \rightarrow arg_{\phi} min \tag{3.42a}$$

$$\begin{split}\sum_{i=1}^{np} \sum_{j=i+1}^{np} (\mathrm{Cov}_{historical}(log(F(t,T_i)), log(F(t,T_j))) \\ - \mathrm{Cov}^{\phi}_{model}(log(F(t,T_i)), log(F(t,T_j))))^2 \rightarrow arg_{\phi} min\end{split} \tag{3.42b}$$

where $np$ is the number of products that compose the panel data of the commodity to be calibrated. In order to find a feasible solution, the optimization is performed considering the following constraints

$$\sigma_1, \sigma_2, k > 0 \tag{3.43a}$$
$$-1 < \rho < 1 \tag{3.43b}$$

However, it is important to highlight that the products on which the calibration is performed are not of the same type. We have month, quarter and year swap contracts. While for the first one we have derived the closed form formula (3.9), the other two typologies are means of month swap contracts. Since the distribution of the mean of log-normals is not known, Kiesel et al. (2009) deal with the problem by implementing moment matching techniques.

[19] Kiesel et al. (2009) calibrate the model on option implied volatilities, while Edoli et al. (2013) use the historical ones.

Differently from Kiesel et al. (2009), in order to handle quarter and calendar swap contracts, we are going to use a numerical integration method, leveraging on the framework promoted by Heiss and Winschel (2008). In the article, an approach based on a *sparse grids integration (SGI) rule* is defined an tested through Monte Carlo. This family of methods are based on the Smolyak (1963) rule that enables to treat multiple dimensions through univariate operators. Heiss and Winschel (2008) have developed an easy to implement method that avoid the computational costs to grow exponentially with the number of dimensions.

When it comes to numerically estimate a function, integrals have to be computed several times. Instead of generating $n$ simulations with equal weight as discretization technique, a number of *nodes* with some specific weights are chosen based on the dimension of the problem ($D$) and on the requested accuracy ($k$)[20]. The nodes and the weights are calculated just once, since they do not depend on the form of the function of interest ($g$). As showed in Heiss and Winschel (2008), in the Monte Carlo framework the integral to be simulated is defined as

$$S_{D,R}[g] = \frac{1}{R}\sum_{r=1}^{R} g(\boldsymbol{x}_r) \tag{3.44}$$

where

- $R$ is the number of simulation based on $R$ random numbers
- $\boldsymbol{x}_r$ are the random numbers.

As well known, under certain conditions, the simulation is unbiased and converges in probability with a rate equal to $\frac{1}{\sqrt{R}}$. There are several approaches and techniques to generate random numbers, however thousands of random numbers would be needed to have a good convergence. This means that, when it comes to evaluate the covariance between two calendars, we would need to simulate 48 streams of random numbers and to use them in every step of the optimization. It is noteworthy that the time consumed

[20] Here the same notation of the mean reversion coefficient of the two-factor model by Kiesel et al. (2009) is used to be coherent with the original article by Heiss and Winschel (2008).

is relevant not only for the scale of the problem dimension, but also because in every simulation we have to calculate the logarithm of the mean of the exponential of a function of the random numbers. This kind of operation is often found in likelihood estimation and the computation costs are well known. So, a Monte Carlo approach repeated in the calibration process would be very heavy. Therefore, Heiss and Winschel (2008) can really reduce the elapsed time of computation and the size of the data to be stored, both when quarter and year contracts are taken into consideration. As Heiss and Winschel (2008), also Di Renzo et al. (2009) performed some tests in order to evaluate the benefit of numerically integration through sparse grids on the traditional Monte Carlo approach. The results from Di Renzo et al. (2009) are astonishing: from an analysis performed on *power-sums of generically correlated Log-Normal RVs*, they succeed to reduce the computational complexity by 99% without any significant loss of accuracy.

Using the formulae from Heiss and Winschel (2008), we can define the integral to be calculated with the sparse grids as

$$A_{D,k}[g] = \sum_{r=1}^{R} g(x_{r,1}, ..., x_{r,D}) w_r \tag{3.45}$$

where

- $A_{D,k}[g]$ is the integral on $D$ dimensions with $k$ accuracy
- $R$ is the number of nodes on which the integral is evaluated
- $x_r$ are the nodes
- $w_r$ are the weights of each node

Therefore, the single commodity calibration is going to be performed through the formulae (3.42) and mainly integrating the methods by Kiesel et al. (2009), Edoli et al. (2013) and Heiss and Winschel (2008), but taking into consideration the step of integration.

### 3.7.2 The Multivariate Calibration

In risk management the correlation among the risk factors is crucial, since the marginal distributions are only a part of the problem, whilst the focus is the joint distribution to be fitted. Moreover, covariances are the parameters that drive the diversification in a portfolio, that is the measure of the *natural hedging* arising from the business before any hedging strategy.

Edoli et al. (2013) approach the two-factor model calibration faced by Kiesel et al. (2009) in a multivariate framework. The aim is to capture the historical covariances among the panels of several commodities. Let us define the parameters for more than one commodity

$$\boldsymbol{\Phi} = (\boldsymbol{\sigma_1}, \boldsymbol{\sigma_2}, \boldsymbol{k}, \boldsymbol{C}) \tag{3.46}$$

where

- $\boldsymbol{\sigma_1}$ is the vector of the volatilities of the short-term factors of the commodities
- $\boldsymbol{\sigma_2}$ is the vector of the volatilities of the long-term factors of the commodities
- $\boldsymbol{k}$ is the vector of the nodes for the long-term factor
- $\boldsymbol{C}$ is the correlation matrix of the short and long-term factors of all the commodities.

Like for the univariate case, we have to find the parameters that minimize the square differences between the theoretical covariances $\boldsymbol{\Sigma_{model}}$ and the historical covariances $\boldsymbol{\Sigma_{historical}}$. In formulae

$$(\boldsymbol{\Sigma_{model}} - \boldsymbol{\Sigma_{historical}})^2 \rightarrow arg_{\boldsymbol{\Phi}} min \tag{3.47}$$

Edoli et al. (2013) calibrate the parameters $\boldsymbol{\Phi}$ in two steps. First, they find the parameters that maximize the fitting commodity by commodity ($\boldsymbol{\sigma_1}$, $\boldsymbol{\sigma_2}$, $\boldsymbol{k}$, the correlation between short and long term factor of each commodity). Then, they seek for the cross correlations that optimize the multivariate distribution.

The optimization has to consider the same constraints (3.43) as the univariate case, but we need also a valid correlation matrix. As defined by Higham (2002), *a correlation* *matrix is a symmetric positive semidefinite matrix with unit diagonal*, so an additional constraint is necessary: the correlation matrix $\boldsymbol{C}$ has to be semidefinite positive[21].

As defined in Wooldridge (2013), a matrix is positive definite if

$$\boldsymbol{x}'A\boldsymbol{x} \; > 0 \text{ for all } n \times 1 \text{ vectors } \boldsymbol{x} \text{ except } \boldsymbol{x} = 0 \tag{3.48}$$

while it is semidefinite positive if

$$\boldsymbol{x}'A\boldsymbol{x} \; > 0 \text{ for all } n \times 1 \text{ vectors} \tag{3.49}$$

This issue is well known in the literature, since every time it is necessary to induce correlation among random variables, the decomposition by Cholesky (1910) is one of the most common techniques to be implemented[22]. In order to decompose the matrix, it has to be semidefinite positive. As shown in Schott (2016), the condition expressed in (3.48) and in (3.49) in terms of the quadratic form $\boldsymbol{x}'A\boldsymbol{x}$ can be explicited also in terms of eigenvalues: a matrix is definite positive if

$$\lambda_i > 0 \; \forall i \tag{3.50}$$

while it is semidefinite positive if

$$\lambda_i \; \geq 0 \; \forall i \text{ with } \lambda_i = 0 \text{ for at least one } i \tag{3.51}$$

where $\lambda_i$ are the eigenvalues of the matrix.

Therefore, if the correlation matrix is negative definite, it will have at least one negative eigenvalue, that, as stated by Simonian (2010), would drive the variance of a portfolio to be negative. In this cases, as shown by Rebonato and Jäckel (1999) using a variance-covariance normal approximation, also the Value at Risk could be negative. Hull (2015), giving a practical interpretation, points out that a correlation matrix[23] has to be semidefinite positive in order to be internally consistent. Suppose to have the following correlation matrix

$$\begin{pmatrix} 1 & 0 & 0.9 \\ 0 & 1 & 0.9 \\ 0.9 & 0.9 & 1 \end{pmatrix} \tag{3.52}$$

[21] In the univariate case, since we deal with a $2 \times 2$ matrix, the constraint $\rho \in (-1; 1)$ is sufficient to grant the semipositive definitiveness.

[22] It might be interesting to note that Turing (1948) considered the Cholesky's method more accurate and convenient than other techniques.

[23] Hull (2015) uses a covariance matrix with all the variances equal to one.

The first variable is highly correlated with the third one, which is highly correlated with the second one. However, the correlation between the first and the second is zero, a configuration that *seems strange*. In fact if we apply to (3.49) a $\boldsymbol{x} = (1, 1, -1)$, the equation will not be satisfied.

An *ill-conditioned* correlation matrix can be found in different situations. Higham and Strabić (2016) talk of the following applications in statistical modeling contexts:

- a correlation matrix computed from empirical or experimental data mainly due to missing observations
- stress testing in finance, when it is necessary to shift one or more elements of a valid correlation matrix
- a correlation matrix that is the result of an aggregation of *small* correlation matrices into a unique *global correlation matrix*.

There are also other causes of not valid correlation matrices such as

- the presence of collinearity among the variables. In this case we have redundancy, since one or more variables are linearly dependent. Practically, perfectly linearly dependent variables are rare, however, when the correlations are empirical estimated, we could find variables that are almost linearly dependent. Coming back to the notation by Schott (2016), this would mean that at least one eigenvalue $\lambda$ of the matrix is very close to zero. In econometrics, different works have been developed to test collinearity through the analysis of the eigenvalues. Belsley (1991), Maddala (1992), Belsley et al. (2004), Callaghan and Chen (2008) and Gujarati and Porter (2008) use the *condition number*[24] and the *condition index*[25] to understand how *small* is an eigenvalue and, so, which are the variables that are linearly dependent. However, they do not provide an objective statistical test, but only

[24]The condition number is given by the ratio between the maximum eigenvalue and the minimum eigenvalue (Gujarati and Porter (2008)).
[25]The condition index is the square root of the condition number (Gujarati and Porter (2008)).

some intervals empirically determined[26]. Moreover, implementing methods such as Cholesky (1910), it is possible that rounding errors trigger the matrix to lose the positive definitiveness.

- the correlation is computed on a too small sample. Goldberger (1991) introduces the term *micronumerosity* to identify this issue in the context of linear regression, since multicollinearity can be induced also by a small sample size.
- a variable is constant. Obviously, this would mean that its variance is zero and so at least one eigenvalue is zero.
- the initial correlation matrix is incomplete, since some elements are unknown. This case has been the object of a specific stream of research such as Grone et al. (1984), Barrett et al. (1989), Lundquist and Johnson (1991), Budden et al. (2007) and Smith (2008). Moreover, Kahl and Günther (2008) present a method to complete the correlation matrix in a multi-dimensional stochastic volatility model, combining Gaussian elimination[27] and graph theory.

Although the approach by Kahl and Günther (2008) is implemented in a similar context of the multivariate extension of the two-factor model, our issue is much more related to the aggregation case mentioned in Higham and Strabić (2016). The $2 \times 2$ correlation matrices of each single commodity can be thought as the *small* correlation matrices. On the other hand, the correlation matrix that relates the short and long-term factors of all the commodities acts as the *global correlation matrix*. In case of ill-conditioned correlation matrix, the challenge is to find the smallest *adjustments* to be applied to the matrix in order to obtain a valid correlation matrix. Since Higham (2002), this issue in finance is known as *the nearest correlation matrix* problem.

[26] In Belsley et al. (2004) weak dependecies are associated with condition index around 5-10, while strong dependencies are associated with condition indexes of 30 or more. In the book the evidence of the experiments suggests to use a threshold between 10 and 30. In Gujarati and Porter (2008) a similar rule of thumb is reported: moderate to strong multicollinearity if the index is between 10 and 30, while sever multicollinearity for values above 30.

[27] The Gaussian elimination is a transformation to triangular form (Section 3.3 of Duff et al. (1989)).

Many methods have been developed to solve the problem of the nearest correlation matrix. Finger (1997) seeks a way to stress some correlations within a correlation matrix without losing the semipositive definitiveness. The part of the matrix to be stressed is successfully changed, but also the other ones are affected in order to ensure the semipositive definitiveness. In the same context, Kupiec (1998) applies shrinkage techniques to adjust the whole correlation matrix to guarantee all its eigenvalues to be positive. However, Rebonato and Jäckel (1999) and Brooks et al. (1998) agree that the approach by Finger (1997) change the other part of the correlation matrix in an uncontrolled way. Moreover, Rebonato and Jäckel (1999) highlight that the method by Kupiec (1998) needs a valid correlation matrix as starting point and that is time consuming. Therefore, Rebonato and Jäckel (1999) propose two new methods: the Hypersphere decomposition and the Spectral decomposition. In the first one, given a correlation matrix $\boldsymbol{C}$, the aim is to find the nearest $\hat{\boldsymbol{C}} = \boldsymbol{B} \cdot \boldsymbol{B}'$ thinking of the rows of $\boldsymbol{B}$ as coordinates lying on a unit hypersphere. In each calibration step, $\theta_{ij}$ are chosen so that the elements of $\boldsymbol{B}$ are $b_{ij} = \cos\theta_{ij} \cdot \prod_{k=1}^{j-1} \sin\theta_{ik}$, for $j \in [1, n-1]$, and $b_{ij} = \prod_{k=1}^{j-1} \sin\theta_{ik}$ for $j = n$. The Spectral decomposition method is based on four main steps to find $\boldsymbol{B}$:

1. calculate the eigenvalues $\lambda_i$ and the right hand side eigenvectors $\boldsymbol{s}_i$ of $\boldsymbol{C}$
2. set all negative $\lambda_i$ to zero and put them into the diagonal matrix $\boldsymbol{\Lambda}^*$
3. multiply the vectors $\boldsymbol{s}_i$ with their associated "corrected" eigenvalues and arrange as the columns of $\boldsymbol{B}^* = \boldsymbol{S} \cdot \sqrt{\boldsymbol{\Lambda}^*}$
4. normalise the row vectors of $\boldsymbol{B}^*$ to unit length to obtain $\boldsymbol{B}$.

Both the methods guarantee to produce a semipositive definite matrix, do not require a valid correlation matrix to start with, and they are fast to implement regardless of the matrix size. However, only the Hypersphere method *allows the determination of a feasible matrix that most closely approximates a target real symmetric (but not positive-semidefinite) matrix in a well-defined and quantifiable sense.* The Spectral decomposition could be used as a starting point for the first methodology. As objective function two alternatives are proposed. The first is to minimize the sum of the squares of the elements of the difference $\boldsymbol{C} - \hat{\boldsymbol{C}}$

$$\sum_{ij}(c_{ij} - \hat{c}_{ij})^2 \tag{3.53}$$

that is the Frobenius norm $\|\boldsymbol{C} - \hat{\boldsymbol{C}}\|_F^2$. The second alternative is to minimize the sum of the squared differences between the eigenvalues

$$\sum_{i}(\lambda_i - \hat{\lambda}_i) \tag{3.54}$$

Bhansali and Wise (2001) and Kercheval (2006) improve their methodology specifying the problem in terms of only $\frac{N \cdot (N-1)}{2}$ variables. Higham (2002) elaborates two methods based on the minimization of weighted Frobenius norms

$$\|W^{\frac{1}{2}}(\boldsymbol{C} - \hat{\boldsymbol{C}})W^{\frac{1}{2}}\|_F \tag{3.55a}$$
$$\|\boldsymbol{H} \circ (\boldsymbol{C} - \hat{\boldsymbol{C}})\|_F \tag{3.55b}$$

where

- $\boldsymbol{W}$ is a positive definite matrix
- $\boldsymbol{H}$ is a symmetric matrix of positive weights
- $\circ$ is the Hadamard product[28].

As stated by the author, the main weakness of the technique is its linear convergence rate, that could require an important amount of time dealing with large matrices.

Edoli et al. (2013) implemented a method that works directly on the Cholesky decomposition of the correlation matrix $\boldsymbol{C}$. In this way, the condition of semidefinite positive is automatically satisfied. Given that from the Cholesky decomposition we have

$$\boldsymbol{C} = \boldsymbol{L}\boldsymbol{L}' \tag{3.56}$$

[28]As reviewed in Schott (2016), the Hadamard product is the element-wise multiplication of two matrices. It can be applied only on matrices having the same size. Let us define $a_{ij}$ as the elements of the $m \times n$ matrix $A$ and $b_{ij}$ as the elements of the $m \times n$ matrix $B$. The Hadamard product between the two matrices is $A \circ B = \begin{pmatrix} a_{11}b_{11} & \cdots & a_{1n}b_{1n} \\ \vdots & & \vdots \\ a_{m1}b_{m1} & \cdots & a_{mn}b_{mn} \end{pmatrix}$

Let us define $\boldsymbol{L}_i$ the $i$-th row of $\boldsymbol{L}$ with $i = 1, ..., 2N$, where $N$ is the number of the commodities. First, given that the correlation matrix diagonal is unitary, we have to consider the following constraint

$$\|\boldsymbol{L}_i\|^2 = 1 \ \forall i = 1, ...2N \tag{3.57}$$

Then, we have also to take in consideration the correlation between short and long-term factors of each commodity, since they have been already calibrated in subsection 3.7.1. Analytically:

$$\|\boldsymbol{L}_{2j-1}\boldsymbol{L}'_{2j}\| = \boldsymbol{C}_{2j-1,2j} \ \forall j = 1, ..., N \tag{3.58}$$

Without this condition, the multivariate calibration would not result consistent with the univariate one, since it would change the marginal distributions of the commodities.

A further constraint of the correlation matrix is that each element must be included in the interval $[-1, 1]$. However, from the Cauchy-Schwartz inequality for the norm, we know that

$$\|\boldsymbol{L}_{2j-1}\boldsymbol{L}'_{2m}\| \leq \|\boldsymbol{L}_{2j-1}\| \cdot \|\boldsymbol{L}'_{2m}\| \tag{3.59}$$

But, given (3.57), we know that the elements of the right hand side of the equation are both 1, giving an upper limit of 1, so that every element of the correlation matrix will be included in $[-1, 1]$.

We can now better specify the optimization objective function (3.47) in terms of $\boldsymbol{L}$

$$(\boldsymbol{\Sigma_{model}} - \boldsymbol{\Sigma_{historical}})^2 \rightarrow arg_{\boldsymbol{L}} min \tag{3.60}$$

In Edoli et al. (2013), details on the starting points to be used for the optimization are not provided, with the exception, obviously, of the correlations already calibrated in the single commodity framework that are included in the constraints formalized in (3.58). We have two main alternatives. The first is to use a global optimization algorithm, whilst the second is to arbitrary choose a specific starting point.

Regarding the global optimization approach, as described in Ugray et al. (2007), there are several methods. In MATLAB Global Optimization Toolbox (2015), we have available the MultiStart Algorithm and GlobalSearch Algorithm. The multistart algorithms, introduced by Locatelli and Schoen (1999), are based on the generation of multiple random starting points in order to avoid the provided solution to be a local minima. The second algorithm, after generating some trial points, applies a merit filter in order to ensure that the remaining points have *high quality*. From an implementation point of view, it is noteworthy that only the multistart algorithm can run in parallel (MATLAB Parallel Computing Toolbox (2015)) and so it is potentially faster, since each starting point is totally independent from the others. Initializing a proper number of starting points, both the algorithms will find an optimal solution. However, this can be time consuming, especially with large matrices.

Coming back to the definition of the two-factor model, we do have some implicit suggestions on how to choose a starting point. From an economical interpretation of the model, like in Kiesel et al. (2009), we know that the second factor *models the long-term uncertainty that is common to all products in the market. This uncertainty*, among other components, can be explained also by *price developments in other commodity markets*. Therefore, it can be said that the long-term factor is the most important in capturing the correlation within the various commodities. Moreover, from an analytical point view, we know that the short-term factor, depending on the size of the mean reverting coefficient $k$, is not relevant for long term time horizon. Therefore, to better specify our starting point, we can select also the correlations arising from the year swaps (calendars) of the different commodities[29]. The so defined initial correlation matrix is then decomposed through Cholesky in order to be a starting point for the Edoli et al. (2013) correlation calibration procedure. However, the starting correlation matrix could be not semipositive definite and so the Cholesky decomposition could be not applicable. In analogy with what have been proposed by Rebonato and Jäckel (1999), a preliminary nearest correlation matrix algorithm could be applied in advance. The approaches discussed above have been used in many contexts, but as pointed out by some authors, they have some limitations or some drawbacks. Qi and Sun (2006) elaborate a Newton-type method that has better performance of the other approaches since, differently from Higham (2002), it is quadratically convergent. Another important characteristic is that the solution is proven to be unique. We are going to use the extended version of the method by Qi and

[29]This point is going to be very important in modeling the long-term dependencies.

Sun (2006), given that it allows to perform a weighted optimization using (3.55a). In this way, we will quickly obtain a valid starting correlation matrix for the procedure by Edoli et al. (2013) that takes into consideration the hypothesis of the two-factor model.

Therefore, the multivariate commodity calibration is performed through formula (3.60), integrating the approach by Edoli et al. (2013) and the numerical integration on sparse grids by Heiss and Winschel (2008). Moreover, a model consistent starting point for the correlation matrix is defined through Qi and Sun (2006), which is used in the algorithm by Edoli et al. (2013). The step of integration in the calculation of the correlations (see subsection 3.5.2) is taken into consideration.

### 3.7.3 Long-Term Dependency Calibration

Formula (3.40) extracts term correlations from the cointegration among the commodities. Given also the term correlation formulae developed in section 3.5, it is possible to insert in the optimization (3.60) the information of long-term dependencies among the commodities. Within the Edoli et al. (2013) framework, we have just to add some products.

In Edoli et al. (2013), we have to consider the constraints (3.57), to guarantee the correlation matrix to have a unitary diagonal and the constraints (3.58), to preserve the correlations between the short and the long-term factors of each single commodity. In order to include the cointegration in the model, first, we have to use the formulae of the term correlations[30] choosing a "long term" value for $T_0$. Then, we have to insert in the objective function (3.60) that the term correlations among the calendars generated by the model are equal to the long term correlations computed in (3.40), obviously considering coherent values of $T_0$ and $h$. As starting point for the correlation matrix, we will use the correlations between the short and long-term factors within each commodity and the correlation according to (3.40) as the correlations among the long-term factors of the commodities. In order to guarantee the initial correlation matrix to be semi-definite positive, but controlling the correlations among the long-term factors, we will use the

[30] See subsection 3.5.2.

method by Qi and Sun (2006) that allows a weighted optimization of the matrix. In the optimization, the term covariances of the calendars will be computed using the numerical integration on sparse grids by Heiss and Winschel (2008).

## 3.8 Summary

We have extended Edoli et al. (2013) to be able to handle the association among the entire forward curves of different commodities considering the presence of one or more cointegrating relations. It is also important that the marginal distribution of the single commodities are not significantly affected by the extension of the model. From an economical point of view, we have developed a way to link factor models to long-run relations (Granger (1981), Engle and Granger (1987), Johansen (1995)), crucial when we have to deal with hedging (Alexander (1999) and Alexander (2001)).

It is noteworthy that we have proposed a different way to deal with the quarter and year swap contracts using numerical integration techniques based on sparse grids (Heiss and Winschel (2008)). Moreover, the two steps procedure in dealing with the nearest correlation matrix problem, based on Qi and Sun (2006) and on Edoli et al. (2013), enhances the flexibility of the calibration, allowing to choose an economical consistent starting point and avoiding to implement a global optimization algorithm.

# Chapter 4

# The Data

*The statistician was no longer responsible for the accuracy or precision of the results of his labors. His business became much less like that of conjurer who is expected to work wonders, and more like that of a chemist who undertakes to assay the proportion of gold in a sample of ore. He need not be ashamed if the assay is low, or elated with pride if it is high.*

Fisher (1947)

## 4.1 Introduction

The data analysed comprehend electricity, gas, coal, EU ETS allowances and oil forward prices. In the calibration we are going to use quoted panel data for all the commodities. The types of product used are month contracts, quarter contracts and year contracts. Depending on data availability, the cointegration analysis is performed on a longer time horizon than the calibration of the rest of the model. As stated in Alexander (2001), *cointegration tests will not produce sensible results if too short a data period is used: they are designed to detect common long-run trends in the variables. The data period has to be sufficiently long for a stochastic trend to be detected.*

Once the calibration is accomplished, as input for the two-factor model, the data as of the end of November 2016 is going to be used. The products are going to be split into equivalent monthly quotations in order to have a full monthly forward curve for each commodity.

The market taken into consideration is going to be Germany. The first reason is that its electricity forward market is one of the most liquid in Europe, while the second is because of its important relevance in the continent. Obviously, the model can be implemented in all the markets where forward contracts prices are available. In case of no available forward curves, like in most countries in South America, the calibration should only rely on historical spot prices, but this issue is beyond the scope of this thesis.

All the prices in the calibration and in the cointegration analysis are going to be treated in natural logarithms. Furthermore, all the data refers to rolling forwards contracts, therefore, all the products experience some "jumps" due to the change of the underlying period. If we take into consideration a one-month forward contract, it will have a change in the level of the price between the last day of a month and the first day of the following month. The same applies for quarter contracts and year contracts. In order to control for this issue, we are going to sterilize the jumps in the prices by adjusting, at end of each period, the series with the difference between the price of the product with the price of the one having as underlying the period after. Since the last product of each typology is not adjusted, we are not going to take them into consideration. It has to be noticed that for electricity, gas and carbon emissions those products have significantly less liquidity then the others.

The selected products are the one month ahead, two months ahead, three months ahead, one quarter ahead, two quarters ahead, three quarters ahead, one year ahead, two years ahead and three years ahead swap contracts.

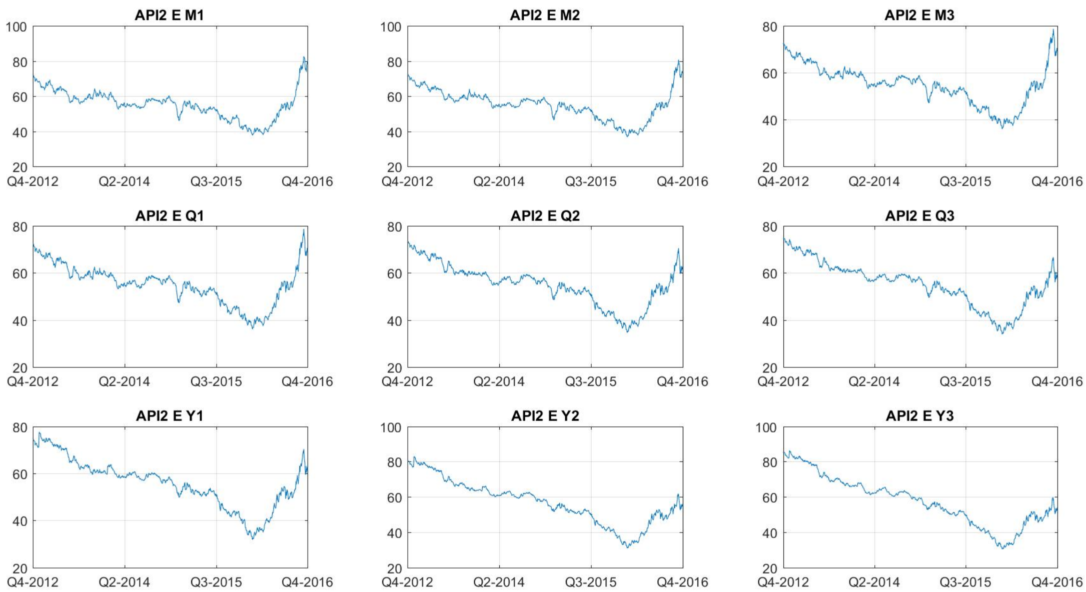


Figure 4.1: API2 prices in €/Ton (levels).

## 4.2 Coal Price. API2

As a reference for coal prices, we are going to use API2, one of the most important coal world indexes. It represents the price of coal delivered into the Amsterdam, Rotterdam, and Antwerp region in the Netherlands. It is quoted in US dollars per metric ton. The daily historical prices of the forward products of API2 are obtained from ICE (2016) and cover the period from December 3, 2012 to November 30, 2016. The data is converted in Euro. The EURUSD historical data is obtained from Bloomberg (2016). The FX forward prices are grouped in periods as for the commodity in order to properly convert the API2 forward prices in Euro. In figure 4.1, the historical prices of all the products are shown.

In figure 4.2, the natural logarithms of the historical prices of all the products after the rolling adjustment are reported.

The down-trend from 2012 to 2015 is mainly driven by a global reduction of commod-

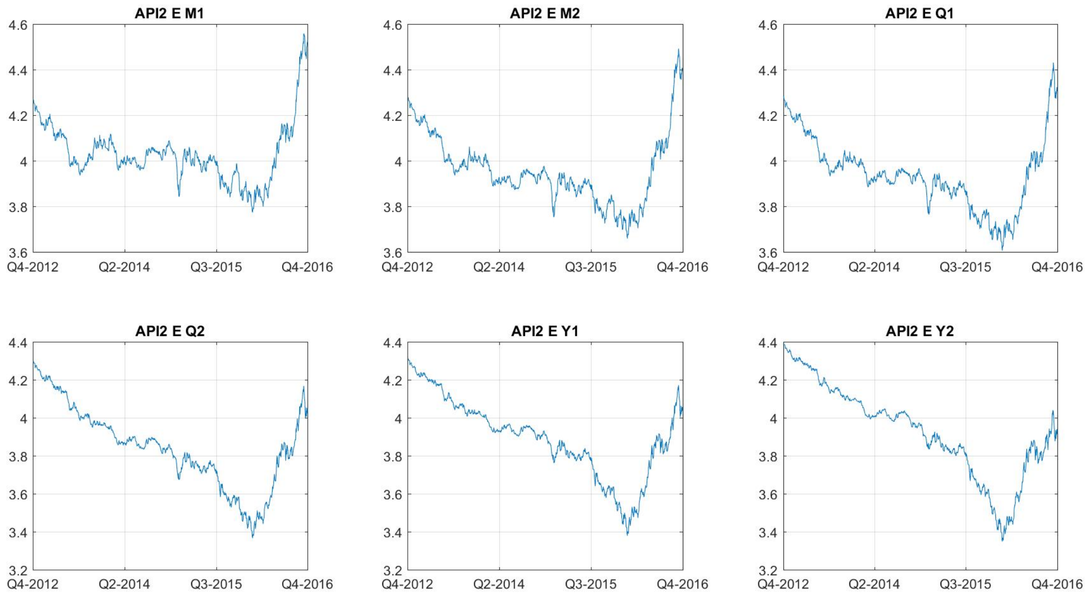


Figure 4.2: API2 prices in €/Ton (natural logarithms and after rolling adjustment).

ity prices. In 2016 the up-trend is mainly due to China. In the country the production of coal has been limited through a reduction of the working days of the mines from 330 to 276. The aim is to restructure the entire industry.

In figure 4.3, the forward curve as of the end of November 2016 is reported. It is the split of the products of the panel into monthly forwards.

## 4.3 Oil Price. Brent

As a reference for oil prices, we are going to use Brent, the most important oil index in Europe. It is quoted in US dollars per barrel. The daily historical prices of the forward products of Brent is obtained from ICE (2016) and cover the period from December 3, 2012 to November 30, 2016. The data is converted in Euro as for API2. In figure 4.4, the historical prices of all the products are shown.

In figure 4.5, the natural logarithms of the historical prices of all the products after

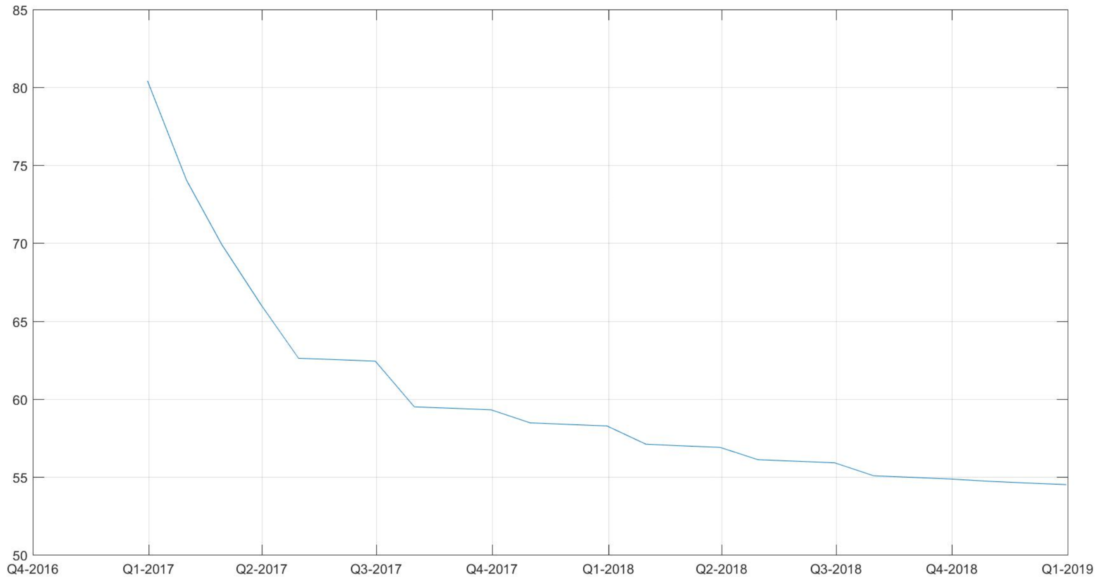


Figure 4.3: API2 forward curve in €/Ton as of the end of November 2016.

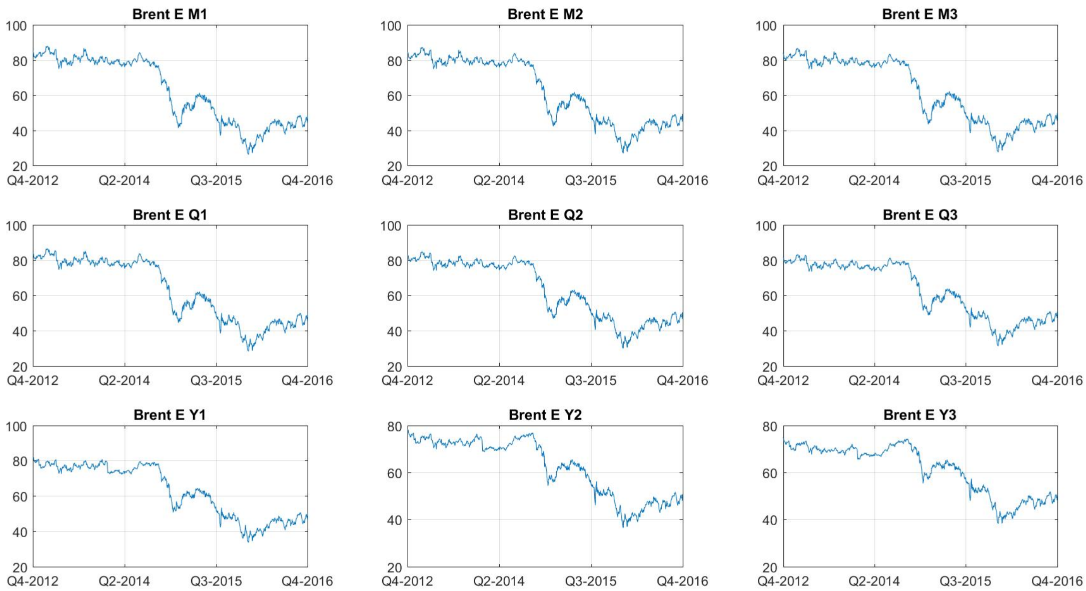


Figure 4.4: Brent prices in €/bbl (levels).

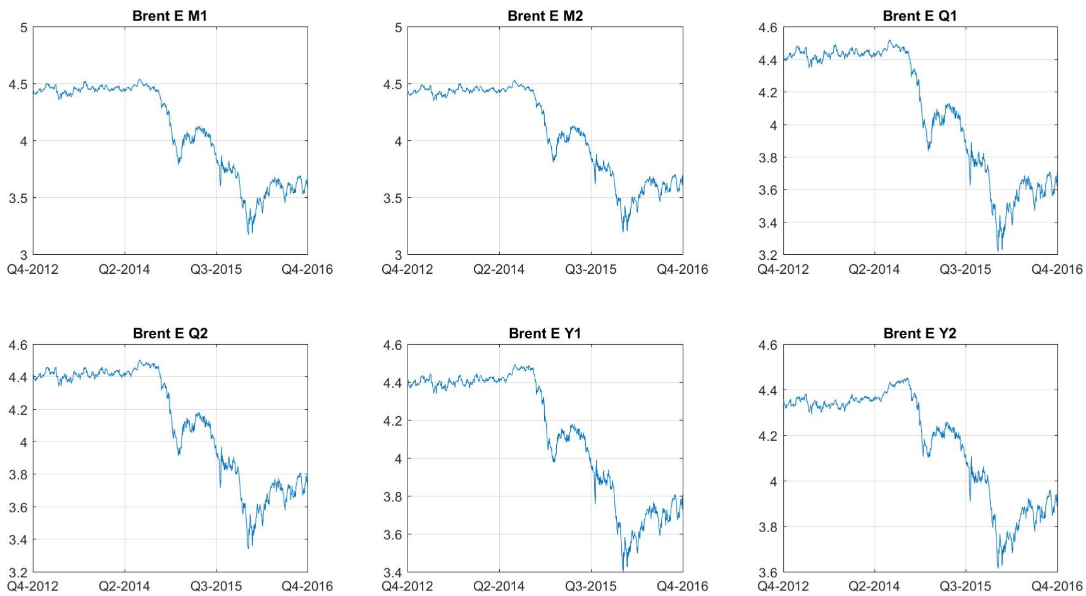


Figure 4.5: Brent prices in €/bbl (natural logarithms and after rolling adjustment).

the rolling adjustment are reported.

The down-trend from to 2013 to the end of 2016 is mainly due to an increase in the world oil supply, driven by the increased production of shale oil in USA. In 2016, the prices are recovering, since they have reached the average break-even of the shale oil producers in USA. Moreover, in the last quarter of 2016, OPEC members have reached an agreement to cap the production of oil.

In figure 4.6, the forward curve as of the end of November 2016 is reported. It is the split of the products of the panel into monthly forwards.

## 4.4 EU ETS price

The daily historical prices of the forward products of EU ETS allowances are obtained from ICE (2016) and cover the period from December 3, 2012 to November 30, 2016. In figure 4.7, the historical prices of all the products are shown.

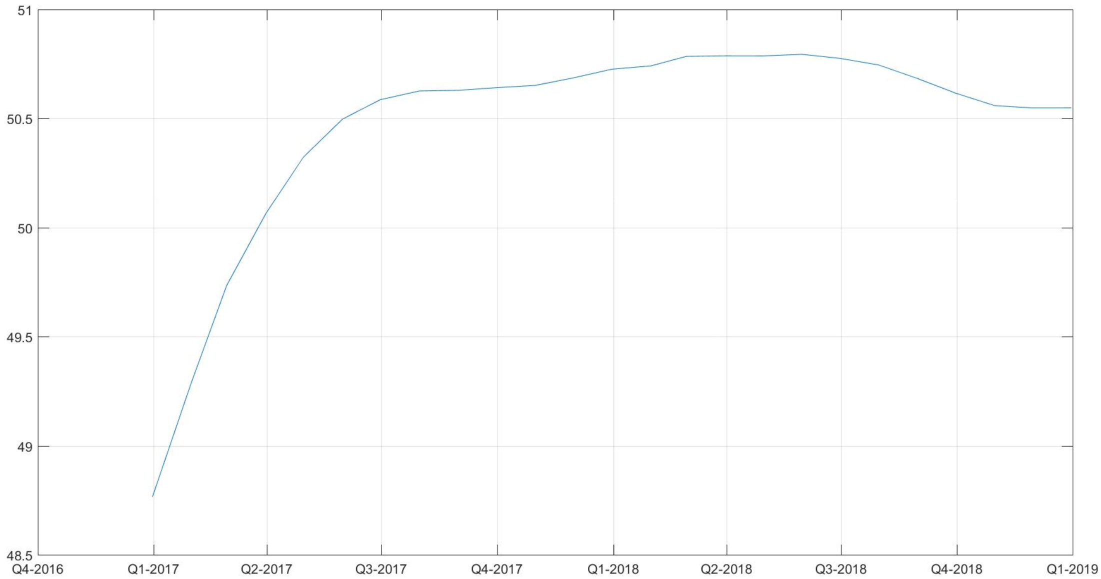


Figure 4.6: Brent forward curve in €/bbl as of the end of November 2016.

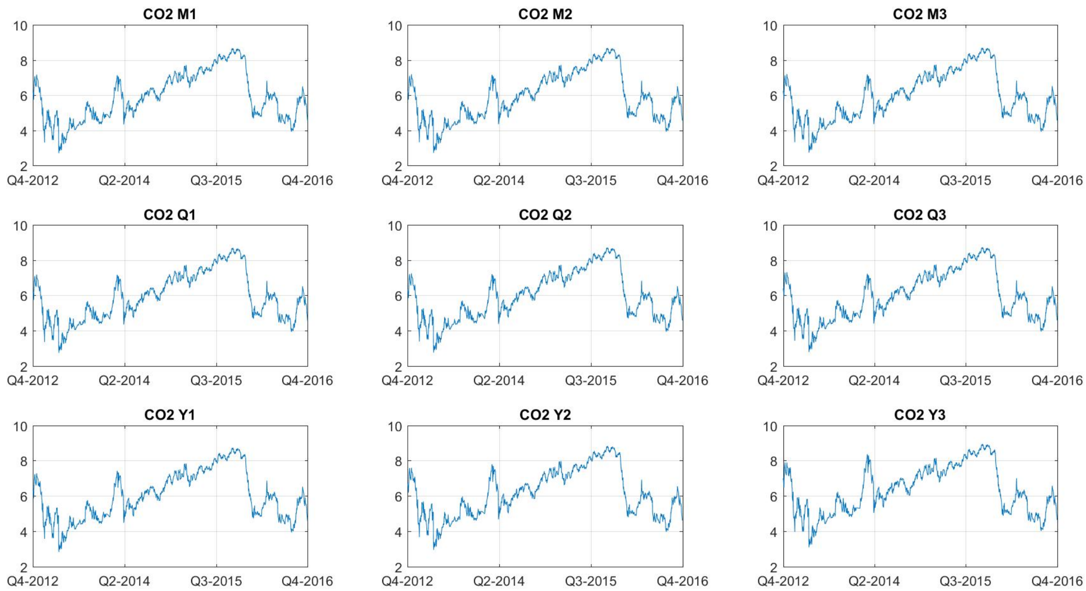


Figure 4.7: EUA prices in €/Ton (levels).

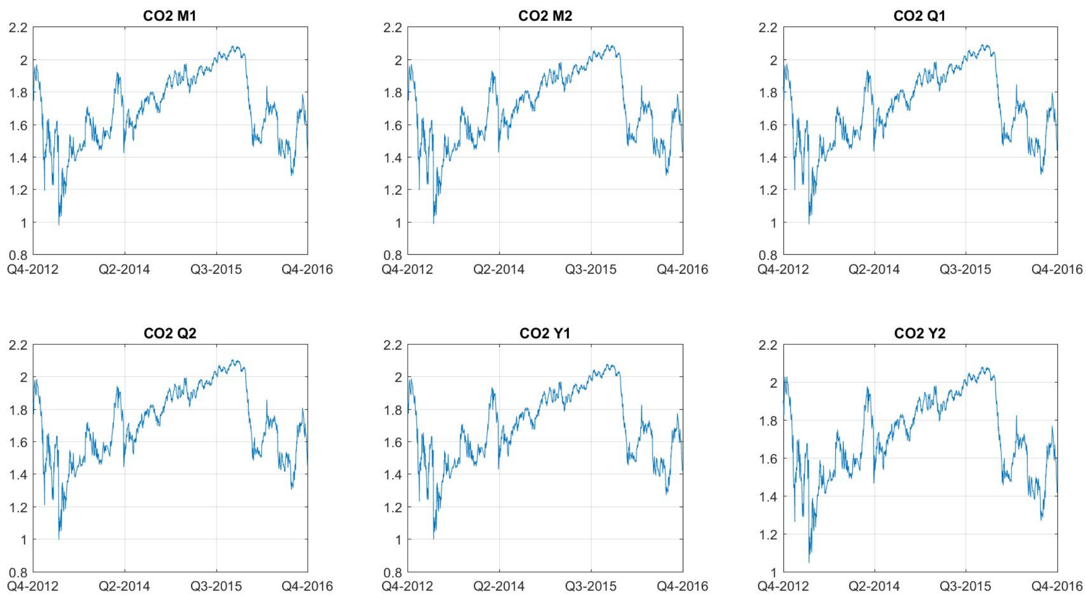


Figure 4.8: EUA prices in €/Ton (natural logarithms and after rolling adjustment).

In figure 4.8, the natural logarithms of the historical prices of all the products after the rolling adjustment are reported.

In figure 4.9 the forward curve as of the end of November 2016 is reported. It is the split of the products of the panel.

## 4.5 Gas price. TTF

As a reference for gas prices, we are going to use TTF (*Title Transfer Facility*), the most important gas index in Europe. The daily historical prices of the forward products of TTF are obtained from ARGUS (2015) from January 3, 2013 to December 31, 2015 and from Heren (2016) until November 30, 2016. In figure 4.10, the historical prices of all the products are shown.

In figure 4.11, the natural logarithms of the historical prices of all the products after the rolling adjustment are reported.

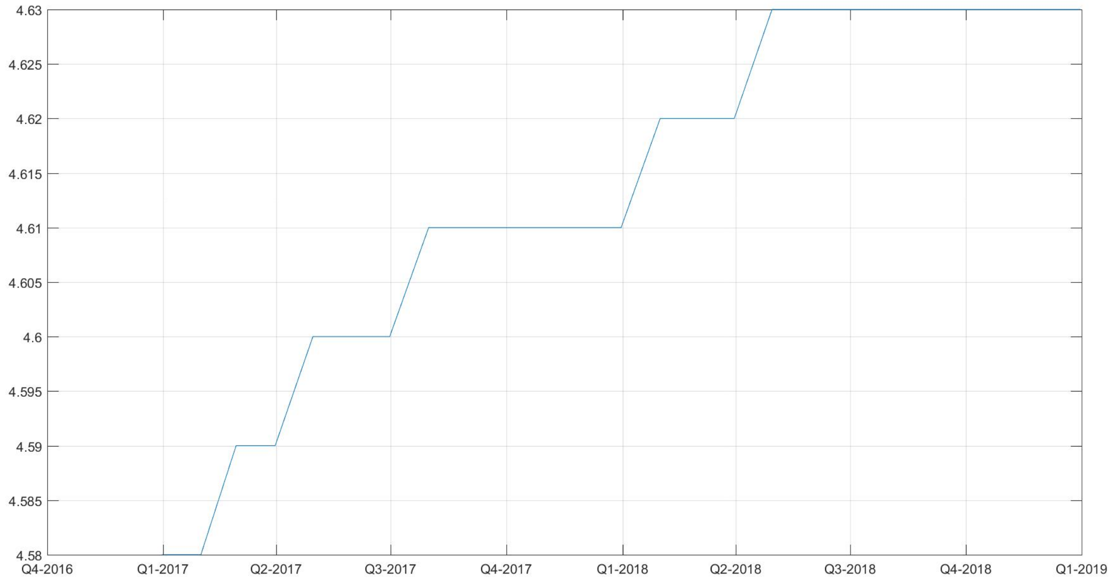


Figure 4.9: EUA forward curve in €/Ton as of the end of November 2016.

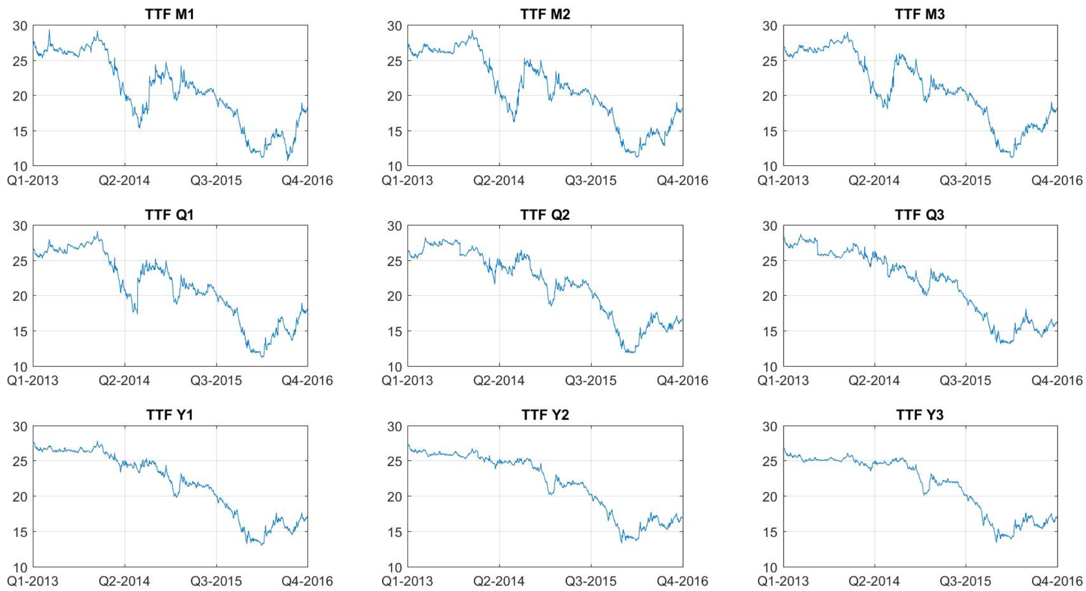


Figure 4.10: TTF prices in €/MWh (levels).

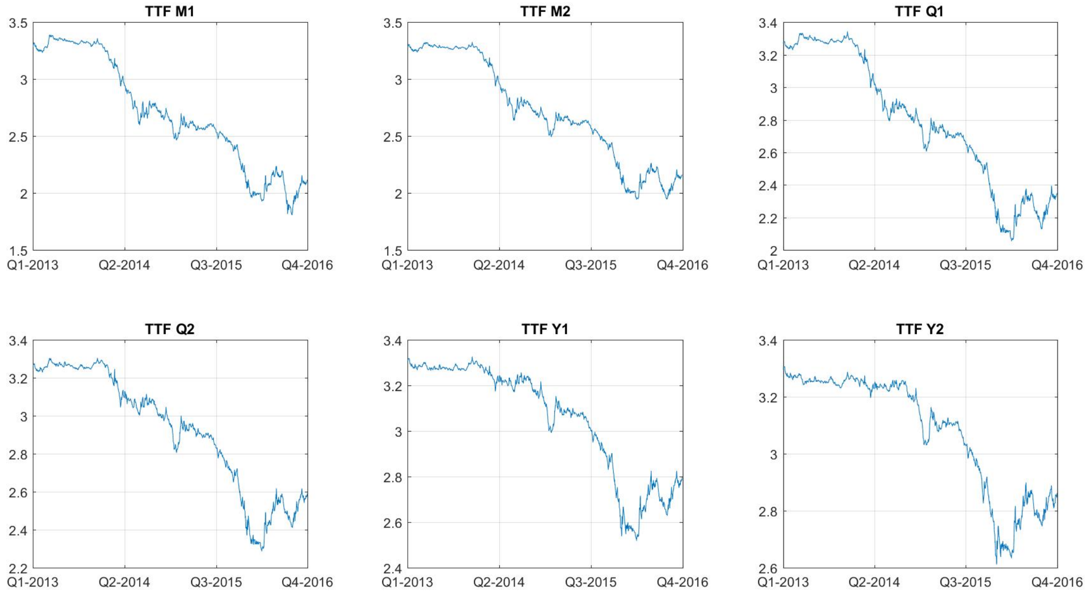


Figure 4.11: TTF prices in €/MWh (natural logarithms and after rolling adjustment).

In figure 4.12, the forward curve as of the end of November 2016 is reported. It is not the simple split of the products of the panel, but it has also been seasonally profiled (source Bloomberg (2016)). As well known in the market, the gas prices have an important seasonal component due to weather and production seasonality.

## 4.6 German Electricity Price

The daily historical prices of the forward products of German electricity is obtained from Bloomberg (2016) and cover the period from December 3, 2012 to November 30, 2016. In figure 4.13, the historical prices of all the products are shown.

In figure 4.14, the natural logarithms of the historical prices of all the products after the rolling adjustment are reported.

In all the products, we can see a down-trend since 2012 until the beginning of 2016. This is due to a general decrease of electricity prices driven by a decreasing cost of the


Figure 4.12: TTF profiled forward curve in €/MWh as of the end of November 2016.

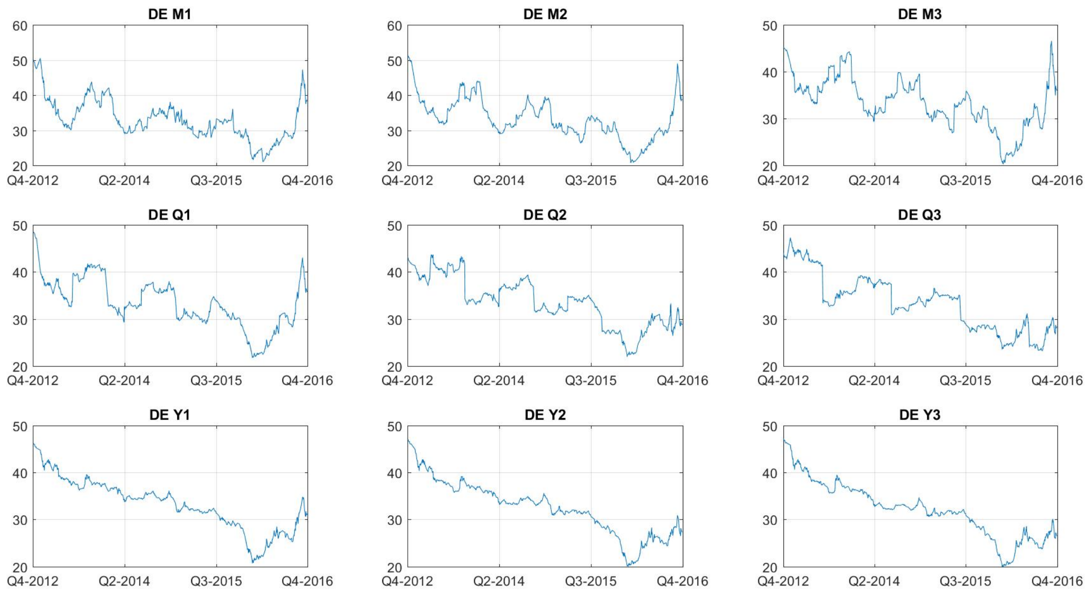


Figure 4.13: German electricity prices in €/MWh (levels).

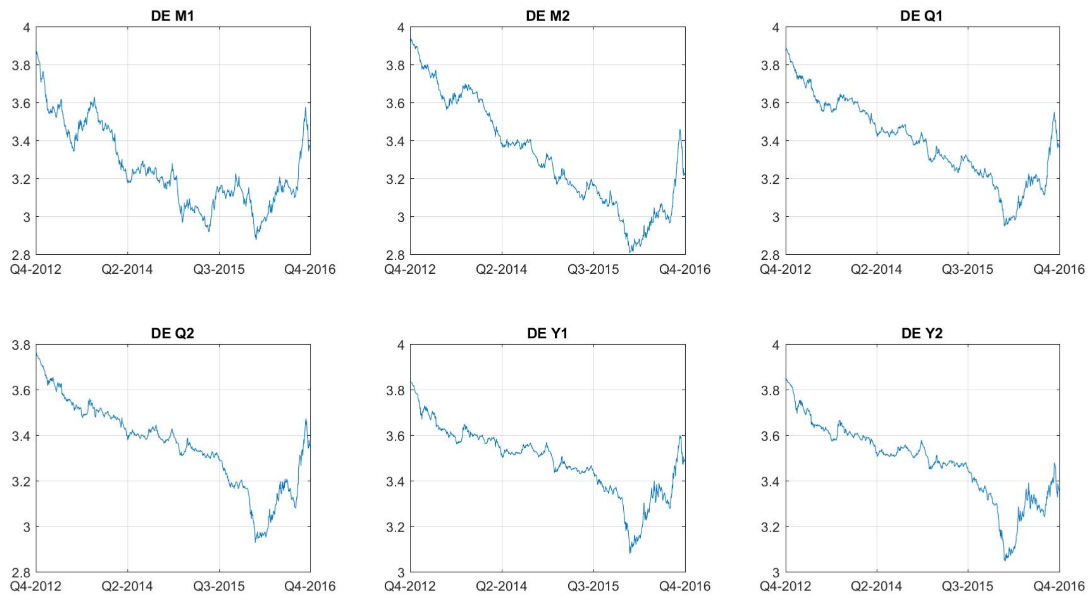


Figure 4.14: German electricity prices in €/MWh (natural logarithms and after rolling adjustment).

fuels and by a stagnant demand. In 2016, we are experiencing a recovery of the prices, especially in the second half of the year. One of the most important reason is the extended outage of several nuclear power plants in France due to longer maintenance periods.

In figure 4.15, the forward curve as of the end of November 2016 is reported. It is not the simple split of the products of the panel, but it has also been seasonally profiled (source Bloomberg (2016)). As well known in the market, the electricity prices have an important seasonal component due to weather and production seasonality.

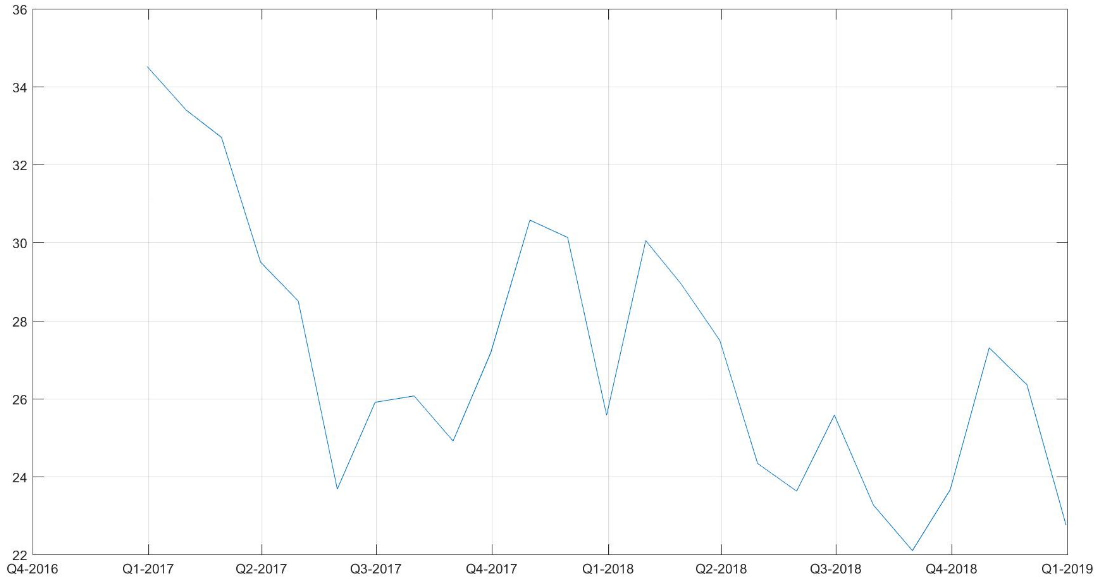


Figure 4.15: German electricity profiled forward curve in €/MWh as of the end of November 2016.

# Chapter 5

# Empirical Results

> *He* [the speculator] *is not so much a prophet (though it may belief in his own gifts of prophecy that tempts him into the business), as a risk-bearer.* [... The speculator] *just as an insurance company makes profits without pretending to know more about an individual's prospects of life or the chances of his house taking fire than he knows himself.*
>
> Keynes (1923)

## 5.1 Introduction

We will evaluate four portfolios using the model calibrated only on the instantaneous correlations, using the framework by Kiesel et al. (2009) and Edoli et al. (2013), and the new proposed model (chapter 3), that includes the long term correlation effect implied in the cointegrating relations, using the framework by Johansen (1995).

Therefore, we are going to analyse the differences between the two approaches in order to properly assess in which way the proposed model could affect the measure of the risk a company is bearing.

The portfolios are:

1. German *Clean Dark Spread*
2. German *Clean Spark Spread*
3. A gas contract indexed to the Brent
4. A portfolio long on gas and oil.

In section 5.2, the parameters result of the calibration within the two models will be compared. In section 5.3, the two models will be applied to the four portfolios.

## 5.2 Calibration of the Model

In this section, we will show the results of the application of the calibration steps as presented in section 3.7.

### 5.2.1 Step 1: Single Commodity Calibration

We are going to use one year history to calibrate the model commodity by commodity, following the methodology by Kiesel et al. (2009), Edoli et al. (2013), but using the formulae of the term correlations computed in subsection 3.5.1 and the numerical integration based on the sparse grids by Heiss and Winschel (2008).

In table 5.1, the results of the calibration of the five commodities from December 2015 to November 2016 are shown. Reading the parameters, we can have some insights on how the two-factor model is interpreting the data. The CO2 (EU ETS) is basically driven by two independent stable factors. The correlation and the mean reversion parameter $k$ are very close to zero. Therefore, the volatility that arises from the short term factor is persistent, since the decay coefficient is very low. One of the reason can be linked to the particularity of the products all expiring in the same month of the year, i.e. December. Thus, the correlations among the products are close to one (see table 5.9). The result can be interpreted also as that CO2 dynamics could almost be described by a one-factor

| Parameters | $\sigma$ short | $\sigma$ long | $k$ short | $\rho$ |
|---|---|---|---|---|
| API2_E | 0.19936 | 0.31048 | 3.3984 | -0.39079 |
| Brent_E | 0.32577 | 0.23931 | 0.49802 | 0.38886 |
| CO2 | 0.41134 | 0.32721 | 0.010858 | -4.5403e-05 |
| TTF | 0.24151 | 0.2628 | 1.7399 | 0.10306 |
| DE | 0.29176 | 0.24532 | 5.7322 | -0.30705 |

Table 5.1: Single Commodity Two-Factor Model Parameters

| $\sigma_{daily}$ | API2_E | Brent_E | CO2 | TTF | DE |
|---|---|---|---|---|---|
| M1 | 0.0177 | 0.0291 | 0.0325 | 0.024 | 0.0192 |
| M2 | 0.0181 | 0.0285 | 0.0325 | 0.0209 | 0.017 |
| Q1 | 0.0184 | 0.0278 | 0.0325 | 0.0205 | 0.0157 |
| Q2 | 0.0186 | 0.026 | 0.0325 | 0.0191 | 0.0163 |
| Y1 | 0.0192 | 0.0231 | 0.0324 | 0.0178 | 0.0173 |
| Y2 | 0.0194 | 0.0193 | 0.0321 | 0.0159 | 0.0174 |

Table 5.2: Products historical volatilities

| $\sigma_{daily}$ | API2_E | Brent_E | CO2 | TTF | DE |
|---|---|---|---|---|---|
| M1 | 0.0181 | 0.0291 | 0.0326 | 0.0228 | 0.0187 |
| M2 | 0.0178 | 0.0284 | 0.0325 | 0.0215 | 0.0161 |
| Q1 | 0.0178 | 0.0278 | 0.0325 | 0.0206 | 0.0155 |
| Q2 | 0.0185 | 0.0259 | 0.0325 | 0.0185 | 0.0158 |
| Y1 | 0.0191 | 0.0231 | 0.0324 | 0.0171 | 0.0162 |
| Y2 | 0.0193 | 0.0193 | 0.0321 | 0.0166 | 0.0162 |

Table 5.3: Products model volatilities

model. On the other hand, we have the German electricity prices (DE), that have a very high $k$, suggesting that there is a short term component lasting only in a very short period. The short term factor is maybe capturing the shocks due to weather effects, temporarily change in the marginal technologies and power plants outages.

After a brief overview of the estimated parameters, let us check how the volatilities of the products are fitted by the model. In table 5.2, the historical daily volatilities are reported, while, in table 5.3, we can find the result of the model. The mean of the absolutes errors can be found in table 5.4. The model is able to capture the volatilities of the products with negligible differences.

In tables 5.5, 5.7, 5.9, 5.11, 5.13 the historical correlations of the commodities panel

| $\sigma_{daily}$ **differences** | **API2_E** | **Brent_E** | **CO2** | **TTF** | **DE** |
|---|---|---|---|---|---|
| Mean of absolutes | 0.0003 | 0 | 0 | 0.0006 | 0.0007 |

Table 5.4: Volatilities: Mean of absolute differences

| **API2_E** | **M1** | **M2** | **Q1** | **Q2** | **Y1** | **Y2** |
|---|---|---|---|---|---|---|
| M1 | 1 | 0.9786 | 0.9608 | 0.9133 | 0.8703 | 0.8091 |
| M2 | 0.9786 | 1 | 0.9876 | 0.9483 | 0.9139 | 0.855 |
| Q1 | 0.9608 | 0.9876 | 1 | 0.9646 | 0.9357 | 0.8793 |
| Q2 | 0.9133 | 0.9483 | 0.9646 | 1 | 0.981 | 0.9447 |
| Y1 | 0.8703 | 0.9139 | 0.9357 | 0.981 | 1 | 0.973 |
| Y2 | 0.8091 | 0.855 | 0.8793 | 0.9447 | 0.973 | 1 |

Table 5.5: API2_E products historical correlations

| **API2_E** | **M1** | **M2** | **Q1** | **Q2** | **Y1** | **Y2** |
|---|---|---|---|---|---|---|
| M1 | 1 | 0.9889 | 0.9687 | 0.9006 | 0.8526 | 0.8337 |
| M2 | 0.9889 | 1 | 0.9948 | 0.9551 | 0.9207 | 0.9065 |
| Q1 | 0.9687 | 0.9948 | 1 | 0.9803 | 0.9556 | 0.9447 |
| Q2 | 0.9006 | 0.9551 | 0.9803 | 1 | 0.995 | 0.9909 |
| Y1 | 0.8526 | 0.9207 | 0.9556 | 0.995 | 1 | 0.9994 |
| Y2 | 0.8337 | 0.9065 | 0.9447 | 0.9909 | 0.9994 | 1 |

Table 5.6: API2_E products model correlations

are reported, while the model ones are in the table 5.6, 5.8, 5.10, 5.12, 5.14.

From table 5.15 we can see that the fitting of the correlation is performing well, except for the German electricity panel. Considering the results of the other commodities, this could be driven by two main kind of causes. From a market perspective, the liquidity of the products is less developed, since we are dealing with a local market. From a

| **Brent_E** | **M1** | **M2** | **Q1** | **Q2** | **Y1** | **Y2** |
|---|---|---|---|---|---|---|
| M1 | 1 | 0.9994 | 0.9981 | 0.9937 | 0.9815 | 0.9556 |
| M2 | 0.9994 | 1 | 0.9993 | 0.9964 | 0.9855 | 0.9614 |
| Q1 | 0.9981 | 0.9993 | 1 | 0.9983 | 0.989 | 0.9664 |
| Q2 | 0.9937 | 0.9964 | 0.9983 | 1 | 0.9939 | 0.9763 |
| Y1 | 0.9815 | 0.9855 | 0.989 | 0.9939 | 1 | 0.9899 |
| Y2 | 0.9556 | 0.9614 | 0.9664 | 0.9763 | 0.9899 | 1 |

Table 5.7: Brent_E products historical correlations

| Brent_E | M1 | M2 | Q1 | Q2 | Y1 | Y2 |
|---|---|---|---|---|---|---|
| M1 | 1 | 0.9999 | 0.9996 | 0.9977 | 0.9893 | 0.9549 |
| M2 | 0.9999 | 1 | 0.9999 | 0.9985 | 0.9911 | 0.9588 |
| Q1 | 0.9996 | 0.9999 | 1 | 0.9992 | 0.9928 | 0.9624 |
| Q2 | 0.9977 | 0.9985 | 0.9992 | 1 | 0.9969 | 0.9727 |
| Y1 | 0.9893 | 0.9911 | 0.9928 | 0.9969 | 1 | 0.988 |
| Y2 | 0.9549 | 0.9588 | 0.9624 | 0.9727 | 0.988 | 1 |

Table 5.8: Brent_E products model correlations

| CO2 | M1 | M2 | Q1 | Q2 | Y1 | Y2 |
|---|---|---|---|---|---|---|
| M1 | 1 | 0.9999 | 0.9999 | 0.9998 | 0.9996 | 0.9994 |
| M2 | 0.9999 | 1 | 0.9999 | 0.9998 | 0.9996 | 0.9993 |
| Q1 | 0.9999 | 0.9999 | 1 | 0.9999 | 0.9997 | 0.9994 |
| Q2 | 0.9998 | 0.9998 | 0.9999 | 1 | 0.9998 | 0.9994 |
| Y1 | 0.9996 | 0.9996 | 0.9997 | 0.9998 | 1 | 0.9996 |
| Y2 | 0.9994 | 0.9993 | 0.9994 | 0.9994 | 0.9996 | 1 |

Table 5.9: CO2 products historical correlations

| CO2 | M1 | M2 | Q1 | Q2 | Y1 | Y2 |
|---|---|---|---|---|---|---|
| M1 | 1 | 1 | 1 | 1 | 1 | 0.9999 |
| M2 | 1 | 1 | 1 | 1 | 1 | 1 |
| Q1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Q2 | 1 | 1 | 1 | 1 | 1 | 1 |
| Y1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Y2 | 0.9999 | 1 | 1 | 1 | 1 | 1 |

Table 5.10: CO2 products model correlations

| TTF | M1 | M2 | Q1 | Q2 | Y1 | Y2 |
|---|---|---|---|---|---|---|
| M1 | 1 | 0.9514 | 0.9141 | 0.8613 | 0.8456 | 0.7884 |
| M2 | 0.9514 | 1 | 0.9797 | 0.9416 | 0.9249 | 0.8701 |
| Q1 | 0.9141 | 0.9797 | 1 | 0.9667 | 0.9526 | 0.8999 |
| Q2 | 0.8613 | 0.9416 | 0.9667 | 1 | 0.9806 | 0.9472 |
| Y1 | 0.8456 | 0.9249 | 0.9526 | 0.9806 | 1 | 0.9749 |
| Y2 | 0.7884 | 0.8701 | 0.8999 | 0.9472 | 0.9749 | 1 |

Table 5.11: TTF products historical correlations

| **TTF** | **M1** | **M2** | **Q1** | **Q2** | **Y1** | **Y2** |
|---|---|---|---|---|---|---|
| M1 | 1 | 0.998 | 0.9926 | 0.9597 | 0.8892 | 0.8075 |
| M2 | 0.998 | 1 | 0.9983 | 0.9757 | 0.9166 | 0.8435 |
| Q1 | 0.9926 | 0.9983 | 1 | 0.9867 | 0.9382 | 0.8732 |
| Q2 | 0.9597 | 0.9757 | 0.9867 | 1 | 0.9819 | 0.9408 |
| Y1 | 0.8892 | 0.9166 | 0.9382 | 0.9819 | 1 | 0.9879 |
| Y2 | 0.8075 | 0.8435 | 0.8732 | 0.9408 | 0.9879 | 1 |

Table 5.12: TTF products model correlations

| **DE** | **M1** | **M2** | **Q1** | **Q2** | **Y1** | **Y2** |
|---|---|---|---|---|---|---|
| M1 | 1 | 0.786 | 0.7935 | 0.6308 | 0.6453 | 0.6454 |
| M2 | 0.786 | 1 | 0.8337 | 0.7634 | 0.722 | 0.7036 |
| Q1 | 0.7935 | 0.8337 | 1 | 0.7946 | 0.8453 | 0.7726 |
| Q2 | 0.6308 | 0.7634 | 0.7946 | 1 | 0.8075 | 0.8056 |
| Y1 | 0.6453 | 0.722 | 0.8453 | 0.8075 | 1 | 0.8471 |
| Y2 | 0.6454 | 0.7036 | 0.7726 | 0.8056 | 0.8471 | 1 |

Table 5.13: DE products historical correlations

| **DE** | **M1** | **M2** | **Q1** | **Q2** | **Y1** | **Y2** |
|---|---|---|---|---|---|---|
| M1 | 1 | 0.9531 | 0.8748 | 0.6834 | 0.6236 | 0.6134 |
| M2 | 0.9531 | 1 | 0.9804 | 0.8723 | 0.8309 | 0.8236 |
| Q1 | 0.8748 | 0.9804 | 1 | 0.9516 | 0.9242 | 0.9192 |
| Q2 | 0.6834 | 0.8723 | 0.9516 | 1 | 0.9969 | 0.9958 |
| Y1 | 0.6236 | 0.8309 | 0.9242 | 0.9969 | 1 | 0.9999 |
| Y2 | 0.6134 | 0.8236 | 0.9192 | 0.9958 | 0.9999 | 1 |

Table 5.14: DE products model correlations

theoretical point of view, this could be an indication that the panel should be described by an additional "middle term" factor. As anticipated before, the opposite consideration can be made on the CO2.

Summarizing, the volatilities of all the commodities are captured very well, whilst for the correlations, the German electricity fitting should be enhanced.

| **Correlation differences** | **API2_E** | **Brent_E** | **CO2** | **TTF** | **DE** |
|---|---|---|---|---|---|
| Mean of absolutes | 0.0185 | 0.0024 | 0.0003 | 0.0253 | 0.0975 |

Table 5.15: Products Correlations: Mean of absolute differences

| **Y1** | $H$ | **Test** | **Critical Value (5%)** | $p$ **value** |
|---|---|---|---|---|
| Rank 0 | 1 | 97.9767 | 76.9721 | 0.001 |
| Rank 1 | 1 | 57.2971 | 54.0779 | 0.0251 |
| Rank 2 | 0 | 35.048 | 35.1929 | 0.0519 |
| Rank 3 | 0 | 13.8454 | 20.2619 | 0.3366 |
| Rank 4 | 0 | 5.0517 | 9.1644 | 0.3247 |

Table 5.16: Product Y1: Johansen cointegration test

### 5.2.2 Step 2: Co-integration Analysis

As explained in section 3.6, it is important to take into consideration the long-term dependencies among the commodities. In the long-run, the instantaneous or daily correlations have less relevance that the physical relation that lies beneath the observed daily movements. Johansen (1995) framework is implemented and the trace test is chosen, using 5% as significance level. The MATLAB Econometrics Toolbox (2015) libraries are used to perform the analysis.

It is crucial to choose the most suitable products in order to investigate the cointegrating relations. It is intuitive that, being the cointegration a long-run measure, we should take into consideration the products with the longest expiry. The calendars are the best candidates. We will investigate the cointegrating relations of both the two available calendars (Y1, Y2) in order to better understand the long term dynamics. Differently from the analysis in subsection 5.2.1, we are going to use the whole available time series to have better chance in capturing the long-run co-movements.

#### 5.2.2.1 Estimation of Cointegrating Relations

In table 5.16, the result of the Johansen co-integration analysis performed on Y1 is shown. We can strongly refuse the hypothesis that there are one or less cointegrating relations. With 5% significance level, we cannot refuse the hypothesis of two cointegrating relations, however, the *p value* is just above 5%, so the result is not so strong.

In table 5.17, the cointegrating vectors for rank equal 2 are shown. In both the vectors, the coal and the electricity have a strong weight relatively to the others. This is realistic,

| Y1 | First | Second |
|---|---|---|
| API2_E Y1 | 16.6546 | -15.4912 |
| Brent_E Y1 | 9.6025 | 1.9671 |
| CO2 Y1 | 4.5637 | -7.5613 |
| TTF Y1 | -11.0369 | -4.0663 |
| DE Y1 | -24.0699 | 18.4089 |

Table 5.17: Product Y1: Cointegrating Vectors

| Y1 | First | Second |
|---|---|---|
| API2_E Y1 | 0.0016 | 0.0014 |
| Brent_E Y1 | 0.0004 | 0.0014 |
| CO2 Y1 | -0.0001 | 0.0046 |
| TTF Y1 | 0.0019 | 0.0007 |
| DE Y1 | 0.0018 | 0.0009 |

Table 5.18: Product Y1: Adjustments speed

since the coal is the marginal technology in Germany, as we have seen in subsection 2.1.2. The first vector simultaneously represent the spread coal-electricity and coal-gas, both important in detecting which is the marginal power plant. The second relation can be identified as the clean dark spread, since here both the gas and the oil lose weight in favour of the CO2.

The historical value of the portfolios composed by the cointegrating vectors are reported in figure 5.1 and in figure 5.2.

If we perform the cointegration analysis on the products Y2, we obtain different results. In table 5.19, we can see that the test is better discriminating: the rank 3 is by far the one that we have to take into consideration. Analysing the cointegrating relations in 5.20, we can interpret the first vector as a spread between the electricity and the fuels used in its production, the coal and the gas. Moreover, it represents the spread between the oil and the other two fuels, relation that could quantify the guiding role of the oil in the commodities prices. The second vector mainly represents the clean dark spread, but with a more realistic reduced weight of the coal with respect to the electricity. The third vector is mainly focused on the spread between the gas and the electricity (spark spread), since gas power plants are rarely the marginal ones in Germany.

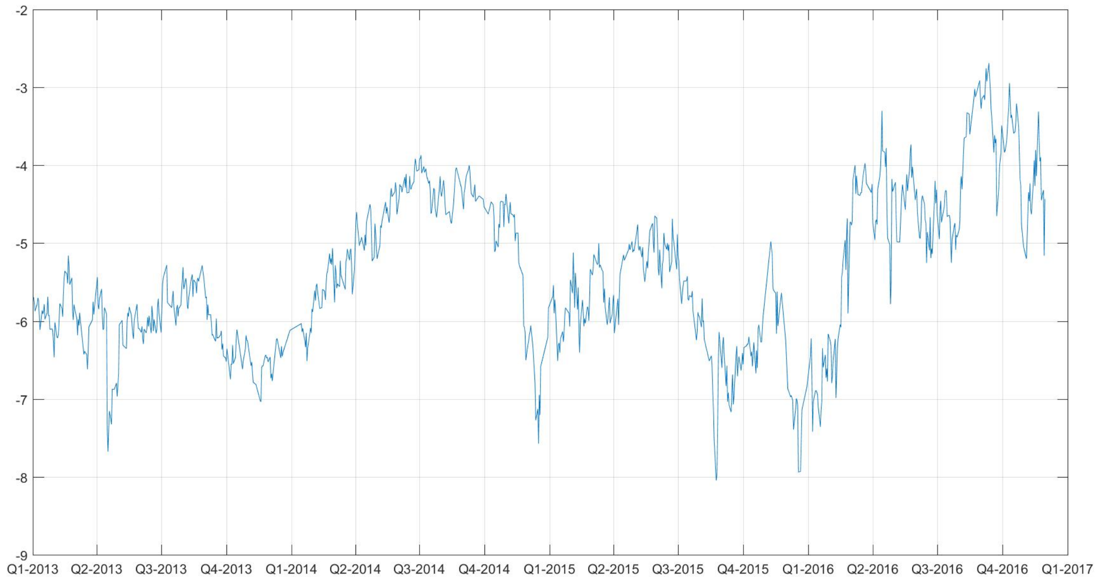


Figure 5.1: Product Y1: First Cointegrated Relation.

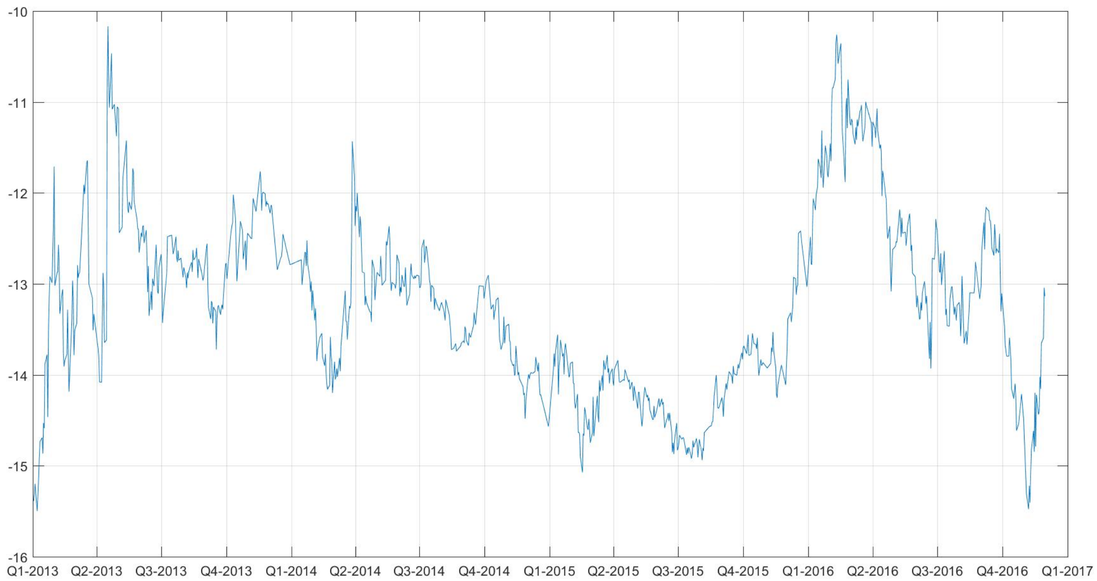


Figure 5.2: Product Y1: Second Cointegrated Relation.

| Y2 | $H$ | Test | Critical Value (5%) | $p$ value |
|---|---|---|---|---|
| Rank 0 | 1 | 120.3971 | 76.9721 | 0.001 |
| Rank 1 | 1 | 75.408 | 54.0779 | 0.001 |
| Rank 2 | 1 | 37.9732 | 35.1929 | 0.0245 |
| Rank 3 | 0 | 14.0652 | 20.2619 | 0.3182 |
| Rank 4 | 0 | 5.9063 | 9.1644 | 0.1982 |

Table 5.19: Product Y2: Johansen cointegration test

| Y2 | First | Second | Third |
|---|---|---|---|
| API2_E Y2 | -15.4414 | -23.0026 | 0.348 |
| Brent_E Y2 | 23.9415 | -5.1999 | 6.8673 |
| CO2 Y2 | -6.5455 | -7.4206 | 3.5462 |
| TTF Y2 | -34.0387 | 4.2351 | -18.9173 |
| DE Y2 | 29.8554 | 33.5022 | 15.1763 |

Table 5.20: Product Y2: Cointegrating Vectors

| Y2 | First | Second | Third |
|---|---|---|---|
| API2_E Y2 | 0.0014 | -0.0003 | -0.001 |
| Brent_E Y2 | -0.0005 | -0.0002 | -0.0014 |
| CO2 Y2 | 0.0035 | 0.0019 | -0.0049 |
| TTF Y2 | 0.0018 | -0.0008 | -0.0003 |
| DE Y2 | 0.001 | -0.0014 | -0.0011 |

Table 5.21: Product Y2: Adjustments speed

Looking at the adjustment speeds in table 5.18 and in table 5.21, we can understand which are the commodities that adjust more when the economy diverges from the long term equilibrium. In both cases, in correspondence of its highest weight, the Brent has the lowest adjustment factor. This means that basically, after a shock in the level of the spreads, the other commodities are affected by a stronger correction than the oil. In the Y2 case, we can also see that in the second relation (clean dark spread) the electricity and the CO2 have the highest adjustment factors. If the commodities, traded in an international context, spike due to some shock, it is probable that the generators of electricity will raise the price of their output in order to offset the growing costs of the fuels. The same consideration stands for the third relation, where the adjustment of the electricity is much higher than the adjustment of the gas.

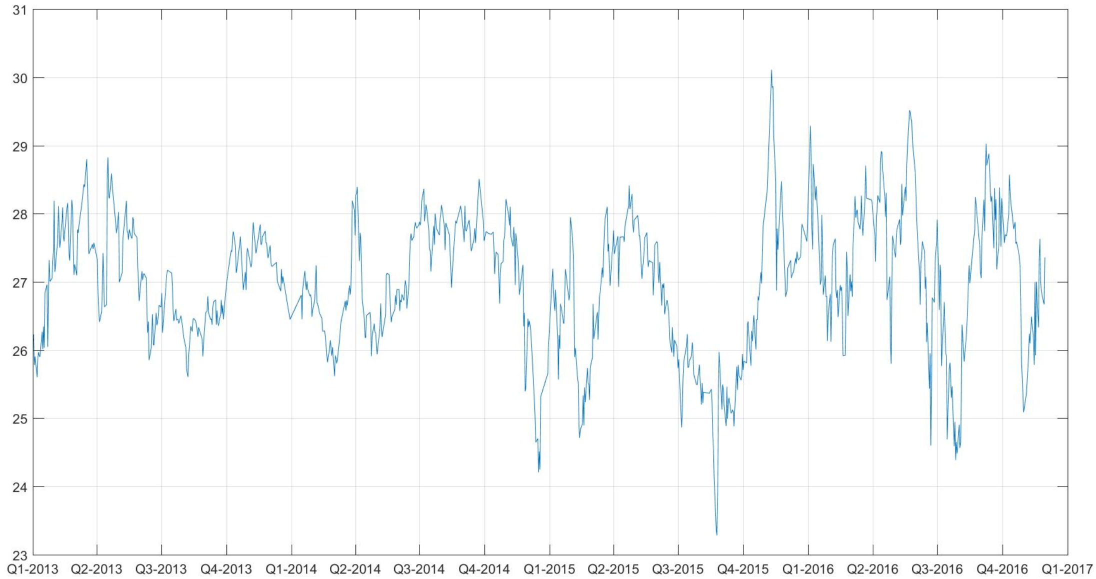


Figure 5.3: Product Y2: First Cointegrated Relation.

The historical value of the portfolios composed by the cointegrating vectors are reported in the figures 5.3, 5.4 and 5.5.

#### 5.2.2.2 Long Term Correlations

Now that the cointegration analysis has been performed, we are ready to apply the methodology shown in section 3.6 to translate the information of the error correction matrix in long term correlations. Since we are going to evaluate exposures until the end of 2018, we choose the time step $h$ that for a Y2 is equivalent to a tenor of two years and one month. In table 5.22, the daily correlation of the Y2 of the commodities are shown, while in table 5.23, the term correlation matrix retrieved through the methodology in section 3.6 is reported. In table 5.24, the difference between the long term correlations and the daily ones is computed.

With the exception of the CO2, in the longer term all the correlations among the products are increased. This trend seems coherent. The correlations among the fuels are


Figure 5.4: Product Y2: Second Cointegrated Relation.


Figure 5.5: Product Y2: Third Cointegrated Relation.

| Correlations | API2_E Y2 | Brent_E Y2 | CO2 Y2 | TTF Y2 | DE Y2 |
|---|---|---|---|---|---|
| API2_E Y2 | 1 | 0.3109 | 0.485 | 0.5335 | 0.6864 |
| Brent_E Y2 | 0.3109 | 1 | 0.3181 | 0.5153 | 0.1834 |
| CO2 Y2 | 0.485 | 0.3181 | 1 | 0.5498 | 0.6212 |
| TTF Y2 | 0.5335 | 0.5153 | 0.5498 | 1 | 0.599 |
| DE Y2 | 0.6864 | 0.1834 | 0.6212 | 0.599 | 1 |

Table 5.22: Product Y2: Daily Correlations

| Correlations | API2_E Y2 | Brent_E Y2 | CO2 Y2 | TTF Y2 | DE Y2 |
|---|---|---|---|---|---|
| API2_E Y2 | 1 | 0.6188 | -0.4605 | 0.6963 | 0.8968 |
| Brent_E Y2 | 0.6188 | 1 | 0.125 | 0.9667 | 0.7607 |
| CO2 Y2 | -0.4605 | 0.125 | 1 | 0.0817 | -0.088 |
| TTF Y2 | 0.6963 | 0.9667 | 0.0817 | 1 | 0.8509 |
| DE Y2 | 0.8968 | 0.7607 | -0.088 | 0.8509 | 1 |

Table 5.23: Product Y2: Long Term Implied Correlations

| Differences | API2_E Y2 | Brent_E Y2 | CO2 Y2 | TTF Y2 | DE Y2 |
|---|---|---|---|---|---|
| API2_E Y2 | 0 | 0.3079 | -0.9455 | 0.1628 | 0.2105 |
| Brent_E Y2 | 0.3079 | 0 | -0.1931 | 0.4514 | 0.5773 |
| CO2 Y2 | -0.9455 | -0.1931 | 0 | -0.4681 | -0.7092 |
| TTF Y2 | 0.1628 | 0.4514 | -0.4681 | 0 | 0.2519 |
| DE Y2 | 0.2105 | 0.5773 | -0.7092 | 0.2519 | 0 |

Table 5.24: Product Y2: Long Term Correlations minus Daily Correlations

higher, since they can be used as substitutes in many economic activities. The relative prices are the key, as in the theory by Leontief (1936) and Hicks (1939). About the correlations with electricity, this is due to the input-output relation, as stated in the seminal work by Granger (1981) and deeply studied through the structural models (see subsection 3.2.1). Regarding electricity, it is noteworthy that the fuel hierarchy in the correlation is the same. The coal has the highest correlations, followed by gas and oil. Once again, this could be explained by the German production mix, marginally based on coal power plants. The CO2 has a strange behaviour as from being a positive correlated asset with the others, in the long term its relation turns negative with the coal, and almost independent with the others. This could have several reasons. First of all, the supply of the CO2 depends on EU regulation, while the fuels depends on physical production

| Factors | A_S | A_L | B_S | B_L | C_S | C_L | T_S | T_L | D_S | D_L |
|---|---|---|---|---|---|---|---|---|---|---|
| API_S | 1 | -0.39 | -0.21 | -0.03 | -0.28 | -0.24 | -0.06 | -0.13 | 0.04 | -0.42 |
| API_L | -0.39 | 1 | 0.18 | 0.35 | 0.14 | 0.6 | -0.01 | 0.59 | -0.43 | 0.8 |
| Bre_S | -0.21 | 0.18 | 1 | 0.39 | 0.3 | 0.29 | 0.03 | 0.21 | -0.15 | 0.21 |
| Bre_L | -0.03 | 0.35 | 0.39 | 1 | -0.03 | 0.31 | 0.01 | 0.57 | -0.23 | 0.32 |
| CO2_S | -0.28 | 0.14 | 0.3 | -0.03 | 1 | 0 | 0.15 | 0.23 | -0.25 | 0.22 |
| CO2_L | -0.24 | 0.6 | 0.29 | 0.31 | 0 | 1 | 0.09 | 0.57 | -0.23 | 0.86 |
| TTF_S | -0.06 | -0.01 | 0.03 | 0.01 | 0.15 | 0.09 | 1 | 0.1 | 0.08 | 0.2 |
| TTF_L | -0.13 | 0.59 | 0.21 | 0.57 | 0.23 | 0.57 | 0.1 | 1 | -0.24 | 0.72 |
| DE_S | 0.04 | -0.43 | -0.15 | -0.23 | -0.25 | -0.23 | 0.08 | -0.24 | 1 | -0.31 |
| DE_L | -0.42 | 0.8 | 0.21 | 0.32 | 0.22 | 0.86 | 0.2 | 0.72 | -0.31 | 1 |

Table 5.25: Correlation Matrix of the factors (without long term dependency)

processes (see section 2.5). Second, the fuels have storage costs, while for CO2 the only "storage cost" could be represented by the interest rates. Finally, a study by Koenig (2011) shows that in presence of a constant marginal technology, electricity, fuels and CO2 prices decouple. Only when there is a price configuration in which the price of CO2 could incentive a switch in technologies, the correlation grows. Since in Germany, given also the growing production of the renewable energies, the coal is nearly always the marginal technology (see figure 2.3), the point by Koenig (2011) is reasonable.

### 5.2.3 Step 3: Multivariate Model with Long-Term Dependency

We have calibrated the commodity in a standalone environment in subsection 5.2.1 and we have computed the long term correlations among the products. Now we are going to calibrate the multivariate two-factor model, as in Edoli et al. (2013), and, then, we are going to see what are the effects on the calibration of including the long-term dependencies (see subsection 5.2.2).

In table 5.25, the correlation matrix of the long and short term factors of the commodities using only the daily correlations is reported, while, in table 5.26, we can find the one adding in the optimization the cointegration outcomes.

Analysing the differences in table 5.27, we can find three clear effects:

1. With the exception of the CO2, all the long-term factors correlations increase. This

| Factors | A_S | A_L | B_S | B_L | C_S | C_L | T_S | T_L | D_S | D_L |
|---|---|---|---|---|---|---|---|---|---|---|
| API_S | 1 | -0.39 | 0.2 | -0.21 | 0.59 | 0.66 | 0.67 | -0.16 | 0.65 | -0.18 |
| API_L | -0.39 | 1 | -0.12 | 0.81 | 0.21 | -0.9 | -0.35 | 0.7 | -0.64 | 0.89 |
| Bre_S | 0.2 | -0.12 | 1 | 0.39 | 0.39 | 0.18 | 0.02 | 0.42 | 0 | 0.04 |
| Bre_L | -0.21 | 0.81 | 0.39 | 1 | 0.47 | -0.62 | -0.07 | 0.98 | -0.45 | 0.9 |
| CO2_S | 0.59 | 0.21 | 0.39 | 0.47 | 1 | 0 | 0.62 | 0.51 | 0.44 | 0.53 |
| CO2_L | 0.66 | -0.9 | 0.18 | -0.62 | 0 | 1 | 0.57 | -0.49 | 0.69 | -0.72 |
| TTF_S | 0.67 | -0.35 | 0.02 | -0.07 | 0.62 | 0.57 | 1 | 0.1 | 0.8 | 0.01 |
| TTF_L | -0.16 | 0.7 | 0.42 | 0.98 | 0.51 | -0.49 | 0.1 | 1 | -0.33 | 0.85 |
| DE_S | 0.65 | -0.64 | 0 | -0.45 | 0.44 | 0.69 | 0.8 | -0.33 | 1 | -0.3 |
| DE_L | -0.18 | 0.89 | 0.04 | 0.9 | 0.53 | -0.72 | 0.01 | 0.85 | -0.3 | 1 |

Table 5.26: Correlation Matrix of the factors (with long term dependency)

| Factors | A_S | A_L | B_S | B_L | C_S | C_L | T_S | T_L | D_S | D_L |
|---|---|---|---|---|---|---|---|---|---|---|
| API_S | 0 | 0 | 0.41 | -0.18 | 0.87 | 0.9 | 0.73 | -0.03 | 0.61 | 0.23 |
| API_L | 0 | 0 | -0.3 | 0.46 | 0.07 | -1.5 | -0.34 | 0.12 | -0.21 | 0.09 |
| Bre_S | 0.41 | -0.3 | 0 | 0 | 0.09 | -0.11 | -0.01 | 0.21 | 0.15 | -0.17 |
| Bre_L | -0.18 | 0.46 | 0 | 0 | 0.5 | -0.93 | -0.08 | 0.41 | -0.22 | 0.58 |
| CO2_S | 0.87 | 0.07 | 0.09 | 0.5 | 0 | 0 | 0.47 | 0.28 | 0.69 | 0.31 |
| CO2_L | 0.9 | -1.5 | -0.11 | -0.93 | 0 | 0 | 0.47 | -1.06 | 0.92 | -1.58 |
| TTF_S | 0.73 | -0.34 | -0.01 | -0.08 | 0.47 | 0.47 | 0 | 0 | 0.72 | -0.19 |
| TTF_L | -0.03 | 0.12 | 0.21 | 0.41 | 0.28 | -1.06 | 0 | 0 | -0.09 | 0.13 |
| DE_S | 0.61 | -0.21 | 0.15 | -0.22 | 0.69 | 0.92 | 0.72 | -0.09 | 0 | 0 |
| DE_L | 0.23 | 0.09 | -0.17 | 0.58 | 0.31 | -1.58 | -0.19 | 0.13 | 0 | 0 |

Table 5.27: Correlations differences (with minus without long term dependency)

is coherent with the results of the cointegration analysis and correctly represents how the market works. Moreover, we are able to capture the CO2 dynamic that cannot be seen with a standard model.

2. The correlations among the short term factors are all increased.

3. The correlations between short and term factors of the same commodity have not changed.

Therefore, in terms of factors correlation, the results are quite satisfying.

Let us check how the term correlations among the products of the commodities are affected. In the charts, the blue line is the Q1 correlation function, the orange line is the Y1 correlation function. The blue diamond is the observed Q1 daily correlation, while the orange diamond is the observed Y1 daily correlation. The black star $*$ is the estimated long term correlation and the black circle $\circ$ is the long term correlation based on the calibrated model.

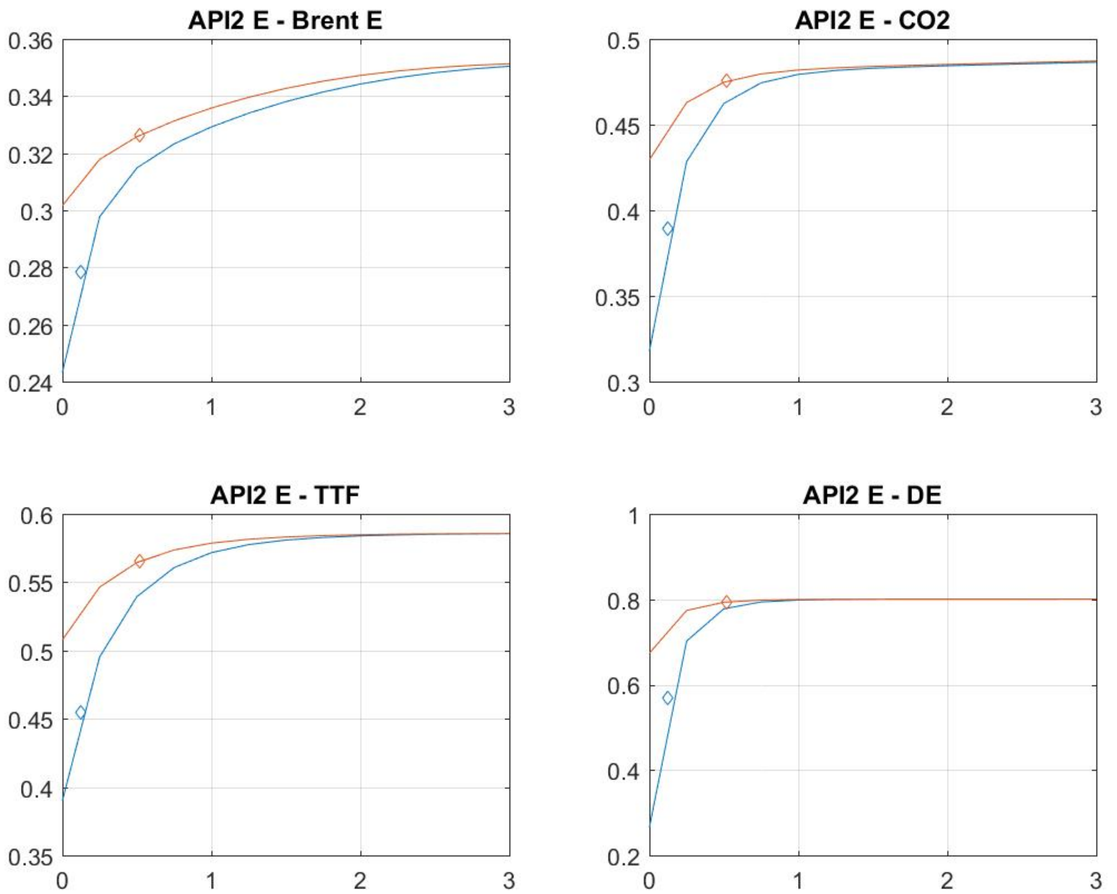


Figure 5.6: API2: Correlations without long-term dependency

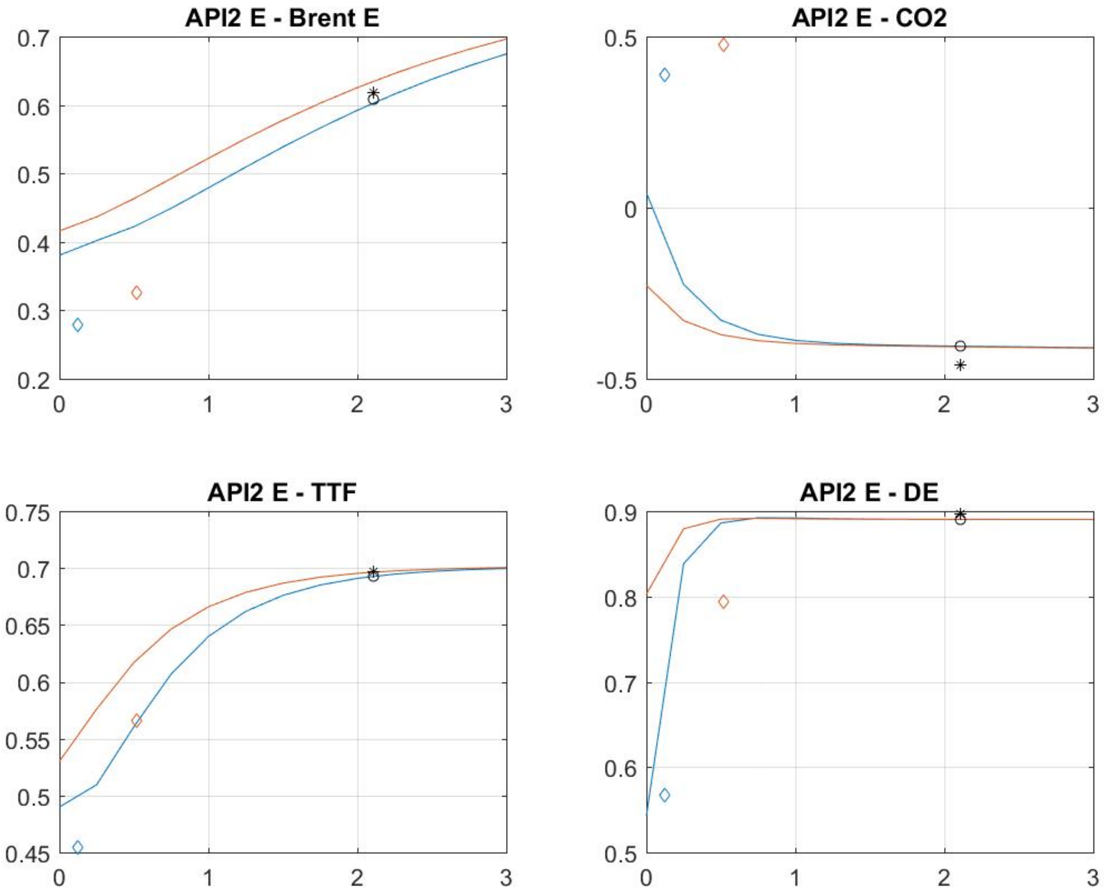


Figure 5.7: API2: Correlations with long-term dependency

From figures 5.6 and 5.7, we can see that considering the long-term dependency has totally changed the correlations trends. The API2-Brent correlation is increased and has changed in shape. The API2-TTF and the API2-DE correlations have shifted up, since they have increased without changing too much their shapes. As expected, the API2-CO2 is totally reversed. However, we have paid the price of losing precision on the daily correlations.

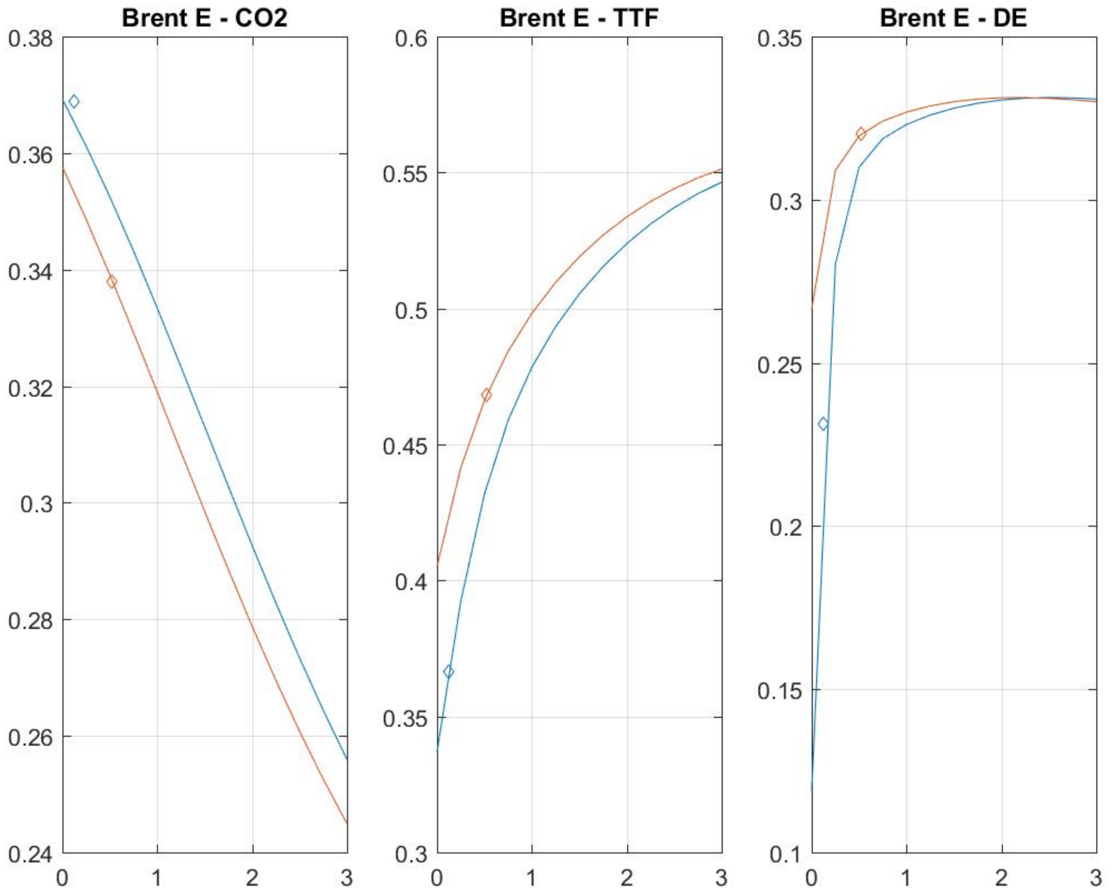


Figure 5.8: Brent: Correlations without long-term dependency

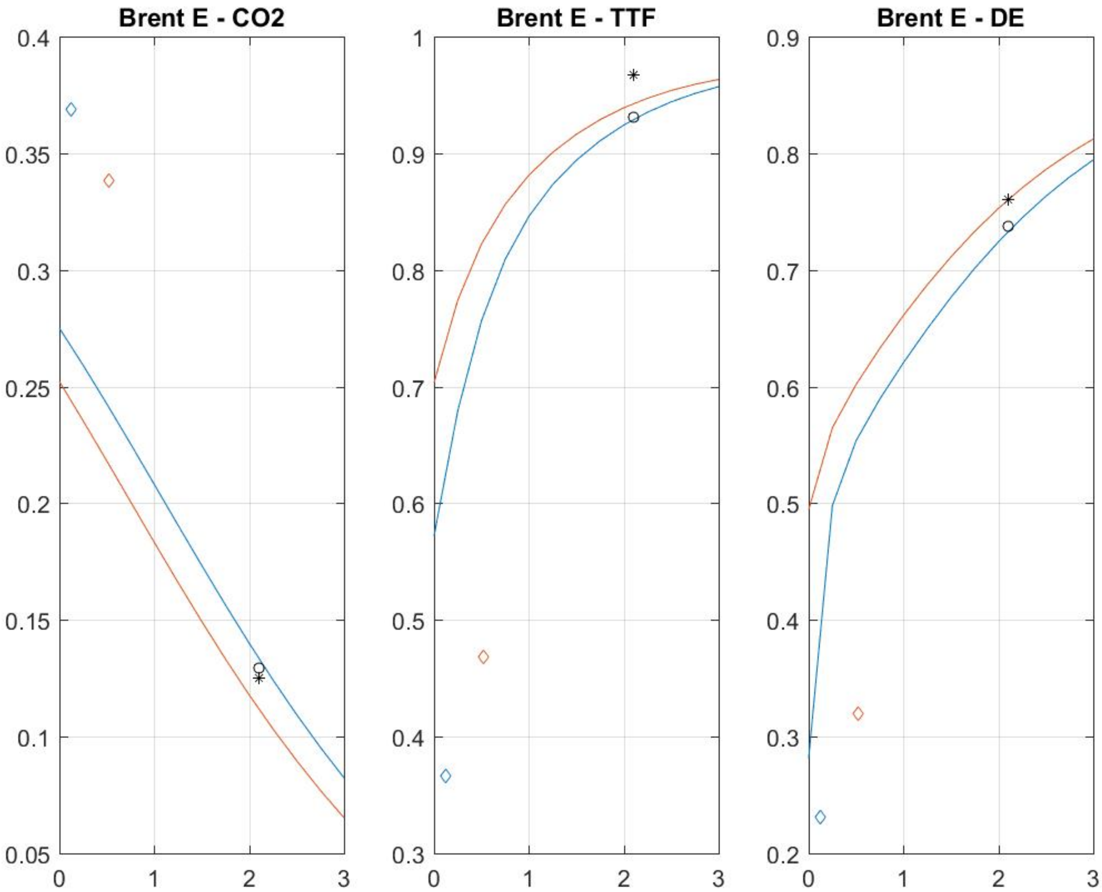


Figure 5.9: Brent: Correlations with long-term dependency

From figures 5.8 and 5.9, we can derive considerations similar to the API2. It is noteworthy that the correlation between Brent and TTF has been affected by a huge increase. This is very realistic, since the indexation of gas contract to the Brent is a standard market price, especially for long term agreements (see section 2.4).

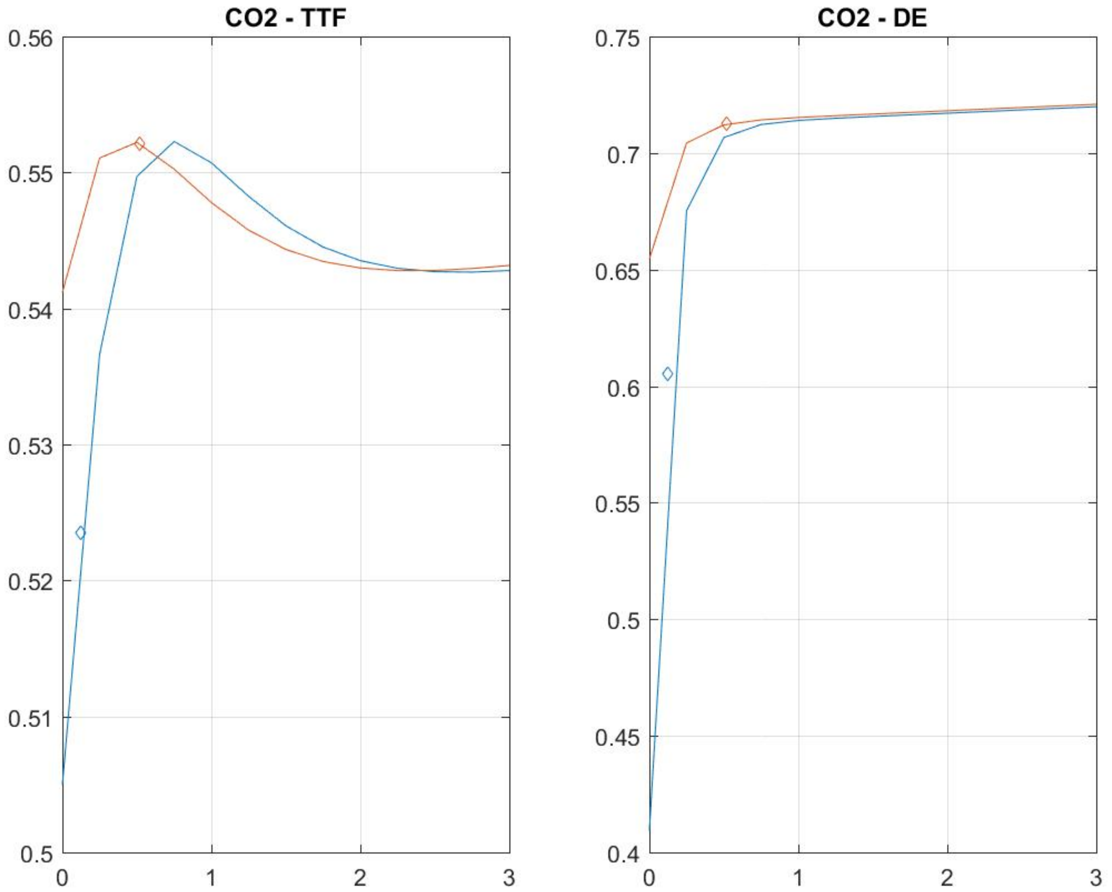


Figure 5.10: CO2: Correlations without long-term dependency

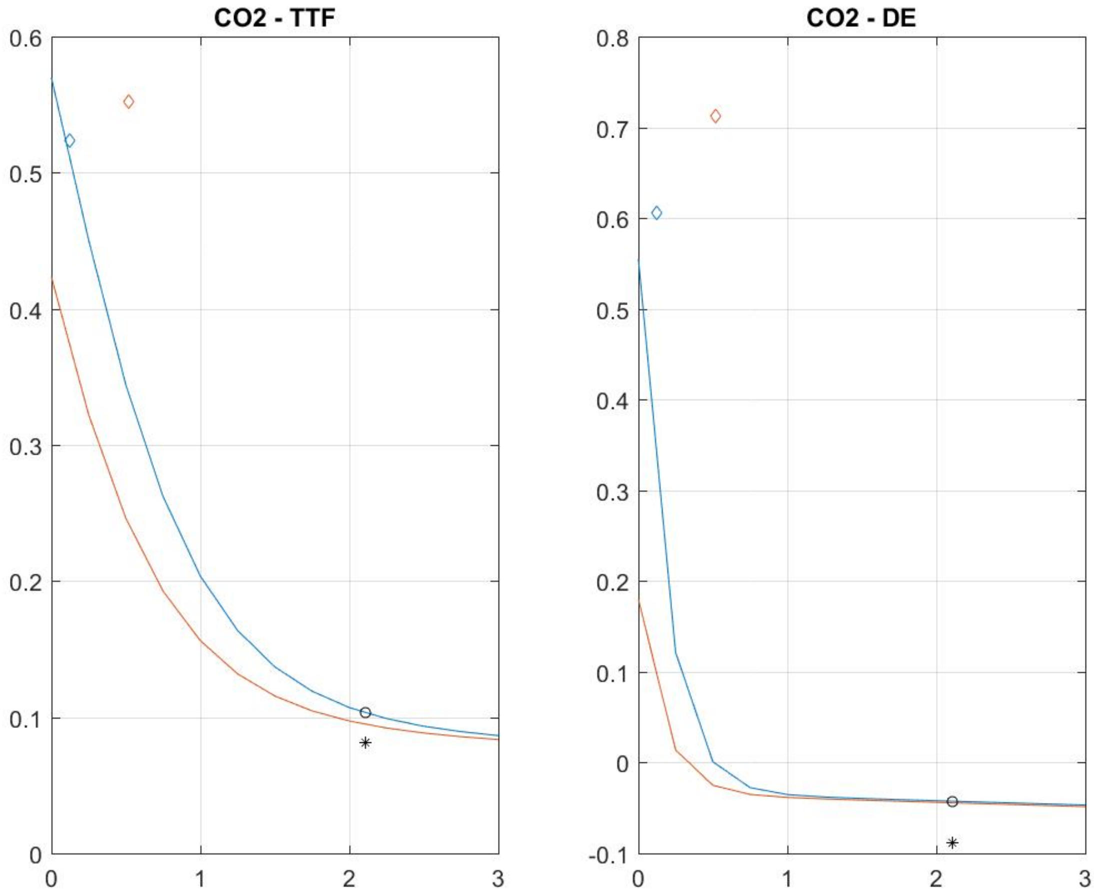


Figure 5.11: CO2: Correlations with long-term dependency

From figures 5.10 and 5.11, we can see that, considering the long-term dependency, the CO2 tends to be not correlated to both gas and German electricity.

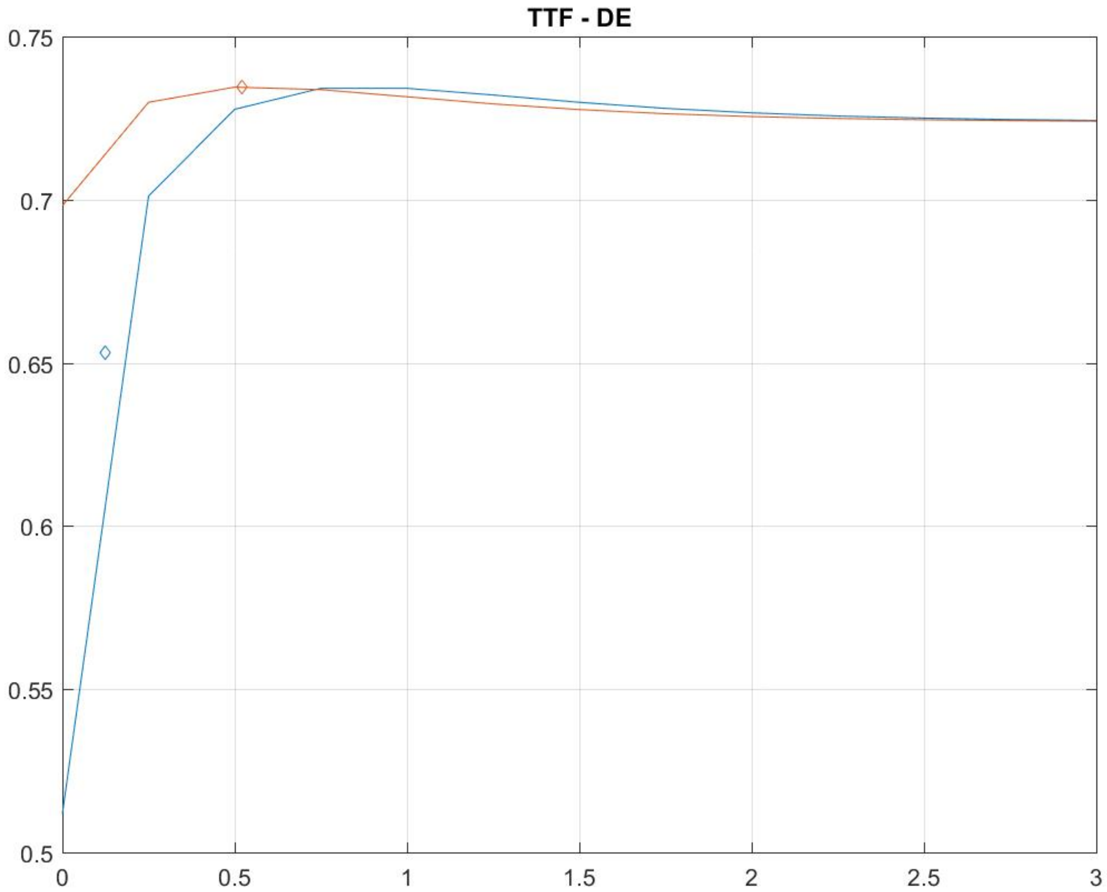


Figure 5.12: TTF: Correlations without long-term dependency

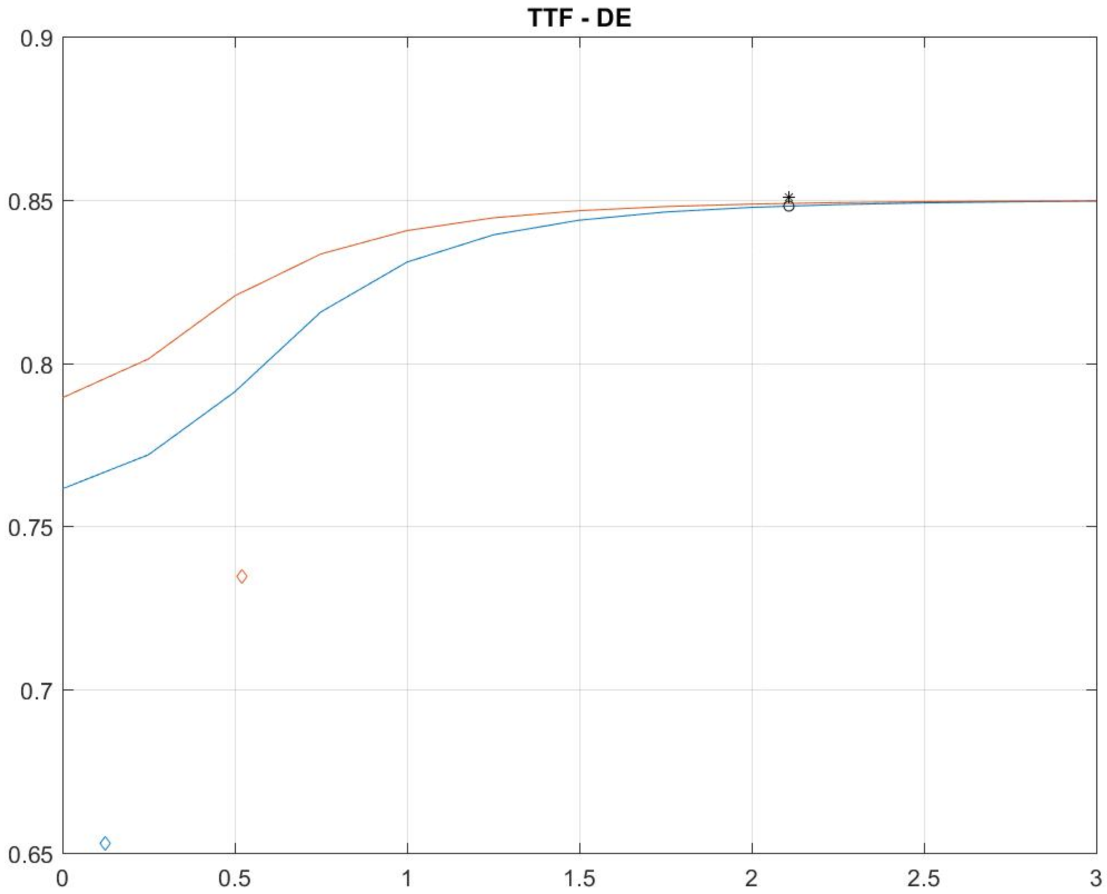


Figure 5.13: TTF: Correlations with long-term dependency

Finally, from figures 5.12 and 5.13, we see that the correlation between gas and electricity price is shifted up.

## 5.3 Some Risk Management Applications

Four case studies will be analysed in this section. The exposures are monthly and refer to the year 2018, considering as valuation date the end of November 2016. The differences between the mean and the 5% and the 95% percentiles are used as risk measures. 10,000 scenarios are simulated for each month until 2019, using Latin Hypercube Sampling (Stein (1987)).

### 5.3.1 Clean Dark Spread

The first portfolio is a long position on a German *Clean Dark Spread*. A *Clean Dark Spread* is the difference between the price of electricity in a given market and the cost of coal and its emissions. Therefore, it is a long position on power, a short position on coal and a short position on CO2. According to Platts (2016), the formula for the German *Clean Dark Spread* 35% efficiency clean dark spreads takes into consideration an energy conversion factor of 6.978 (converting 1 metric ton of coal into MWh), a fuel efficiency factor (coal) of 35% and an emissions intensity factor of 0.973 mtCO2/MWh. Taking into consideration the API2 index as a reference for coal cost, the *Clean Dark Spread* formula is

$$\begin{aligned} Clean\ Dark\ Spread = {} & Baseload\ Power\ Price(€/MWh) \\ & - \frac{API2(USD/Ton)}{FX_{EURUSD}} \frac{1}{6.978} \frac{1}{0.35} - 0.973CO2(Ton/MWh) \end{aligned} \tag{5.1}$$

We are going to consider a monthly exposure of 1 TWh.

In table 5.28 and in table 5.29, the monthly standalone risks without and with long-term dependency are reported. In figure 5.14, the values are plotted in a chart. Long-term dependency is not only decreasing the risk of each monthly exposure, but it is also reducing its growth rate. In figure 5.15, we can see that difference of the risk between the two-factor model and the two-factor model with long-term dependency has a clear up-trend.

| **€mln** | $P_5 - \mu$ | $P_{95} - \mu$ |
|---|---|---|
| Jan-2018 | -10.95 | 9.72 |
| Feb-2018 | -11.1 | 9.43 |
| Mar-2018 | -11.24 | 9.39 |
| Apr-2018 | -11.96 | 8.99 |
| May-2018 | -12.07 | 9.1 |
| Jun-2018 | -12.15 | 9.54 |
| Jul-2018 | -12.54 | 9.25 |
| Aug-2018 | -13.07 | 9.22 |
| Sep-2018 | -12.92 | 9.62 |
| Oct-2018 | -12.52 | 10.28 |
| Nov-2018 | -12.91 | 10.21 |
| Dec-2018 | -14.2 | 9.91 |

Table 5.28: CDS: risk by month without Long Term Dependency

| **€mln** | $P_5 - \mu$ | $P_{95} - \mu$ |
|---|---|---|
| Jan-2018 | -6.56 | 6.64 |
| Feb-2018 | -6.55 | 6.5 |
| Mar-2018 | -6.57 | 6.21 |
| Apr-2018 | -7.01 | 5.73 |
| May-2018 | -7.33 | 5.85 |
| Jun-2018 | -7.22 | 6.27 |
| Jul-2018 | -7.39 | 5.93 |
| Aug-2018 | -7.9 | 5.91 |
| Sep-2018 | -7.89 | 6.2 |
| Oct-2018 | -7.55 | 7.36 |
| Nov-2018 | -7.68 | 7 |
| Dec-2018 | -8.57 | 6.3 |

Table 5.29: CDS: risk by month with Long Term Dependency

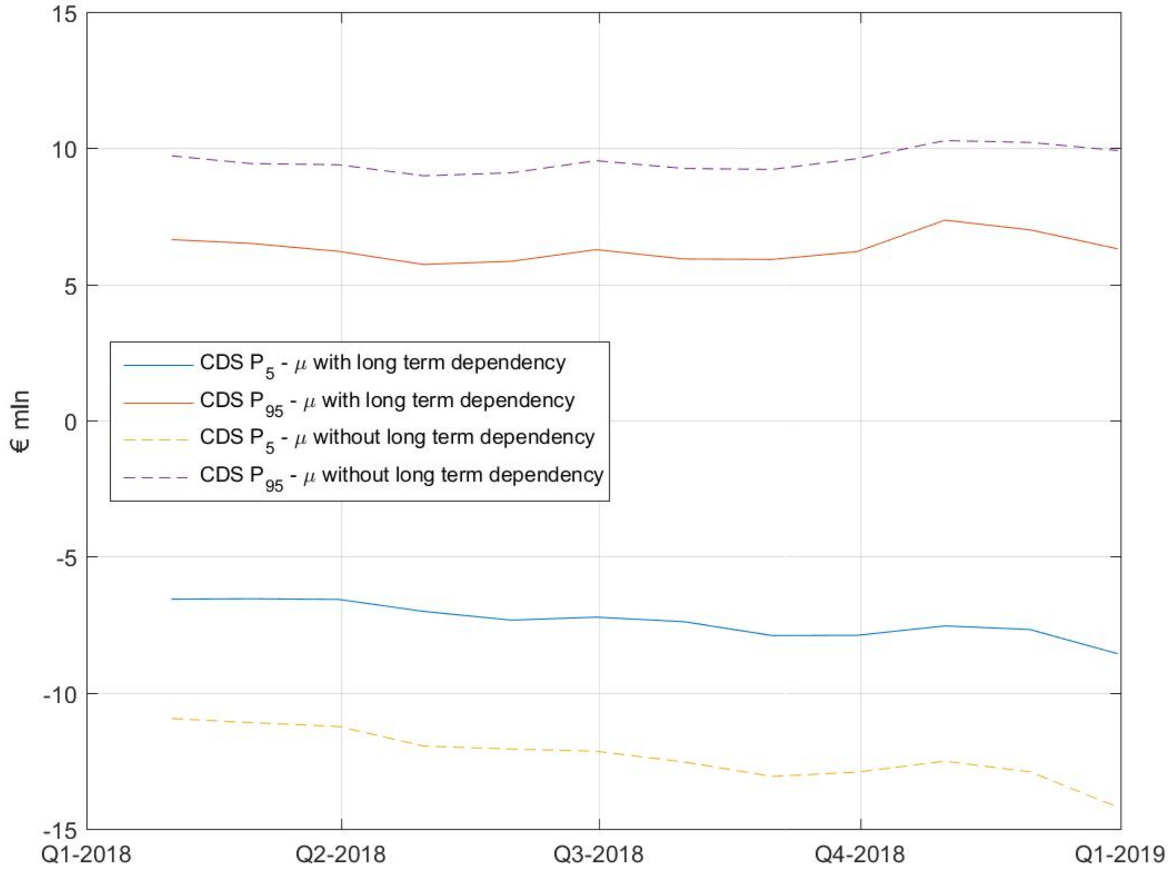


Figure 5.14: CDS: risk by month with Long-Term Dependency

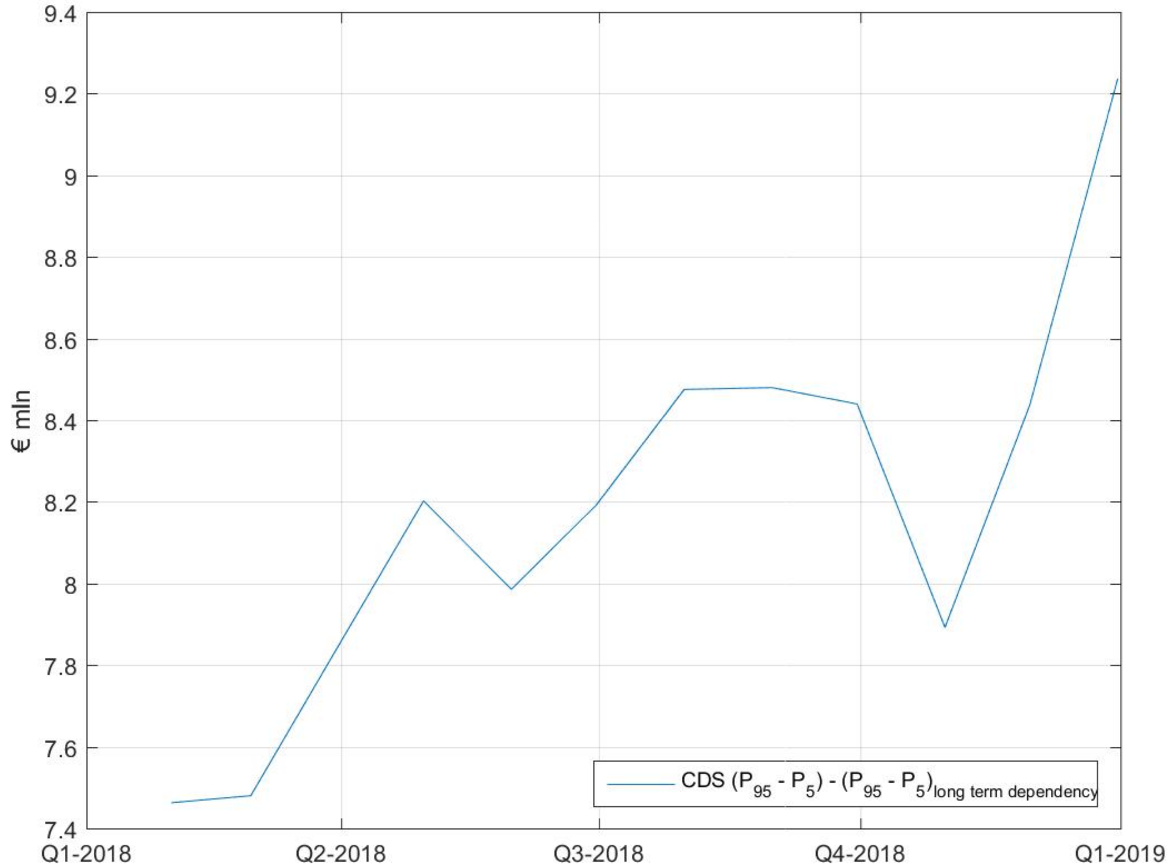


Figure 5.15: CDS: differences in the monthly risk between the model without Long-Term Dependency and the model with Long-Term Dependency

In figure 5.16, the risk of the two models is shown in order to retrieve the effects of the long-term dependency. The results are quite impressive, since the risk is reduced by more than one third. This is caused mainly by the higher API2-DE correlation with respect to the standard model, after considering the long-term dependency.

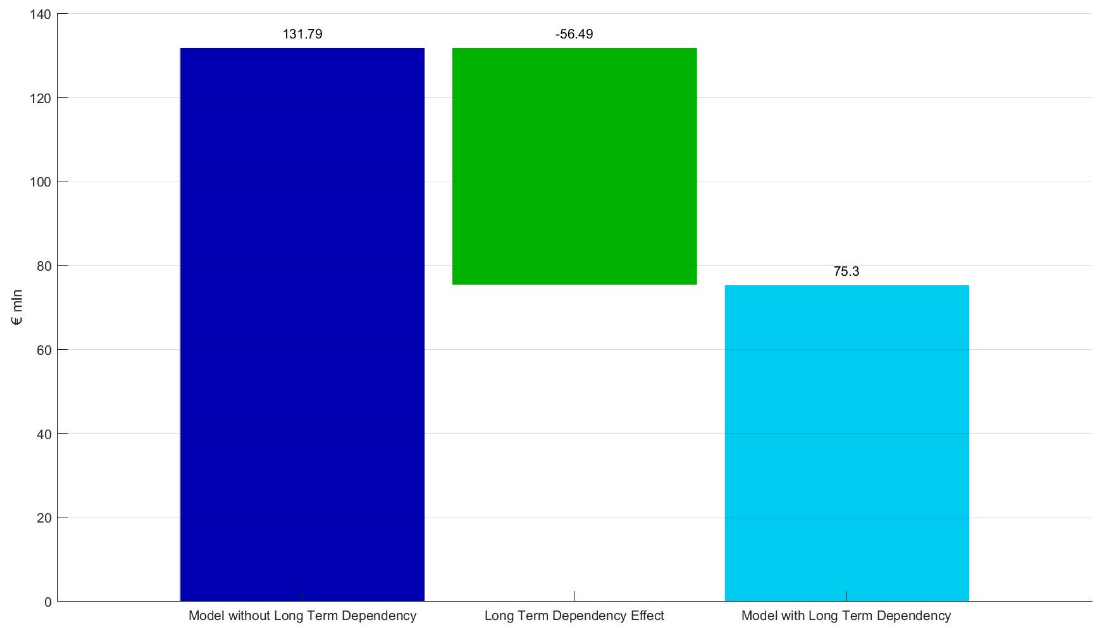


Figure 5.16: CDS: Overall risk without and with Long-Term Dependency

### 5.3.2 Clean Spark Spread

The second portfolio is a long position on a German *Clean Spark Spread*. A *Clean Spark Spread* is the difference between the price of electricity in a given market and the cost of gas and its emissions. Therefore, it is a long position on power, a short position on gas and a short position on CO2. According to Platts (2016), the formula for the 50% efficiency *Clean Spark Spread* uses an emission intensity factor of 0.053942 tCO2e/MMbtu. Taking into consideration the TTF index in €/MWh as a reference for gas cost, the *Clean Spark Spread* formula is

$$\begin{aligned} &\textit{Clean Spark Spread} \\ &\quad = \textit{Baseload Power Price}(€/MWh) \\ &\qquad - (TTF(€/MWh)\frac{1}{0.50} - 3.4121411565 \times 0.053942 CO2(Ton/MMbtu)) \end{aligned} \tag{5.2}$$

| €mln | $P_5 - \mu$ | $P_{95} - \mu$ |
|---|---|---|
| Jan-2018 | -16.32 | 12.86 |
| Feb-2018 | -16.09 | 12.57 |
| Mar-2018 | -17.35 | 13.04 |
| Apr-2018 | -16.8 | 12.28 |
| May-2018 | -16.52 | 12.1 |
| Jun-2018 | -16.81 | 12.67 |
| Jul-2018 | -17.36 | 12.54 |
| Aug-2018 | -16.8 | 12.35 |
| Sep-2018 | -18.15 | 13.15 |
| Oct-2018 | -17.95 | 14.01 |
| Nov-2018 | -19.67 | 14.68 |
| Dec-2018 | -21.64 | 14.52 |

Table 5.30: CSS: risk by month without Long Term Dependency

| €mln | $P_5 - \mu$ | $P_{95} - \mu$ |
|---|---|---|
| Jan-2018 | -14.46 | 10.9 |
| Feb-2018 | -14.48 | 10.6 |
| Mar-2018 | -15.79 | 11.27 |
| Apr-2018 | -15.34 | 10.88 |
| May-2018 | -14.93 | 10.64 |
| Jun-2018 | -14.73 | 10.84 |
| Jul-2018 | -15.49 | 10.93 |
| Aug-2018 | -15.44 | 10.59 |
| Sep-2018 | -16.27 | 11.23 |
| Oct-2018 | -15.71 | 11.61 |
| Nov-2018 | -17.35 | 12.11 |
| Dec-2018 | -19.35 | 12.68 |

Table 5.31: CSS: risk by month with Long Term Dependency

where 3.4121411565 is how many MMbtus are needed to generate 1 MWh according to unit converter of IEA (2016c).

As for the *Clean Dark Spread*, we are going to consider a monthly exposure of 1 TWh.

In table 5.30 and in table 5.31, the monthly standalone risks without and with long-term dependency are reported. In figure 5.17, the values are plotted in a chart. Like in the precedent case, the differences between the standard model and the new model risks increase with time (figure 5.18).

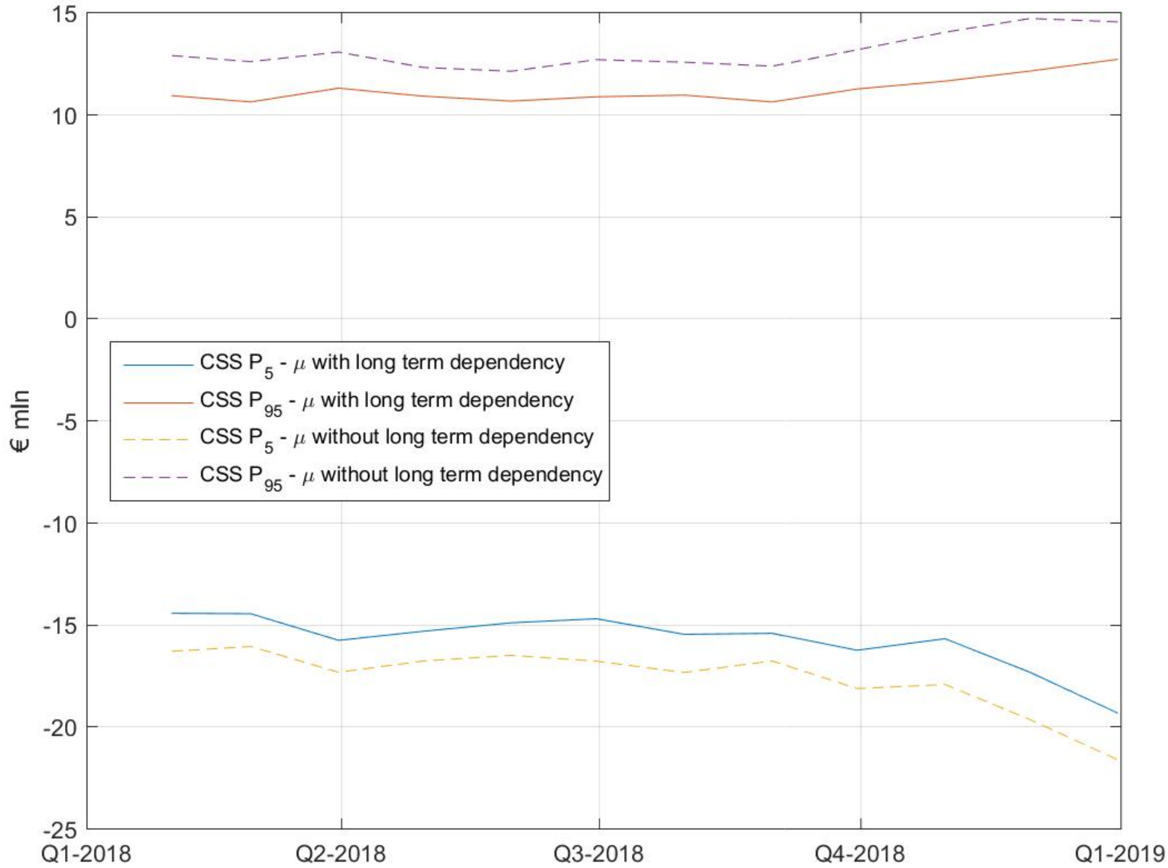


Figure 5.17: CSS: risk by month with Long-Term Dependency

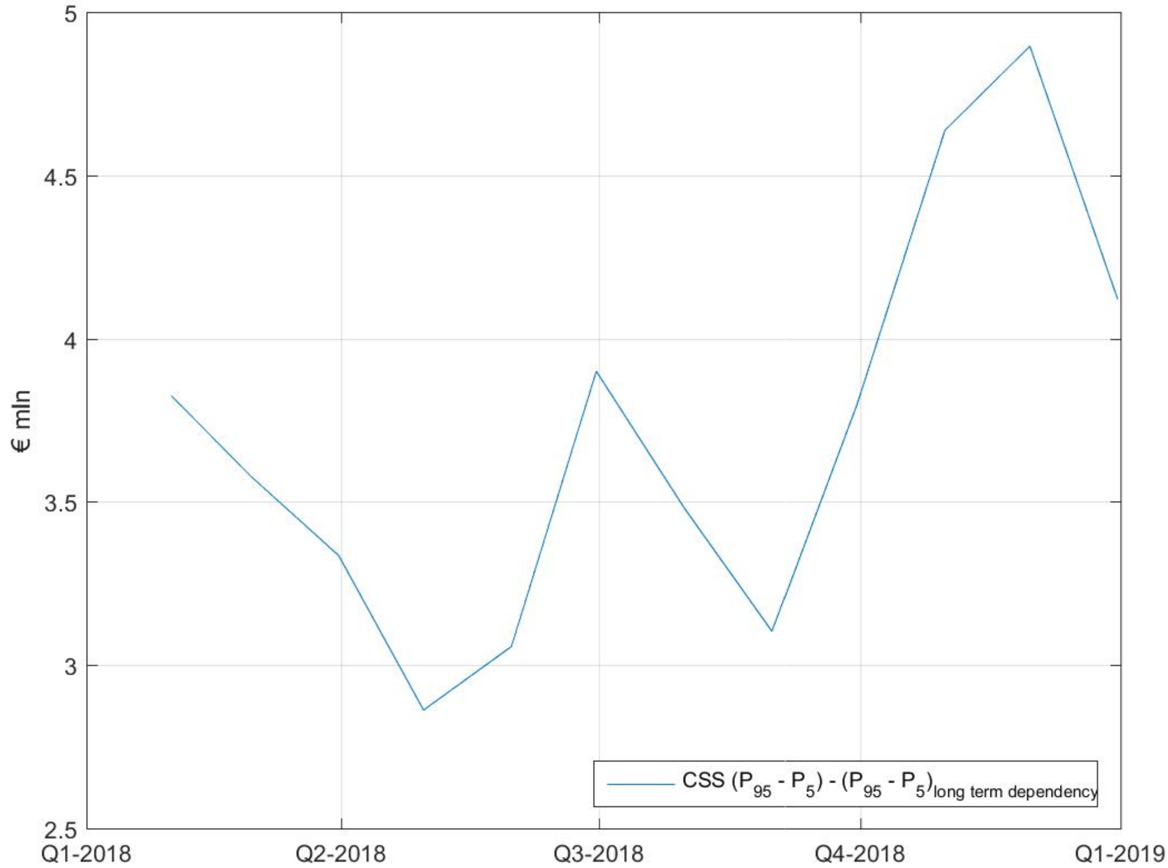


Figure 5.18: CSS: differences in the monthly risk between the model without Long-Term Dependency and the model with Long-Term Dependency

In figure 5.19, the risk of the two models is shown. As for the *Clean Dark Spread*, the risk is significantly reduced. However, the impact is less pronounced, since the correlation between TTF and DE with the long-term dependency is increased, but with less strength than the API2-DE one. This makes sense given that a *Clean Dark Spread*, in a market with coal as marginal fuel, is better naturally hedged in comparison with other types of generation. As a confirmation, in both the models, the *Clean Spark Spread* has a higher risk in the monthly and in the overall exposures.

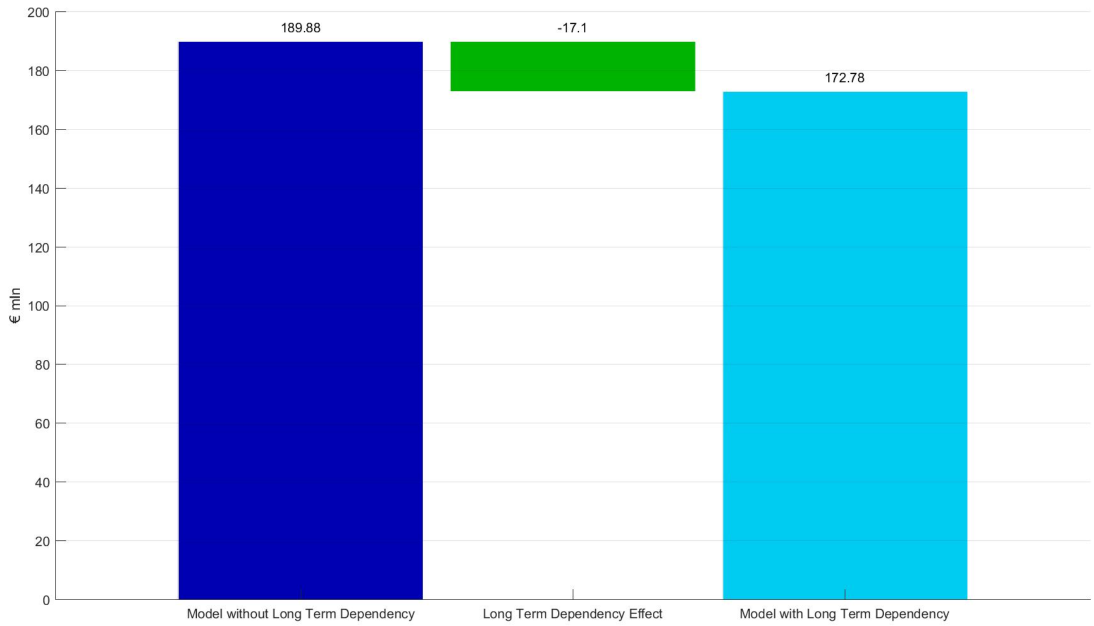


Figure 5.19: CSS: Overall risk without and with Long-Term Dependency

### 5.3.3 Gas contract indexed to Oil

The third portfolio is an example of a gas contract indexed to Brent. As gas volume, we are going to use the same quantity of gas needed for the *Clean Spark Spread*. Therefore, we will have a long position on TTF and a short position to the Brent. As conversion factor between the two, we simply apply the average ratio between the two forward curves ($\frac{Brent}{TTF} = 2.97$).

In table 5.32 and in table 5.33, the monthly standalone risks without and with long-term dependency are reported. In figure 5.20, the values are plotted in a chart. Also in this case, we have a growing difference between the risk of the standard model and the new model (figure 5.21).

| **€mln** | $P_5 - \mu$ | $P_{95} - \mu$ |
|---|---|---|
| Jan-2018 | -27.08 | 22.79 |
| Feb-2018 | -27.5 | 22.6 |
| Mar-2018 | -28.62 | 23.78 |
| Apr-2018 | -29.02 | 23.38 |
| May-2018 | -30.04 | 23.23 |
| Jun-2018 | -30.94 | 23.75 |
| Jul-2018 | -30.29 | 24.48 |
| Aug-2018 | -31.02 | 24.2 |
| Sep-2018 | -31.35 | 25.44 |
| Oct-2018 | -31.63 | 25.76 |
| Nov-2018 | -32.4 | 26.93 |
| Dec-2018 | -32.59 | 27.77 |

Table 5.32: OIL_INDEX: risk by month without Long Term Dependency

| **€mln** | $P_5 - \mu$ | $P_{95} - \mu$ |
|---|---|---|
| Jan-2018 | -20.04 | 15.82 |
| Feb-2018 | -20.51 | 16.02 |
| Mar-2018 | -20.43 | 16.34 |
| Apr-2018 | -21.13 | 15.9 |
| May-2018 | -21.38 | 15.89 |
| Jun-2018 | -21.88 | 16.1 |
| Jul-2018 | -21.76 | 16.25 |
| Aug-2018 | -22.91 | 16.12 |
| Sep-2018 | -22.46 | 16.44 |
| Oct-2018 | -22.64 | 16.84 |
| Nov-2018 | -22.52 | 17.34 |
| Dec-2018 | -22.53 | 17.39 |

Table 5.33: OIL_INDEX: risk by month with Long Term Dependency

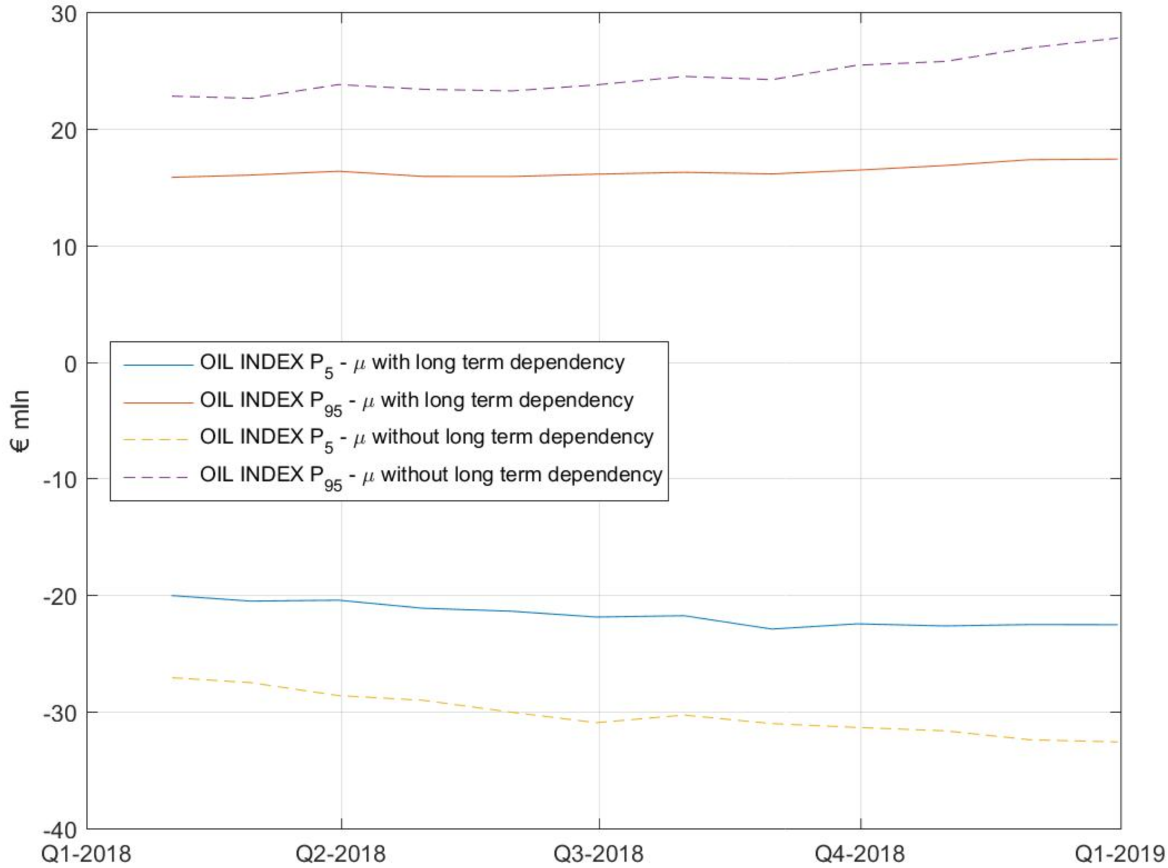


Figure 5.20: Oil Index: risk by month with Long-Term Dependency

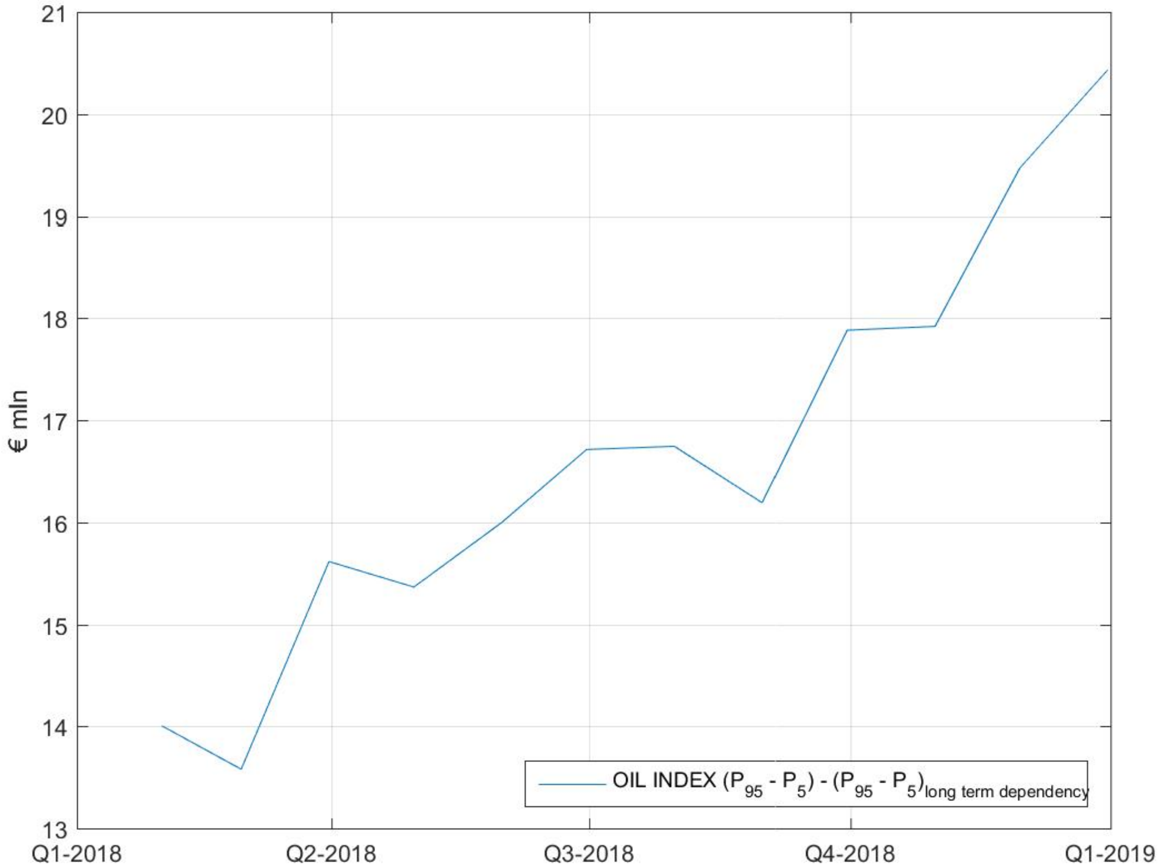


Figure 5.21: Oil Index: differences in the monthly risk between the model without Long-Term Dependency and the model with Long-Term Dependency

In figure 5.22, the risk of the two models is shown. The long-term dependency reduces the risk by more than one fourth, given that the term correlation between TTF and Brent is strongly increased.

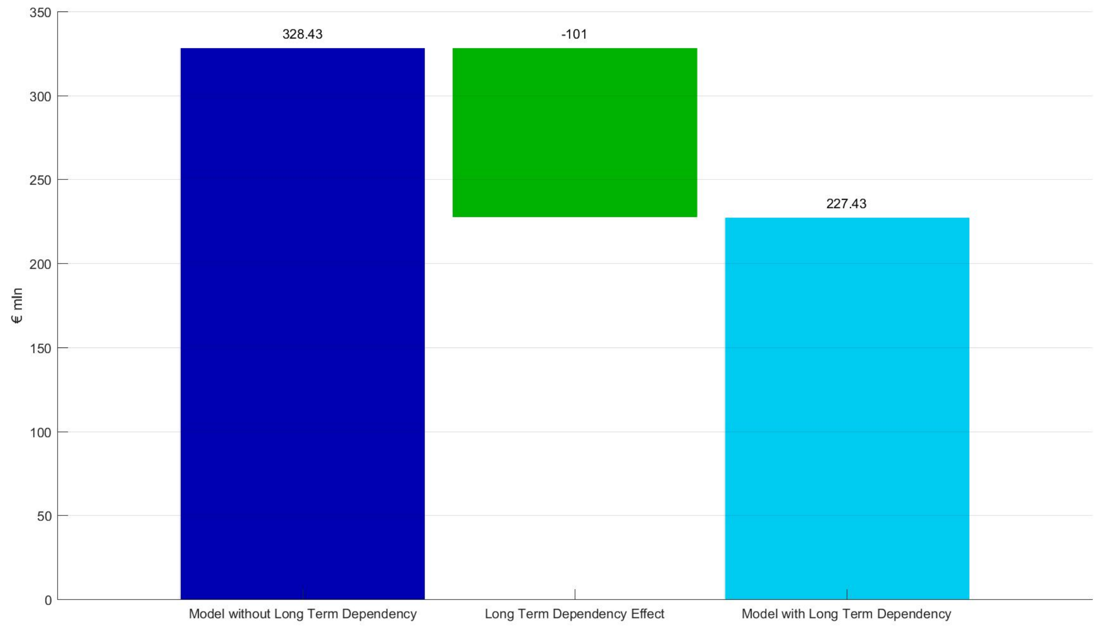


Figure 5.22: Oil Index: Overall risk without and with Long-Term Dependency

### 5.3.4 Gas and Oil

The fourth portfolio is a long position on TTF and a long position on Brent. Their volumes are halved with respect to the the third portfolio. The aim to compose this last combination is to consider a portfolio completely long, since all the others are mixed combination of long and short positions.

In table 5.34 and in table 5.35, the monthly standalone risks without and with long-term dependency are reported. In figure 5.23, the values are plotted in a chart. Differently from the other cases, here the long-term dependency is increasing the risk at a higher rate than the standard model (figure 5.24).

| **€mln** | $P_5 - \mu$ | $P_{95} - \mu$ |
|---|---|---|
| Jan-2018 | -15.37 | 21.54 |
| Feb-2018 | -15.41 | 21.93 |
| Mar-2018 | -16 | 22.88 |
| Apr-2018 | -15.89 | 23.19 |
| May-2018 | -15.94 | 23.29 |
| Jun-2018 | -16.14 | 24.04 |
| Jul-2018 | -16.26 | 24.27 |
| Aug-2018 | -16.34 | 24.85 |
| Sep-2018 | -16.81 | 25.89 |
| Oct-2018 | -17.26 | 25.99 |
| Nov-2018 | -17.81 | 26.99 |
| Dec-2018 | -18.23 | 27.62 |

Table 5.34: OIL_GAS: risk by month without Long Term Dependency

| **€mln** | $P_5 - \mu$ | $P_{95} - \mu$ |
|---|---|---|
| Jan-2018 | -16.56 | 24.17 |
| Feb-2018 | -16.69 | 24.29 |
| Mar-2018 | -17.4 | 25.68 |
| Apr-2018 | -17.16 | 25.99 |
| May-2018 | -17.26 | 26.54 |
| Jun-2018 | -17.44 | 26.89 |
| Jul-2018 | -17.6 | 27.61 |
| Aug-2018 | -17.62 | 27.72 |
| Sep-2018 | -18.32 | 28.48 |
| Oct-2018 | -18.68 | 29.17 |
| Nov-2018 | -19.34 | 30.33 |
| Dec-2018 | -19.61 | 31.35 |

Table 5.35: OIL_GAS: risk by month with Long Term Dependency

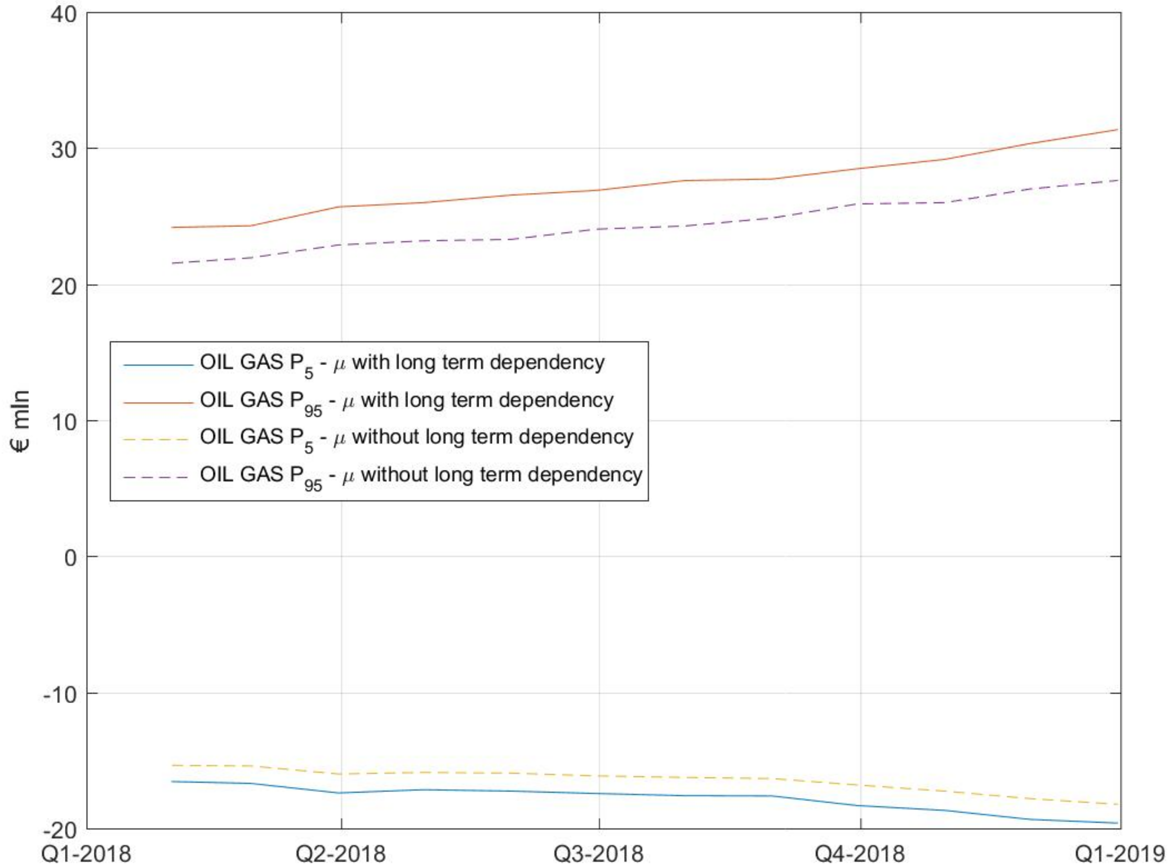


Figure 5.23: Oil and Gas: risk by month with Long-Term Dependency

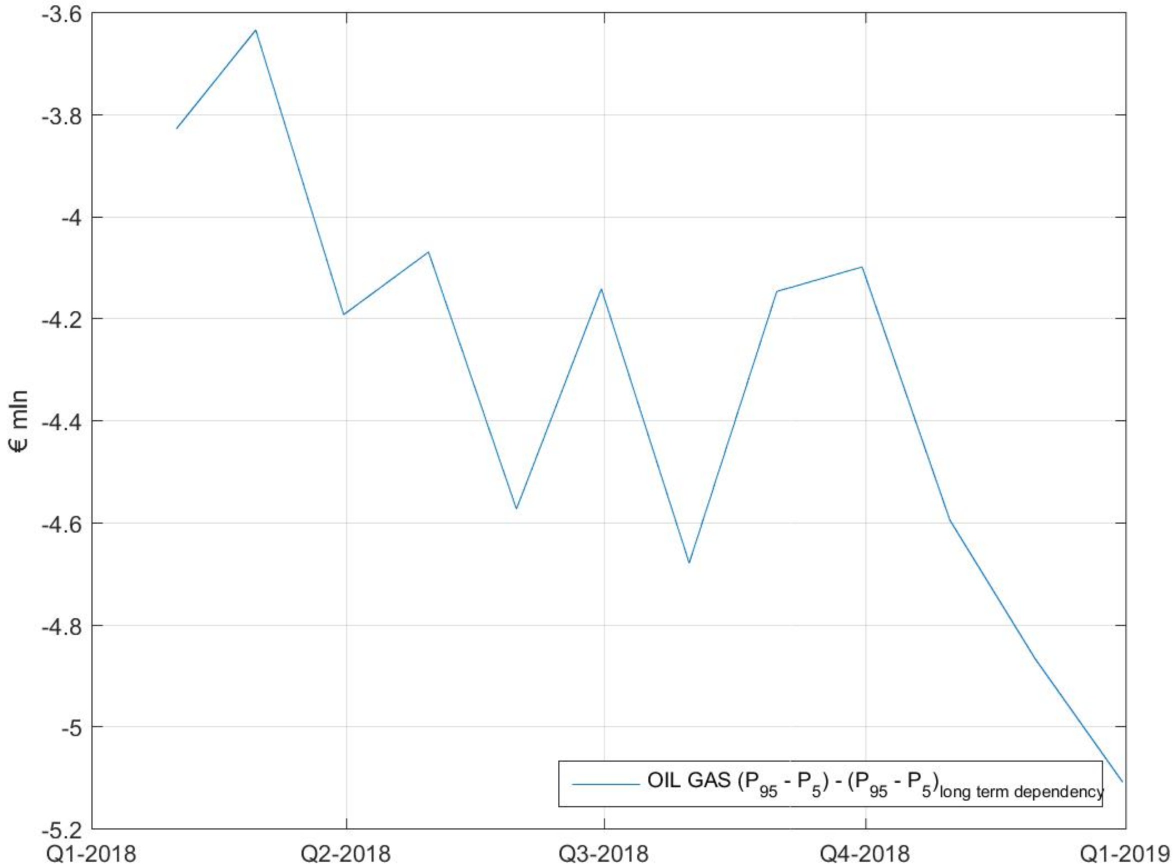


Figure 5.24: Oil and Gas: differences in the monthly risk between the model without Long-Term Dependency and the model with Long-Term Dependency

In figure 5.25, the risk of the two models is shown. Here the risk is increased, since differently from the indexed contract, we have two long positions.

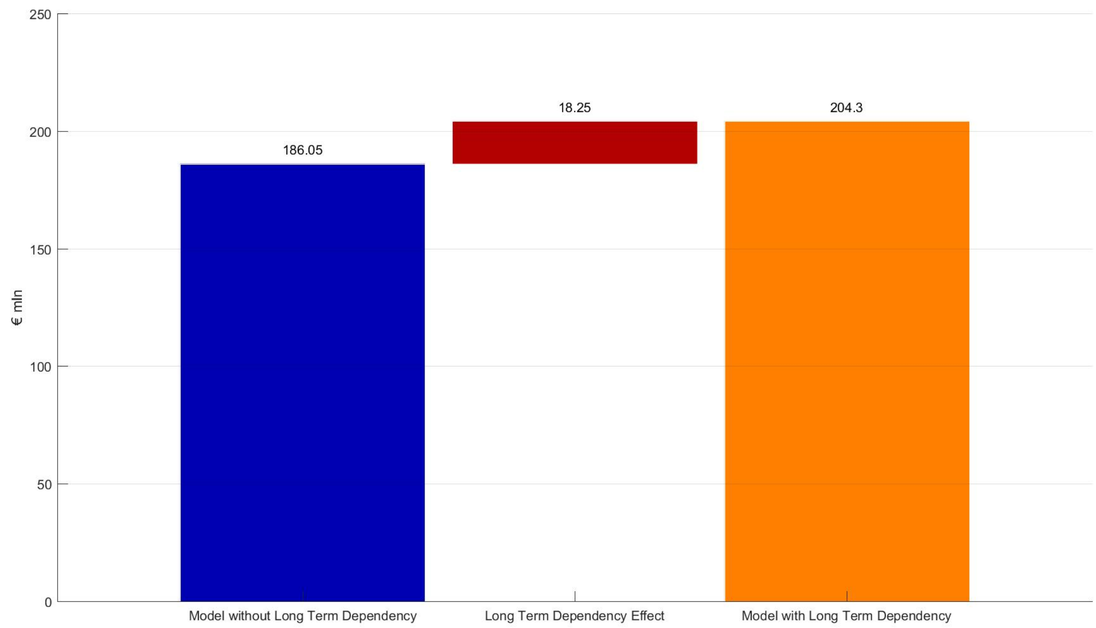


Figure 5.25: Oil and Gas: Overall risk without and with Long-Term Dependency

## 5.4 Summary

The empirical results show that the correlations among the commodities change both in size and in shape. Consistently with economic fundamentals, electricity, oil, coal and gas associations become more powerful in the long term. On the other hand, the CO2 correlations are reduced, becoming negative. This can be explained by its specific characteristic that significantly differs from the other commodities. The allowances are not a physical assets and their market is driven by EU regulators. Moreover, as stated in Koenig (2011), if CO2 price level is not enough high to trigger a switch in the marginal technology, electricity, fuels and CO2 prices can decouple, reducing, in this way, the long term correlation. It is noteworthy that such a dynamic is not captured by a standard two-factor model.

The risk of the exposures computed with the proposed model are decreased in the case of *Clean Dark Spread*, *Clean Spark Spread* and of the oil indexed gas contract. This is due to the higher correlations that allow diversification to increase. For the same reason, the last portfolio, being long both on oil and gas, results to have a higher risk.

Finally, we have seen that the risk measured with long-term dependency, is not only different in level, but has also a different rate of growth. In the first three cases, the risk measured with the standard two-factor model grows with higher rates, whilst in the fourth case the opposite occurs.

# Conclusions

*... apprendere ciò che è stato fatto da altri in passato, non deve servire a far adorare feticci mummificati, ma a proseguire gli sviluppi in cui quei contributi, pur superati e rielaborati continuamente, continueranno a vivere come apporti all'evoluzione del pensiero umano.*

De Finetti (1959)

In this thesis, a two-factor model with Long-Term Dependency has been developed and applied to commodities, including electricity. Edoli et al. (2013) are able to treat the correlations among the commodities, however, as stated in Alexander (1999) and in Alexander (2001), the correlation is just a short term measure. By including cointegration, it is possible to extend the model in order to be consistent and resilient in a long term environment. The calibration of the model has been divided into three main steps.

The first step is to calibrate univariate models as in Kiesel et al. (2009) and Edoli et al. (2013). The main contribution is to develop closed form formulae for term correlations. Furthermore, to reduce the computational complexity, numerical integration based on sparse grids (Heiss and Winschel (2008)) is implemented to treat quarter and year swap contracts.

The second step is to perform a cointegration analysis. Following the guidelines by Alexander (2001), the Johansen framework is implemented (Johansen (1995), Johansen (2000)). The main contribution is to extract from the error correction matrix the information on long term correlations.

The third step is to work in a multivariate environment, as in Edoli et al. (2013), including long term correlations. The main contribution is to develop closed form formulae for cross commodity term correlations, in order to include in the calibration the information retrieved with the cointegration analysis. Moreover, a two steps procedure is implemented to obtain a valid global correlation matrix: the method by Qi and Sun (2006) is used to find a starting point for the algorithm proposed by Edoli et al. (2013). Such a procedure let us avoid the use of a global optimization algorithm, reducing the dimension of the problem. It is noteworthy that through the term correlation formulae it is easy to work in monthly simulation framework, although the calibration has been made on daily data.

The empirical results show that the Long-Term Dependency in a risk model can have very important consequences in the risk management activity. Companies, especially utilities, have to plan their strategy on a time horizon that goes beyond the year and can reach three or five years. In considering an investment, as for a power plant, the valuation has to take into consideration cash flows that cover the whole life of the structure. When it comes to choose between two or more business strategies, possible investments, feasible hedging strategies, it is common to support the decision with a risk-return framework. The Long-Term Dependency could spot out that the risk a company think is bearing, or is going to bear, using a model based only on daily correlations, is overestimated (see cases in subsections 5.3.1, 5.3.2 and 5.3.3) or underestimated (see subsection 5.3.4). This could jeopardize the hierarchy among the choices and could suggest the company to hedge an exposure instead of another one. The measured risk also differs in its growth rate with respect to time. In the first three cases, long-term dependency curbs the increase in risk, whilst, in the last one, it widens the distributions with time. Those results are coherent with a recent paper by Gatarek and Johansen (2016), where the cointegration is used to find the *optimal hedging strategy*. The authors prove that *cointegration plays an important role in hedging. It allows for the possibility that the hedging portfolio has a risk that is bounded in the horizon $h$, as opposed to the unhedged risk.*

The designed model can also help market participants and regulators to discover, and/or quantify, the *economic forces* (Engle and Granger (1987)) that are hidden for standard models. We have seen that in the long term, CO2 becomes independent with electricity, oil and gas, whilst the correlation turns negative with coal. This point can be explained by the fundamentals of the market and is coherent with the functioning of the system marginal price, as shown in Koenig (2011).

The proposed model could also be applied in hedge accounting. As shown in Juhl et al. (2012), cointegration could affect both the hedge horizon and the hedge effectiveness test.

Finally, long-term dependency can help to find the hedging that exploit the long-run economic relations in order to increase the efficiency and the resilience of the implemented strategies. On the other hand, if it is not taken into consideration, the correlation structure could be misspecified and hedging strategies could destroy diversification. Therefore, the model should be implemented in finding the *optimal hedging strategy*, like in Gatarek and Johansen (2016), in order to find portfolios that are consistent and resilient in the long run. In the analysis, the hedging costs should be considered, since they could significantly change due to a different hedging time horizon. This is important given that bid-ask spreads are higher for long term contracts.

# Bibliography


Adämmer, P., Bohl, M. T., and Gross, C. (2016). Price Discovery in Thinly Traded Futures Markets: How Thin is Too Thin? *The Journal of Futures Markets*, 36(9):851–869.

Alexander, C. (1999). Optimal Hedging Using Cointegration. *Philosophical Transactions: Mathematical Physical and Engineering Sciences*, 357(1758):2039–2058.

Alexander, C. (2001). *Market Models: A Guide to Financial Data Analysis*. Wiley, UK.

Álvaro Cartea and Villaplana, P. (2008). Spot price modeling and the valuation of electricity forward contracts: The role of demand and capacity. *Journal of Banking and Finance*, 32(12):2502–2519.

ARGUS (2015). Argus Media group.

Asche, F., Osmundsen, P., and Sandsmark, M. (2006). The UK Market for Natural Gas, Oil and Electricity: Are the Prices Coupled? *The Energy Journal*, 27(2):27–40.

Barlow, M. T. (2002). A diffusion model for electricity prices. *Mathematical Finance*, 12:287–298.

Barrett, W. W., Johnson, C. R., and Lundquist, M. (1989). Determinantal formulae for matrix completions associated with chordal graphs. *Linear Algebra and its Applications*, 121:265–289.

Belsley, D. A. (1991). A Guide to using the collinearity diagnostics. *Computer Science in Economics and Management*, 4(1):33–50.

Belsley, D. A., Kuh, E., and Welsch, R. E. (2004). *Regression Diagnostics - Identifying Influential Data and Sources of Collinearity*. Probability and Statistics. Wiley-Interscience, Hoboken, New Jersey. First published in 1980.

Bencivenga, C., Sargenti, G., and D'Ecclesia, R. L. (2010). Energy markets: crucial relationship between prices. In Corazza, M. and Pizzi, C., editors, *Mathematical and Statistical Methods for Actuarial Sciences and Finance*, chapter 3, pages 23–32. Springer, Verlag Berlin Heidelberg.

Bhansali, V. and Wise, M. B. (2001). Forecasting portfolio risk in normal and stressed markets. *Journal of Risk*, 4(1):91–106.

Black, F. (1986). Noise. *The Journal of Finance*, 41(3):529–543.

Black, F. and Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy*, 81(3):673–654.

Bloomberg (2016). Bloomberg Professional. Accessed: 9 December 2016.

Bosco, B., Parisio, L., Pelagatti, M., and Baldi, F. (2010). Long-run relations in european electricity prices. *The Journal of Applied Econometrics*, 25(5):805–832.

Breitung, J. (2002). Nonparametric tests for unit roots and cointegration. *The Journal of Econometrics*, 108(2):343–363.

Breitung, J. and Taylor, A. R. (2003). Corrigendum to "Nonparametric tests for unit roots and cointegration" [J. Econom. 108 (2002) 343-363]. *The Journal of Econometrics*, 117(2):401–404.

Brennan, M. J. (1958). The supply of storage. *American Economic Review*, 48(1):50–72.

Brennan, M. J. and Schwartz, E. S. (1985). Evaluating Natural Resource Investments. *The Journal of Business*, 58(2):135–157.

Brigo, D. and Mercurio, F. (2001). *Interest Rate Models - Theory and Practice*. Springer Finance. Springer, Berlin Heidelberg New York.

Brigo, D. and Mercurio, F. (2006). *Interest Rate Models - Theory and Practice. With Smile, Inflation and Credit*. Springer Finance. Springer, Berlin Heidelberg New York.

Brooks, C., Scott-Quinn, B., and Walmsley, J. (1998). *Adjusting VAR models for the impact of the EURO*. ISMA Centre, University of Reading.

Budden, M., Hadavas, P., Hoffman, L., and Pretz, C. (2007). Generating valid 4x4 correlation matrices. *Applied Mathematics E-Notes*, 7:53–59.

Bunn, D. W. and Fezzi, C. (2009). A vector error correction model of the interactions among gas, electricity and carbon prices: an application to the cases of Germany and the United Kingdom. In Gullì, F., editor, *Markets for Carbon and Power Pricing in Europe: Theoretical Issues and Empirical Analyses, 2009, 2nd Edition*, New Horizons in Environmental Economics, chapter 6, pages 145–159. Edward Elgar, Cheltenham, UK, Northampton, MA, USA.

Callaghan, K. J. and Chen, J. (2008). Revisiting the Collinear Data Problem: An Assessment of Estimator 'Ill-Conditioning' in Linear Regression. *Practical Assessment Research and Evaluation*, 13(5).

CarbonBrief (2016a). Mapped: How Germany generates its electricity.

CarbonBrief (2016b). Mapped: The global coal trade.

Cholesky, A.-L. (1910). Sur la résolution numérique des systèmes d'équations linéaires. In *Bulletin*, number 39 December 2005. la Société des amis de la bibliothèque et de l'historie de l'Ecole polytechnique.

Clewlow, L. and Strickland, C. (1999a). *A Multi-Factor Model for Energy Derivatives*. QFRC Research Paper Series 28. University of Technology, Sydney.

Clewlow, L. and Strickland, C. (1999b). *Valuing Energy Options in a One Factor Model Fitted to Forward Prices*. QFRC Research Paper Series 10. University of Technology, Sydney.

Cordano, D. (2015). Ucraina e Sicurezza Energetica. *Sistema di informazione per la sicurezza della Repubblica.*

Crowder, W. J. and Hamed, A. (1993). A cointegration test for oil futures market efficiency. *The Journal of Futures Markets*, 13(8):933–941.

Davison, M., Anderson, C. L., Marcus, B., and Anderson, K. (2002). Development of a hybrid model for electrical power spot prices. *IEEE Transactions on power systems*, 17(2):257–264.

De Finetti, B. (1959). La probabilità e la statistica nei rapporti con l'induzione, secondo i diversi punti di vista. In Springer, editor, *Induzione e Statistica - Lectures given at a Summer School of the Centro Internazionale Matematico Estivo (C.I.M.E.) held in Varenna (Como), Italy, June 1-10, 1959*, Science and Business Media, chapter 1, pages 1–122. Verlag Berlin Heidelberg.

De Jong, C. and Schneider, S. (2009). Cointegration between gas and power spot prices. *The Journal of Energy Markets*, 2(3):27–46.

Deutsche Bank (2013). *Oil and Gas for Beginners* . Markets Research. Deutsche Bank, London, UK.

Di Renzo, M., Imbriglio, L., Graziosi, F., and Santucci, F. (2009). Smolyak's algorithm: a simple and accurate framework for the analysis of correlated log-normal power-sums. *IEEE Communications Letters*, 13(9):673–675.

Dickey, D. A. and Fuller, W. A. (1979). Distribution of the Estimators for Autoregressive Time Series With a Unit Root. *The Journal of American Statistical Association*, 74(366):427–431.

Dickey, D. A. and Fuller, W. A. (1981). Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root. *Econometrica*, 49(4):1057–1072.

Doeblin, W. (1940). Sur l'équation de Kolmogoroff. In *Pli cacheté à l'Académie des Sciences*, number 331 (2000). B. Bru et M. Yor, CRAS, Paris.

Duff, I. S., Erisman, A. M., and Reid, J. K. (1989). *Direct methods for sparse matrices*. Monographs on Numerical Analysis. Oxford Science Publications, Great Clarendon Street, Oxford OX2 6DP.

Edison, T. (1880). The Electric Light. *New York Herald January, 4 1880*, page 6.

Edoli, E., Tasinato, D., and Vargiolu, T. (2013). Calibration of a multifactor model for the forward markets of several commodities. *Optimization*, 62(11):1553–1574.

Emery, G. W. and Liu, Q. W. (2002). An Analysis of the Relationship between Electricity and Natural-Gas Futures Prices. *The Journal of Futures Markets*, 22(2):95–122.

Engle, R. F. and Granger, C. W. J. (1987). Co-Integration and Error Correction: Representation, Estimation, and Testing. *Econometrica*, 55(2):251–276.

European Commission (2016). The EU Emissions Trading System (EU ETS).

Finger, C. C. (1997). A methodology for stress correlation. In *RiskMetrics Monitor. Fourth Quarter 1997*, pages 3–11.

Fisher, R. A. (1947). Development of the Theory of Experimental Design. In *Proceedings of the International Statistical Conferences*, volume 3, pages 434–439, Washington.

Frydenberg, S., Onochie, J. I., Westgaard, S., Midtsund, N., and Ueland, H. (2014). Long-term relationships between electricity and oil, gas and coal future prices-evidence from Nordic countries, Continental Europe and the United Kingdom. *OPEC Energy Review*, 38(2):216–242.

Gatarek, L. T. and Johansen, S. (2016). Optimal hedging with the cointegrated vector autoregressive model allowing for heteroscedastic errors. *Working Paper*.

Gibson, R. and Schwartz, E. S. (1990). Stochastic Convenience Yield and the Pricing of Oil Contingent Claims. *The Journal of Finance*, 45(3):959–976.

Girma, P. B. and Paulson, A. S. (1999). Risk Arbitrage Opportunities in Petroleum Futures Spreads. *The Journal of Futures Markets*, 19(8):931–955.

Goldberger, A. S. (1991). *A Course in Econometrics*. Harvard University Press, Cambridge, Massachusetts.

Granger, C. W. J. (1969). Investigating Causal Relations by Econometric Models and Cross-spectral Methods. *Econometrica*, 37(3):424–438.

Granger, C. W. J. (1981). Some Properties of Time Series Data and Their Use in Econometric Model Specification. *Journal of Econometrics*, 16(1):121–130.

Grone, R., Johnson, C. R., Sá, E. M., and Wolkowicz, H. (1984). Positive definite completions of partial Hermitian matrices. *Linear Algebra and its Applications*, 58:109–124.

Gujarati, D. N. and Porter, D. C. (2008). *Basic Econometrics*. Economics. McGraw-Hill Irwin, 1221 Avenue of the Americas, New York, NY, 10020, 5th edition.

Gyöngy, I. (1986). Mimicking the one-dimensional marginal distributions of processes having an ito differential. *Probability Theory and Related Fields*, 71(4):501–516.

Heiss, F. and Winschel, V. (2008). Likelihood approximation by numerical integration on sparse grids. *Journal of Econometrics*, 144(1):62–80.

Hendry, D. F. and Juselius, K. (2000). Explaining Cointegration Analysis: Part 1. *The Energy Journal*, 21(1):1–42.

Heren (2016). ICIS - Heren.

Hicks, J. R. (1939). *Value and Capital*. Oxford University Press, London, 2nd edition.

Higham, N. J. (2002). Computing the nearest correlation matrix-a problem from finance. *Numerical Analysis*, 22:329–343.

Higham, N. J. and Strabić, N. (2016). Bounds for the Distance to the Nearest Correlation Matrix. *Journal on Matrix Analysis and Applications*, 37(3):1088–1102.

Hilliard, J. E. and Reis, J. (1998). Valuation of Commodity Futures and Options Under Stochastic Convenience Yields,Interest Rates, and Jump Diffusions in the Spot. *The Journal of Financial and Quantitative Analysis*, 33(1):61–86.

Hull, J. C. (2015). *Options, Futures, and Other Derivatives*. Pearson, New York, 9th edition.

ICE (2016). Intercontinental Exchange.

IEA (2016a). Coal: Medium-Term Market Report 2016.

IEA (2016b). International Energy Agency.

IEA (2016c). International Energy Agency Unit Converter.

IEA (2016d). Key Natural Gas Trends.

IHS (2016). Drivers of European Power Prices: Detailed review of the six largest markets.

International Gas Union (2016). Wholesale Gas Price Survey 2016 Edition: A Global Review of Price Formation Mechanisms.

Itô, K. (1944). Stochastic Integral. *Proceedings of the Japan Academy, Series A, Mathematical Sciences*, 20(8):519–524.

Joëts, M. and Mignon, V. (2012). On the link between forward energy prices: A nonlinear panel cointegration approach. *Energy Economics*, 34(4):1170–1175.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control*, 12(2-3):231–254.

Johansen, S. (1991). Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models. *Econometrica*, 59(6):1551–1580.

Johansen, S. (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Advanced Texts in Econometrics. Oxford University Press, Great Clarendon Street, Oxford OX2 6DP.

Johansen, S. (2000). Modelling of cointegration in the vector autoregressive model. *Economic Modelling*, 17(3):359–373.

Juhl, T., Kawaller, I. G., and Koch, P. D. (2012). The Effect of the Hedge Horizon on Optimal Hedge Size and Effectiveness When Prices Are Cointegrated. *The Journal of Futures Markets*, 32(9):837–876.

Kahl, C. and Günther, M. (2008). Complete the Correlation Matrix. In Breitner, M. H., Denk, G., and Rentrop, P., editors, *From Nano to Space: Applied Mathematics Inspired by Roland Bulirsch*, chapter 4, pages 229–244. Springer, Verlag Berlin Heidelberg.

Kaldor, N. (1939). Speculation and economic stability. *The Review of Economic Studies*, 7(1):1–27.

Kanamura, T. and Ohashi, K. (2007). A structural model for electricity prices with spikes: measurement of spike risk and optimal policies for hydropower plant operation. *Energy Economics*, 29(5):1010–1032.

Kercheval, A. N. (2006). On Rebonato and Jäckel's parametrization method for finding nearest correlation matrices. *Department of Mathematics.*

Keynes, J. M. (1923). Some aspects of commodity markets. *The Manchester Guardian Commercial, Reconstruction Supplement, 29 March.*

Kiesel, R., Schindlmayr, G., and Börger, R. H. (2009). A two-factor model for the electricity forward market. *Quantitative Finance*, 9(3):279–287.

Koenig, P. (2011). *Modelling Correlation in Carbon and Energy Markets*, volume 1107 of *EPRG Working Paper, Electricity Policy Research Group.* University of Cambridge, Cambridge, UK.

Kupiec, P. (1998). Stress Testing in a Value at Risk Framework. In Dempster, M., editor, *Risk Management: Value at Risk and Beyond, 2002*, chapter 3, pages 76–99. Cambridge University Press, The Edinburgh Building, Cambridge CB2 2RU, United Kingdom. First published in *Journal of Derivatives, Fall, 1998*, pages 7-23.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., and Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root. *Journal of Econometrics*, 54(1-3):159–178.

Lanne, M., Saikkonen, P., and Lütkepohl, H. (2002). Comparison of unit root tests for time series with level shifts. *The Journal of time series analysis*, 23(6):667–685.

Leontief, W. (1936). Composite Commodities and the Problem of Index Numbers. *Econometrica*, 4(1):39–59.

Levin, A., Lin, C.-F., and Chu, C.-S. J. (2002). Unit root tests in panel data: asymptotic and finite-sample properties. *The Journal of Econometrics*, 108(1):1–24.

Locatelli, M. and Schoen, F. (1999). Random Linkage: a family of acceptance/rejection algorithms for global optimisation. *Mathematical Programming*, 85(2):379–396.

Lucas, A. (1998). Inference on cointegrating ranks using lr and lm tests based on pseudo-likelihoods. *Econometrics Reviews*, 17(2):185–214.

Lundquist, M. E. and Johnson, C. R. (1991). Linearly constrained positive definite completions. *Linear Algebra and its Applications*, 150:195–207.

Lütkepohl, H. (1991). *Introduction to Multiple Time Series Analysis*. Verlag Berlin Heidelberg, springer edition.

Lütkepohl, H. and Krätzig, M. (2004). *Applied Time Series Econometrics*. Themes in Modern Econometrics. Cambridge, UK, cambridge university press edition.

Lyle, M. R. and Elliott, R. J. (2009). A "simple" hybrid model for power derivatives. *Energy Economics*, 31(5):757–767.

Madaleno, M., Moutinho, V., and Mota, J. (2015). Time Relationships among Electricity and Fossil Fuel Prices: Industry and Households in Europe. *International Journal of Energy Economics and Policy*, 5(2):525–533.

Maddala, G. S. (1992). *Introduction to Econometrics*. Macmillan Publishing Company, New York, 2nd edition.

MATLAB Econometrics Toolbox (2015). *MATLAB Release 2015b*. The Mathworks, Inc., Natick, Massachusetts, United States.

MATLAB Global Optimization Toolbox (2015). *MATLAB Release 2015b*. The Mathworks, Inc., Natick, Massachusetts, United States.

MATLAB Parallel Computing Toolbox (2015). *MATLAB Release 2015b*. The Mathworks, Inc., Natick, Massachusetts, United States.

Miltersen, K. R. and Schwartz, E. S. (1998). Pricing of Options on Commodity Futures with Stochastic Term Structures ofConvenience Yields and Interest Rates. *The Journal of Financial and Quantitative Analysis*, 33(1):33–59.

Moosa, I. A. and Al-Loughani, N. E. (1995). The effectiveness of arbitrage and speculation in the crude oil futures market. *The Journal of Futures Markets*, 15(2):167–186.

Murray, M. P. (1994). A Drunk and Her Dog: An Illustration of Cointegration and Error Correction. *The American Statistician*, 48(1):37–39.

Niu, S., Jia, Y., Wang, W., He, R., Hu, L., and Liu, Y. (2013). Electricity consumption and human development level: A comparative analysis based on panel data for 50 countries. *Electrical Power and Energy Systems*, 53:338–347.

Panagiotidis, T. and Rutledge, E. (2007). Oil and gas markets in the UK: Evidence from a cointegrating approach. *Energy Economics*, 29(2):329–347.

Pedroni, P. (2001). Purchasing Power Parity Tests in Cointegrated Panels. *Review of Economics and Statistics*, 83(4):727–731.

Pedroni, P. (2004). Panel Cointegration: Asymptotic and Finite Sample Properties of Pooled Time Series Tests with an Application to the PPP Hypothesis. *Econometric Theory*, 20(3):597–625.

Pérez-Arriaga, I. J. (2013). *Regulation of the Power Sector*. Power Systems. Springer, London Heidelberg New York Dordrecht.

Peroni, E. and McNown, R. (1998). Noninformative And Informative Tests Of Efficiency In Three Energy Futures Markets. *The Journal of Futures Markets*, 18(8):939–964.

Phillips, P. C. B. and Perron, P. (1988). Testing for a unit root in time series regression. *Biomètrica*, 75(2):335–346.

Pirrong, C. and Jermakyan, M. (2008). The price of power: The valuation of power and weather derivatives. *Journal of Banking and Finance*, 32(12):2520–2529.

Platts (2016). Methodology and Specifications Guide European Electricity Assessments and Indices.

Qi, H. and Sun, D. (2006). A Quadratically Convergent Newton Method for Computing the Nearest Correlation Matrix. *SIAM Journal on Matrix Analysis and Applications*, 28(2):360–385.

Rebonato, R. and Jäckel, P. (1999). The most general methodology for creating a valid correlation matrix for risk management and option pricing purposes. *Journal of Risk*, 2(2):17–27.

Rittler, D. (2012). Price discovery and volatility spillovers in the European Union emissions trading scheme: A high-frequency analysis. *The Journal of Banking and Finance*, 36(3):774–785.

Roncoroni, A., Fusai, G., and Cummins, M. (2015). *Handbook of Multi-Commodity Markets and Products: Structuring, Trading and Risk Management*. Wiley Finance Series. Wiley, Chichester, UK.

Saikkonen, P. and Lütkepohl, H. (2002). Testing for a unit root in a time series with a level shift at unknown time. *Econometric Theory*, 18(2):313–348.

Samuelson, P. A. (1965). Proof That Properly Anticipated Prices Fluctuate Randomly. *Industrial Management Review*, 6(2):41–49.

Schott, J. R. (2016). *Matrix Analysis for Statistics*. Wiley Series in Probability and Statistics. Wiley, Hoboken, New Jersey, 3rd edition.

Schwartz, E. S. (1997). The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging. *The Journal of Finance*, 52(3):922–973.

Schwartz, E. S. (1998). Valuing long-term commodity assets. *Journal of Energy Finance and Development*, 3(2):85–99.

Schwarz, T. V. and Szakmary, A. C. (1994). Price Discovery in Petroleum Markets: Arbitrage, Cointegration, and the Time Interval of Analysis. *The Journal of Futures Markets*, 14(2):147–167.

Serletis, A. and Herbert, J. (1999). The message in North American energy prices. *Energy Economics*, 21(5):471–483.

Shreve, S. (2004). *Stochastic Calculus for Finance II. Continuous-Time Models*. Springer Finance. Springer, Berlin Heidelberg New York.

Simon, D. P. (1999). The Soybean Crush Spread: Empirical Evidence and Trading Strategies. *The Journal of Futures Markets*, 19(3):271–289.

Simonian, J. (2010). The most simple methodology to create a valid correlation matrix for risk management and option pricing purposes. *Applied Economics Letters*, 17:18:1767–1768.

Sims, C. A. (1980). Macroeconomics and Reality. *Econometrica*, 48(1):1–48.

Smith, R. L. (2008). The positive definite completion problem revisited. *Linear Algebra and its Applications*, 429(7):1442–1452.

Smolyak, S. (1963). Quadrature and interpolation formulas for tensor products of certain classes of functions. *Soviet Mathematics Doklady*, 4:240–243.

Stein, M. (1987). Large Sample Properties of Simulations Using Latin Hypercube Sampling. *Technometrics*, 29(2):143–151.

Turing, A. M. (1948). Rounding-off errors in matrix processes. *The Quarterly Journal of Mechanics and Applied Mathematics*, 1(1):287–308.

Ugray, Z., Lasdon, L., Plummer, J., Glover, F., and Martí, J. K. R. (2007). Scatter Search and Local NLP Solvers: A Multistart Framework for Global Optimization. *INFORMS Journal on Computing*, 19(3):328–340.

Vasicek, O. (1977). An Equilibrium Characterization of the term structure. *Journal of Financial Economics*, 5(2):177–188.

Westgaard, S., Estenstad, M., Seim, M., and Frydenberg, S. (2011). Co-integration of ICE Gas oil and Crude oil futures. *Energy Economics*, 33(2):311–320.

Wooldridge, J. M. (2013). *Introductory Econometrics: A Modern Approach*. Cengage Learning, South-Western 5191 Natorp Boulevard Mason, OH 45040 USA, 5th edition.

Working, H. (1949). The theory of price of storage. *American Economic Review*, 39(6):1254–1262.

World Coal Association (2016). What is coal?

Wårrel, L. (2006). Market integration in the international coal industry: a co-integration approach. *The Energy Journal*, 27(1):99–118.

Wright, B. D. and Williams, J. C. (1989). A theory of negative prices of storage. *Journal of Futures Markets*, 9(1):1–13.

Yang, J., Bessler, D. A., and Leatham, D. J. (2001). Asset Storability And Price Discovery In Commodity Futures Market: A New Look. *The Journal of Futures Markets*, 21(3):279–300.