JAMHUURIYADDA DIMOQRADIGA SOOMAALIYA

-WASAARADDA WAXBARASHADA IYO BARBAARINTA-

BUUGGA XISAABTA EE LABAAD

DUGSIYADA SARE EE TABABARKA MACALLIMIINTA

MUQDISHO, 1976

HORDHAC

Buuggani waxa weeye buugga labaad ee xisaabta ah, looguna talagalay macalimiinta tababarka ku soo qaadatay hal sano dugsiga Tababarka ee Macallimiinta ee ku yaalla Xalane. Macalimiintaasi muddo laba sannadood ah ayey buuggan aqoon dirsi ahaan ku baran doonaan.

Waxa buuggan qoray Guddi macalimiin ah oo loo saaray sannadkii 1976. Guddigaasina waxa uu ka koobnaa Jaallayaasha kala ah:- Cali Iid Ibrahim Axmed, Faarax Maxamed Shire, Xirsi Caynab Maxamed, iyo Xuseen Cumar Geelle. Waxa buugga sawirrada u sameeyey Maxamed Cabdalle Axmed, Waxana garaacay Cabdi Xirsi Qanyare. Dhammaantoodna waa ay mahadsan yihiin.

Sidii buugga kowaad, buuggan waxa lagu koobay fikradaha xisaabeed ee ku jira manhajka saddexda sannadood ee u horreeya dugsiyada Sare; hase yeeshee, mawaadiicda buuggan ku jirta si tafatiran oo saafan ayaa looga hadlay. Kii kowaadse guud ahaan ayuunbaa loo taataabtay mawaadiicdiisa. Waxa kale oo jirta in manhajka saddexda sannadood ee u horreeya dugsiga sare aan sidiisii loo raacin, sababta oo ah waxa jirta in mawaadiic manhajka dugsiga sare ku jirta la iska dhaafay, qaar kale oo aan ku jirinna lagu soo daray, sida manhajka xisaabta ee Tababarka macallimiinta ku xusan.

Ujeedada buuggan lagu leeyahay waxa weeye in uu aqoonta xisaabeed ee ardayga Xalane hal sano ku soo jiray si qoto dheer kor ugu qaado. Barashada fikradaha buuggan ku jiraa waxa ay hirgeli kartaa oo keliya haddii fikradihii xisaabeed ee buugga kowaad si dhab ah hore loogu soo dhuuxay.

Mawaadiicda buugga ku jirta waxa loo habeeyey loona taxay sidii ay guddidu istidhi waay ay ugu habbon tahay. Hase yeeshee, waa ay dhici kartaa in taxa iyo horsamaanta mawaadiicdaanay ahayn sidii ugu haboonayd. Markaa barayaasha loogu talagalay buuggan waxa lagu baraarujinayaa in ay u raacaan buugga sida ugu hawl yar ee ay u maali karaan fikradaha lagu bandhigay buugga.

Xifaaleyn kasta oo ku saabsan buuggaanna aad ayaa loo soo dhaweynayaa.

BASHIIR FAARAX KAAHIYE
MAAMULAHA XAFIISKA MANAAHIJTA W.W.B.

URUR

Fikradda urur ee walaxyo waxa weeye mid ku caan ah xisaabta iyo nolosha guud ahaanba. Fikrad xisaabeedda urur sida ay ugu dheehan tahay nolosha waxa tusaalayaal badan lagaga bixiyey buugga kowaad ee xisaabta iyada oo la qaatay tusaalooyin badan oo laga dhadhansanayo macnaha urur sida ereyada kadin, guuto, raxan, iwm.

TUSAALE 1: Xayn ari ah i waa urur. Waxa uu ururku ka kooban yahay waa neefaf ari ah. Neefafkaasna waxa loo yaqaan ku-tirsanayaasha ururka; sababtoo ah xaynta waxa abuuray ama sameeyey neefaf la isu ururiyey loona arkay inay samaynayaan wax dhan ama idil.

TUSAALE 2: Tirooyinka mutuxan ee 1,2,3,5,7,11,........ waxa ay suubiyaan urur, tiro kasta oo mutuxanina waa ku-tirsane urur.

Sida runtu tahay way adag tahay in qeexid kooban oo sugan aynu siino ereyga urur, hase yeeshee haddii aynu isku dayno inaynu si kooban u qeexno fikrad xisaabeedda urur waxa aynu u suubinaynaa sida soo socota:

QEEXID: Urur waa wax idil oo ka kooban walaxyo gaar ahaaneed oo loo yaqaan ku-tirsanayaasha ururka.

Ururada waxa lagu isticmaalaa xaruufta afka ee waaweyn si ay ugu taagnaadaan ururada. Xaraf kasta oo ka mid ah xuruufta waaweyn ayaa u taagnaan kara urur. Matalan waxa laga yaabaa in xarafka A uu u taagan yahay ururka ardayda ee fasalka kowaad ee dugsiyada sare. Summad ahaanna waxa loo qori karaa A = {ardayda fasalka kowaad ee dugsiyada sare }. Sidii aynu horeba ugu soo sheegnay buugga kowaad ee xisaabta, tidicyo ayaa lagu xiraa ururada. Isla markaas xuruufta afka ee yar-yar waxa aynu ku isticmaalnaa inay u taagnaadaan ku-tirsanayaasha urur. Bal ka soo qaad in X ay u taagan tahay urur, m-na ay u taagan tahay

ku-tirsane ka mid ah ku-tirsaneyaasha ururka X, kolkaa summad ahaan waxaynu u qori karaa m $\in$ X (m waa ku-tirsane X). Sidoo kale bal ka soo qaad in X ay tahay urur, hase yeeshee m ay tahay ku-tirsane urur kale, kolkaa waxaynu niraahnaa m ma aha ku-tirsane X; summad ahaan waxa aynu u qoraa m $\notin$ X (m ma aha ku-tirsane X).

TUSAALE: 1 Haddii A ay tahay ururka tirooyinka dhabanka ah, kolkaa 2 $\in$ A laakiin 7 $\notin$ A.

Marka aynu urur sugayno laba dariiqo ayuun baa mid la isticmaalaa. Labada dariiqo waxa ay kala yihiin dariiqada taxidda (ama dariiqada roostar) ama dariiqada astaan qeexida. Haddaba haddii aynu ku horayno dariiqada taxidda, bal aan fiirino sida urur loogu sugo dariiqadan iyada ah. Runtii dariiqada taxiddu aad bay u fududahay sababtoo ah si hawl yar ayeynu ugu taxnaa ku-tirsanayaasha ururka tidicyada dhexdooda. Matalan haddii urur ay ku jiraan ku-tirsanayaasha a,b,c,..... kolkaa ururka waxa aynu u qorna

A = $\{a, b, c, \ldots\ldots\}$. Run ahaantiina waxa aynu ka hadlaynaa, xooggana aynu saaraynaa waa ku-tirsanayaasha ah a, b, c, ...

TUSAALE 2:

A = $\{1, 2, 3, 4\}$ waxa ay muujinaysaa in ururka A uu leeyahay ku-tirsanayaasha 1, 2, 3, iyo 4.

TUSAALE 3: X = $\{a, b, c\}$ waxa ay muujinaysaa in ururka X uu leeyahay ku-tirsanayaasha a, b iyo c.

TUSAALE 4: B $\{1, 2, 3, 5, 7, 11, \ldots\}$ waxa ay muujinaysaa in ururka B uu leeyahay dhammaan ku-tirsanayaasha tirooyinka mutuxan.

TUSAALE 5: H = $\{\{1, 2\}, \{1, 3\}, \{2,3\}\}$ waxa ay muujinaysaa in ururka H uu leeyahay ku-tirsanayaasha ah ururada $\{1,2\}$, $\{1,3\}$, iyo $\{2, 3\}$.

Astaan - Bal iimkana aan milicsano sida dariiqada qeexidda loogu sugo urur. Dariiqadan iyada ahi waxa weeye dariiqo ku sal leh sifeyn. Dariiqadan macneheedu waxa weeye sheegista la sheegayo in ku-tirsanihii kasta ee ka mid ah ururka uu raali gelinayo ka tirsanaantiisa runta ah ee ururkaa isaga ah. Dariiqada qeexidda waxa aynu ku isticmaalaa summadda: oo macneheedu tahay "ee ama oo " sida ka muuqata tusaalooyinkan soo socda:

TUSAALE 6: B = $\{X : X$ waa tiro mutuxan $\}$ waxa ay muujinaysaa in ururka B uu yahay dhammaan ku-tirsanayaasha X ee X ay tahay tiro mutuxan. Tan oo macneheedu yahay B waxa weeye dhammaan tirooyinka mutuxan.

TUSAALE 7: A = $\{X : X^2 - 3X + 2 = 0\}$ waxa ay muujinaysaa in ururka A uu yahay dhammaan ku-tirsanayaasha X ee X ay tahay mid raaligelinaysa isleegta ah $X^2 - 3X + 2 = 0$. Halkan waxa inooga caddaan ah in A = $\{1,2\}$.

LAYLI

Tax ku-tirsanayaasha ku jira dhammaan ururadan soo socda adiga oo N u qaadanaya inay u taagan tahay ururka abyoonayaasha togan, B-na u qaadanaya inay u taagan tahay ururka tirooyinka mutuxan.

1. A = $\{X: X^2 = 25\}$
2. H = $\{X : 3X + 2 = 0$ ama $2X + 3 = 0\}$
3. G = $\{X : X^2 - 4X + 3 = 0$ iyo $2X^2 - 3X + 1 = 0\}$
4. O = $\{X: X \in N$ iyo X oo dhaban ah$\}$
5. F = $\{X : X \in B$ iyo X oo uu u qaybsamo 3$\}$
6. C = $\{X : X \in N$ iyo $X \notin B\}$

7. $K = \{X : X^3 = 27\}$
8. $L = \{X : X^3 = 8\}$
9. $S = \{X : 4X - 2 = 0\}$
10. $N = \{X : X^2 - 2X + 1\}$

HORMO-URUR IYO ISLEEGAANTA URURADA

QEEXID: Ururka A waxa weeye hormo-ururka B, haddii ku-tirsanihii kasta ee ku jira A uu isla markaasna ku jiro ururka B. Haddii ururka B ay ku jiraan ku-tirsanayaal aan ku jirin ururka A, kolkaa ururka A waxa aynu niraahnaa waa hormo-ururka quman ee B. Hormo-ururnimada waxa aynu ku muujinaa summadda C.

TUSAALE 1: Haddii $A = \{1, 3, 4\}$, B-na $= \{1, 2, 3, 4, 5, 6\}$ kolkaa A waxa weeye hormo-urur quman ee B. Weedhan kore waxa si gaaban summad ahaan loogu qori karaa A C B.

TUSAALE 2: Haddii $X = \{1, 3\}$, y-na $= \{X: X^2 - 4X + 3 = 0\}$ kolkaa X waxay hormo-urur u tahay Y; waxaana loo qori karaa X C Y. Ogow in X aanay ahayn hormo-urur quman ee Y.

Waxa aynu niraahnaa urur A waxa uu le'eg yahay urur B haddii iyo haddii oo keliya labada ururba ay ku jiraan isla ku-tirsanayaal (ku-tirsanayaal isku mid ah); waxana summad ahaan loo qoraa A = B. Haddaba si aynu u muujino in laba urur, kaba sóo qaad X iyo Y, ay isle'eg yihiin waxa inagu filan in aynu tusno in XCY isla markaasna in YCX.

TUSAALE 3: Haddii $A = \{1, 2, 3\}$, B-na $= \{3, 1, 2\}$ kolkaa A = B

TUSAALE 4: Haddii A $\{1, 2, 3\}$, B-na $= \{X: X$ waa tiro mutuxan kana yar 6$\}$ kolkaa A = B.

Waxa aad kala ilaalisaa inaad isku qaladid marka ay laba urur isle'eg yihiin iyo marka ay laba urur isku dhigmaan. Way nagii hore u soo qeexnay isle'egaanta laba urur, bal imminkana aan ka bixino qeexid gaaban isudhignaanta ee laba urur.

QEEXID: Labada urur A iyo B waa ay isu dhigan yihiin haddii isku beegnaan mid-mid ahi ay ka dhexayso ku-tirsanayaasha ururada. Ama si kaleba labada urur A iyo B waa ay isu-dhigan yihiin haddii qiimayaasha labada urur ay isle'eg yihiin. Sumad ahaanna waxa loo qoraa A $\sim$ B.

TUSAALE 5: Haddii $A = \{1, 2, 3\}$ kolkaa qiimaha A waa 3. Waxaba iska dhici karta in aad ku aragto weedhan qormada n (A) = 3. Macnaha qormadanna waxa weeye ku-tirsanayaasha ururka A waa 3.

TUSAALE 6: Hadii $A = \{1, 2, 3\}$, B-na $= \{5, 6, 7\}$ kolkaa A iyo B waa ay isu dhigmaan; sababtoo ah n (A) = 3 isla markaas n(B) = 3. Kolka mar haddii n(A) = n(B) = 3 markaa A iyo B waa ay isu dhigmaan.

LAYLI:

1. Ururadan hoos ku taxan sheeg kuwa kooban iyo kuwa tirobeelka ah. Haddaba qaado kuwa kooban oo kala sheeg kuwa isu dhigma iyo kuwa isle'eg.

b) Ururka ka kooban labada tiro ee ugu horaysa tirsiimada kisiga ah.

t) Ururka ka kooban dhammaan tirooyinka tirsiimada kisiga ah. ee ka yar 5.

j) Ururka ka kooban dhammaan tirooyinka tirsiimada kisiga ah.

x) Ururka ka kooban xarfaha ereyga "MUQDISHO".

kh) Ururka ka kooban ku-tirsanayaasha kala ah 1 iyo 3.

2. U fiirso weedhahan soo socda dabeedna sheeg in weedh kastaa ay tahay run ama been:

b) $x \in \{x, y, s\}$ t) $x \subset \{x, y, s\}$ j) $\{x\} \in \{x, y, s\}$

x) $\{x\} \subset \{x, y, s\}$

3. Bal ka soo qaad in G ay tahay dhammaan ururka afar geeslayaasha <u>kalgalsan</u> (cyclic quadrilaterals)
in B ay tahay dhammaan ururka barbaroolayaasha,
in Q ay tahay dhammaan ururka afargeeslayaasha,
in R ay tahay dhammaan ururka laydiyaasha,
in S ay tahay dhammaan ururka laba jibbaaranayaasha,
in T ay tahay dhammaan ururka kooraha,
in F ay tahay dhammaan ururka qandhaasaha,

Haddaba kuwan kuwee baa sax ah?

b) $S \subset R$
t) $F \subset B$
j) $R \subset B \subset Q$
x) $T \subset B$
kh) $S \subset R \subset B \subset T \subset Q$
d) $I \subset G \subset Q$

<u>URURRO GAAR AHAANEED</u>

Ururka dhammaan ay ku jiraan ku-tirsanayaasha laga sheekeenayaa ee mas'alo gaar ahaaneed ayaa la yiraahdaa urur guud; sida badanna waxa loo taagaa summadda $\cup$. Ogow in ururka guud aanu ahayn wax aan isbeddelin; hase yeeshee waxa uu isla beddelaa hadba ku-tirsanayaasha ururkaa aad ka hadlaysid. Haddii aynu isniraahno qeexid kooban ka bixiya ururka guud waxa uu noqon karaa sida soo socota:

<u>QEEXID</u> : Ururka guud ee hadba la haystaa waxa weeye isku dhammida ku-tirsanayaasha hadba loo qaato inay yihiin ku-tirsanayaasha ururkaa.

<u>QEEXID:</u> Ururka aanay ku jirin wax ku-tirsanayaali waxa loo yaqaan urur madhan , waxana loo taagaa summadda ah $\emptyset$

Ogow in $\emptyset$ ay tahay hormo-ururka urur kasta. Waxa kale oo aad maskaxda ku haysaa in summadaha $\cup$ iyo $\emptyset$ aanay ahayn ku-tirsanayaal; hase yeeshee ay u taagan yihiin urur gaar ahaaneed.

<u>QEEXID</u> : Laba urur A iyo B waa kala edeg haddii iyo haddii oo keliya aanay A iyo B lahayn ku-tirsanayaal ka dhexeeya.

<u>QEEXID:</u> Urur duleedka urur kasta A marka loo eego ama loo fiiriyo urur guud oo la isla ogyahay waxa weeye ururka ku-tirsanayaasha ururka guud ee aan ku jirin ama ka mid ahayn urur A. Summad ahaanna waxa loo qoraa A'. Kolka
$A' = \{x : x \in \cup \text{ iyo } x \notin A\}$

<u>TUSAALE 1:</u> $\emptyset$ iyo $\{\emptyset\}$ waxa weeye laba wax oo aad u kala gedisan ama u kala jaad ah. $\emptyset$ waxa weeye urur madhan. $\{\emptyset\}$ iyana waxa weeye urur uu ku jiro hal ku-tirsane oo ah $\emptyset$.

<u>TUSAALE 2:</u> Fax dhammaan hormo-ururrada ururkan $\{a, b, c\}$.
Immisaa hormo-ururrada-ah kuwa quman?

<u>FURFURIS:</u> Hormo-ururada la rabaa waa
$\{a,b,c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$
$\{a\}$, $\{b\}$, $\{c\}$, $\emptyset$

Toddoba kuwan ka mid ihi waa hormo-ururro quman. Ta keliya ee aan ahayn hormo-urur quaman waa $\{a,b,c\}$.
Ogow in urur kastaa uu isu noqon karo hormo-urur.

<u>TUSAALE 3:</u> Fax dhammaan hormo-ururrada ururkan $\{1,2\}$?

<u>FURFURIS:</u> Hormo-ururrada la rabaa waa $\{1,2\}$, $\{1\}$, $\{2\}$, $\emptyset$.

Waxa aad moodaa in labada tusaale ee 2 iyo 3 ay inagu hogaaminayaan jid lagu helo tirada hormo-ururrada-urur. Haddii aad dib ugu noqotid tusaalaha labaad waxa aad arki kartaa in tirsi ahaan ay dhan yihiin sideed. Haddaba 8 = 2x 2 x 2 ama 2^3. Bal si fiican u dheeho ku-tirsanayaasha tusaalaha. Runtii 3 ku-tirsane oo ah a, b iyo c ayaa ku jira tusaalaha. Haddaba ma kuu muuqan kartaa in jibbaarka salka 2 uu u taagan yahay

ku-risanayaasha ururka. Sidoo kale hormo-ururrada ururka tusaalaha saddexaad waa $4 = 2x2$ ama 2^2. Isla markaas ku-tirsanayaasha ururkani waa 2. Kuwaas oo kala ah 1 iyo 2. Haddaba ma kuu muuqan kartaa in jibbaarka salka 2 uu u taagan yahay ku-tirsanayaasha ururka?

TUSAALE 4: Immisa hormo-urur ayuu leeyahay urur ay ku jiraan n ku-tirsane?

FURFURIS:

Marka aynu dhisayno hormo-urur waxa aynu uga fekeraa in aynu fiirinayno kolkiisa mid kasta oo ka mid ah ku-tirsanayaasha ururka ee n innaga oo ku-tirsane kasta u qoodaynayna laba suure oo ah in aynu haysan karo ama diidi karo. Haddaba marka aynu dariiqadan raacno waxa jiri kara :

$2 \times 2 \times 2 \times 2 \times 2 \ldots\ldots \times 2 = 2^n$ hormo-urur oo suuragal ah.

TUSAALE 5: Haddii ururka guud uu yahay dhammaan ururka abyoonayaasha oo $A = \{ x : x$ waa abyoone dhaban ah $\}$, kolkaa $A' = \{ x : x$ waa abyoone kisi ah $\}$.

TUSAALE: Waa maxay urur duleedka $\emptyset$? Isla markaas waa maxay urur duleedka U ?

QEEXID ahaan $\emptyset' = U'$, $U' = \emptyset$.

XISAABFALADA URURRADA

Sal ka soo qaad inaad haysatid labada urur ee kala ah $B = \{a, b, c, d, e\}$ iyo $F = \{a, e, i, 0, u\}$. Haddii aad si fiican u dheehatid labada urur ee kor lagumagacaabay waxa aad arki kartaa in labadooduba ay yihiin hormo-ururro ka mid ah dhammaan ururka ku-tirsanayaasha xarfaha afka Soomaaliga. Haddaba ururka ku-tirsanayashiisu ay ku jiraan ururka B ama ururka F ama labadooduba waa ururka $D = \{a, b, c, d, e, i, 0, u\}$. Kolka ururka jaadkan

ah ayaa loo yaqaan isutagga ururrada B iyo F. Ururka ku-tirsanayaashiisu ay ku jiraan B iyo F waa ururka $E = \{a, e\}$. Ururka E waxa loo yaqaan dhextaalka B iyo F.

QEEXID: Isutagga laba urur A iyo B waxa weeye ururka ku-tirsanayaashiisu ay ugu yaraan ku jiraan labada urur midkood ama labadoodaba.

Isutagga laba urur A iyo B waxa lagu asteeyaa ama loo taagaa summadda ah $A \cup B$, loona akhriyo isutagga A iyo B. $A \cup B$ waxa qormo urur loogu qori karaa

$$A \cup B = \{x : x \in A \text{ ama } x \in B, \text{ ama labadoodaba}\}$$

QEEXID: Dhextaalka laba urur A iyo B waxa weeye ururka ku-tirsanayaashiisu ay ku jiraan labada urur ee A iyo B.

Dhextaalka laba urur A iyo B waxa lagu asteeyaa summadda ah $A \cap B$, loona akhriyo dhextaalka A iyo B. $A \cap B$ waxa qormo urur loogu qoraa $A \cap B$ $\{x : x \in A$ iyo $x \in B\}$

Ogow in haddii $x \in A$ ama $x \in B$, kolkaa in $x \in A \cup B$. Isla markaas haddii $x \in A$ oo weliba $x \in B$ markaana $x \in A \cap B$. Rogga weedhahan kore isna waa run. Taas oo ah haddii $x \in A \cup B$, kolkaa $x \in A$ ama $x \in B$ ama x way ku jiri kartaaA iyo B. Sidoo kale haddii $x \in A \cap B$ kolkaa $x \in A$ islamarkaas $x \in B$.

TUSAALE 1: Haddii $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$
Kolkaa $A \cup B \{1, 2, 3, 4, 5, 6, 8\}$, $A \cap B = \{2, 4\}$.

TUSAALE 2: Haddii A ay tahay dhammaan ururka eyda, isla markaasna B ay tahay dhammaan ururka mukulaalaha ama bisadaha markaa $A \cup B$ waa ururka xayawaanka ah ee ah eyda ama mukulaalaha. Hase yeeshee $A \cap B = \emptyset$.

TUSAALE 3: Haddii A ay tahay urur kasta, kolkaa $A \cup A' = U$, isla markaas $A \cap A' = \emptyset$

TUSAALE 4:

Adiga oo dib ugu noqonaya weydiinta saddexaad ee laylis ku saabsan hormo-ururrada,kuwee baa weedhan sax ah.

b) S = R $\cap$ F (t) R $\subseteq$ C $\cap$ B (j) B = C $\cap$ T

Labada weedhood ee hore waa sax; laakiin ta saddexaad ma aha.

LAYLI

1.- Waxa lagu siiyey A = $\{3,5,7,9,11\}$, B = $\{3,4,5,6,7,8\}$, C = $\{2,4,6,8,10\}$; ururkaaga guud waxa weeye $\{2,3,4,5,6,7,8,9,10,11\}$. Raadi

b) A $\cup$ B (t) B $\cap$ C (j) A' $\cap$ C

x) A $\cap$ (B $\cup$ C) (kh) A' $\cup$ (B' $\cup$ C') (d) (A' $\cup$ B') $\cup$ C

r) (B $\cup$ C)' (s) (A $\cap$ B) $\cup$ (A $\cap$ C)

2.- Waxa lagu siiyey in E ay tahay dhammaan xisaab yahanada afka ingiriiska ku hadla, in G ay tahay dhammaan xisaab yahanada afka jarmalka ku hadla, in R ay tahay dhammaan xisaab yahanada afka Ruushka ku hadla, iyo in U ay tahay dhammaan xisaab yahanada aduunka. Haddaba ereyo ku sharax ururrada soo socda:

b) E $\cup$ G (t) R $\cap$ G (j) R $\cap$ E' (x) G' $\cup$ E

kh) (E $\cap$ R)' (f) E $\cup$ (G $\cap$ R) (s) (E' $\cup$ R') $\cap$ G

3.- Waxa lagu siiyey in ururka guud uu yahay dhammaan ururka abyoonayaasha togan, in A ay tahay dhammaan ururka abyoonayaasha togan ee ka yar ama leeg 6, in E ay tahay dhammaan ururka abyoonayaasha togan ee dhabanka ah, iyo in M ay tahay dhammaan ururka abyoonayaasha togan ee ah dhufsanayaasha 3; kolka u raadi tibaaxo fudud oo ku tibaaxan A, E iyo M ururradan soo socda:

b) $\{3,6\}$ (t) $\{1,3,5\}$ (j) Dhammaan abyoonayaasha togan ee ah dhufsanayaasha 6.

x) Dhammaan abyoonayaasha togan ee ka weyn 6.

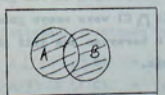
kh) Ururka ay ku jiraan dhammaan dhufsanayaasha 3 iyo dhammaan abyoonayaasha kisiga ah.

4. Ururka ka kooban dhammaan hormo-ururradaurur lagu siiyey, ayaa loo yaqaan urur jibbaar. Sumaad shaanna waxa loo qoraa B(A). Haddaba tus in tirada ku-tirsanayaasha B(A) ay yihiin 2^n, n-na waxa weeye tirada ku-tirsanayaasha A. Haddii A = $\{a,b,c\}$ T = $\{b,c,d\}$, kolkaa qor ku-tirsanayaasha ururrada B (A), B(T) iyo B(A $\cap$ T).

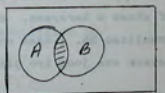
JAANKUSYADA FEN

Had iyo jeer waxa aad u muhiim ah in aad sawir ahaan ku muujiso ururrada si aad u fustid una xaqiijisid xidhiidhka ka dhexeeya ururro. Dariiqo sawir ahaan loogu muujin karo xidhiidhkaas ayaa loo yaqaan jaantuska fen. Jaantuska fen ururka guud waxa lagu muujiyaa dhammaan ururka baraha ee laydi ku dhex jira. Ururrada kale ee ururka guud ku dhex jirana waxa lagu muujaa urur baroodyo ku dhex jira gobolo xiran ama oodan ama si kaleba goobooyin ku dhex jira laydiga. Adiga oo haraynaya ama hoosaynaya bededka habboon, dhammaan racaynaha ururrada waxa lagu muujin karaa sawiro. Bal hadda aan tusaalooyin ku muujino fikradan:

TUSAALE 1:



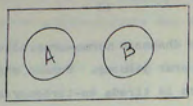
A $\cup$ B



A $\cap$ B

Sh. 1

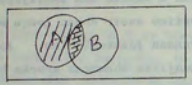
TUSAALE 2:



Ururro kala edeg ah (ama $A \cap B = \emptyset$)

Sh.2

TUSAALE 3: Xaqiiji xidhiidhka ah $A \cup (A \cap B) = A$

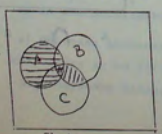


Sh.3

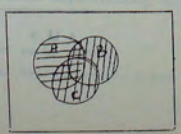
Jaantuskan isaga ah gobolka A waxa loo hareeyey joog ahaan, gobolka $A \cap B$-na waxa loo hareeyey jiif ahaan. $A \cup (A \cap B)$ waxa weeye gobolka joog ahaan u haraysan ama jiif ahaan u haraysan ama joog iyo jiifba u haraysan. Kolka $A \cup (A \cap B) = A$.

TUSAALE 4: Xaqiiji xidhiidhka $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Sh.4, $B \cap C$ waa gobolka joog ahaan u haraysan, A-na waa gobolka jiif ahaan u haraysan. Haddaba $A \cup (B \cap C)$ waxa weeye gobolka ku muujisan Sh. 4 kaas oo joog ahaan u haraysan ama jiif ahaan u haraysan ama joog iyo jiifba u haraysan.



Sh.4



Sh. 5

Sh.5, $A \cup B$ waa gobolka joog ahaan u haraysan, $A \cup C$-na waa gobolka jiif ahaan u haraysan. Haddaba $(A \cup B) \cap (A \cup C)$ waa gobolka joog ahaan iyo jiif ahaanba u haraysan. Haddaba mar haddii gobolada ku beegan $A \cup (B \cap C)$ iyo $(A \cup B) \cap (A \cup C)$ ay midaalsan yihiin, kolkaa waa sax in $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

LAYLI

1.- Adiga oo u qaadanaya in A iyo B aanay ahayn ururro kala edeg ah, ku muuji ururradan soo soodaa jaantuska fen:

b) A' (t) $A' \cap B'$ (j) $(A \cup B)'$ (x) $A' \cap B$

kh) $A \cup B'$ (d) $A' \cup B'$ (r) $(A \cap B)'$

Ka soo dheeg jaantuskaaga laba xidhiidh oo arur.

2.- U qaado in $A \cap B \cap C \neq \emptyset$, dabeedna ku muuji ururradan soo soodaa isla jaantus ama si kalsha ururrada jaantus keliya kuwada muuji:

b) $A \cap B \cap C$ (t) $A \cap B \cap C'$ (j) $A \cap B' \cap C$

x) $A' \cap B \cap C$ (kh) $A \cap B' \cap C'$ (d) $A' \cap B \cap C'$

r) $A' \cap B' \cap C$ (s) $A' \cap B' \cap C'$

Kolka fududee ururka

$(A \cap B \cap C) \cup (A \cap B \cap C') \cup (A \cap B' \cup C) \cup (A' \cap B \cap C) \cup$
$(A \cap B' \cap C') \cup (A' \cap B \cap C') \cup (A' \cap B' \cap C) \cup (A' \cap B' \cap C')$

[Waxa aad u qaadan kartaa in $A \cap B \cap C = A \quad (B \cap C)$]

3.- Isticmaal jaantuska fen si aad u xaqiijisid in:

b) $A \cup (A' \cap B) = A \cup B$

t) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

j) $(A \cup B) \cap (A' \cup C) \cap (B \cup C) = (A \cup B) \cap (A' \cup C)$

XEEGARKA XISAABTALKA

Si aynu ugu sasne dhidibada midaalnimada aljebrada ururrada waxa aynu adeegsan karaa jaantuska fen ama waxa aynu raaci karaa dariiqadan soo socda:

Tusaale 1: Muuji ama tus in $A \cup (A \cap B) = A$.

Dariiqada aynu raacaynaa waxa ay ku sal leedahay hubaasha ah haddii $X \subseteq Y$ isla markaasna $Y \subseteq X$ kolkaa waxa caddaan ah in $X = Y$. Doodeenu waxa ay u qaybsan tahay laba.

I. Bal ka soo qaad in $x \in A$
Kolkaa $x \in A \cup (A \cap B)$
Markaa $A \subseteq A \cup (A \cap B)$ - - - - - - - - - (1)

II. Bal ka soo qaad in $x \in A \cup (A \cap B)$
Kolkaa $x \in A$ ama $x \in A \cap B$
Taasi waxa u sii kala bixi karta
$x \in A$, ama $x \in A$ iyo $x \in B$
:. $x \in A$

Haddaba $A \cup (A \cap B) \subseteq A$ - - - - - - - - -(2)

Haddaba jeedeeyooyinka (1) iyo (2) waxa ay inna siinayaan in $A \cup (A \cap B) = A$

Tusaale 2: Tus in $(A \cup B)' = A' \cap B'$.

Waxa aynu raacaynaa dariiqadii aynu tusaalaha hore ku isticmaalay.

I. Bal u qaado in $x \in (A \cup B)'$
Kolkaa $x \notin A \cup B$
:. $x \notin A$ iyo $x \notin B$
:. $x \in A'$ iyo $x \in B'$
:. $x \in A' \cap B'$

Haddaba $(A \cup B)' \subseteq A' \cap B'$ - - - - - - - - - - (1)

II. Sidoo kale $A' \cap B' \subseteq (A \cup B)'$ - - - - - - - - (2)
Kolka jedeeyooyinka (1) iyo (2) waxa ay inna siinayaan in $(A \cup B)' = A' \cap B'$.
B

Bal hadda aan taxno xeerarka saldhigga u ah aljebrada ururrada. Xeerarkan qaarkood dareen ahaan ayey u muuqan karaan, kuwa kale se aad caan uma aha. Dhisnaanta xeer kasta waxa loo sugi karaa dariiqooyinka lagu isticmaalay tusaalooyinka kore.

XEERARKA KALA HORMARINTA

(1b) $A \cup B = B \cup A$ (1t) $A \cap B = B \cap A$

XEERARKA HORMOGELINTA

(2b) $A \cup (B \cup C) = (A \cup B) \cup C$ (2t) $A \cap (B \cap C) = (A \cap B) \cap C$

XEERARKA KALA DaIGGA

(3b) $A \cup (B \cup C) = (A \cup B) \cup C$ (3t) $A \cap (B \cup C) = (A \cap B) \cap C$

XEERARKA ISKU NOQOD

(4b) $A \cup A = A$ (4t) $A \cap A = A$

XEERARKA MUUGISTA

(5b) $A \cup (A \cap B) = A$ (5t) $A \cap (A \cup B) = A$

XEERARKA URUR-DULEEDNIMADA

(6b) $A \cup A' = U$ (6t) $A \cap A' = \emptyset$

XEER URUR-DULEEDNIMADA LABAALE

(7) $(A')' = A$

XEERKA DIMOORGAN

(8b) $(A \cup B)' = A' \cap B'$ (8t) $(A \cap B)' = A' \cup B'$

XEERARKA KU LUG LEn $\emptyset$ IYO U

(9b) $U \cup A = U$ (9t) $\emptyset \cap A = \emptyset$
(10b) $\emptyset \cup A = A$ (10t) $U \cap A = A$
(11b) $\emptyset' = U$ (11t) $U' = \emptyset$

XEERARKA nORMO-URURRADA

12. naddii $A \subset B$, $B \subset C$ kolkaa $A \subset C$

13. naddii $A \subset B$, $A \subset C$ kolkaa $A \subset B \cap C$

14. naddii $A \subset B$ kolkaa $A \subset B \cup C$ [C waa urur kasta oo la iska qaato]

15. $A \subset B$ haddii iyo haddii oo keliya oo $B'CA'$

TUSAALE 3: Tus in haddii $A \subset B$, $A \subset C$ kolkaa $A \subset B \cap C$

Bal ka soo qaad in $x \in A$

Kolkaa $x \in B$ iyo $x \in C$

:. $x \in B \cap C$

:. $A \subset B \cap C$

TUSAALE 4: Tus in $A \subset B$ haddii iyo haddii oo keliya oo $B'CA'$

I. Siin $B'CA'$, u qaado in $x \in A$

Kolkaa $x \notin A'$

:. $x \notin B'$ mar haddii $B'C A'$

:. $x \in B$

:. $A \subset B$

II. Siin $A \subset B$, u qaado in $x \in B'$

Kolkaa $x \in B$

:. $x \notin A$ mar haddii $A \subset B$

:. $x \in A'$

:. $B' \subset A'$

Haddaba $A \subset B$ haddii iyo haddii oo keliya oo $B' \subset A'$

LAYLI

1. Tus in $A \cup B = B \cup A$ iyo in $A \cap B = B \cap A$
2. Tus in $A \cap (A \cup B) = A$
3. Tus in $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
4. Tus in $(A \cap B)' = A' \cup B'$
5. Tus in (b) haddii $A \subset B$, $B \subset C$ kolkaa $A \subset C$

(t) haddii $A \subset B$ kolkaa $A \subset B \cup C$ [C waxa weeye ururogaan la isaga qaatay]

(j) $A \subset B$ haddii iyo haddii oo keliya oo $A \cap B' = \emptyset$

FUDUDAYNTA TIBAAXAHA KU LUG LEn URURRADA

Inaga oo isticmaalayna xeerarka saldhigga u ah aragtida urur, tibaaxaha ku lugta lehururradatartiib tartiib ayaa loo fududayn karaa ciida tibaaxaha aljebra caadiga ah loo fududeeyo. Bal tusaalayaal aan ku muujino ujeedadeena.

TUSAALE 1: Tus in $A \cup (A' \cap B) = A \cup B$

$A \cup (A' \cap B) = (A \cup A') \cap (A \cup B)$..... (xeerka 3b)

$= U \cap (A \cup b)$ - - - - - - - (xeerka 6b)

$= A \cup B$ - - - - - - - - - (xeerka 10t)

TUSAALE 2: Fududee $\{A \cap (A' \cup B)\} \cup \{B \cap (A' \cup B')\}$

$\{A \cap (A' \cup B)\} \cup \{B \cap (A' \cup B')\}$

$= \{(A \cap A) \cup (A \cap B)\} \cup \{(B \cap A') \cup (B \cap B')\}$--------(xeerka 3t)

$= \{\emptyset \cup (A \cap B)\} \cup \{(B \cap A') \cup \emptyset\}$ - - - - - - - - - - (xeerka 6t)

$= \{A \cap B) \cup \{(B \cap A') \cup \emptyset\}$ - - - - - - - - - - - (xeerka 10b)

$= (A \cap B) \cup \{\emptyset \cup (B \cap A)\}$ - - - - - - - - - - - - (xeerka 1b)

$= (A \cap B) \cup (B \cap A')$ - - - - - - - - - - - - - (xeerka 10b)

$= (B \cap A) \cup (B \cap A')$ - - - - - - - - - - - - - (xeerka 1t)

$= B \cap (A \cup A')$ - - - - - - - - - - - - - - (xeerka 3t)

$= B \cap U$ - - - - - - - - - - - - - - - - (Xeerka 6b)

$= U \cap B$ - - - - - - - - - - - - - - - - (xeerka 1t)

$= B$ - - - - - - - - - - - - - - - - - (xeerka 10t)

TUSAALE 3: Tus in $(A \cup B \cup C)' = A' \cap B' \cap C'$ iyo in $(A \cap B \cap C)' = A' \cup B' \cup C'$

Guud mar ahaan u tus jedeeyooyinkan.

$(A \cup B \cup C)' = \{A \cup (B \cup C)\}'$

$= A' \cap (B \cup C)'$ - - - - xeerka dimoorgan ee 1

$= A' \cap (B' \cap C')$ - - - -xeerka dimoorgan ee 1

$= A' \cap B' \cap C'$

Inaga oo mataanka (dual) jedeeyadan caadanayna waxa aynu kolkiiba helaynaa $(A \cap B \cap C)' = A' \cup B' \cup C'$

Jedeeyooyinkani waxa ay guud mar ahaan u noqonayaan

$(A_1 \cup A_2 \cup A_3 \cup A_4 \text{------} \cup A_n)' = A_1' \cap A_2' \cap A_3' \cap A_4' \text{----} \cap A_n'$

iyo

$(A_1 \cap A_2 \cap A_3 \cap \text{-------} \cap A_2)' = A_1' \cup A_2' \cup A_3' \cup \text{------} \cup A_n'$

<u>TUSAALE 4</u>: Fududee $(A \cap B) \cup (A \cap B') \cup (A' \cap B) \cup (A' \cap B')$.

$(A \cap B) \cup (A \cap B') \cup (A' \cap B) \cup (A' \cap B')$

$= \{A \cap (B \cup B')\} \cup \{A' \cap (B \cup B')\}$

$= (A \cap U) \cup (A' \cap U)$

$= A \cup A'$

$= U$

<u>TUSAALE 5</u>: Fududee $A' \cup B' \cup C' \cup (A \cap B \cap C)$

$A' \cup B' \cup C' \cup (A \cap B \cap C) = (A \cap B \cap C) \cup (A \cap B \cap C)$

$= U$

<u>TUSAALE 6</u>: Fududee $(A \cap B) \cup (A' \cap C) \cup (B \cap C)$

$(A \cap B) \cup (A' \cap C) \cup (B \cap C)$

$= (A \cap B) \cup (A' \cap C) \cup \{(A \cap B \cap C) \cup (A' \cap B \cap C)\}$

$= \{(A \cap B) \cup (A \cap B \cap C)\} \cup \{(A' \cap C) \cup (A' \cap B \cap C)\}$

$= (A \cap B) \cup (A' \cap C)$

<u>TUSAALE 7</u>: Tus in $(A \cap B \cap C) \cup (A' \cap C) \cup (B' \cap C) = C$

$(A \cap B \cap C) \cup (A' \cap C) \cup (B' \cap C) = \{(A \cap B) \cup A' \cup B'\} \cap C$

$= \{(A \cap B) \cup (A \cap B)'\} \cap C$

$= U \cap C$

$= C$

<u>LAYLI</u>

1. Fududee

(b) $(A \cap B) \cup (A \cap B \cap C')$

(t) $A \cup B' \cup (A' \cap B)$

(j) $\{(A \cup B' \cup C) \cap (A' \cap B)\} \cup (A \cap B \cap C)$

(x) $(A \cap B' \cap C') \cup (A \cap B' \cap C' \cap D) \cup (A \cap C')$

(kh) $(A \cap B) \cup (A \cap B' \cap C)$

(d) $(A \cap B \cap C) \cup (A \cap B' \cap C) \cup (A \cap C')$

2. Fududee

(b) $(X \cup Y) \cap (X \cup Y)$

(t) $(X \cup Y \cup S') \cap (X' \cap Y' \cap S)$

(j) $\{X \cup (X \cap Y')\} \cap \{X \cup (Y \cap S)\}$

(x) $(X' \cup Y) \cap \{X \cup Y \cup (X \cap Y)\}$

(kh) $\{(X \cup Y) \cap (X' \cup Y')\} \cup (X \cap Y)$

(d) $\{(X \cup Y) \cap (Y \cup S) \cap (X' \cup S')\} \cup (X \cap Y \cap S') \cup (Y \cap S)$

3. Fududee

(b) $(A \cup b)' \cup (A' \cup B)'$

(t) $(A' \cap B)' \cup (A \cup B)'$

(j) $\{(A \cap B) \cup (A' \cap B')\}'$ (x) $\{(A \cap B) \cup (A \cap B') \cup (A' \cap B)\}'$

(kh) $(A \cup B \cup C)' \cup (A' \cap B)'$

(d) $\{(A \cap C) \cup (B \cap D)\}' \cup \{(A' \cap B) \cup (C' \cap D)\}'$

KU ISTICMAALKA DHEEGIDDA ALJEBRADA MACNAWI

Hab dhiska xisaabeed waxa uu ka kooban yahay urur ku-tirsanayaal ah, hal ama wax ka badan oo xisaabfallo ah kuna lug leh ku-tirsanayaasha ururka, iyo hawraaro ku saabsan astaamaha ku-tirsanayaasha marka loo fiiriyo xisaabfalladaa la qaatay.

Marka la rabo in la sugo ama la dejo astaamaha habdhiska xisaabeed waxa la adeegsadaa fikrado dareen ahaan (intuitively) la fahmi karo, hase yeeshee aan xisaab ahaan loo qeexi karayn. Qaar ka mid ah tusaalayaasha tibxaha maqeexsame (undefined terms) waa urur, tiro, bar iyo xarriiq. Waxase jiro fikrado xisaabeed oo la qeexi karo; waxaana ka mid ah isutag, dhextaal, isledegyada, iwm.

Qaar ka mid ah astaamaha tirooyinka maangalka ah ayeynu ay run yihiin. Hawraaraha ku saabsan u u qaadanaa in qaadashooyinkaas ayaa la yiraa dhardhaarro. Bal hadda dheeho tusaalahan ku saabsan sida aynu u gaarno u qaa-dashada ku saabsan isledegaanta. Waxaynu niri laba urur way isledegyihiin haddii iyo haddii oo keliya ay labada ururba ku jiraan ku-tirsanayaal isku mid ahi. Haddaba hawraarta ah A=B waxay innoo tilmaamaysaa in aynu ku isticmaalayno laba ...ur magac oo kala duwan isla urur keliya. Isla markaas hawraarta waxa aynu ka fahmi karaa in aynu magacyada laftooda isku beddeli karo.

Haddii fikradan aynu ku fidino ururka tirooyinka maangal ah, hawraarta ah a= b oo a, b ∈ R [R waxay u taagan tahay ururka tirooyinka maangalka ah] waxay inoo tilmaamaysaa in a iyo b ay yihiin laba magac oo isla tiro keliya u taagan, iyo in kolba kii la doono lagu beddeli karo ka kale. Xeerkan ku beddelidda waxa si rasmi ah loogu soo gaabin karaa sida soo socota:

Haddii a, b ∈ R, oo weliba a = b markaas tibixdii kasta ee magacaabaysa tiro maangal ah, a waxa lagu beddeli karaa b; ama sidaas oo kale b waxa lagu beddeli karaa a.

Hawraartan iyada ah ayaa la yiraa dhardhaarka ku beddelidda. Dhardhaarka ku beddelidu isagung markiisa waxa uu inna abaarsiinayaa astaamaha isledegaanta ee soo socda; kuwaas oo a, b iyo c ay yihiin tirooyin maangal ah ama si kooban ha a, b, c ∈ R.

1. Astaanta isku noqod oo lagu muujin karo:
   a = a
2. Astaanta wanqareno oo lagu muujin karo:
   Haddii a = b, markaa b=a
3. Astaanta dhaxidda oo lagu muujin karo:
   Haddii a = b, b = c markaas a = c.

Xidhiidhkii kasta ee leh saddexdan astaamood waxa la yiraa xidhiidhka isu dhignaanta.

LAMMAANEEYAYAASHA XISAABFAL

Marka laba tiro oo maangal ah la isu geeyo jadeeyadu waxa ay noqonaysaa tiro kale oo maangal ah. Sidoo kale marka laba tiro oo maangal ah la kala jaro jadeeyadu waxa ay noqonaysaa tiro kale oo maangal ah. Hadda isugeynta iyo kala goyntuba waxa weeye xeerar raacaymeed oo marka lagu isticmaalo ku-tirsa-nayaasha ururka tirooyinka maangalka ah dhaliya jadeeyo lafteedu ka mid ah ururkaas. Sidoo kale xisaabfalka isutaggu marka lagu isticmaalo hormo-ururo urur guud waxa uu dhaliyaa jadeeyo lafteedu ah hormo-ururo ururkaa guud. Haddaba xisaabfallada +, -, iyo ∪ ee kor lagaga sheekeeyey waxa ay tusaale u yihiin lammaaneeyaha xisaabfal ee hoos aynu si rasmi ah ugu qeexi doono.

QEEXID: Haddii lagu siiyo ururka ku-tirsanayaasha M markaa lammaaneeyaha xisaabfal * ee ururka M waa xeerka

racaynta ee ku toosiya lammaanihii horsan ee kasta kaba dhig a, b ∈ M, ku-tirsane madi ah c ∈ M.
Summad ahaan waxa aynu u qoraa c = a * b

TUSAALE 1:

Bal ka soo qaad in A ay u taagan tahay dhammaan ururka abyoonayaasha togan. Marka xisaabfalka isugeynta ee A waa lammaaneeyaha xisaab-fal; sababtoo ah haddii a, b ∈ A markaas c ∈ A.
Tusaalahan c = a + b.

TUSAALE 2:

Bal ka soo qaad in X ay u taagan tahay dhammaan ururka abyoonayaasha togan ee kisiga ah. Haddaba xisaabfalka isugeynta ee ururka X ma aa lammaaneeye xisaabfal; sababtoo ah haddii a, b ∈ X markaa c ∉ X.
Tusaalahan c = a + b. Tusaalahaani waxa uu innoo sheegayaa in aanad heleyn tiro kisi ah haddii aad isu geysid laba tiro oo kisi ah.

TUSAALE 3:

Bal ka soo qaad in M ay u taagan tahay dhammaan ururka abyoonayaasha. Haddaba xisaabfalka qaybinta ee M ma aha lammaaneeye xisaabfal; sababtoo ah c = a ÷ b had iyo jeer ma noqon karto ku-tirsane M marka a, b ∈ M.
Haddii aynu qaadano tirooyin a = 2, b = 3 kolkaa c =0.6666...
Taas oo aan ahayn ku-tirsane M.

LAYLI

Kuwee baa ah kuwa soo socda lammaaneeyayaasha xisaabfal ee ururada hoos lagugu siiyey?

1. - Isku dhufashada dhammaan ururka abyoonayaasha togan.
2. - Kala goynta dhammaan ururka abyoonayaasha togan.
3. - Kala goynta ee dhammaan ururka abyoonayaasha.
4. - Isku dhufashada dhammaan ururka abyoonayaasha togan ee dhabanka ah.

5. - Kala goynta dhammaan ururka tirooyinka lakabka ah .
6. - Dhextaalka dhammaan hormo-ururada urur ee urur guud.

Hore waxa aynu u sheegnay in aynu isku koobayno barashada dhardhaarada, astaamaha iyo aragtiinaha saldhigga u ah ururka tirooyinka maangalka ah sababtoo ah gadaal ayeynu ka baran doonaa dhardhaarada iyo xeerarka saldhigga u ah ururka tirooyinka maangalka ah. Bal hadda aan taxno dhardhaarada tirooyinka maangalka ah innaga oo ku fiirinayna siyaabaha loogu istiicmaalo lammaaneeyayaasha xisaabfal ee isu geynta iyo isku dhufashada.

1. - Dhardhaarka oodnaanta ee isu geynta
dhammaan tirooyinka a, b ∈ R, a + b ∈ R,
kolkaa a + b waa madi [R waxay u taagan tahay tirooyinka maangalka ah]
2. - Dhardhaarka hormogelinta ee isugeynta
Dhammaan tirooyinka a, b, c ∈ R,
(a + b) + c = a + (b + c)
3. - Dhardhaarka aasal madoorshaha ee isugeynta waxa jirta tiro madi ah 0 ∈ R si tiro kasta a ∈ R ay u raaligeliso
a + 0 = 0 + a = a
4. - Dhardhaarka isweydaarka ee isugeynta tiradii kasta a ∈ R waxa ay leedahay tiro kale oo madi ah - a ∈ R, loona yaqaan tabaniga a, si ay a + (-a) = (-a) + a = 0 .
5. - Dhardhaarka kala hormarinta ee isugeynta
Dhammaan tirooyinka a, b ∈ R,
a + b = b + a
6. - Dhardhaarka oodnaanta ee isku dhufashada
Dhammaan tirooyinka a, b ∈ R, ab ∈ R.
7. - Dhardhaarka hormogelinta ee isku dhufashada
Dhammaan tirooyinka a, b, c ∈ R,
(a b) c = a (bc)

8. - Dhardhaarka asal madoorshaha ee isku dhufashada waxa jirta tiro madi ah $|E$ R si tiro kasta a E R ay u raaligeliso

$$a. \ | = | \ .a = a$$

9. - Dhardhaarka isweydaarka ee isku dhufashada tiradii kasta a E R, 0 mooyaane, waxa ay leedahay tiro kale $\frac{1}{a} E$ R, loona yaqaan rogaalka a, si ay $a.\frac{1}{a} = \frac{1}{a}.a = 1$

10. Dhardhaarka kala hormarinta ee isku dhufashada
Dhammaan tirooyinka a, b E R,

$$a\ b = ba$$

11.-Dhardhaarka kala dhigga
Dhammaan tirooyinka a, b, c E R,

$a\ (b + c) = ab + ac$
isla markaas
$(a + b)\ c = ac + bc$

Kow iyo tobankan astaamood ee kor ku taxan ayaa loo yaqaan astaamaha badka. Haddaba ururkii kasta ee ku-tirsanayaal ah lehna astaamahaan ayaa la yiraahdaa bad.

Weliba waxa aynu u baahan nahay in aynu raacino dhardhaarada tirooyinka maangalka ah tibaaxo summado ku qoran oo iyana magacaaba tirooyin maangal ah. Tibaaxahan summadaha ku qoran ujeedada aynu ka leenahay waxa weeye in aynu ku dabaqno ama waafaqsiinoba lammaaneeyaha xisaabfal. Bal hadda cheeho tusaalahan. $a + b + c$ uma qaadan karo lammaaneeye xisaabfal sababtoo ah waxa ay ka kooban tahay isugeynta saddex ku-tirsane. Kolka a+b+c waxa aynu u qeexi karaa in ay la mid tahay amase la macno tahay (a+b) +c taas oo ku dabaqan ama waafaqsanba lammaaneeye xisaabfal. Marka bal aan qaadano taxa qaar ka mid ah qeexidyada aljebrada hoose, innaga oo u arkeyna ama u qaadanaynaba in dhammaan doorsoomayaashu yihiin tirooyin maangal ah.

QEEXID: Dhammaan tirooyinka a, b, c E R,

$$a + b + c = (a + b) + c$$
$$a + b + c + d = (a + b + c) + d$$
$$a + b + c + d + e = (a + b + c + d) + e$$
$$\vdots$$

QEEXID: Dhammaan tirooyinka a, b, c E R,

$$a\ b\ c = (a\ b)\ c$$
$$a\ b\ c\ d = (a\ b\ c)\ d$$
$$a\ b\ c\ d\ e = (abcd)\ e$$
$$\vdots$$

QEEXID: Dhammaan tirooyinka a, b E R,

$$a - b = a + (-b)$$

QEEXID: Dhammaan tirooyinka a, b E R,

$$\frac{a}{b} = a.\frac{1}{b} \quad (b \neq 0)$$

QEEXID: Dhammaan tirooyinka a, b E R,

$$-ab = -(ab)$$

LAYLI

Hawraartii kasta ee ka mid ah weydiimaha 1-20 waxa lagu caddayn karaa ama lagu barixi karaa mid ka mid ah astaamaha xidhiidhka isleegaanta ama astaamaha tirooyinka maangalka ah. Haddaba magacaw astaanta barixi karta hawraar kasta. U qaado doorsoomayaasha oo dhammi in ay yihiin tirooyin maangal ah.

TUSAALEYAAL:

b) $x + y = x + y$

c) $5 + s$ E R

Furfuris

b) Astaanta isku noqod

t) Dhardhaarka oodnaanta ee isugeynta.

1. - $0 + (x + y) = x + y$
2. - Haddii $x-7 = 3$, kolkaa $3 = x - 7$
3. - $x + 17$ E R

4. haddii $x = y$, $y = s + 7$, markaa $x = s + 7$
5. $(x+y) \cdot \frac{1}{(x+y)} = 1 \quad (x \neq -y)$
6. $(x + y)(m+q) = (x+y)m + (x+y)q$
7. $(x + 2y) + [-(x+2y)] = 0$
8. haddii $(x+y) = s$, $s = (x-7)$, kolkaa $(x+y) = (x-7)$
9. $(xy) \in R$
10. $(\frac{x}{y} + s) + \frac{m}{q} = \frac{x}{y} + (s + \frac{m}{q}) \quad (y, q \neq 0)$
11. $3x - (y+5) = 3x - (5+y)$
12. $\frac{x-7}{4} = \frac{x-7}{4}$
13. $[(x + 3)(x-2)](x+2) = (x+3)[(x-2)(x+2)]$
14. $\frac{x}{y} \cdot \frac{y}{s} = \frac{y}{s} \cdot \frac{x}{y} \quad (y, s \neq 0)$
15. $x(x-7y + 2) = (x-7y+2)x$
16. $(x+\sqrt{2}) \cdot \frac{1}{(x+\sqrt{2})} = 1 \quad (x \neq -\sqrt{2})$
17. haddii $4 = y + 5$, kolkaa $y - 5 = 4$
18. $(x+y) + 2 \in R$
19. $\sqrt{2}(x+3) = \sqrt{2} \cdot X + \sqrt{2} \cdot 3$
20. $(X + Y) + [-(x+y)] = 0$

Sheeg qeexidda sugaysa ama caddaynaysa in tibaaxdii kasta ee ka mid ah weydiimaha 21-26 ay u taagan tahay tiro maangal ah haddii dhammaan doorsoomayaashu ay yihiin tirooyin maangal ah.

21. $x + y + 2 = (x+y) + 2$
22. $3.x.y = (3.x).y$
23. $\frac{5}{x} = 5 \cdot \frac{1}{x} \quad (x \neq 0)$
24. $3x - 7y = 3x + (-7y)$
25. $(x+y)(x+s)(x-y) = [(x-y)(x+s)](x-y)$
26. $(2x+3) - (x+4) = (2x+3) + [-(x+4)]$

QAAR KA MID AH ARAGTIIMAHA SALDHIGGA U AH URURKA TIROOYINKA MAANGALKA AH

Waxa la filayaa haatan in waxoogaa waayoaragnimo ah loo yeeshay sida si fudud loogu caddeeyo aragtiimaha dheegidda. Hase yeeshee mar haddii fahamka ama garashada caddaynuhu ay

ka mid tahay dhinacyada ugu adag barashada xisaabta waxa lagama maarmaan ah in aynu dib ugu noqno caddaymaha qaar ka mid ah aragtiimaha aasaasiga ah iyaga oo ay raacsan yihiin saafidda caddaymuhu. Aragtiimo kale ayeynu gadaal ku taxi doonaa si ay u noqdaan tixraac loogana faa'iidaysto caddaymaha aragtiimaha guud ahaan.

Hawraarta aragtiini waxa ay ka kooban tahay laba qaybood:

1) Qaybta loo yaqaan afeefta, inta badanna uu ku jiro ereyga" haddii"; qaybtaa iyada ahna waxa had iyo jeerba loo qaataa inay tahay run.

2) Qaybta labaad oo loo yaqaan go'aan, uuna ku jiro ereyga "markaas ama kolkaas; qaybtan waxa ay si loojig ah uga soo mulaaxsantaa ama uga soo dheegantaa afeefta.

Dariiqada dheegidda ee caddayntu waxay ku bilaabataa afeefta aragtiinta ama hawraar kale oo la og yahay in ay run tahay. Had iyo jeerba waa in aad maskaxda ku haysaa in go'aanku yahay himilada la rabo in la gaaro, lana doonayo in hawraar kasta markeeda lagu sugo ama lagu caddeeyo qeexid, dhardhaar ama aragtiin hore loo caddeeyey. Bal hadda aan saafno caddaynta aragtiinta mararka qaarkood loo yaqaan xeerka isugeynta.

ARAGTIIN:

Haddii $a, b, c \in R$, $a = b$ markaas $a + c = b + c$

OGOW:

Aragtiintanu sida ay u dhigan tahay waa in ay kaaga caddaataa in " $a, b, c \in R$, $a = b$" ay tahay afeefta isla markaasna in " $a + c = b + c$ " ay tahay go'aanka la rabo in la soo dheego.

CADDAYN

Hawraaro	Garaadayn
(1) $a,b,c \in R$, $a = b$	(1) Afeef

(Go'aanku waa in uu ka soo malaaxsamaa afeefta; laakiin waxa muuqata in go'aanka ay ku jiraan wadaro iyada oo haba yaraatee aanay wadaro si cad uga muuqan afeefta. Iyada oo go'aanka sarid lagala baxayo waxa aynu qoraynaa.....)

Hawraaro	Garaadayn
(2) (a + c) ∈ R	(2) Oodnaanta isugeynta (laakiin go'aanku waxa weeye hawraar ah isldegaanshaha laba wadarood. Dabeedna .....)
(3) a + c = a + c	(3) Astaanta isku noqod ee isleegaanta

(Haddaba mar haddii aynu qaadanay in a = b waxa aynu a ku beddeli karnaa b dhinaca midig ee isleegta si aynu u helno....)

(4) a + c = b + c	(4) Isku beddelid

(Marba haddii aynu gaarnay himiladii go'aanka, caddayntu way dhan tahay).

Marar badan ayaa waxa dhici kara in xiriirka ka dhexeeya afeefta iyo go'aanku uu galo mugdi ama aanu si cad u muuqanin . Sidaa darteed ayaa laga yaabaa in ay adkaato sidii hawraaraha isku xiga loo ratibi lahaa iyada oo aad waliba ka soo mulaaxsamaysa ta ka horeysa. Bal hadda fiiri caddaynta aragtiinta soo socota:

Aragtiin

Haddii a, b ∈ R markaas a (-b) = - ab

OGOW: Si aynu u saafno hawraarta kore waa in aynu garanaa in -b, a(-b), iyo -(ab) ay gebi ahaantoodba yihiin tirooyin maangal ah; laakiin runtii aad uma cadda xiriirkoodu. Mas'aladan muftaaxeedu waxa weeye taranta a(-b). Mar haddii -b aanay ku jirin afeefta waa sandule in aynu caddaynta soo gelino.

CADDAYN

Hawraaro	Garaadayn
1) a, b ∈ R	(1) Afeef
2) -b ∈ R, b + (-b) = 0	(2) Xeerka isweydaarka isugeynta
(Imminka ayeynu samayn karnaa taranta a (-b))	
3) a [b+ (-b)] = a. 0	(3) Xeerka isku dhufashada
4) a.b + a.(-b) = a.0	(4) Xeerka kala dhigga
5) ab + a(-b) = .0	(5) Xeerka isirka eber.
6) ab ∈ R	(6) Dhardhaarka oodnaanta isku dhufashada

(7) -ab ∈ R, ab + (-ab) = 0	(7) Xeerka isweydaarka isugeynta
(8) a (-b) = -ab	(8) Isweydaarka isugeynta waa madi.

Bal hadda aan taxno qaar ka mid ah aragtiimaha laga helo aljebrada hoose. Aragtiimaha 1 ilaa 8 a, b, c ∈ R. (R waa tiro maangal ah).

Xeerka isugeynta

Aragtiin 1: Haddii a, b, c ∈ R , a = b markaas a + c = b + c

Xeerka isku dhufashada

Aragtiin 2: Haddii a, b, c ∈ R, a = b markaas ac = bc

Xeerka Isirka eber

Aragtiin 3: Dhammaan tirooyinka a ∈ R, a. 0 = 0

Xeerka isujaridda isugeynta

Aragtiin 4: Haddii a, b, c, ∈ R, ac = bc markaas a = b

Xeerka Isu jaridda isku dhufashada

Aragtiin 5: Haddii a, b, c ∈ R, ac = bc markaas a = b

Aragtiin 6:

$$-a + (-b) = -(a+b)$$

Aragtiin 7:

$$a.(-b) = (-a).b = -(a.b)$$

Aragtiin 8: $(-a).(-b) = a.b$

Aragtiimaha 9 ilaa 16, a, b, c, d ∈ J [J waa abyoone]

Aragtiin 9

$\frac{a}{b} = \frac{c}{d}$ haddii iyo haddii oo keliya oo a.d = b.c (b,d≠0)

Aragtiin 10: Xeerka salka u ah jajabyada

$$\frac{a.c}{b.c} = \frac{a}{b} \quad \text{weliba} \quad \frac{a}{b} = \frac{a.c}{b.c} \quad (b, c \neq 0)$$

Aragtiinta 11:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad (c \neq 0)$$

Aragtiinta 12:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a.c}{b.d} \quad (b, d \neq 0)$$

Aragtiinta 13:

$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \quad (b,c,d \neq 0)$$

Aragtiinta 14:

$$\frac{1}{\frac{a}{b}} = \frac{b}{a} \quad (a, b \neq 0)$$

Aragtiinta 15:

$$\frac{a}{b} = \frac{-a}{-b} = -\frac{-a}{b} = -\frac{a}{-b} \quad (b \neq 0)$$

Aragtiinta 16:

$$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b} = -\left(\frac{-a}{-b}\right) \quad (b \neq 0)$$

Maadaama caddaynta aragtiimuhu ay adag tahay bal aan caddayno aragtiimo kale si caddaynta dheegidda aragtiimuhu ay innoogu sii fududaato.

ARAGTIIN

Haddii a + b = 0 markaas b = - a

CADDAYN:

Haddii aynu - a u geyno labada dhinac ee afeefta waxa aynu helaynaa -a + (a+b) = -a+0. Haddii aynu isticmaalno dhardhaarka hormogelinta ee isugeynta waxa ay hawraarteenu noqonaysaa. (-a+a) + b = - a + 0. Mar haddii - a + a = 0 marka aynu saadano dhinaca bidix, isla markaasna -a + 0 = -a marka aynu saadano dhinaca midig waxa aynu helaynaa 0 + b = - a . Sidaa darteedna b = - a. Mar haddii b ay u taagan tahay tiro alaale tiradii kasta ee marka loo geeyo a jedeeyadu ay noqonayso eber, isla markaasna aynu caddaynay in b = - a, run ahaantii waxa aynu caddaynay in isweydaarka isugeyntu uu yahay madi. Aragtiintan soo socota ee ku saabsan isku dhufashada isuguna waxa uu caddaynayaa in isweydaarka isku dhufahsadu yahay madi.

Aragtiin

Haddii ab = 1, a ≠ 0, markaas b = $\frac{1}{a}$

Caddaynta aragtiintan layli ahaan ayaa lagaaga tegey.

ARAGTIIN:

Haddii a = b markaas - a = - b

CADDAYN:

Mar haddii a = b waxa aynu ku dari karnaa dhinacii kasta ee isleegta afeefta -a, taas oo u daiganta a + (-a) = b + (-a) inna siinaysana 0 = b + (-a), marba haddii a + (-a) =0. Si aynu u gaarno go'aenka aynu rabno waxa aynu ku dari karnaa dhinacii kasta ee isleegta 0=b + (-a) - b. Waayo? Dabeedna waxa aynu helaynaa - b + 0 = - b + [b + (-a)] ; iyada oo tan loo soo gaabin karo - b = -a waayo? Haddaba si aynu ugu dabaqno jedeeyadan go'aankeena waxa aynu isleegta u dambaysa u qori karnaa - a = - b.

ARAGTIIN:

Haddii ab = 0, markaas a = 0 ama b = 0

CADDAYN:

Haddii a = 0, aragtiinta waa la caddeeyey. Haddiise a ≠ 0 markaa a waxa ay leedahay isweydaar marka loo fiiriyo isku dhufashada kaasoo ah $\frac{1}{a}$. Haddii dhinacii kasta ee isleegta ab = 0 aynu ku dhufanno $(\frac{1}{a})$ waxa aynu helaynaa $(\frac{1}{a}).(ab)=0$, innaga oo isticmaalayna xeerka hormagelinta, hubaalaha $(\frac{1}{a}).a = 1$, 1.b = b marka la qaato dhinaca bidix, iyo $(\frac{1}{a}).0=0$ marka la qaato dhinaca midig waxa aynu helaynaa go'aankii la rabay.

LAYLI

Weydiimaha 1 ilaa 10 ku caddee ama ku garaadee hawraartii kasta mid ka mid ah aragtiimaha 1 ilaa 16. Dhammaan doorsoomayaasha ku jira weydiimuhu waxa ay u taagan yihiin tirooyin maangal ah. Haddii aragtiinta magac la siiyey,

magacaas ayaad ku bixin kartaa jawaabtaada.

TUSAALAYAAL

b) $(x+y) \cdot 0 = 0$

t) $\frac{x}{4} + \frac{y}{4} = \frac{x+y}{4}$

FURFURIS

b) Aragtiinta 3aad ama xeerka isirka eber $a.0 = 0$

t) Aragtiinta 4aad $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

1. Haddii $x + 4 = 7$, kolkaa $x + 4 + (-4) = 7 + (-4)$

2. Haddii $10x = 30$ kolkaa $10 \times (\frac{1}{10}) = 30 (\frac{1}{10})$

3. $\frac{(x+5)}{2}.0 = 0$

4. $-2x-1(-3y) = -(2x+3y)$

5. $(-2x)(-3y) = (2x)(3y)$

6. Haddii $\frac{x}{4} = \frac{y}{3}$, kolkaa $3x = 4y$

7. $\frac{\frac{1}{2}}{\frac{3}{2}}$

8. $\frac{x+2}{3} \cdot \frac{x-1}{5} = \frac{(x+2)(x-1)}{3.5}$

9. Haddii $7(x+y) = 3(x-y)$, kolkaa $\frac{x+y}{3} = \frac{x-y}{7}$

10. Haddii $x+(y-2) = y+4$ kolkaa

$x + (y-2) + [-(y-2)] = y + 4 + [-(y-2)]$

Weydiimaha 11 ilaa 16 ku sug ama ku caddee hawraar kasta oo ku jirta caddaymaha soo socda dhardhaar, qeexid, ama aragtiin hore loo caddeeyey. Runtii caddaymaha soo socda aad uma taftitira; nase yeeshee talaabooyinka la qaaday way ku filan yihiin caddaynta weydiin kasta.

11. Aragtiinta 2aad

(b) a,b,c $\in$ R, $a = b$

(t) $ac \in R$

(j) $ac = ac$

(x) $ac = bc$

12. ARAGTIINTA 3aad

Dhammaan tirooyinka $a \in R$, $a.0 = 0$.

(b) $a \in R$

(t) $0 + 0 = 0$

(j) $a(0+0) = a(0)$

(x) $a.0 + a.0 = a.0$

(kh) $a.0 + a.0 + [-(a.0)] = a.0 + [-(a.0)]$

(f) $a.0 + 0 = 0$

(g) $a.0 = 0$

13. ARAGTIINTA 11aad

Haddii a, b, c $\in$ j, $c \neq 0$, markaas $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

(j waa abyoone)

(b) $\frac{a}{c} + \frac{b}{c} = a.\frac{1}{c} + b.\frac{1}{c}$

(t) $a.\frac{1}{c} + b.\frac{1}{c} = (a+b)\frac{1}{c}$

(j) $(a+b) \cdot \frac{1}{c} = \frac{a+b}{c}$

(x) $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

14. ARAGTIINTA 12aad

Haddii $\frac{a}{b}, \frac{c}{d} \in Q$, markaas $\frac{a}{b} \cdot \frac{c}{d} = \frac{a.c}{b.d}$

[Q waa tiro lakab ah]

(b) $\frac{a}{b} \cdot \frac{c}{d} = \frac{a}{b} \cdot \frac{c}{d}$

(t) $= a.\frac{1}{b} \cdot c.\frac{1}{d}$

(j) $= a.\frac{1}{b} . c \frac{1}{d} . b. d. \frac{1}{b.d}$

(x) $= a.c. \frac{1}{bd} \cdot \left(\frac{1}{b} . b\right)\left(\frac{1}{d} . d\right)$

(kh) $= a.c. \frac{1}{b.d} .1.1$

(f) $= a.c.\ \frac{1}{b.a}$

(g) $= \frac{a}{b} \cdot \frac{c}{d} = \frac{a.c}{b.d}$

15. ARAGTIINTA 5aad

Haddii a, b, c $\in$ R , c $\neq$ 0, ac = bc, markaas a = b

(b) a, b, c $\in$ R, ac = bc

(t) $\frac{1}{c} \in$ R

(j) $ac.\frac{1}{c} = bc \cdot \frac{1}{c}$

(x) $a\ (c.\frac{1}{c}) = b\ (c.\frac{1}{c})$

(kh) a.1 = b.1

(d) a = b

16. ARAGTIINTA 16aad

Haddii $\frac{a}{b}$, $\frac{b}{a} \in$ Q, markaas $\frac{1}{\frac{a}{b}} = \frac{b}{a}$ [Q waa tiro lakab ah]

(b) $\frac{a}{b}$, $\frac{b}{a} \in$ R

(t) $\frac{1}{\frac{a}{b}} \in$ R

(j) $\frac{a}{b} \cdot \frac{1}{\frac{a}{b}} = 1$

(x) $\frac{a}{b} \cdot \frac{b}{a} = \frac{a.b}{b.a}$

(kh) $\frac{a.b}{b.a} = 1$

(d) $\frac{a}{b} \cdot \frac{1}{\frac{a}{b}} = \frac{a}{b} \cdot \frac{b}{a}$

(r) $\frac{1}{\frac{a}{b}} = \frac{b}{a}$

17. Caddee in $-\frac{a}{b} = -\frac{a}{b} = \frac{a}{-b}$

18. Haddii c $\neq$ 0, caddee in $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

19. Adiga oo u qaadanaya in b $\neq$ 0, caddee in

$(\frac{a}{b}) = c$ haddii iyo haddii oo keliya oo a = bc.

20. Adiga oo u qaadanaya in b $\neq$ 0, caddee in

$(\frac{a}{b}) = c + (\frac{d}{b})$ haddii iyo haddii oo keliya oo a=bc+d

QOOL (GROUP) IYO XERO

Qool waxa weeye fikradda ugu muhiimsan uguna fudud dhiamayaasha xisaabeed ee aljebrada macnawi ururka ku-tirsanayaasha G$=\{a,b,c,\ldots\}$ iyo xisaabfalka * ayaa la yiraa waxa ay samaynayaan qool haddii dhardhaarada soo socda la raaligeliyo.

1. Astaanta oodnaanta ee G

   * waa lammaaneeye xisaabfal. Macnee haddii a,b $\in$ G markaa a*b $\in$ G.

2. Xeerka hormogelinta G

   a*(b*c) = (a*b) *c marka dhammaan a, b, c $\in$ G.

3. Jiritaanka asal madoorshaha G

   Waxa jira ku-tirsane e $\in$ G si a * e = e * a = a midkii kasta a $\in$ G,

4. Jiritaanka weydaarka G

   Tiradii kasta a $\in$ G waxa ay leedahay ku-tirsane $a^{-1} \in$G si ay u raaligeliso

   $a*a^{-1} = a^{-1} * a = e$. a^{-1} ayaa loo yaqaan weydaarka a.

OGOW IN: (b) in laga tegi karo summadda lammaaneeyaha xisaabfal marka aanay wax khalkhal ah keenaynin . Haddaba xeerka hormogelinta waxa loo qori karaa a(bc) = (ab)c marka a, b, c $\in$ G.

(t) Haddii ku-tirsanayaasha qoolka G ay weliba raaligeliyaan xeerka ab = ba marka a, b $\in$ G, kolkaas qoolka waxa aynu u bixinaynaa qoolka kala hormarineed (commutative group).

(j) Asal madoorshaha e uu yahay madi. Sababtoo ah haddii e_1 iyo e_2 ay labaduba yihiin asal madoorshayaal markaas $e_1\ e_2 = e_2$ mar haddii e_1 ay tahay asal madoorshe, iyo $e_1\ e_2 = e_1$ ay tahay asal madoorshe. Kolka $e_1 = e_2$.

(x) Weydaarka ku-tirsane a $\in$ G uu yahay madi.
Sababtoo ah haddii **a** ay leedahay laba weydaar x iyo y markaas

$ax = xa = e$ ---------- (1)

iyo $ay = ya = e$ ---------- (2)

Hadda iyada oo la isticmaalayo (1) waxa aynu helaynaa

:. $yax = ye$

$yax = y$

Sidoo kale iyada oo la isticmaalayo (2) waxaynu helaynaa

:. $yax = ex$

$yax = x$

kolka $x = y$

(kh) Waxa markiiba (x) laga heli karaa in haddii a,b ay yihiin ku-tirsanayaasha qoolka G, markaas isle'egta $ax = b$ ay leedahay furfur madi ah marka loo eego G. Marka isle'egta dhinac walba lagaga dhufto a^{-1} waxa aynu helaynaa in

$a^{-1}ax = a^{-1}b$

$x = a^{-1}b$

(d) Aynu a.a u qorayno a^2, a.a.a na a^3, iwm. sida aljebrada caadiga ah. Ma dhibana in aynu muujino in $a^m a^n = a^n a^m = a^{m+n}$

iyo $(a^m)^n = a^{mn}$

m iyo n waa abyoonayaal togan.

(r) Tirada ku-tirsanayaasha qool , G ay koobnaan karaan ama tirobeel noqon karaan. Haddii qool kooban G ay ku jiraan n ku-tirsane markaas waxa aynu oranaynaa qoolka G waa horsiimada n-aad .

TUSAALE 1: Dhammaan ururka abyoonayaasha j ee xisaabfalka isugeynta waa qool. Sababtoo ah:

I. Xisaabfalka isugeynta ee ururka j waa lammaaneeye xisaabfal.

II. $a + (b+c) = (a + b) + c$ marka a, b, c $\in$ J

III. $a + 0 = 0 + a = a$ tiradii kasta a $\in$ J; sidaa darteedna 0 waa asal ma doorshe.

IV. $a + (-a) = (-a) + a = 0$ tiradii kasta a$\in$J; sidaa darteedna ku-tirsanihii kasta a $\in$ J waxa uu leeyahay weydaar madi ah $(-a) \in J$.

TUSAALE 2: Dhammaan ururka tirooyinka lakabka ah, Q oo uu eber ka reeban yahay ama ka baxsan yahay waa qool marka loo eego xisaabfalka isku dhufashada. Sababtoo ah:

I. Xisaabfalka isku dhufashada ee ururka Q waa lammaaneeye xisaabfal.

II. $a(bc) = (ab)c$ marka dhammaan a, b, c $\in$ Q

III. $a.1 = 1.a = a$ tiradii kasta a$\in$ Q sababtoo ah 1 waa asal madoorshaha isku dhufashada.

IV. $a.\frac{1}{a} = \frac{1}{a} . a = 1$ tiradii kasta a $\in$ Q; sidaa darteed ku-tirsane kasta a $\in$ Q waxa uu leeyahay weydaar $\frac{1}{a} \in Q$.

Ogow in 0 laga reebayo sababtoo ah 0 ma laha weydaar marka la haysto dhammaan ururka tirooyinka lakabka ah lana isticmaalo xisaabfalka isku dhufashada.

TUSAALE: 3

Ururka J ee dhammaan abyoonayaasha ee xisaabfalka isku dhufashadu ma aha qool.

Sababtoo ah haddii a $\in$ J, waxa dhici karta in a aanay yeelanin weydaar $a^{-1} \in J$. Tusaale ahaan $2 \in J$, hase yeeshee ma jiro ku-tirsane $x \in J$ si $2.x = x.2 = 1$.

Urur ku-tirsanayaal ahi waxa uu samaynayaa xero haddii:

I. Ku-tirsanayaasha ururku ay samaynayaan qool kala hormarineed marka la qaato xisaabfalka isugeynta.

III. Ku-tirsanayaasha ururka ay raaligelinayaan xeerka hormogelinta isku-dhufashada.

IV. Ku-tisanayaasha ururku ay raaligelinayaan xeerka kala dhigga isku dhufashada ee isugeynta .

LAYLI

1. Tus in ururka abyoonayaashu aanay samaynin qool marka la haysto xisaabfalka isku dhufashada.

2. Tus in ururka ka kooban labada ku-tirsane + 1 iyo -1 uu samaynayo qool kala hormarineed marka la haysto xisaab falka isku dhufashada.

3. Tus in dhammaan ururka abyoonayaasha dhabanka ahi uu yahay qool marka la haysto xisaabfalka isugeynta.

4. Ma yahay dhammaan ururka abyoonayaasha kisiga ahi qool marka la haysto xisaabfalka isku dhufashada?

5. Tus in dhammaan ururka horse-ururada ee urur guud aanu ahayn qool marka la haysto xisaabfalka isu tagga (U).

TIROOYINKA KAKAN

Haddii aad naqtiin ahaan ugu naqotid buugii kowaad ee xisaabta, waxa aad arki doontaa in abla-ablaynta habdhiska tirada si-ffiican oo balaaran loogula joex joexay buugaas isaga ah. Buugaas isaga ah waxa aynu ku sheegnay in tirooyinka kakan loo kala qaybin karo laba qaybood oo waaweyn; kuwaas oo ah tirooyinka maangalka ah iyo tirooyinka maangadka ah. Hase yeeshee gaar-ahaan waxa aynu hadda wixii ka horreeyey si tafatiran uga hadlaynay qaybta tirooyinka maangalka ah. Bal se hadda aan u soo joosano guud ahaan tirooyinka kakan innaga oo isku deyi doona in aynu aii balaarino fikradda tirooyinka kakan gaar ahaanna xisaab fallada la xiriira, astaamaha salka u ah iyo xiriirka ka dhexeeya tirooyinka maangadka ah iyo kuwa maangalka ah.

QEEXID: Isutegga ururka tirooyinka maangalka ah iyo ururka tirooyinka maangadka ah ayaa sameeya ururka tirooyinka kakan oo ku-tirsanayaashiisu yihiin sansaanka a + bi, a iyo b-na waa tirooyin maangal ah.

Tirada kakan ee a + bi waxa aad marar badan ku arki doontaa iyada oo ku qoran ama u qoran sidii lammaane horsan oo tirooyin maangal ah ama sansaankanba (a,b). Inta aynaan qeexin xisaabfallada salka,u ah ama kuba lug leh tirooyinka kakan bal aan raadraacno astaamaha xubinta maangadka a+ ee i. Qaybta maangalku ah ee tirada a + bi waa a; qaybta maangadka ahina waa b.

Haddaba waxa isweydiin leh sida jibbaarada i loo fududayn karo. Hore ayaynu u soo sheegnay in ay $i = \sqrt{-1}$. Haddaba haddii aynu laba jibbaaro dhinacii kasta ee isle'egtan waxa aynu helaynaa in $i^2 = -1$. Waxa kale oo aad xiriirkan kala soo bixi kartaa in $i^{-1} = i$. Kolka innaga oo ka faa'iiday-sanayna xiriirkan waxa aynu dhisi karaa jibbaarada sare ee i iyada oo lagu isticmaalayo xisaabfalka isku dhufashada. Jibbaarada ugu horreeya ee i waxa weeye: $i^3 = -i$,

$i^4 = 1$, $i^5 = i$, $i^6 = -1$, $i^7 = -1$, $i^8 = 1$, iwm. Halkan waxa kaaga muuqan kara in guud ahaan i^n, n waa tirsiimo, lagu soo gaabin karo 1, i, -1, ama -i. Fiiro gaar ah bal sii habka meertada ah ee jedeeyooyinka kore adiga oo dhuganaya ama milicsanayaba in $i^n = 1$ mar allaale markii n ay u qaybsamayso 4. Halkan waxa aynu ka soo dheegan karnaa in $i^n = i^{4r+s} = i^s$. Tibaaxdan u dambaysa ee ay jibbaaradu saaran yihiin macneheedu waxa weeye markii n loo qaybiyo 4 waxa ay noqona saa in 4 ay u qaybsanto n oo aanu haraa soo bixin, ama in haraa soo baxo. Haraagaas oo naqon kara 1, 2, 3 oo keliya. Ogow in halkan qaybshaheenu yahay 4, la qaybshuhuna yahay n, qaybtuna tahay r, haraaguna yahay s. Haddaba $4\sqrt{n} = 4.\ r + s$. Mar haddii salkeenu uu ahaa i, n-na ahayd jibbaar, kolkaa tibaaxdeenu ahayd $i^n = i^{4n+s} = i^s$.

Marka fududaynta i^n waxa lagu gaari karaa iyada oo 4 loo qaybiyo n; dabeedna jedeeyada lagu tibaaxo ama lagu metelo 1, i, -1, ama -i haddii uu haraagu noqdo 0, 1, 2, ama 3 sida ay u kala horreeyaan. Tan macneheedu waxa weeye haddii haraagu uu noqdo 0 kolkaa jedeeyada i^n waxa ay noqonaysaa 1, haddii haraagu uu noqdo 1 kolkaana jedeeyadu waxa ay noqonaysaa i, haddii uu 2 noqdona waxa ay noqonasaa -1, haddiise uu noqdo 3 kolkaana waxa ay noqonaysaa -i.

TUSAALE 1: $i^{35} = i^{4.8+3} = i^3 = -i$
TUSAALE 2: $i^{36} = i^{4.9+0} = i^0 = 1$
TUSAALE 3: $i^{37} = i^{4.9+1} = i^1 = i$
TUSAALE 4: $i^{38} = i^{4.9+2} = i^2 = -1$

QEEXID: Labo tiro oo kakani (a+bi) iyo (c+di) way isle'eg yihiin haddii iyo haddii oo keliya oo ay a = c, b = d.

Afarta xisaabfal ee saldhigga u ah tirooyinka kakan waxa lagu qeexi karaa isle'egyadan soo socda marka a + bi iyo c + di ay yihiin labadii tiro ee kasta ee tiro kakan ah:

ISUGEYN: $(a+bi) + c+di) = (a+c) + (b+d)i$
KALA GOYN: $(a+bi) - (c+di) = (a-c) + (b-d)i$
ISKU DHUFASHO: $(a+bi)(c+di) = (ac-bd) + (ad+bc)i$
ISU QAYBIN $= \frac{(a+bi)}{(c+di)} = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$

Waxa se isu qaybinta u shardi ah in aanay c iyo d noqonin eber labadooduba.

QEEXID: Dhammaan tirooyinka kakan ee (a+bi) xistiga (a+bi) waxa weeye (a-bi). Sidoo kale xistiga (a-bi) waxa weeye (a+bi).

TUSAALE: 1: $(2+3i) + (4-i) = 6+2i$
TUSAALE 2: $(3+7i) - (-4+2i) = +7+5i$
TUSAALE 3: $(2+5i)(3-i) = (2.3-5(-1),\ 2(-1)i+5.3i)$
$= (6+5,\ -2i + 15i)$
$= (11+13i)$

TUSAALE 4: $\frac{2+5i}{1+2i}$

$= \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$

$= \frac{(2.1+5.2)+(5.1-2.2)i}{1^2+2^2}$

$= \frac{(2+10)+(5-4)i}{1+4}$

$= \frac{12+i}{5} = \frac{12}{5} + \frac{1}{5}i$

TUSAALE 5: (b) Waa maxay xistiga (3+4i)?
(t) Waa maxay xistiga (3-4i)?
FURFURIS: (b) Xistiga (3+4i) waa (3-4i).
(t) Xistiga (3-4i) waa (3+4i).

Isu qaybinta tirooyinka kakan waxa si fudud loogu furfuri karaa isticmaalka xistiga. Bal hadda dheeho sida tusalaha 4aad ee isu qaybinta loogu furfuri karo isticmaalka xistiga.

TUSAALE 6: Ku fududee $\frac{2+5i}{1+2i}$ isticmaalka xistiga.

FURFURIS: Sarreyaha iyo hooseeyaha jajabka $\frac{2+5i}{1+2i}$ ku dhufo xistiga 1+2i kolkaa waxa aynu helaynaa

$$\frac{(2+5i)\ (1-2i)}{(1+2i)\ (1-2i)}$$

Dabeedna ku isticmaal sarreeyaha iyo hooseeyahaba qeexidda isku dhufashada tirooyinka kakan. Marka waxa aynu helaynaa

$$\frac{(\ (2.1) - 5(-2)\ ) + (2.(-2) + 5.1)i}{(\ (1.1-2.(-2)) + (1.(-2) + 2.1)i}$$

$$= \frac{(2-(-10)\ ) + (-4+5)\ i}{(1\ -\ (-\ 4) + (-2 + 2\ )\ i}$$

$$= \frac{(12) + (1)i}{(5) + (0)i} = \frac{12+i}{5} = \frac{12}{5} + \frac{1}{5}\,i$$

Xiriirka ka dhexeeya ururka tirooyinka maangalka ah iyo ururka tirooyinka kakani waa iska caddaan sababtoo ah haddii aynu b-kala mid dhigno eber waxa aynu helaynaa a oo keliya oo ah tiro maangal ah. Sidaa darteed waxa innoo muuqan karta in ururka tirooyinka maagalka ahi uu yahay hormo-urur ururka tirooyinka kakan. Sidoo kale haddii aynu kala mid dhigno eber waxa innoo muuqanaysa in dhammaan tirooyinka bi ay abuurmayaan; ururkan isaga ahna waxa loo yaqaan ururka asliga ah ee tirooyinka maangadka ah.

ASTAAMAHA TIROOYINKA KAKAN

Bal hadda aan fiiro gaar ah siino weyddiinta ah ma sameeyaan ururka tirooyinka kakani bad. sidii tirooyinka maangalka ahi ay u samaynayeen badka? Runtii

jawaabtu waa haa; astaamihii gaar ahaaneed ee badku uu u baahnaana waa kuwan soo socda. Sida badan xarafka weyn ee C ayaa loo qaataa inuu u taagnaado ururka tirooyinka kakan. Waa inagii hore u soo sheegnay in tiro kasta oo kakan loo qori karo summad ahaan sansaanka lammaane horsan; kolka, marka aynu astaamaha taxayno waxa aynu isticmaali doonaa sansaanka lammaanaha horsan si muujinta astaammu ay inoogu fududaato.

Haddii (a,b), (c,d), iyo (e,f) $\in$ C, markaas astaamaha soo socdaa waa run.

I. Oodnaanta isku dhufashada iyo isu-geynta .

b) (a,b). (c,d) $\in$ C

t) (a,b) + (c,d) $\in$ C

II. Isugeynta iyo isku dhufashaduhe waa ay raaligeliyaan kala hormarinta

(b) (a,b) + (c,d) = (c,d) + (a,b)

(t) (a,b). (c,d) = (c,d). (a,b)

III. Isugeynta iyo isku dhufashaduba way raaligeliyaan hormogelinta.

b) ((a,b) + (c,d)) + (e,f) = (a,b) + ((c,d) + (e,f))

t) ((a,b(, (c,d)) . (e,f) = (a,b). ((c,d) . (e,f))

IV. Xeerka kala dhigga

(a,b).((c,d) + (e,f)) = (a,b). (c,d)) + ((a,b) . (e,f))

V. Asal ma doorshaha isugeyta waxa jirta tiro kakan (0,0) si ay tiro kasta oo kakani (a,b) u raaligeliso isle'egta ah (a,b) + (0,0) = (a,b)

VI. Waydaarka isugeynta

Tiro kasta oo kakani (a,b) waxa ay leedahay tiro kale oo kakani (-a,-b) si ay (a,b)+ (-a,-b) = (0,0).

VII. Asal madoorshaha isku dhufashada. Waxa jirta tiro kakan (1,0) si ay tiro kasta oo kakan (a,b) ay u raali geliso isle'egta ah (a,b). (1,0) = (a,b).

VIII. Weydaarka isku dhufashada.

Tiro kasta oo kakani (a,b) waxa ay leedahay tiro kale oo kakan (x,y) si ay (a,b). (x,y) = (1,0).

Astaamaha badidoodu waxa ay si toos ah uga yimaadeen ama ugu dabaqan yihiinba qeexidaha tirooyinka kakan, kuwo kale oo ka mid ah astaamahan si hawl yar baa loo caddayn karaa; hase yeeshee waxa aad mooddaa inay yar adag tahay caddaynta sideedaad. Bal kolka aanu isku dayno in aynu caddayno astaantaa ku lug leh weydaarka isku dhufashada.

CADDAYN

HAWRAAR	XAQIIJIN
1. a, b, x, y $\in$ R	1. Qeexidda tirooyinka kakan
2. (a,b).(x,y) = (1,0)	2. Afeef
3. (ax-by, ay + bx) = (1,0)	3. Qeexidda isku dhufashada tirooyinka kakan.
4. ax - by = 1	4. Qeexidda isle'eggaanshaha tirooyinka kakan

Haddii lammaanahan isle'egyada aynu habdhiis ahaan u furfuro, waxa aynu helaynaa:

$$x = \frac{a}{a^2+b^2}, \text{ iyo } y = \frac{-b}{a^2+b^2}$$

Kolka, mar haddii la tusi karo in isle'egyada hawraarta afraad (4) ay yihiin isle'egyo toosan oo madaxbanaan, ururfurfurku waa in uu noqdo mid madi ah; weliba haddii (a,b) ay leedahay weydaar isku dhufasho, waa inuu noqdaa $\left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$./

Tiradan kakani waxa ay jiri kartaa haddii iyo haddii oo keliya oo $a^2+b^2 \neq 0$. Mideed kaleeto $a^2=0$ waxa ay run tahay haddii iyo haddii oo keliya oo a= 0, b = 0. Hadda tiro kasta oo kakani waxa ay leedahay weydaar marka laga reebo (0,0).

Marka aynu isgarab dhigno astaamaha tirooyinka maangalka ah iyo kuwa tirooyinka kakan waxa aynu helaynaa in ay ku kala duwan yihiin oo keliya astaamaha horsanaantan sababtoo

ah ma oran karo tiro kakani waxa ay ka weyn tahay ama ka yar tahay tiro kale oo kakan . Waxa aynu oran karnaa oo keliya laba tiro oo kakani way isle'eg yihiin ama isma le'ega.

LAYLI

Ku tibaax weyddiimaha 1 ilaa 12 i, -1, -i, ama 1.

1. i^5 2. i^6 3. $2i^4$ 4. $5i^8$ 5. $-i^{11}$ 6. $-i^{10}$
7. $6i^2$ 8. $10i^5$ 9. $-3i^{14}$ 10. $-7i^{12}$ 11. $-i^{123}$ 12. i^{346}

Ku qor tibixaale kasta oo ka mid ah weyddimaha 13ilaa 20 sansaanka a + bi.

13. $2i^6 + 3i^5 - 4i^3 + 10$ 17. $3i^7 + 3i^5 - 2i^2 + 7$
14. $5i^9 + 7i^8 - 2i^6 + 4i^3$ 18 $4i^8 + 2i^7 + 4i^2 - 3i$
15. $7i^{14} - 8i^{13} - 2i^8 + i^7$ 19. $2i^9 - i^8 - 3i^7 + i^6 - 5i^5 + 4i^4 + 2i^2$
16. $3i^5 - 2i^6 + 8i^9 - 5i^{10}$ 20. $4i^{13} + 5i^{12} + 2i^{11} + 3i^{10} - 2i^8 + 3i^6$

Qor wadarta, faraqa iyo taranta tirooyinkan kakan ee soo socda:

1. (4+2i) + (6-3i)
2. (3-7i) +(-1+4i)
3. (-5-2i) +(3-4i)
4. (-6-3i) + (-1-4i)
5. (5+3i) - (2+i)
6. (3-2i) - (4-i)
7. (-6-2i) - (-5-3i)
8. (-7-5i) - (-8-4i)
9. (5+i) (2+6i)
10. (2-3i) (8+7i)
11. (-6-4i) (2-5i)
12. (-4-7i) (-3-2i)
13. (3-7i) + (2+5i)
14. (-12+3i) - (7-5i)
15. (4+8i) (2-3i)
16. (5 +i) - 2 (3+5i) +6(-2-i)
18. $(3\sqrt{2}+2i)$ $(3\sqrt{2}-2i)$
19. (i+5) (i-2) + (2+3i) (1+i)
20. $(3-2i)^3$

21. Haddii (2+5i) (3+2i) (1-i) = (a+bi), raadi a iyo b?
22. Haddii (3+i) (8+5i) = (19+xi), raadi x?

U fududee mid kasta oo ka mid ah kuwa soo socda sansaanka ah (a+bi).

23. $\sqrt{-16} + 3 - \sqrt{-9} - 7$

24. $\sqrt{-25-i}$

25. $(3+\sqrt{-3})\ (2-\sqrt{-3})$

Qor xistiga tiro kasta oo kakan oo soo socota:

1. (3+vi) 2. (5+7i) 3. (2-i) 4.(6-2i)

5. (-3+4i) 6. (-5+10i) 7. $(3+\sqrt{2i})$ 8. $(5+\sqrt{5i})$

9. (2+3i). 10. $(-5-\frac{1}{7}i)$

Ku tibaax qaybta weyddiimahan tirooyin kakan ee sansaankan ah (a x bi).

1. $\frac{4+2i}{1+i}$ 2. $\frac{5-i}{3+2i}$ 3. $\frac{3-7i}{2-3i}$ 4. $\frac{1-8i}{4-2i}$

5. $\frac{8+2i}{3i}$ 6. $\frac{7-5i}{2i}$ 7. $\frac{1}{i}$ 8. $\frac{2}{3i}$

9. $\frac{2+i\sqrt{3}}{2-i\sqrt{3}}$ 10. $\frac{\sqrt{5}-i\sqrt{3}}{\sqrt{3}+i\sqrt{5}}$

1. U qor asal madoorshaha isugeynta ee ururka tirooyinka kakan sansaanka ah a + bi.
2. U qor asal ma doorshaha isku dhufashada ee ururka tirooyinka kakan sansaanka ah a+bi.
3. Qor weydaarka isugeynta a + bi
4. Qor weydaarka isku dhufashada ee a+bi, haddii a, b $\neq 0$

JIBBAARO

QEEX: a^m waa taranta m isir oo mid waliba yahay a. macnee $\overbrace{a.a.a...a}^{m}$ / Maadaam m ay tahay tirada isirada, ma noqon karto tabane ama jajab, waxa se ay tahay abyoone togan. Inaga oo qeexdaa isticmaaleyna ayeynu dheegi karaa xeerarka asaaska u ah jibbaarada.

1. $a^m . a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$, $a \neq 0$ haddii $m > n$

   $= \frac{1}{a^{n-m}}$ haddii $m < n$
3. $(a^m)^n = a^{mn}$
4. $(ab)^m = a^m b^m$
5. $(a/b)^m = a^m/b^m$, $b \neq 0$

Caddeymaha Xeerarka

1. $a^m.a^n = a^{m+n}$, $m,n \in A^+ = \{1,2,3,...\}$

Caddayn: $a^m = \overbrace{a.a.a....a}^{m}$

$a^n = \overbrace{a.a.a...a}^{n}$

$\therefore a^m.a^n = \overbrace{(a.a.a...a)}^{m}\ \overbrace{(a.a.a...a)}^{n}$

$= \overbrace{a.a.a...a}^{m+n}$

$= a^{m+n}$

Xeerkan waxa lagu dabiqi karaa tiro kasta oo $\overbrace{\text{xoogaga}}^{ah}$ a

Marka $a^m.a^n.a^r = a^{(m+n)}.a^r = a^{(m+n+r)}$

$a^m.a^n.a^r... = a^{m+n+r+...}$

2. $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$ haddii $m > n$

   $= \frac{1}{a^{n-m}}$ haddii $m < n$ $\quad m,n \in A^+$

Caddayn : $a^m = \overbrace{a.a.a...a}^{m}$

$a^n = \overbrace{a.a.a...a}^{n}$

$\therefore \dfrac{a^m}{a^n} = \dfrac{\overbrace{a.a.a...a}^{m}}{\underbrace{a.a.a \ldots a}_{n}}$

(i) $m > n$

Maadaam $m = (m-n) + n$, m-da isir ee sarreeyaha ayaa loo qaybin karaa laba kooxood, kooxda hore waxa weeye (m-n) isir, ta danbena waa n isir.

$$\therefore \frac{a^m}{a^n} = \frac{\overbrace{(a.a.a...a)}^{m-n} \cdot \overbrace{(a.a.a...a)}^{n}}{\underbrace{a.a.a...a}_{n}}$$

$$= \overbrace{a.a.a...a}^{m-n}$$

$$= a^{m-n}$$

(ii) $m < n$

Maadaam $n = (n-m) + m$, n-da isir ee hooseeyaha ayaa loo qaybin karaa laba kooxood, kooxda hore waxa weeye (n-m) isir, ta danbena m isir.

$$\therefore \frac{a^m}{a^n} = \frac{\overbrace{a.a.a...a}^{m}}{\overbrace{(a.a.a...a)}^{n-m}\ \overbrace{(a.a.a...a)}^{m}}$$

$$= \frac{1}{\overbrace{a.a.a...a}^{n-m}}$$

$$= \frac{1}{a^{n-m}}$$

3. $(a^m)^n = a^{mn}$, $m,n \in A^+$

Caddeyn:

$$(a^m)^n = \overbrace{a^m.a^m.a^m...a^m}^{n}$$

$$= \overbrace{a^{m+m+m+\ldots}}^{\text{n tibxood}} \text{ (xeerka koowaad)}$$

$$= a^{mn}$$

4. $(ab)^m = \overbrace{ab.ab.ab...ab}^{m}$

$$= \overbrace{(a.a.a...a)}^{m}\ \overbrace{(b.b.b...b)}^{m}$$

$$= a^m.b^m$$

5. Xeerka 5aad $(a/b)^m = a^m/b^m$, $b \neq 0$ caddeyntiisa waxan u dhaafayaa ardeyga.

TUSAALOOYIN (Jibbaaradu dhammaantood waa abyoonayaal togan)

Tusaale 1: Fududee $\dfrac{3m^3n^2.4mn^3}{18m^2n^6}$

Furfuris: $\dfrac{3m^3n^2.4mn^3}{18m^2n^6} = \dfrac{3.4.m^{3+1}n^{2+3}}{18m^2n^6}$

$$= \frac{2m^{(3+1-2)}}{3n^{6-(2+3)}} = \frac{2m^2}{3n}$$

Tusaale 2: Fududee $\dfrac{2^2.8.8^4}{2^6.27^2} = \dfrac{2^2.2^3.2^4}{2^6.(3^3)^2} = \dfrac{2^5.2^4}{2^6.3^6} = \dfrac{1}{2.3^2} = \dfrac{1}{81}$

Tusaale 3: Fududee: $\dfrac{10.2^7-48.2^4}{48.2^5-2^8} = \dfrac{5.2.2^7-3.2^4.2^4}{3.2^4.2^5-2^8}$

$$= \frac{5.2^8 - 3.2^8}{3.2^9 - 2^8}$$

$$= \frac{2^8(5-3)}{2^8(6-1)} = \frac{2}{5}$$

LAYLI

Fududee:

1. $a^4.a^2$

2. $3a^2.4a^4$

3. $\frac{a^7}{a^3}$

4. $\frac{2a^5}{3a^2}$

5. $2(a^3)^4$

6. $(2a^3)^3$

7. $(2a^2b)^2 \cdot 2ab^2$

8. $\frac{2a^3b \cdot (2ab^2)^2}{16\ a^4\ b^4}$

9. $\frac{(3a^2b)^3.2(ab^2)^2}{18\ (a^2b^2)^3}$

10. $(\frac{a^3b}{c^2})^2 \cdot (\frac{b^3c}{a^2}) \cdot (\frac{c^3a}{b^2})$

11. $\frac{6.10^3.5}{15.20^3}$

12. $\frac{(4.3^2)^3 \cdot 6}{18.(3.2^2)^4}$

Dhammeystir weedhahan soo socda:-

13. $2^6 + 2^3 = 2^3$ ( )

14. $4^2+2^2 = 2^2$ ( )

15. $9^2+ 27 = 3^3$ ( )

16. $3.9^3+9.3^3 = 3^5$ ( )

Fududee:

17. $\frac{4.3^5 - 3.3^3}{11.\ 3^6}$

18. $\frac{3^6 + 3^5 + 3^4}{3^6 + 3^4 + 3^2}$

19. $\frac{2.3^2 + 3.2^2}{3.4^2-4.3^2}$

Waxa aynu qeexnay a^m haddii $m \in A^+ = \{1,2,3,...\}$, haddaba aan qeexno a^m haddii $m = 0, m \in A^-$, $m \in$ {tirooyinka lakab} - {Abyoonayaasha}.

Sidaas darteed waa in aan qeexno tibaaxaha ay ka mid yihiin a^0, a^{-m}, $a^{m/n}$ $m,n \in A$, $n \neq 0$, $n \neq 1$ tibaaxahan waxaynu qeexi karnaa marka aynu u qaadano in weedha $a^m.a^n = a^{m+n}$ ay run tahay $m,n \in M$, M = ururka tirooyinka maangalka ah.

B. Jibbaarka Eber:

Si aan u qeexno a^0

$a^0.a^m = a^{0+m}$

Haddii aan haddaba dhinac kasta u qaybino a^m waxa aynu heli in $a^0 = 1$ haddii $a \neq 0$.

t. Jibbaarada taban

Si aan u qeexno a^{-n}

$a^{-n}.a^n = a^{-n+n}$

$= a^0$

$= a^0$

$= 1$

$\therefore\ a^{-n} = \frac{1}{a^n}$ (dhinac kasta u qaybi $a^n \neq 0$)

Maadaam $a \in M$, $a \neq 0$.

$5^0 = 1$, $(\frac{-2}{3})^0 = 1$, $(x^2)^0 = 1$

Ogow: $(2x)^0 \neq 2x^0$, maadaam $(2x)^0 = 1, 2x^0 = 2.1 = 2$

weliba $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$(\frac{3}{4})^{-2} = \frac{1}{(\frac{3}{4})^2} = (\frac{4}{3})^2 = \frac{16}{9}$

$(-3)^{-4} = \frac{1}{(-3)^4} = \frac{1}{81}$.

Ogow $(2x)^{-3} \neq 2x^{-3}$, maadaam $(2x)^{-3} = \frac{1}{(2x)^3} = \frac{1}{8x^3}$

Laakiin $2x^{-3} = 2.\ \frac{1}{x^3} = \frac{2}{x^3}$

Tusaalooyin (jibbaaradu waxa ay katirsan yihiin ururka abyoonayaasha A = $\{0, \pm 1, \pm 2, ...\}$

Tusaale 1: Fududee:

$(3a^2b)^{-1} \cdot 2\ (ab^{-3})^{-2}$

$(3a^ab)^{-1} \cdot 2\ (ab^{-3})^{-2} = 3^{-1}a^{2}b^{-1}.2.a^{-2}b6$

$= \frac{2}{3} a^{2-2} . b^{6-1}$

$= \frac{2}{3} a^{0} b^{5}$

$= \frac{2}{3} .1.b^{5}$

$= \frac{2}{3} b^{5}$

Tusaale 3: Haddii $a = 4.10^{-2}$, $b = 5^{-1}.10^{3}$ doon qiimaha (1) ab^2 (ii) a^3b^{-2} (una dhig sansaanka ah $m.10^{n}$, $1 < m < 10$, $n \in A$ $\{0, \pm 1, \pm 2, \ldots\}$

(1) $ab^2 = 4.10^{-2}.(5^{-1}.10^{3})^2$

$= 4.10^{-2}.5^{-2}.10^{6}$

$= 4.5^{2} . 10^{4}$

$= \frac{4}{5^2}.10^{4}$

$= \frac{4}{5^2} \cdot \frac{2^2}{2^2} \cdot 10^{4} = \frac{16.10^{4}}{10^{2}} = 16.10^{2} = 1.6 \times 10^{3}$

(ii) $a^3b^{-2} = (4.10^{-2})^3 (5^{-1}.10^{3})^{-2}$

$= (4^3.10^{-6}) (5^2 10^{-6})$

$= 4^3.5^2.10^{-12}$

$= \frac{4^3.5^2.10^{-12}.2^2}{2^2}$

$= \frac{4^3.10^2.10^{-12}}{4} = 4^2 10^{-10} = 16 \times 10^{-10}$

$= 16 \times 10^{-1} \times 10^{-9}$

$= 1.6 \times 10^{-9}$

LAYLI Fududee, (Jawaabahana kor ku qor jibbaaro togan)

1. $(x^{-4})^{0}$
2. $\frac{(x^{-1}y^{-1})}{x^{2}}$
3. $\frac{(2x)^{-1}}{2x^{2}}$
4. $\frac{2x^{2}}{(3x)^{-3}}$
5. $\frac{3x^{0}}{2x^{-2}}$
6. xy^{-3}
7. $2x^{-1}y$
8. $2x^{-2}y^{2}$
9. $\frac{(2x)^{-2}}{3x^{2}y^{-2}}$

10. $a^{2}b^{-2} . (2a^{-1}b)^{2}$
11. $(\frac{x}{y})^{-1} . (\frac{y}{x})^{-1} . (\frac{m}{x})^{-1}$
12. $(a^{-}b^{2})^{-2} + (a^{2}b^{-1})^{-2}$
13. $a^{2} \frac{(2x^{-1})^{-2}}{(4ax)^{-1}}$
14. $(3x^{-2}y^{3})^{-1} + (2xy^{2})^{-2}$
15. $\frac{(2a^{2})^{-2} . 3 (b^{2})^{-1}}{(2a^{-1})^{-3}b^{-2}}$
16. $\frac{ab^{-1} - a^{-1}b}{b^{-1} - a^{-1}}$
17. $\frac{1 - a^{2} x^{-2}}{a^{-1} - x^{-1}}$
18. $\frac{1}{x^{-1}+y^{-1}} + \frac{1}{x^{-1}-y^{-1}}$
19. $\frac{x^{2} + x^{-2} + 2}{x^{2} - x^{-2}}$
20. $\frac{x^{2} - xy^{-1} - 2y^{-2}}{2x^{2} - xy^{-1} + 2y^{-2}}$

Dhammeystir weedhahan soo socda

21. $x + x^{-1} = x^{-1}$ ( )
22. $x^{2} + 2 + x^{-2} = x^{-2}$ ( )
23. $x^{2} - 2 + x^{-2} = x^{-2}$ ( )
24. $x^{2} - x^{-2} = (x - x^{-1})$ ( )
25. $x^{3} + x^{-3} = (x + x^{-1})$ ( )
26. $x^{2} + 1 x^{-2} = (x + 1 + x^{-1})$ ( )
27. $1 - 3^{-1} = 3^{-1}$ ( )
28. $4^{2} + 4^{-2} = 2^{-4}$ ( )
29. $4^{-2} - 8^{-2} = 2^{-6}$ ( )
30. $2.3^{-2} + 3.2^{-2} = 2.3^{-2}$ ( )

Fududee:

31. $\frac{2^{-2} . 6^{4}}{3^{-2} . 4^{-4}}$

32. $\frac{3^2.9^{-4}}{3^{-4}}$

33. $\frac{2^3}{15^{-5}}.\frac{6^{-2}}{5^5}$

34. $\frac{4.2^{-2} - 2^{-4}}{3.2^{-1}}$

35. $\frac{2^{-2}.3 - 6^{-1}}{3^{-1}.7.2^{-1}}$

Jibbaaro jajabyo ah (fractional exponents)

Si aan u qeexno $a^{\frac{1}{n}}$, $n \in A - \{0\} = \{\pm 1, \pm 2, \ldots\}$

$a^{\frac{1}{n}}.a^{\frac{1}{n}}.a^{\frac{1}{n}}$ ... ilaa n isir = $(a^{\frac{1}{n}} + \frac{1}{n} + \frac{1}{n} + \ldots$ ilaa n tibxood)

$= a^{n.\frac{1}{n}} = a$

$\therefore (a^{\frac{1}{n}})^n = a$

$\therefore a^{\frac{1}{n}} = \sqrt[n]{a}$

j) Si aan u qeexno $a^{m/n}$,

$a^{m/n}.a^{m/n}.a^{m/n}$... ilaa n isir = $a^{(m/n+m/n+\ldots \text{ilaa n tibxood})}$

$= a^{n.\,m/n} = a^m$

$\therefore (a^{m/n})^n = a^m$

$\therefore a^{m/n} = \sqrt[n]{a^m}$

Matalan $16^{\frac{1}{2}} = \sqrt{16} = 4$

ama $8^{2/3} = (8^{\frac{1}{3}})^2 = (\sqrt[3]{8})^2 = 2^2 = 4$

$8^{2/3} = (8^2)^{\frac{1}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

$(729)^{2/3} = (3^6)^{2/3} = 3^4 = 81$

$a + 2\sqrt{ab} + b = a + 2a^{\frac{1}{2}}b^{\frac{1}{2}} + b = (a^{\frac{1}{2}} + b^{\frac{1}{2}})^2$

Tusaalooyin

Tusaale 1:

Fududee $\frac{(32)^{3/5}.(2/3)^{-2}}{\sqrt{5^{\frac{1}{16}}}^{\,1}}$

$= (2^5)^{3/5}.\frac{2^{-2}}{3^{-2}} + (\frac{81}{16})^{\frac{1}{2}}$

$= (2^5)^{3/5}.\frac{2^{-2}}{3^{-2}} + (\frac{3^4}{2^4})^{\frac{1}{2}}$

$= 2^3.\frac{3^2}{2^2}.\frac{2^2}{3^2} + 2^3 = 8$

Tusaale 2: Haddii x = 8, y = $\frac{1}{9}$, doon qiimayaasha

(i) $x^{-\frac{1}{3}} + y^{-\frac{1}{2}}$ (ii) $(\frac{1}{3}xy)^{2/3}$

(1) $x^{-\frac{1}{3}} + y^{-\frac{1}{2}} = (8)^{-\frac{1}{3}} + (\frac{1}{9})^{-\frac{1}{2}}$

$= (2^3)^{-\frac{1}{3}} + (\frac{1}{3^2})^{-\frac{1}{2}}$

$= 2^{-1} + \frac{1}{3^{-1}}$

$= \frac{1}{2} + 3 = 3\frac{1}{2}$

(2) $(\frac{1}{3}xy)^{2/3} = (\frac{1}{3}.8.\frac{1}{9})^{2/3} = (\frac{1}{3}.2^3.\frac{1}{3})^{2/3}$

$= (\frac{2^3}{3^3})^{2/3}$

$= \frac{2^2}{3^2} = \frac{4}{9}$

Tusaale 3: Fududee:

$\frac{x^{2/3}y^{2/3} + x^{\frac{1}{3}}y}{x^{4/3}y^{\frac{1}{3}} + xy^{2/3}}$

$\frac{x^{2/3}y^{2/3} + x^{\frac{1}{3}}y}{x^{4/3}y^{\frac{1}{3}} + xy^{2/3}} = \frac{x^{\frac{1}{3}}y^{2/3}(x^{\frac{1}{3}} + y^{\frac{1}{3}})}{xy^{\frac{1}{3}}(x^{\frac{1}{3}} + y^{\frac{1}{3}})} = \frac{y^{2/3 - \frac{1}{3}}}{x^{1-\frac{1}{3}}} = \frac{y^{\frac{1}{3}}}{x^{2/3}}$

LAYLI

Doon qiimaha

1. $2^{\frac{1}{2}}.2^{\frac{1}{2}}$
2. $2^{\frac{1}{3}}.2^{-\frac{1}{3}}$
3. $3^{\frac{1}{2}} + 3^{-\frac{1}{2}}$
4. $3^{-\frac{1}{2}}.3$
5. $8 + 8^{\frac{1}{3}}$
6. $3^{-3/2} + 3^{\frac{1}{2}}$
7. $2^{\frac{1}{2}}.3^{-\frac{1}{2}} + 2^{-\frac{1}{2}}\,3^{\frac{1}{2}}$
8. $4^{\frac{1}{2}}$
9. $27^{\frac{1}{3}}$
10. $8^{-\frac{1}{3}}$
11. $9^{-\frac{1}{2}}$
12. $8^{-2/3}$
13. $16^{3/4}$
14. $3^{\frac{1}{2}}.27^{\frac{1}{2}}$
15. $2^{-\frac{1}{2}}.8^{\frac{1}{2}}$
16. $(x^2)^{\frac{1}{2}}$
17. $(4x^2)^{-\frac{1}{2}}$
18. $2(x^3)^{-\frac{1}{3}}$
19. $(\frac{x^2}{4})^{-\frac{1}{2}}$
20. $3^{\frac{1}{3}}.(\frac{1}{48})^{-1/4}.(108)^{-1/4}$
21. $36^{\frac{1}{2}}.\sqrt[3]{\frac{3}{2}} + 81^{1/6}$
22. $8^{2/3}.(\frac{1}{2})^{-2}.(64)^{-5/6}$

23. Haddii x = 16, y = 9, doon qiimayaasha.

(i) $x^{\frac{1}{2}} y^{-\frac{1}{2}}$

(ii) $x^{-\frac{1}{2}} + y^{-\frac{1}{2}}$

(iii) $(x+y)^{-\frac{1}{2}}$

24. Haddii x = 4, y = 27, doon qiimayaasha

(i) $(x^2 y^{2/3})^{1/4}$

(ii) $(2xy)^{-\frac{1}{3}}$

(iii) $\left(\frac{12x}{y}\right)^{\frac{1}{2}}$

(iv) $\sqrt{\frac{y}{2y^{\frac{1}{3}} + x^{-1}}}$

TUSAALE 1:Doon qiimaha x haddii

$2^{x+3} + 2^{x+2} + 2^{x+1} = \frac{7}{8}$

FURFURIS: $2^{x+3} + 2^{x+2} + 2^{x+1} = \frac{7}{8}$

:. $2^{x+1} \ (2^2+2^1+1) = \frac{7}{8}$

:. $2^{x+1} \ (4+2+1) = \frac{7}{8}$

:. $2^{x+1} \ (7) = \frac{7}{8}$

:. $2^{x+1} = \frac{1}{8} = \frac{1}{2^3} = 2^{-3}$

:. $x+1 = -3$

Markaa $x = -4$.

TUSAALE 2: Doon qiimaha x haddii $4^x - 3.2^{x+1} + 2 = 0$

Furfuris: $4^x - 3.2^x + 2 = 0$

:. $2^{2x} - 3.2^x + 2 = 0$

:. $(2^x)^2 - 3.2^x + 2 = 0$

Ka dhig $y = 2^x$

:. $y^2 - 3y + 2 = 0$

:. $(y-1)(y-2) = 0$

:. $y = 1$ ama $y = 2$

:. $2^x = 1$ ama $2^x = 2$

:. $2^x = 2^0$ ama $2^x = 2^1$

Markaa $x = 0$ ama $x = 1$

LAYLI Doon qiimaha x.

1. $2^x = 8$
2. $3^{2x} = 3^{-2}$
3. $(3^x - \frac{1}{3})(3^x - 1) = 0$
4. $(5^x - \frac{1}{25})(5^x - 25) = 0$
5. $3^{2x} - 4.3^x + 3 = 0$
6. $2^{2x} - 5.2^x + 4 = 0$
7. $4^x - 9.2^x + 8 = 0$
8. $9^x - 10.3^x + 9 = 0$
9. $(\frac{1}{3})^{x-1} = 1$
10. $3.2^x = 24$
11. $\frac{2^{x2}}{2^{2x}} = \frac{8}{1}$

LOGARDAMYADA

Isle'egta $2^3 = 8$ macneheedu waxa weeye "3 waa jibbaarka saaran (raised) salka 2 si ay u dhalato tirada 8".

Logardam waa jibbaar, tusaalahan sarena, 3 waa logardamka 8 marka salku yahay 2, waxana loo qoraa:

$$\text{Log}_2 8=3$$

Labada isle'eg $2^3 = 8$ iyo $\log_2 8 = 3$ wa ay midaalsan yihiin (identical).

Guud ahaanna isle'egta $y = a^x$ waxa ay la midaalsan tahay isle'egta $\log_a y = x$; macnee

$y = a^x$ haddii iyo haddii oo keliya oo ay $\log_a y = x$ u fiirso in isle'egta danbe, x ay tahay logardamka, isle'egta horena x ay tahay jibbaarka saaran a.

QEEX: Logardamka tiro ee sal ogaali waa jibbaarka ku kacsan salka si ay tiradu u dhalato.

Labada isle'eg ee aynu tixgeliney midba ta kale ayaa loo beddeli karaa.

Tusaale 1: $\log_3 x = \frac{5}{2}$, doon x

Furfuris: $\log_3 x = \frac{5}{2}$ waxa ay u dhigantaa $3^{5/2} = x$

$$3^2 \cdot 3^{\frac{1}{2}} = x$$

$$9\sqrt{3} = x$$

Tusaale 2: Haddii $\log_x 27 = \frac{3}{2}$, doon x.

Furfuris: $\log_x 27 = \frac{3}{2}$ waxa ay u dhigantaa

$$x^{3/2} = 27$$

$$x^{3/2} = 3^3$$

$$(x^{3/2})^{2/3} = (3^3)^{2/3}$$

$$x = 3^2 = 9$$

Tusaale 3: Qiimee $\log_{\frac{1}{3}} 81$

Furfuris. Ka dhig $\log_{\frac{1}{3}} 81 = x$, markaa

$\text{Log}_{\frac{1}{3}} 81 = x$ waxa ay u dhigantaa $(\frac{1}{3})^x = 81$

$(3^{-1})^x = 3^4$

$3^{-x} = 3^4$

$\therefore \quad -x = 4 \;/\; x = -4$

LAYLI

1. U qor sansaan logardam

(i) $2^3 = 8$ (iii) $3^2 = \frac{1}{9}$ (v) $b = c^x$

(ii) $2 = 4^{\frac{1}{2}}$ (iv) $5^a = x$

2. U qor sansaan jibbaar

(i) $\text{Log}_2 16 = 4$ (iv) $\text{Log}_x 1 = 0$

(ii) $\text{Log}_3 \frac{1}{9} = -2$ (v) $\text{Log}_{16} 1/4 = -\frac{1}{2}$

(iii) $\text{Log}_{25} 5 = \frac{1}{2}$ (vi) $\text{Log}_q r = n$

Doon qiimaha x, haddii:

3. $\text{Log}_2 x = 3$
4. $\text{Log}_3 x = 4$
5. $\text{Log}_{\sqrt{2}} x = 6$
6. $\text{Log}_{\sqrt{3}} x^2 = 8$
7. $\text{Log}_{\sqrt{2}} x^3 = 6$
8. $\text{Log}_{\frac{1}{2}} x = 4$
9. $\text{Log}_{\frac{1}{3}} x = -2$
10. $\text{Log}_{\frac{1}{\sqrt{2}}} x^2 = -6$
11. $\text{Log}_{\frac{1}{16}} x^3 = -3/4$
12. $\text{Log}_{\sqrt[3]{27}} x = -2/3$
13. $\text{Log}_x 8 = 3$
14. $\text{Log}_x 27 = 3/4$
15. $\text{Log}_x \frac{1}{16} = -4$
16. $\text{Log}_x \sqrt{2} = 1/4$
17. $\text{Log}_x 2^{3/2} = -3/4$
18. $\text{Log}_x 1/16 = -2/3$
19. $\text{Log}_x 1/27 = -3/5$
20. $\text{Log}_x 1 = 0$
21. $\text{Log}_2 8$
22. $\text{Log}_5 125$
23. $\text{Log}_2 1/4$
24. $\text{Log}_{\frac{1}{2}} 4$
25. $\text{Log}_3 1/9$
26. $\text{Log}_{\sqrt{3}} 9$
27. $\text{Log}_{\sqrt{21}} 8$
28. $\text{Log}_9 \frac{1}{27}$
29. $\text{Log}_{\frac{1}{3}} 81$
30. $\text{Log}_{\frac{1}{9}} 27$

Fiiro gaar ah(F.G.): Maadaam $2 = 2^1$, $\text{Log}_2 2 = 1$

Sideo kale $\text{Log}_3 3 = 1$, $\text{Log}_{10} 10 = 1$

guud ahaanna maadaam $a = a^1$,

$\text{Log}_a a = 1$, $a \in M$, $a \neq 0$.

Weliba maadaam $1 = 2^0$, $\text{Log}_2 1 = 0$

Sidoo kale $\text{Log}_3 1 = 0$, $\text{Log}_{10} 1 = 0$ guud ahaanna

Maadaam $1 = a^0$, $a \neq 0$, $\text{Log}_a 1 = 0$

Qeexda logardamku waxa ay inna garan siineysaa in

$\text{Log}_3 27 = 3$, $\text{Log}_3 9 = 2$

$\therefore \text{Log}_3 27 + \text{Log}_3 9 = 3+2 = 5 = \text{Log}_3 3^5 =$

$\text{Log}_3 (27.9)$

Sidoo kale $\text{Log}_3 27 - \text{Log}_3 9 = 3-2 = 1 = \text{Log}_3 3 = \text{Log}_3 (\frac{27}{9})$

Weliba $\text{Log}_2 2^3 = 3 = 3.1 = 3\ \text{Log}_2 2$.

Kuwan sare waa tusaalooyin gaarahaaneed oo ka yimi xeerar guud. Xeerarkaas guud waa kuwa ku xusan aragtiinkan.

ARAGTIIN: Haddii $x, y \in M^+$, $a > 0$, $a \neq 1$

$m, n \in A = \{0, \pm 1, \pm 2, \ldots\}$, $n \neq 0$

markaa

1. $\text{Log}_a x + \text{Log}_a y = \text{Log}_a xy$.
2. $\text{Log}_a x - \text{Log}_a y = \text{Log}_a \frac{x}{y}$.
3. $\text{Log}_a (x^n) = n\ \text{Log}_a x$.
4. $\text{Log}_a \sqrt[n]{x^m} = \frac{m}{n}\ \text{Log}_a x$.

Caddaymaha Xeerarka

1. $\text{Log}_a xy = \text{Log}_a x + \text{Log}_a y$

Caddayn

Ka dhig $\text{Log}_a x = m$, $\text{Log}_a y = n$

Markaa $x = a^m$, $y = a^n$

$xy = a^m \cdot a^n = a^{m+n}$

$\therefore \text{Log}_a xy = m+n = \text{Log}_a x + \text{Log}_a y$

2. $\text{Log}_a \frac{x}{y} = \text{Log}_a x - \text{Log}_a y$

Caddeyn:

Ka dhig $\text{Log}_a x = m$, $\text{Log}_a y = n$

Markaa $x = a^m$, $y = a^n$

$\frac{x}{y} = \frac{a^m}{a^n} = a^{m-n}$

$\therefore \text{Log}_a \frac{x}{y} = m - n = \text{Log}_a x - \text{Log}_a y$

3. $\text{Log}_a x^n = n\ \text{Log}_a x$

Caddeyn:

Ka dhig $\text{Log}_a x = m$

Markaa $x = a^m$

$x^n = (a^m)^n = a^{mn}$

$\therefore \text{Log}_a x^n = mn = n\ \text{Log}_a x$

In kasta oo ay lagama maarmaan tahay in xeerarkan loo xusuusnaado sansaanka ay u qoran yihiin, waxa iyana lagama maarmaan ah in loo xusuusnaado looguna dhaqmo sansaankan hoos ku yaalana;

1. $\text{Log}_a x + \text{Log}_a y = \text{Log}_a xy$
2. $\text{Log}_a x - \text{Log}_a y = \text{Log}_a \frac{x}{y}$
3. $n\ \text{Log}_a x = \text{Log}_a x^n$

Markaa tibaaxaha $\log_a 2 + \text{Log}_a 12$, $\text{Log}_a 3 + \text{Log}_a 8$, iyo $\log_a 4 + \log_a 6$ waa isku wada qiime, mid kastaabana wuxuu le'eg yahay $\text{Log}_a 24$. Sidoo kale tibaaxaha $\text{Log}_a 12 - \text{Log}_a 2$, $\text{Log}_a 18 - \text{Log}_a 3$, iyo $\text{Log}_a 24 - \text{Log}_a 4$ ayana waa isku wada qiime, mid kastaabana wuxuu le'eg yahay $\text{Log}_a 6$.

Tusaale 1: Ka dhig $\text{Log} \frac{18^{\frac{1}{2}} . 12^{\frac{1}{2}}}{6}$ tibaax ay ku jiraan Log 2 iyo Log 3.

Furfuris: $\text{Log} \frac{18^{\frac{1}{2}} . 12^{\frac{1}{2}}}{6} = \text{Log} \frac{(3^2.2)^{\frac{1}{2}} . (2^2.3^{\frac{1}{2}})}{2.3}$

$= \text{Log} \frac{3.2^{\frac{1}{2}} . 2.3^{\frac{1}{2}}}{2.3}$

$= \text{Log}\ (2^{\frac{1}{2}} . 3^{\frac{1}{2}})$

$= \text{Log}\ 2^{\frac{1}{2}} + \text{Log}\ 3^{\frac{1}{2}}$

$= \frac{1}{2}\ \text{Log}\ 2 + \frac{1}{2}\ \text{Log}\ 3$

Tusaale 2: Ka dhig Log $\frac{a\sqrt{b}}{c^{2/3}}$ tibaax leh Log_a a, Log b, Log c

Furfuris: Log $\frac{a\sqrt{b}}{c^{2/3}}$ = Log $(a\sqrt{b})$ - Log $c^{2/3}$

= Log_a + Log $\sqrt{b}$ - $\text{Log}_a c^{2/3}$

= Log a + ½ Log b - 2/3 Log c

Tusaale 3: Haddii $\text{Log}_{10}2$ = 0.30, $\text{Log}_{10}3$ = 0.48, xisaabi qiimayaasha:

(i) $\text{Log}_{10}6$ (ii) $\text{Log}_{10}432$ (iii) $\text{Log}_{10}0.375$ (iv) $\text{Log}_{10}5$

Furfuris: (1) $\text{Log}_{10}6$ = Log_{10} (2.3) = $\text{Log}_{10}2$ + $\text{Log}_{10}3$

= 0.30 + 0.48 = 0.78

(ii) $\text{Log}_{10}432$ = Log_{10} $(2^4.3^3)$

= $\text{Log}_{10}2^4$ + $\text{Log}_{10}3^3$

= 4 $\text{Log}_{10}2$ + 3 $\text{Log}_{10}3$

= 1.2 + 1.44

= 2.64

(iii) $\text{Log}_{10}0.375$ = Log_{10} $\frac{375}{1000}$

= Log_{10} 3/8

= $\text{Log}_{10}3$ - $\text{Log}_{10}8$

= $\text{Log}_{10}3$ - $\text{Log}_{10}2^3$

= $\text{Log}_{10}3$ - 3 $\text{Log}_{10}2$

= 0.48 - 0.90

= -0.42 = (9.58-10)

1. U tibaax mid kasta Log 2 iyo Log 3

(i) Log 12 (ii) Log 18 (iii) Log 1/6

Iiv) Log $\sqrt{2/3}$ (v) Log $(2^2.3^{½})$

(vi) Log $(3^n.2^{-n})$

2. Haddii $\text{Log}_{10}2$ = 0.3010, $\text{Log}_{10}3$ = 0.4771, xisaabi (

(i) $\text{Log}_{10}5$ (ii) $\text{Log}_{10}6$ (iii) Log_{10} 0.1875

(jawaabtaada qurubku ha noqdo togane)

3. Haddii $\text{Log}_2 3$ = 1.585 , doon (tuse uma baahnid) qiimayaasha

(i) $\text{Log}_2 6$ (ii) Log_2 2/9 (iii) $\text{Log}_2 72$

(Qurubku ha noqdo togane)

4. U qor mid kasta sidii logardam hal tiro

(i) Log 2 + Log 3	(vi) Log a + Log b
(ii) 4 Log 2	(vii) Log (ab) - 3/2 Log b
(iii) ½ Log 3	(viii) ½ (Log x - Log y)
(iv) -2 log 3	(ix) ½ Log (ab) - ½ Log a
(v) 3 Log 3 - 2 Log 2	(x) Log x + ½ Log y

5. Haddii 10 yahay salka Logardamka, u qor mid kasta sidii Log hal tiro

(i) 1 + Log a + Log b

(ii) Log (ab) - 2 Log b - 1

(iii) 1-2 Log a

(iv) 2- (Log a + 2 Log b)

(v) 2 Log (ab) - 3 Log a + 2

6. U qor mid kasta sidii xaddiyo Logardam wadartood ama faraqood.

Tusaale:

Log_b $(\frac{mn}{r})^{½}$

Furfuris: Log_b $(\frac{mn}{r})^{½}$ = ½ Log_b $\frac{mn}{r}$

= ½ $\left[\text{Log}_b m + \text{Log}_b n - \text{Log}_b r\right]$

(i) $\text{Log}_b xy$

(ii) Log_b mnL

(iii) $\text{Log}_b \frac{x}{y}$

(iv) $\text{Log}_b x^5$

(v) $\text{Log}_b x^{½}$

(vi) $\text{Log}_b x^{⅓}$

(vii) $\text{Log}_{10}\ 2\pi\sqrt{\frac{L}{g}}$

(Ix) $\text{Log}_b\ (\frac{mn^2}{L})^{\frac{1}{2}}$

(x) $\text{Log}_b (xy)^{1/4}$

7. U gor sidii Logardam keliya oo weheliyihiisu yahay 1.

Tusaale : $\frac{1}{2}(\text{Log}_b x - \text{Log}_b y)$

Furfuris: $\frac{1}{2}(\text{Log}_b x - \text{Log}_b y) = \frac{1}{2}(\text{Log}_b \frac{x}{y}) = \text{Log}_b (\frac{x}{y})^{\frac{1}{2}}$

(i) $\text{Log}_b x + \text{Log}_b y$

(ii) $2\ \text{Log}_b x + 3\ \text{Log}_b y$

(iii) $3\ \text{Log}_b x + \text{Log}_b y - 2\ \text{Log}_b n$

(iv) $\text{Log}_{10}(x-2) + \text{Log}_{10} x - 2\ \text{Log}_{10} n$

(v) $\frac{1}{2}(\text{Log}_{10} x - 3\ \text{Log}_{10} y - 5\ \text{Log}_{10} n)$

LAYLI

Tusaale: Haddii $3.2^{1-x} = 4.5^x$, doon qiimaha x

Furfuris: Haddii $3.2^{1-x} = 4.5^x$, markaa

$\text{Log}\ 3.2^{1-x} = \text{Log}\ 4.5^x$

$\text{Log}\ 3 + \text{Log}\ 2^{1-x} = \text{Log}\ 2^2 + \text{Log}\ 5^x$

$\text{Log}\ 3 + (1-x)\ \text{Log}\ 2 = 2\ \text{Log}\ 2 + x\ \text{Log}\ 5$

$\therefore\ \text{Log}\ 3 + \text{Log}\ 2 - x\ \text{Log}\ 2 = 2\ \text{Log}\ 2 + x\ \text{Log}\ 5$

$\text{Log}\ 3 - \text{Log}\ 2 = x(\text{Log}\ 2 + \text{Log}\ 5)$

$\text{Log}\ 3/2 = x\ \text{Log}\ (2.5)$

$\text{Log}\ 3/2 = x\ \text{Log}\ 10$

$\frac{\text{Log}\ 3/2}{\text{Log}\ 10} = x$

Haddii uu salka Logardamku yahay 10

markaa $x = \frac{\text{Log}_{10} 3 - \text{Log}_{10} 2}{\text{Log}_{10} 10}$

$x = \text{Log}_{10} 3 - \text{Log}_{10} 2$

$= 0.4771 - 0.3010 = 0.1761$

Ogow: $\text{Log}\frac{a}{b} \neq \frac{\text{Log}\ a}{\text{Log}\ b}$

Furfur isle'egyadan:

1. $\text{Log}_x 2 + \text{Log}_x 4 + \text{Log}_x 8 = 12$
2. $\text{Log}_3 x + \text{Log}_3 x^2 + \text{Log}_3 x^3 = 3/2$
3. $\text{Log}_x 48 + \text{Log}_x 24 - 2\ \text{Log}_x 6 = 5/2$
4. $\text{Log}_3(xy) = 6,\ \text{Log}_3(\frac{x}{y}) = 2$
5. $\text{Log}_2 (x^2y^3) = -1,\ \text{Log}_2(\frac{x}{y^2}) = 3$
6. $\text{Log}_2(2x-5) - \text{Log}_2 (x+3) = 0$
7. $\text{Log}_2(5x+1) - \text{Log}_2 (3x-5) = 2$

FANSAARADA JIBBAARKA IYO KUWA LOGARDAMKA

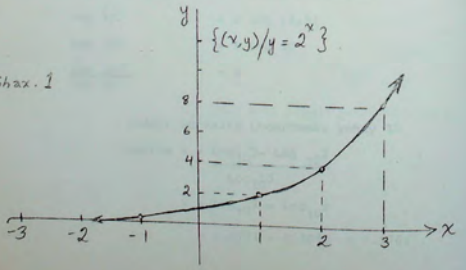
x kasta oo maangal ah waxa la xiriira hal tiro b^x ($b \in M$, $b > 0$). Sidaa darteed isle'egta $y = b^x$ ($b > 0$)....(1) waxa ay qeexdaa fansaar. Maadaam $1^x = 1\ \forall\ x \in M$, isleegta(1) waxa ay qeexdaa fansaar madoorsoome ah, haddii $b = 1$. Haddii $b \neq 1$, isle'egta (1) waxa ay qeexdaa fansaar jibbaar (exponential function). fansaarada jibbaarka waxa si dhab ah loo dersi karaa haddii la tixgeliyo garaafyadooda. Waxa aynu halkan marka hore ku dersi doonaa garaafka $y = b^x$, ($b > 1$).

Matalan, garaafka $y = 2^x$ waxa lagu heli karaa iyaga-oo marka hore qiimayaal kala gedisan la siiyo x dabeednaila xisaabiyo (compute) qiimayaasha ku aaddan ee y. Jadeeyooyinki waxa ay ka muuqdaan tusaha iyo garaafka hoos ku yaal.

$\{(x,y) \mid y = 2^x\}$

x	-3	-2	-1	0	1	2	3	
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	

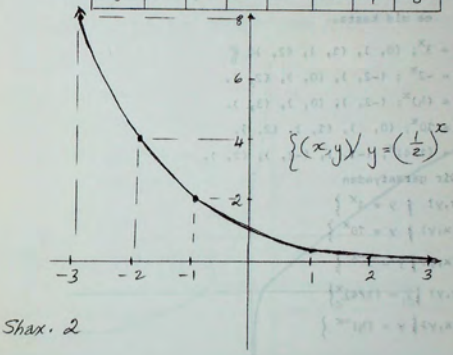
Shax. 1



Marka labaad aan derisno garaafka $\{(x,y) \mid y = b^x,\ 0 < b < 1\}$ matalan, $\{(x,y) \mid y = (\frac{1}{2})^x\}$

$\{(x,y) \mid y = (\frac{1}{2})^x\}$

x	-3	-2	-1	0	1	2	3
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$



Shax. 2

Shax. 2

Garaafka fansaarka ku qeexan $y = (\frac{1}{2})^x$ midig buu hoos u aadaa. Garaafka fansaarka $y = 2^x$-na bidix buu hoos u aadaa. Sidaas darteed fansaarka $\{(x,y) \mid y = b^x,\ b > 1\}$ waa fansaar kordhaya, kan kale $\{(x,y) \mid y = b^x,\ 0 < b < 1\}$ isna waa fansaar dhinmaya. Labada jeerba, horaadku waa dhammaan ururka tirooyinka maangalka ah M; macnee $H = \{x \mid x \in M\}$, danbeedkuna waa $\{y \mid y \in M,\ y > 0\}$.

LAYLI Doon xubinta labaad ee lamaanayaashan horsan ee mid kasta.

1. $y = 3^x$; (0,), (1,), (2,).
2. $y = -2^x$; (-2,), (0,), (2,).
3. $y = (\frac{1}{3})^x$; (-3,), (0,), (3,).
4. $y = 10^x$; (0,), (1,), (2,).
5. $y = (1/4)^x$, (-1,), (-2,), (2,).

Sawir garaafyadan

6. $\{(x,y) \mid y = 4^x\}$
7. $\{(x,y) \mid y = 10^x\}$
8. $\{(x,y) \mid y = 3^{-x}\}$
9. $\{(x,y) \mid y = (1/4)^x\}$
10. $\{(x,y) \mid y = (\frac{1}{3})^{-x}\}$

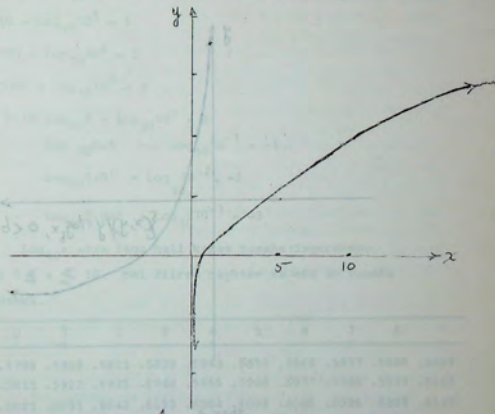
Fansaarada Logardamka

Fansaarka jibbaarka $\{(x,y) \mid y = b^x,\ (b > 0,\ b \neq 1)\}$ ee aynu garaafkiisa soo aragnay isweydaarkiisu waa $\{(x,y)\ x = b^y,\ b > 0,\ b \neq 0,\ x > 0\}$...(2)

F.G. Shardiga $x > 0$ waxa loo sameeyey si y ay u noqoto maangal, maxaa yeeley majirto tiro maangal ah y oo b^y aaney togneyni.

Garaafyada fansaarada ku qeexan isle'eg (2) aan washero innaga oo tixgelineyna tusaalaha $\{(x,y) \mid x = 10^y\ (x > 0)\}$

x	0.01	0.1	1	10	100	1000
y	-2	-1	0	1	2	3



Shax.3

Isle'egta (2) waxa loo qori karaa sidan:

$\{(x,y) \mid y = \text{Log}_b x\ (x > 0,\ b > 0,\ b \neq 1)\}$... (3)

Fansaarada ku qeexan isle'egta (3) ayaa la yiraahaa fansaarada logardamka.

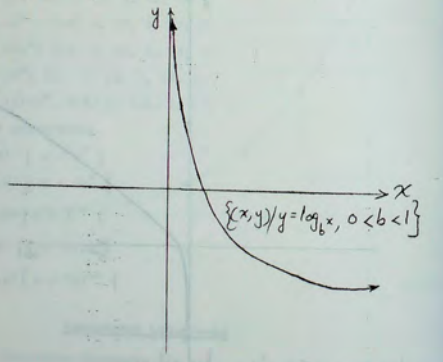
Pansaarka logardamku wuxuu leeyahay astaamahan soo socda:

1. Horaadku waa ururka tirooyinka maangalka ah ee togan, $\{x \mid x \in M,\ x > 0\}$, danbeedkuna waa dhammaan ururka tirooyinka maangalka ah M.

2. Haddii $b > 1$, markaa $\text{Log}_b x < 0$, marka $x < 1$, $\text{Log}_b x = 0$, marka $x = 1$, $\text{Log}_b x > 0$ marka $x > 1$.

3. Haddii $0 < b < 1$, markaa $\text{Log}_b x > 0$ marka $x < 1$, $\text{Log}_b x = 0$ marka $x = 1$, $\text{Log}_b x < 0$ marka $0 < b < 1$.

Garaafka fansaarka logardam marka $0 < b < 1$ waa sidan:



Shax.4

Isle'egyada (2) iyo (3) waa isle'egyo kala gedisan oo fansaar keliya wada sugaya sida ay isle'egyada $x = y - 1$ iyo $y = x + 1$ fansaar keliya u wada sugayaan. Sidaa awgeed weedhaha jibbaabaran iyo kuwa ku qoran sansaan logardam waa La isu bedeli karaa sidii aynu hore u soo sheegnay.

LOGARDAMYADA CAADIGA AH

Qiimayaasha $\text{Log}_{10} x$ ayaa la yiraahaa logardamyada caadiga ah, macnee $\text{Log}_{10} x$ waa jibbaarka la saari doono 10 si loo helo x. Weydiinta aynu qaybtan kaga jawaabeynaa waa, Waa maxay $\text{Log}_{10} x$, haddii $x \in M^+$?

Marka ugu horreysa haddii x ay tahay jibbaar abyoone ah oo saaran 10, $\text{Log}_{10} x$ waxa loo sugi karaa sidan:

$\text{Log}_{10} 10 = \text{Log}_{10} 10^1 = 1$

$\text{Log}_{10} 100 = \text{Log}_{10} 10^2 = 2$

$\text{Log}_{10} 1000 = \text{Log}_{10} 10^3 = 3$

Sidoo kale $\text{Log}_{10} 1 = \text{Log}_{10} 10^0 = 0$

$\text{Log}_{10} 0.1 = \text{Log}_{10} 10^{-1} = -1$

$\text{Log}_{10} 0.01 = \text{Log}_{10} 10^{-2} = -2$

$\text{Log}_{10} 0.001 = \text{Log}_{10} 10^{-3} = -3$

$\text{Log}_{10} x$ waxa laga heli karaa tusaha logardamka haddii $1 \leq x \leq 10$. bal fiirso qaybtan ka mid ah tusaha logardamka.

X	0	1	2	3	4	5	6	7	8	9
3.8	.5798	.5809	.5821	.5832	.5843	.5855	.5866	.5877	.5888	.5899
3.9	.5911	.5922	.5933	.5944	.5955	.5966	.5977	.5988	.5999	.6010
4.0	.6021	.6031	.6042	.6053	.6064	.6075	.6085	.6096	.6107	.6117
4.1	.6128	.6138	.6149	.6160	.6170	.6180	.6191	.6201	.6212	.6222
4.2	.6232	.6243	.6253	.6263	.6274	.6284	.6294	.6304	.6314	.6325
4.3	.6335	.6345	.6355	.6365	.6375	.6385	.6395	.6405	.6415	.6425
4.4	.6435	.6444	.6454	.6464	.6474	.6484	.6493	.6503	.6513	.6522
4.5	.6532	.6542	.6551	.6561	.6571	.6580	.6590	.6599	.6609	.6618
4.6	.6628	.6637	.6646	.6656	.6665	.6675	.6684	.6693	.6702	.6712

Tiro kasta oo ku taal joogutaxa madaxa ay kaga taal x waxa ay u taagan tahay labada rug-cudoon ee ugu horreya x; tiro kasta oo ku taal dhinac u taxa ka horjeeda x-na waxa ay u taagan

tahay ru-cudoonka seddexaad ee x. Godadka (digits) ku yaal isgoyska dhinac-u-tax iyo joogutax ayaa sameeya logardam x. Matalan, si loo helo $\text{Log}_{10}4.25$ waxa la eegi isgoyska dhinac u taxa ka horjeeda 4.2 ee ku hoos yaal x iyo joog u taxa ay madaxa kaga taal tirada 5, kolkaa waxa aynu arki in

$\text{Log}_{10}4.25 = 0.6284$

Sidoo kale $\text{Log}_{10}4.02 = 0.6042$

$\text{Log}_{10}4.49 = 0.6522$

Matalan waxa aynu rabnaa in aan helo $\text{Log}_{10}x$ iyada oo $0 < x < 1$ ama $x > 10$. Marka ugu horreysa tirada aan ku qoro qormo saynis; macnee waxaan tirada u qoreynaa sansaan ah $m.10^n$ oo $1 < m < 10$, $n \in A = \{0, \pm 1, \pm 2, \ldots\}$

Tusaale: $\text{Log}_{10}42.5 = \text{Log}_{10}4.25\text{x}10^1 = \text{Log}_{10}4.25 + \text{Log}_{10}10^1$

$= 0.6284 + 1 = 1.6284$

$\text{Log}_{10}425 = \text{Log}_{10}(4.25 \times 10^2) = \text{Log}_{10}4.25 + \text{Log}_{10}10^2$

U fiirso in qaybta jajab toban laha ee

$= 0.6284 + 2 = 2.6284$

U fiirso in qaybta jajab toban laha ee logardamku ay had iyo jeer tahay 0.6284 qaybta abyoonaha ahina ay tahay jibbaarka saaran 10 marka tirada lagu qoro qormo saynis.

$\text{Log}_{10}x$ wuxuu ka koobmaa laba qaybood, qayb abyoone ah (oo la yiraaho abyan) iyo qayb ah jajab tobanle oo aan tabaneyn lana yiraaho Qurub. Kolkaa tusaha qiimayaasha $\text{Log}_{10}x$; $1 < x < 10$ waa tusaha qurubka $\text{Log}_{10}x$, $\forall\ x > 0$.

Si aan u helo $\text{Log}_{10}43700$, marka hore waxa aynu qori $\text{Log}_{10}43700 = \text{Log}_{10}(4.37 \times 10^{4.37})$, dabeedna waxa aynu ka eegnaa tusaha Log_{10} oo le'eg 0.6405.

$\therefore \text{Log}_{10}\ 43700 = 4.6405$

Imikana waxa aad tixgelisaa tusaale sansaankan leh

$\text{Log}_{10}x$, $0 < x < 1$ sida $\text{Log}_{10}\ 0.00402$

Marka hore aan tirada ku qoro qormo saynis

$\therefore \text{Log}_{10}\ 0.00402 = \text{Log}_{10}\ (4.02 \times 10^{-3})$, dabeedna aan tusaha ka baadhno $\text{Log}_{10}\ 4.02$, waxa aynu heli in $\text{Log}_{10}4.02 = 0.6042$. Haddii aynu isugeyno 0.6042 iyo abyanaha -3 waxa aynu heli in $\text{Log}_{10}0.00402 = -2.3958$ oo aanu qurubka logardamku ahayn 0.6042,

sidii uu ahaan jirey marka $x > 1$, seddexda ruq-cudoon ee ugu horreeya x-na ay yihiin 402. Si haddaba aan taas uga badhbaadno waxa caado ah in logardamka loo qoro sansaan uu qurubku togan yahay. Matalan tusaalahan waxa aynu qori

$\text{Log}_{10}0.00402 = 0.6042 - 3$

$= 0.6042 + (7-10)$

$= 7.6042 - 10$ (oo qurubku togan yahay)

Waxa kale oo la qori karaa

$\text{Log}_{10}0.00402 = 6.6042 - 9$

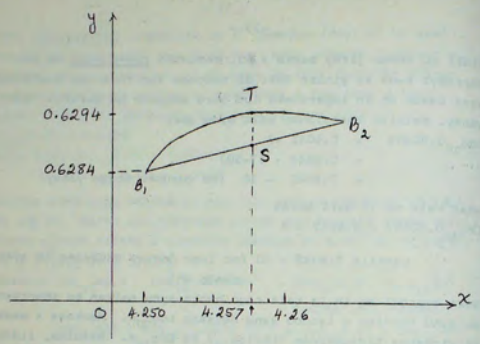
Laakiin 7.6042 - 10 (oo laga gooyey dhufsane 10 ayaa caado ah)

Waxa suurgal ah in la kala rogo habka aynu halkan ku sharraxney oo aynu raadino x iyadoo aynu heysano $\text{Log}_{10}x$. Markaas x waxa la yiraahaa Lidlogardam (Lidlog$_{10}$) ka $\text{Log}_{10}x$. Matalan, lidlog 1.6395 waxa lagu heli karaa iyada oo laga baadho qurubka 0.6395 tusaha Log_{10} oo dabeedna la arko in lidlog_{10} uu yahay 4.36. Kolkaa $\text{lidlog}_{10}1.6395 = 4.35 \times 10^1 = 43.6$.

Haddii aan rabno in aan helo logardamka caadiga ah ee tiro aan ku jirin tusaha(sida log 3712), ama aan rabno x, iyada oo $\text{Log}_{10}x$ aanu u ku jirin tusaha, waxa caado ah in aan isticmaallo habka loo yaqaan DHEXBEEGIDDA TOOSAN. Tusaha Logardamku waa urur ka kooban lamaanayaal horsan, tiro kasta x waxa la xiriira $\text{Log}_{10}x$, dabeedna waxa aynu haysanaa (x, $\text{Log}_{10}x$) oo tuse ku muujisan. Dulalaatiga oo aan inagu fileyn darteed, tusaha waxa ku yaal 3 god (digit) oo ay leedahay x iyo 4 ay leedahay $\text{Log}_{10}x$.

Habka dhexbeegidda toosan ayaa inna awood siinaya in aan tusaha ka helo logardamyada tirooyin 4 god ah.

Aan joomateri ahaan u fiirino fikradda dhexbeegidda toosan. Qayb ka mid ah garaafka y = $\text{Log}_{10}x$ ayaa ku yaal shaxan 5. Aan u sticmaalo in xarriiqda toosan ee isku xireysa baraha B_1iyo B_2 ay tahay xoodka mara barahaa dhexdooda. Haddii aynu heysano garaaf weyn oo y = $\text{Log}_{10}x$, qiimaha $\log_{10}$ 4.257 waxa lagu heli karaa isticmaalidda qiimayaasha ordineytka RT ee xoodka (curve) marka x = 4.257. Maadaam aanu tuse-qiimayaal keligii inaga kaalmeyneyn taas, waxa aynu isticmaali doonaa qiimaha ordineytka RS ee xarriiqda toosan.



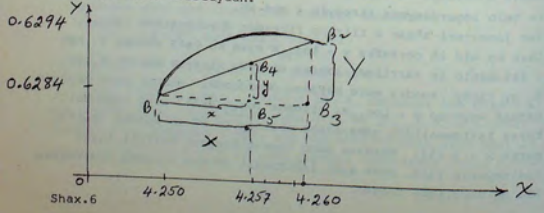
Shax.5

Taas waxa toos looga heli karaa ururka tirooyinka ah ee ku yaal tusaha logardamka.

Tixgeli shaxan 6. oo $B_2\ B_3$ iyo $B_4\ B_5$ ay ku qotomaan $B_1\ B_3$.

$\triangle B_1 B_4 B_5 \sim \triangle B_1 B_2 B_3$, .. dhinacyada isku beegani waa ay saamigalsan yihiin, kolkaa $\frac{x}{X} = \frac{y}{Y}$.

Haddii aynu ognahay 3 ka mid ah tirooyinkaa, waa aynu sugi karnaa ta afraad. Aan u qaadano in tirooyinkeena oo idili ay yihiin tirooyin 4 god leh,macnee waxa aynu qaadan 4.250 halkii aynu ka qaadan lahayn 4.25 iyo 4.260 halkii aynu ka qaadan lahayn 4.26. Aan ogaano in tirada 4.257 ay ku dhacdo bar ah 7/10 masaafada ay ku dhacaan baraha 4.250 iyo 4.260 sida ay u kala horreeyaan.



Shax.6

Qiimaha y (0.0010) waa faraqa u dhexeeya logardamyada 0.6284 iyo 0.6294. Qiimayaasha haddii aan ku beddelo isle'egta waxa aynu heli $7/10 = y/0.0010 \Rightarrow y = 7/10\ (0.0010)$ haddii aan isku darno 0.6284 iyo 0.0007, waxa aynu heli logardam 4.257. Macnee $\text{Log}_{10}4.257 = 0.6291$. Lidlogardamka tirona waxa lagu heli karaa habkaas oo kale.

Tusaha $\text{Log}_{10}x$ iyo xeerarka (1) $\text{Log}_a x + \text{Log}_a y = \text{Log}_a xy$

(2) $\text{Log}_a x - \text{Log}_a y = \text{Log}_a \frac{x}{y}$ (3) $\text{Log}_a x^n = n\ \text{Log}_a x$ - haddii wadajir loo isticmaalo waxa ay fududeeyaan xisaabo leh tarano, qaybo, xoogag iyo xidido. Waxa kale oo ay inna awoodsiiyaan furfurida isle'egyo hal doorsoome leh oo doorsoomuhu uu yahay jibbaar inta aynaan u gelin isticmaalka xeerarka sare aan sameyno 3 Hawraarood oo run ah.

(1) Haddii $M = N\ (M,N > 0)$, markaa $\text{Log}_b M = \text{Log}_b N$

(2) Haddii $\text{Log}_b M = \text{Log}_b N$, markaa $M = N$

(3) Haddii $M = N$, markaa $b^m = b^n$

TUSAALE Xisaabi $\frac{(8.21)^{\frac{1}{2}}(2.17)^{2/3}}{(3.14)^3}$

Furfuris: Ka dhig $N = \frac{(8.21)^{\frac{1}{2}}(2.17)^{2/3}}{(3.14)^3}$

$$\therefore \text{Log}_{10} N = \text{Log}_{10} \frac{(8.21)^{\frac{1}{2}}(2.17)^{2/3}}{(3.14)^3}$$

$$= \text{Log}_{10}(8.21)^{\frac{1}{2}} + \text{Log}_{10}(2.17)^{2/3} - \text{Log}_{10}(3.14)^3$$

$$= \tfrac{1}{2}\ \text{Log}\ 8.21 + 2/3\ \text{Log}_{10} 2.17 - 3\ \text{Log}_{10} 3.14$$

$$= \tfrac{1}{2}\ (0.9143) + 2/3\ (0.3365) - 3\ (0.4969)$$

$$= 0.4572 + 0.2243 - 1.4907$$

$$= -0.8092 = 9.1908 - 10$$

$$\therefore N = \text{Lidlog}_{10}\ (9.1908 - 10)$$

$$\therefore N \approx 0.155$$

LAYLI

I. Logardamyadan ka soo saar tusaha $\text{Log}_{10}x$.

1. $\text{Log}_{10}6.73$
2. $\text{Log}_{10}891$
3. $\text{Log}_{10}\ 0.813$
4. $\text{Log}_{10}\ 0.00214$
5. $\text{Log}_{10}\ (2.48 \times 10^{2})$
6. $\text{Log}_{10}\ (5.39 \times 10^{-3})$

Doon Lidlog_{10}

7. $\text{Lidlog}_{10}\ 0.6128$
8. $\text{Lidlog}_{10}\ 0.5647$
9. $\text{Lidlog}_{10}\ (8.8075 - 10)$
10. $\text{Lidlog}_{10}\ 0.2504$
11. $\text{Lidlog}_{10}\ 3.9258$
12. $\text{Lidlog}_{10}\ (3.9722 - 5)$

II. LOGARDAM KASTA : Ku raadi dhexbeegidda toosan

TUSAALE : $\text{Log}_{10}4257$

	x	$\text{Log}_{10}x$	
10	4.250	0.6284	0.0010
7	4.257	?	y
	4.260	0.6294	

$$\frac{7}{10} = \frac{y}{0.0010}$$

$$y = 0.0007$$

$$\therefore \text{Log}_{10}\ 4.257 = \text{Log}_{10}\ 0.6284 + 0.0007$$
$$= 0.6284 + 0.0007$$
$$= 0.6291$$

1. $\text{Log}_{10}\ 4.213$
2. $\text{Log}_{10}203.4$
3. $\text{Log}_{10}\ 8.184$
4. $\text{Log}_{10}0.5123$
5. $\text{Log}_{10}\ 1522$
6. $\text{Log}_{10}\ 0.008351$

III. LIDLOGARDAMYADAN ku raadi dhexbeegidda toosan

TUSAALE : $\text{Lidlog}_{10}0.6446$

	x	$\text{Lidlog}_{10}x$	
0.0010	0.6444	4.410	0.010
0.0002	?	y	
	0.6454	4.420	

$$\frac{0.0002}{0.0010} = \frac{y}{0.010}$$

$$y = 0.002$$

Isugee qiimaha y iyo 4.410

:. $\text{Lidlog}_{10}\ 0.6446 = 4.410 + 0.002$

$= 4.412$

1. $\text{Liglog}_{10}\ 0.5085$
2. $\text{Lidlog}_{10}\ 1.0220$
3. $\text{Lidlog}_{10}\ (8.7055 - 10)$
4. $\text{Lidlog}_{10}\ 0.8087$
5. $\text{Lidlog}_{10}\ 3.0759$
6. $\text{Lidlog}_{10}\ (9.8742 - 10)$

IV. Xisaabi adiga oo kaalmeysanaya logardamyada

Tusaale $\frac{(23.4)\ (0.681)}{4.13}$

Furfuris: Ka dhig $R = \frac{(23.4)\ (0.681)}{4.31}$

Markaa $\text{Log}_{10} R = \text{Log}_{10} 23.4 + \text{Log}_{10} 0.681 - \text{Log}_{10} 4.31$

$= (1.3692) + (9.8331 - 10) - (0.6160)$

$= 0.5863$

:. $\text{Lidlog}_{10}\ 0.5863 = 3.857$

1. $(2.32)\ (1.73)$
2. $\frac{3.15}{1.37}$
3. $(2.3)^5$
4. $\frac{\sqrt[3]{\ }}{8.12}$
5. $\frac{(0.421)^2\ (84.3)}{\sqrt{\ 21.7}}$
6. $(0.0128)^4$
7. $\frac{6.49\ \sqrt[3]{\ 8.21}}{17.9}$
8. $\frac{\sqrt[5]{\ }}{0.0471}$

V. Furfur isle'egta $3^{x-2} = 16$ haddii salka logardamku yahay 10.

Furfuris: $3^{x-2} = 16$:. $\text{Log}_{10} 3^{x-2} = \text{Log}_{10} 16$

:. $x-2\ \text{Log}_{10} 3 = \text{Log}_{10} 16$

$x-2 = \frac{\text{Log}_{10} 16}{\text{Log}_{10} 3}$

$x = \frac{\text{Log}_{10} 16}{\text{Log}_{10} 3} + 2$

:. $x = \frac{1.2041}{0.4771} + 2 = 2.2524$

1. $3^{x+1} = 8$
2. $4^{x^2} = 15$
3. $2^{x-1} = 9$
4. $8^{x^2} = 21$
5. $3^{x+2} = 10$

BEDDELAADDA SALKA LOGARDAMYADA

Matalan waxa aynu heysanaa tuse aan ka heli karo $\log_a x$. Waxa aan rabnaa in aan helo $\text{Log}_b x$ inagaboo aan isticmaalin tuse.

Ka dhig $\text{Log}_b x = m$

:. $x = b^m$

$\text{Log}_a x = \text{Log}_a b^m$

$\text{Log}_a x = m\ \text{Log}_a b$

$m = \frac{\text{Log}_a x}{\text{Log}_a b} = \frac{1}{\text{Log}_a b} \cdot \text{Log}_a x$

$= \text{Log}_b x = \frac{1}{\text{Log}_a b} \cdot \text{Log}_a x$

Tusaale 1 : Qiimee $\text{Log}_2 5$

Furfuris: Maadaam $\text{Log}_y x = \frac{\text{Log}_a x}{\text{Log}_a y}$

$$\text{Log}_2 5 = \frac{\text{Log}_{10} 5}{\text{Log}_{10} 2} = \frac{0.6990}{0.3010} = 2.32$$

TUSAALE 2: Qiime $\text{Log}_3 0.25$

$$\text{Log}_3 0.25 = \frac{\text{Log}_{10} 0.25}{\text{Log}_{10} 3}$$

$$= \frac{9.3979 - 10}{0.4771}$$

$$= - \frac{0.6021}{0.4771}$$

$$= - 1.26 = 8.74 - 10$$

LAYLI

Qiimee

(1) $\text{Log}_2 7$

(2) $\text{Log}_3 0.5$

(3) $\text{Log}_{3.6} 27.8$

(4) $\text{Log}_5 10$

(5) $\text{Log}_3 100$

Abla Ablaynta isku aadinta

1. B T Kutirane kasta oo soocan oo ku jira horaadka B, ku aad soocan ayuu ku leeyahay urur dambeedka T. Isla mar ahaantaas ma jiro kutirsane T oo aan ahayn ku aad kutirsane B. Isku aadkaas oo kale waa mid-mid waana dhammays; waxana la yira fansaar isku beegnaan-mid-mid ah.

2. B T Isku aad mid-mid ah oo aan dhammays ahayn waxa la yiraa fansaar isku begnaan-mid-mid ah oo aan dhammays ahayn .

3. Isku aad badi-mid ah oo dhammays ah waxa la yiraa fansaar isku beegnaan-badi-mid ah.

4. Isku aad badi-mid ah oo aan dhammays ahayn waxa la yiraa fansaar badi-mid ah oo aan dhammaysa ahayn.

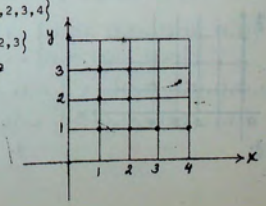
1. Garaafka fansaar

Garaafka fansaar waa urur barood; oo tii kasta ee ka mid ahba ay xubinteeda hore tahay kutirsane urur horaad; xubinteeda dambena tahay kutirsane urur dambeedka. Kutirsane urur horaad wuxuu xubin u noqon kara bar keliya ah oo ka mid ah baraha garaafka.
Haddaba haddii aan doonno in aan hubino in garaaf sawirani yahay fansaar iyo in kale, waxan jeexaynaa xarriiq dhidibka y barbarro la ah; haddii ay xarriiqdaaso bar wax ka badan ka jarto garaafka, markaa xiriirkaasu ma aha.fansaar.

TUSAALE I : Haddii, H = $\{1,2,3,4\}$

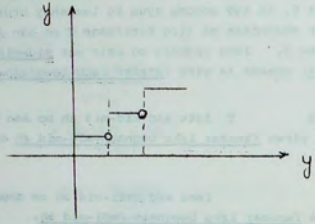
D = $\{1,2,3\}$

Garaafkani ma yahay fansaar?



Jawaabtu waa maya. Xarriiqda ligan ee marta barta dhidibka x yahay (1,0) waxa ay ka gooysaa garaafka hal bar in ka badan.

TUSAALE II: Shaxanka soo socdaa ma yahay garaaf fansaar?



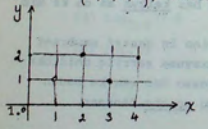
Shax.2

Haa! Barta goobaysani waa ay ka reeban tahay garaafka. Xarriiq kasta oo liganna kama goyn karto garaaf kaas bar keliya wax ka badan.

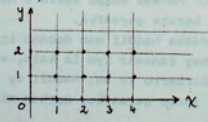
LAYLI

Keebaa garaafyada soo socda ah fansaar?

Horaad : $\{1,2,3,4\}$, Dambeed: $\{-1,0,1,2\}$



1.



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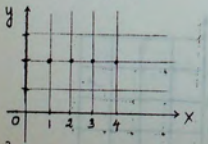
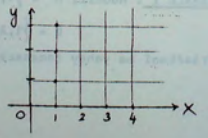


3.



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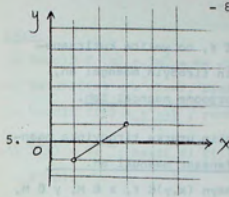

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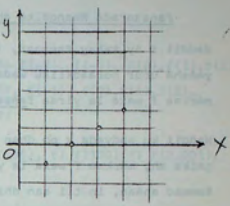

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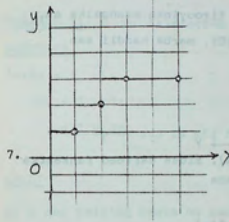

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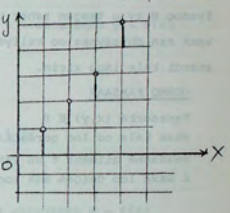
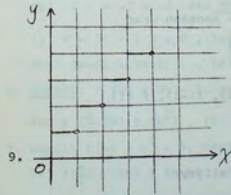


8.



9.

10. Keebaa xiriiradan soo socda ah fansaar?

b = $\{(1,2)\ (1,3)\ (4,-1)\}$ t = $\{(1,-1),\ (2,0),\ (3,1),\ (4,2)\}$

j = $\{(1,-1)\ (2,-1),\ (3,0)\}$ x = $\{(2,0)\ (3,0),\ (4,0)\}$

kh = $\{(2,4)\ ,\ (1,2),\ (4,2),\ (3,0)\}$, d = $\{(1,2)\ (2,2),(3,2),(4,2)\}$

r = $\{(3,0)\ (4,-1),(3,2),(1,1)\}$., s = $\{(4,-1),(3,2)(1,0),(4,2)\}$.

Fansaarada Maangalka ah

Haddii f ay tahay fansaar, (x,y) ∈ f, oo waliba kutirsaneyaasha urur horaadkiisu wada yihiin tirooyin maangal ah, markaa f waxa la yiraa fansaar doorsoome maangal leh.

Haddii ku aadyada x oo dhan ay yihiin ururka tirooyinka maangalka ah, markaa f waxa la yiraa fansaar maangal ah.

Summad ahaan, in tii aan dhigi lahayn (x,y)∈ f, x ∈ M, y ∈ M, iyadoo M ay u taagan tahay ururka tirooyinka maangalka ah, waxa aan dhigaynaa oo kaliya (x,y)∈f, marba haddii aan shardi kale lagu xirin.

QORMO FANSAAR

Fansaarka (x,y) ∈ f

waxa kale oo loo qoraa f: x ⟶ y

maadaama qiimaha f uu yahay f(x), sidaa darteed fansaarka f waxa loo dhigaa ama loo qoraaba

$$f : x \longrightarrow f(x)$$

Haddii aan doonno in aan f u qorno sansaan ururf = { (x,y) / y = f (x) }

Tusaale 1:

Haddii f(x) = 2x+3 raadi f(-4), f(-3), f (1) f(-2), f(0)

FURFURIS :

x kasta ku beddel qiimaha lagu siiyey:

f(-4) = 2(-4)+3 = -8+3 = $-\frac{5}{}$ | f(-2) =2(-2) +3 = $-\frac{1}{}$

f (-3) = 2(-3)+3 = 6+3 = - 3 | f(0) = 2(0)+3 = $\frac{3}{}$

f (1) = 2(1) + 3 = 2 + 3 = $\frac{5}{}$

Qiimayaalkan waxa aan isugu soo ururin karnaa tusahan:

Y	-4	-3	-2	0	1	
f(x)	-5	-3	-1	3	5	

LAYLI

1. Haddii f (x) = $3x^2$ - 4x +1, raadi f(-2), f(-1), f(0),f(1),f(2).
2. Haddiif(x) = x^3-5x-2, raadi f(-2), f(-1), f(0),f(1),f(2).
3. Haddii f(t) = $\frac{t^3+2t}{t-1}$, raadi f(-2)
4. f(x) = $\sqrt{x}$: Raadi a) f(1), b) f(36), t) f(100) j) f(0.001), f(16).

TUSAALE II: U qor isle'egtan 3x+5y = 1 sansaanka y = f(x)

FURFURIS

3x+5y = 1

5y = 1 - 3x

y = $\frac{1-3x}{5}$ waa la mid f(x) = $\frac{1-3x}{5}$

LAYLI

5, U qor isle'eg kasta oo soo socda sansaanka w = f(s)

(b) 4 S = -7 W +2 (t) $\frac{2S+3w}{5}$ = 7

(j) W - 3S + 2 S = 6 , (x) WS+3 = 8S

(kh) 3W+4S = 2S+9 , (d) -16 S + W = - 8 S - 6 W - 18

6. Haddii; f (x) = 3x - 2, raadi:

(a) f ($\frac{a}{2}$)(b) f (a^2) , (t) f (a+2)

7. Haddii f(x) = x^2 - 5, raadi:

(a) f($\frac{a}{2}$) (b) f (a+b) (j) f {f(a) }

FANSAAR TOOSAN

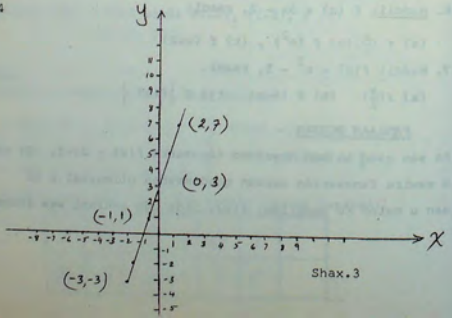
Ka soo qaad in aan haysanno fansaarka f(x) = 2x+3. Si aan u sawiro fansaarkan waxaan qaadanaynaa qiimeyaal x si aan u helno ku aadkiisa, f(x). Sida ugu sahlani waa inaga

oo isticmaalna tuse:

x	2x + 3	f(x) ama y
-1	2(-1) + 3 = -2 + 3 = 1	1
-2	2(-2) + 3 = -4 + 3 = -1	-1
-3	2(-3) + 3 = -6 + 3 = -3	-3
0	2(0) + 3 = 3	3
1	2(1) + 3 = 2 + 3 = 5	5
2	2(2) + 3 = 4 + 3 = 7	7

Baraha: (-1,1), (-2,-1), (-3,-3), (0,3), (1,5), (2,7)
waxayay ka mid yihiin f(x) = 2x+ 3, oo garaafkeedu yahay

Shax.14



Shax.3

Waxa aan aragnaa in garaafka fansaarkaasi yahay xarriiq toosan. Sidaas awgeed fansaarka f(x) = 2x+3 waxa la yiraa "fansaar xarriiqeed"- ama "fansaar toosan". U fiirso f(x) waxa ay ku xiran tahay qiimaha ay x qaadato; sidaas awgeed "x"waxa la yiraa "doorsoomaha madaxa banaan f(x)na waa dabajoog".

QEEXID: Fansaar sansankiisu yahay f(x) = ax +b, iyaga oo a iyo b ay yihiin madoorsameyaal , waxa la yiraa "fansaar xarriiqeed" ama "fansaar toosan"

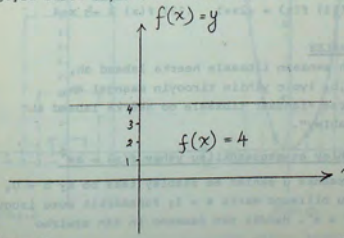
TUSAALE: f(x) = x+2, (a = 1, b = 2)
f (x) = -3x, (a = -3, b = 0)
f (x) = 4 , (a = 0, b = 4)

FANSAAR MA DOORSAME

Haddii aad u fiirsatid fansaarka f(x) = 0.x +4, waxa aad arkaysaa in qiima kasta oo ay "x" qaadataba ay f(x) = 4. Fansaarka caynkaas oo kale ah waxa la yiraa "fansaar ma doorsame". Intii loo qori lahaa f(x) = 0.x+b, waxa lagu soo gaabiyaa f(x) = b.

TUSAALE I: Sawir garaafka f(x) = 4
Furfuris.

Qiima kasta oo x la siiyaba f (x) = 4, garaafkeeduna waa xarriiq dhidibka -x barbarro la ah, dhidibka y-na ka goysa barta (0,4) (Fiiri shax. 4)

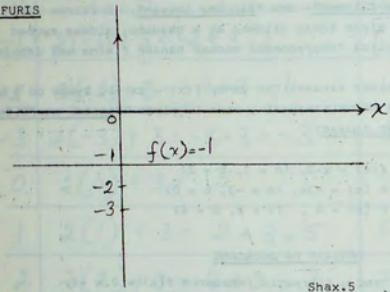


Shax.4

Xusuus!
f (x) = y

TUSAALE II: Sawir garaafka f(x) = -1

FURFURIS



Shax.5

Ogow!

Fansaar ma doorsame waa "fansaar isku beegnaan - badi-mid ah"

LAYLI:

Sawir garaafka fansaarada soo socda sheeg in uu yahay isku-beegnaan badi mid ah ama isku beegnaan mid-mid ah

1) f(x) = 3 (2) f(x) = - 2 (3) f(x) = 2
4) f(x) = 2x+2 (5) f(x) = 2x - 2 (6) f(x) = 2x
7) f(x) -x = 0 (8) f(x) = ½ x + 1 (9) f(x) = $\frac{x+1}{2}$
10) f(x) =-x -1 (11) f(x) = -2x+1 (12) f(x) = -½ x+4

FANSAARKA SAABLEY

Fansaarka leh sansaan tibxaale heerka labaad ah, ax^2+bx+c, oo ay a,b, iyo c yihiin tirooyin maangal ah, $a \neq 0$, ayaa la yiraa "fansaar tibxaale oo heerka labaad ah" ama "fansaarka saabley".

Fansaarka Saabley ee sansaankiisu yahay $f(x) = ax^2$

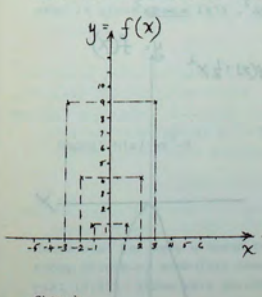
$f(x) = ax^2$ waa sansaanka u sahlan ee saabley taas oo ay b = 0, c = 0. Bal aan ku billawno marka a = 1; fansaarkii wuxu isugu soo ururayaa $f(x) = x^2$. Haddii aan damacno in aan sawirno

waxa aan samaysanaynaa tusaha soo socda oo kale.

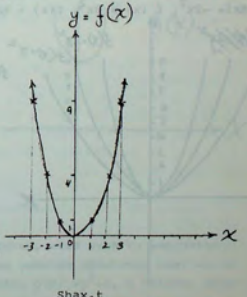
x	x^2	f(x)	
-3	$(-3)^2$	9	
-2	$(-2)^2$	(4)	
-1	$(-1)^2$	1	
0	0^2	0	
1	1^2	1	
2	2^2	4	
3	3^2	9	

Tusahaasi wuxuu la mid yahay inagaoo u qorna sansaan horsiimo lammaaneyaal, sida

{.....(-3,9), (-2,4), (-1,1), (0,0), (1,1), (2,4), (3,9)....} marka aan barahan ku sawirno kulannada kartis" garaafka shaxanka soo socda ayaa soo baxaya.



Shax.b



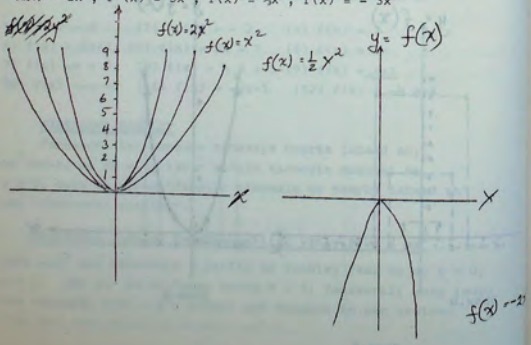
Shax.t

Shax.6

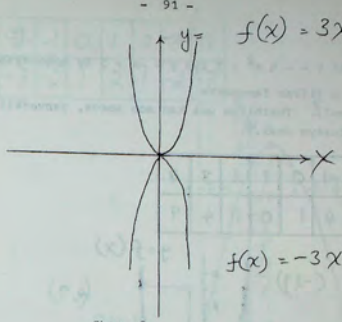
Xarriiqda xoodan ee isku xiraysa baraha shax.b ayaa inna siisay shax.t; waana garaafka fansaarka $f(x) = x^2$ ama $y = x^2$, maadaama ay f(x) iyo y isku mid yihiin. Malaha, waa aad aragta in baraha: (-1,1) iyo (1,1); (-2,4) iyo (2,4); (-3,9) iyo (3,9) qiimaha f(x) uu isla mid yahay; hase yeeshee qiimaha x in midba kan kale tabane u yahay. Haddii fansaar leeyahay astaanta $f(x) = f(-x)$ marka garaafka $y = f(x)$ waxa la yiraa "wuxuu ku wanqaaran yahay dhidibka y". Micnuhu waxa weeye, haddii aan garaafkaas ka laabno dhidibka y, barba bar ku aad ah ayey dul fuulaysaa waana ay isku sargo'naanayaan. (shax.2, (-1,1) iyo (1,1) isku in bay u wada jiraan dhidib -y; baraha isku lammaan ee soo socdaana waa sidoo kale (-2,4) iyo (2,4) ; (-3,9) iyo (3,9) iw.m.).
Fansaarka $f(x) = x^2$ wuxu leeyahay bar ugu qiime yaraguna hooseysa, markii la fiirsho baraha garaafka $f(x) = x^2$ oo dhan.

Ugu dambayn, fansaar $f(x) = x^2$ wuxu inna siiyaa xarriiq xoodan oo kor u furan lana yiraa SAAB.

Isku day in aad bardhigto baro badan si ay kugu suurtowdo in aad sawirto garaafka $f(x)=2x^2$, $f(x) = \frac{1}{2}x$; $f(x) = 2x^2$, $f(x)= -2x^2$, $f(x) = 3x^2$, $f(x) = \frac{1}{2}x^2$, $f(x) = -3x^2$



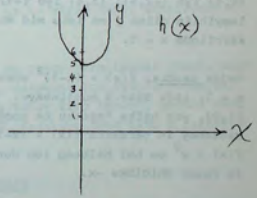
Shax.7



Shax. 8

Waxaad aragtaa in garaafyada $f(x) = 2x^2$, $f(x) = \frac{1}{2}x^2$, $f(x) = 3x^2$ ay barta ugu yar tahay unugga, ayna kor u wada furan yihiin; hase yeeshee garaafka fansaarada $f(x) = -x^2$, $f(x) = -3x^2$ waxa ay u furan yihiin hoos, unuguna waa barta ugu weyn ama ugu sarraysa

Barta ugu weyn ama ugu yar garaafka fansaarka saabley waxa la yiraa gees.



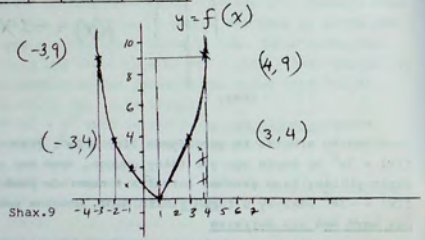
Haddii $h(x) = x^2+5$

Marka h(x) kor ayey u furan tahay, waxa ayna ku wanqaaran tahay dhidib-y, sida f(x); hase yeeshee geeskeedu waxa uu ku yaal (0,5). Sidoo kale garaafka $g(x) = x^2-5$, 5 halbeeg ayey ka hooseeysaa geeska f(x); wuxuna u furan yahay kor; isla markaas waxa uu wanqaaran yahay dhidib -y. Ka warran sida

garaafyada $y = -\frac{1}{3}x^2 + 3$ iyo $y = -x + 5$ ay noqonayaan.

Bal u fiirso fansaarka
$f(x) = (x-1)^2$. Tusihiisu waa kan soo socda, garaafkiisana waxa ku tusaya shax.9.

x	-2	-1	0	1	2	3	4
f(x)	9	4	1	0	1	4	9



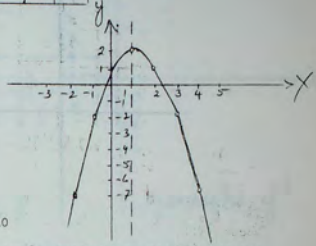
Shax.9

Kor ayuu u furan yahay; barta ugu yarna waa (1,0). Baraha (4,9) iyo (-2,9); (3,4) iyo (-1,4); (2,1) iyo (0,1) waxa ay leeyihiin qiima y oo isku mid ah; waxa ayna isku in u jiraan xarriiqda x = 1.

Hadda saabka, $f(x) = (x-1)^2$ wuxu ku wanqaaran yahay xarriiqda x = 1, sida Shax.9 muujinaayo. Waliba barta ugu yar garaafka, (1,0), waa halka "saabku ka gooyo dhidib-x. Marka waa ay inna caddahay in garaafka $f(x) = (x-1)^2$ u la mid yahay garaafka $f(x) = x^2$ oo hal halbeeg loo durkiyay xagga midig, marka la raaco dhidibka -x.

Xarriiqda x = 1 waxa kale oo ay tahay dhib - wanqaaranka garaafka fansaarka $f(x) = -(x-1)^2+2$, (fiiri Shax.10; hase yeeshee geeska garaafkan waa (1,2).

x	-2	-1	0	1	2	3	4
f(x) ama y	-7	-2	1	2	1	-2	-7



Shax.10

Fansaarkan saabley hoos ayey u furan tahay geeskeeduna waa ugu weyn yahay.

GABAGABO

Garaafka fansaar $f(x) = a(x-h)^2+k$, $a \neq 0$, wuxu ku wanqaaran yahay xarriiqda x = h, geeskeeduna waa barta (h,k). Haddaba k waxa aan niraahnaa qiimahá gees. Sansaanka iyo hadba dhinaca uu u furmayo garaafku waxa ay ku xiran tahay weheliyaha x^2, oo ah a.

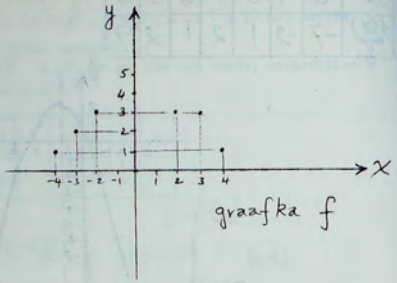
Haddii a $>$ 0 (amase a ay tahay tiro togan) geesku waxa uu noqonayaa barta ugu yar, saabkuna kor ayuu u furmayaa. Haddii a $<$ 0 (amase a ay tahay tiro taban), geesku wexa uu noqonayaa barta ugu weyn, saabkuna hoos ayuu u furmayaa.

WEYDAARKA FANSAAR

Horay waxa aan u soo araqnay in uu fansaar yahay xiriir gaar ah. Weydaarka xiriirna, $\bar{g}^1$, waxa weeye xiriirkii g oo horaadkiisii iyo dambeedkiisii la isku beddelay. Sidoo kale weydaarka f, oo loo qoro f^{-1}, waa xiriirka dhasha marka la isku beddelo horaadka iyo dambeedka f. Bal u fiirso fansaarkan:

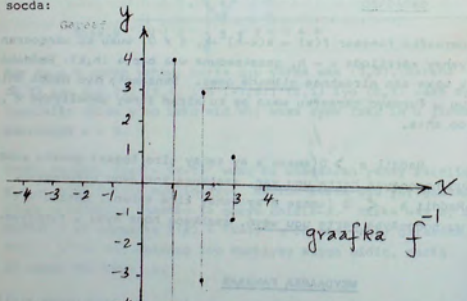
$f = \{(2,3), (-2,3), (-3,2), (3,2), (4,1), (-4,1)\}$

Sawirka garaafkan waa ka hoos ku muujisan:



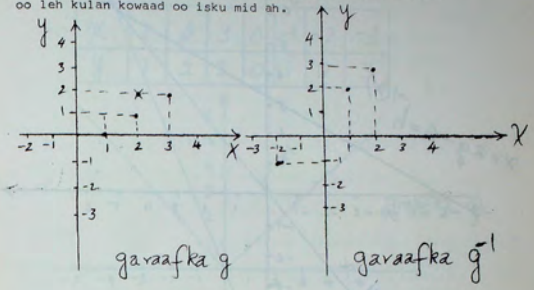
Weydaarka fansaarka f waa xiriir
$f^{-1} = \{(3,2), (3,-2), (2,-3), (2,+3), (1,4), (1,-4)\}$.

f^{-1} waa xiriir, mana aha fansaar, garaafkiisuna waa kan soo socda:



Mar labaad dheeho fansaarka
$g = \{(1,0), (2,1), (3,2), (0,-1), (-1,-2)\}$
Weydaarka fansaarkaa waa xiriir $g^{-1} = \{(0,1),(1,2),(2,3)$ $(-1,0), (-2,-1)\}$.

g^{-1} waliba waa fansaar, maadaama ayna jirin laba barood oo leh kulan kowaad oo isku mid ah.



TUSAALE: Ka soo qaad in fansaarka
$h = \{(x,y) / y = 2x - 6\}$, h^{-1} ma yahay fansaar? Ku sawir h iyo h^{-1} isku sallax.

FURFURIS:

h: waxa aan aragnaa in $y = 2x-6$ ay tahay fansaar toosan, sawirkeedana waxa inagaga filan in la helo laba barood oo keliya ; dabadeedna la isku xiro barahaas.

$y = 2x - 6$
Aan qaadanno barahan (0,-6),(1,-4),(2,-2),

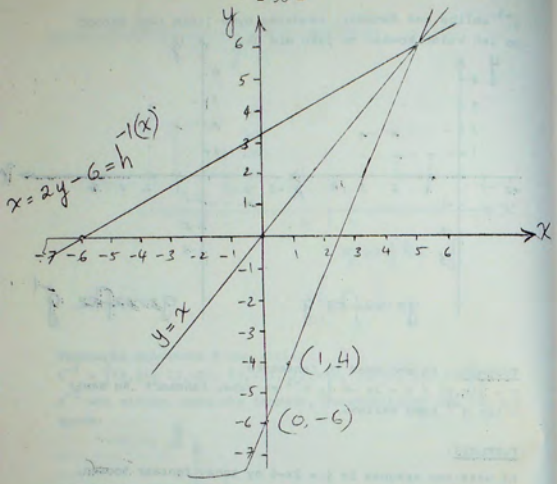
x	0	1	2
y	-6	-4	-2

h^{-1}: waxa ay la mid tahay inaga oo doorsoomaha madaxa banaan ka dhigna y oo isle'egta x ku tibaaxna.

$h^{-1} = \{(y,x) / y = 2y \quad 6\}$,

h^{-1} iyana waa fansaar toosan.
$x = 2y - 6$

y	0	1	+2
x	-6	-4	+2



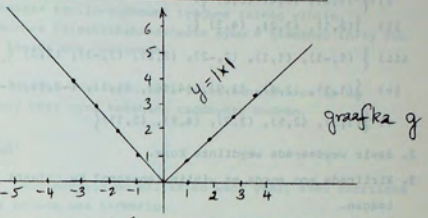
Waa ay muuqata in h^{-1} ay tahay fansaar. Maadaama fansaar toosani yahay fansaar isku beegnaan - mid-mid ah, weydaarkiisuna waa fansaar. Fansaarka y = x waxa la yiraa fansaar asal madoorshe ah. Weydaarkiisu waa isla isagii. Haddaba garaafka fansaar kasta iyo weydaarkiisu waxa ay ku wanqaaran yihiin xarriiqda y = x.

TUSAALE II. Sawir garaafka g = $\{(x,y) / y = |x|\}$ iyo $\bar{g}^1$.

FURFURIS:

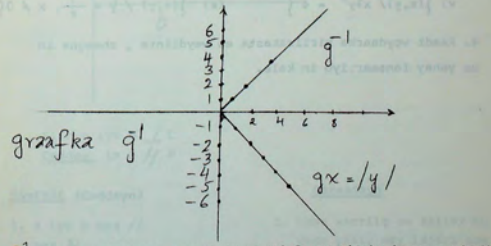
1. y = |x|. Samee tuse si ay kuugu fududaato bardhigidda baraha.

x	1	2	3	0	-1	-2	-3
y	1	2	3	0	1	2	3



2. Si aad u sawirtid g^{-1}, marka hore isku beddel x iyo y ee fansaarka g.

x	1	2	3	0	-1	-2	-3
y	1	2	3	0	1	2	3



g^{-1} ma aha sfansaar. g waxa ay ahayd fansaariisku beegnaan-badi-mid ah, markaa weydaarkeedu ma aha fansaar.

LAYLI

1. Raadi weydaarka xiriir kasta ee soo socda, dabadeedna sheeg weydaarku in uu yahay fansaar ama in aanu ahayn.

i) $\{(1,2), (3,4) (5,6)\}$.

ii) $\{(2,1), (3,2), (4,3)\}$.

iii) $\{(1,-1), (1,1), (2,-2), (2,2), (3,-3), (3,3)\}$.

iv) $\{(1,1), (2,4), (3,9), (4,16), (1,1), (-2,4), (-3,9), (4,16)\}$

v) $\{(1,3), (2,5), (3,7), (4,9), (5,11)\}$

2. Sawir weydaarada weydiinta kore.

3. Xiriirada soo socda ma yihiin fansaaro? Ku tijaabi xarriiq taagan.

i) $\{(x,y) / y = 2x + 4\}$

ii) $\{(x,y) / y = /2x/\}$.

iii) $\{(x,yO / y = 4x^2\}$

iv) $\{(x,y) / y = \frac{x}{|x|}\}$

v) $\{(x,y)/ x^2+y^2 = 4\}$

(vi) $\{(x,y) / y = (x,2)^2\}$

(vii) $\{(x,y) / y = (x-2)^2+1$

(viii) $\{x,y) / y = (x+1)^2$,

(ix) $\{(x,y) / y = -(x-1)^2$

(x) $\{(x,y) / y = \frac{1}{x}, x \neq 0\}$

4. Raadi weydaarka xiriirakasta ee weydiinta , sheegna in uu yahay fansaar iyo in kale.

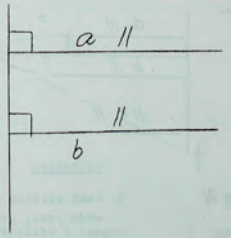
JOOMATARI

Buuggii kowaad waxa aynu ku soo qaadanay in haddii, laba xarriiq oo barbarro ah u gudbane jaro in ay lammaane xagallo-gudeed talantaali ah ay isleeg yihiin, iyo lammaane xagllo-gudbooni iyaguna isleeg yihiin. Hase-yeeshee isleekaanshahaaawaxa aynu u qaadanay in ay run yihiin innaga oon caddeynin.

Haatan waxa aynu tusi isleekaanshahaasu in uu run yahay waxa aynu kaashana caddaynta dadban.

ARAGTIIN:

Laba xarriiq oo sallax ku wada yaal, isku xarriiqna ku wada qotoma waa barbarro.



Siin a iyo b ⊥ C
Caddee in a // b

Hawraar (Caddayn)

1. a iyo b waa // ama ⊥
2. Ka soo qaad in aanay ⊥
3. Markaa waxay ku kulmi bar sida B.

Garaadayn

1. Laba xarriiq oo sallax ku wada yaal, way isjari ama waa barbarro.
2. U qaadasho
3. Xarriiqyo aan barbarro ahayni sallaxna ku wada yaal, way is jaraan.

4. Markaa waxa jiri laba xarriiq oo sallax ku wada yaal, bar debada ahna ka wada yimid, qotona xarriiq u wada ah.

4.

5. Hase-yeeshe tani caqliga ma gelayso

5. Bar debadda ah sida B, xarriiq qudha ah oo qoton u ah xarriiq kale ayaa laga soo jeexi karaa.

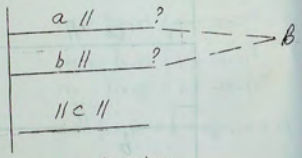
6.:. a iyo b ma kulmi karaan, markaa a // b

6. Xarriiqyo sallax ku wada yaal,oo inkastoo la fidiyo aan kulmayni waa // .

Xigasho: Laba xarriiq oo sallax ku wadayaal isku xarriiqna la barbarro ahi, waa barbarro.

Siin a iyo b waxay b // c

Caddayn in a // b.



Hawraar

1. a iyo b waa // ama ✗

2. Ka soo qaad in ay is jaraan, // .

3. Markaa, bar ayay ku kulmi sida B

4. Markaa, waxa jira laba x xarriiq oo bar ka soo wada dusay, barbarrana u ah xarriiq seddexaad.

Garaadayn

1. Laba xarriiq oo sallax ku wada yaal, waa // ama ✗

2. U qaadasho.

3. Xarriiqyo aan barbarro ahayn, ~~if~~ sallaxna ku wada yaal, way is jaraan

4.

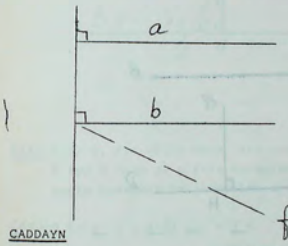
5. Hase-yeeshe tani ma jirto.

5. Bar lagu siiyay xariiq qudha ayaa laga jeexi karaa //-na u ah xarriiq kale.

6.:. a iyo b ma kulmi karaan markaa a // b

6. Xarriiqyo sallax ku wada yaal, inkastoo la fidiyana aan kulmayni waa //.

ARAGTIIN: Haddii xarriiqi qotome u tahay laba xarriiq oo barbarro ah, mid ahaan, markaa ka kalena waa u qotome.



Siin a // b, c ⊥ a barbar D, c haddana waxay ka jartay b barta E.

Caddee in: c ⊥ b barta E

CADDAYN

Hawraar

1. Sawir xarriiqa f isagoo maraya E qotome-na ha u ahaado c

2. a ⊥ c

3. :. f // a

4. Hase-yeeshe b // a

Garaadayn

1. Qotome waa lagu taagi karaa bar xarriiq ku taal.

2. Siin

3. Laba xarriiq oo sallax ku wada yaal, isku xarriiqna qotome u ahi, waa //

4. Siin

5. b iyo f waa in ay is dul dhacaan

6. Hase-yeeshe f ⊥ c

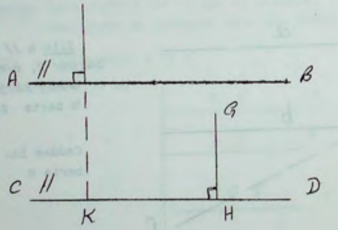
7. :. b ⊥ c

5. Qumaatiga bar lagu siiyay xarriiq qudha ayaa laga sawiri karaa oo la // ah xarriiq lagu siiyay.

6. Dhisme

7. Xarriiq wuxuu qaadan astaamaha xarriqa, u u dul dhacoo idil.

<u>XIGASHO:</u> Laba xarriiq oo barbarroah, haddii mid waliba leeyahay qotome, markaa qotomaduna waa barbarro.



Siin AB // CD; EF ⊥ AB ; GH ⊥ CD

Caddee in EF // GH

<u>CADDAYN</u>

<u>Hawraar</u>	<u>Garaadayn</u>
1. Fidi EF si u CD u ka jaro barta K	1. Dhisme
2. EF ⊥ AB	2. Siin
3. :. EF oo la fidiyay waxay ⊥ u tahay CD	3. Haddii xarriiq laba xarriiq oo // ah mid qotome u yahay, ka kalana waa u ⊥.

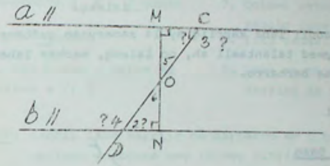
4. Hase-yeeshe GH ⊥ CD

5. :. EF // GH

4. Siin

4. Laba xarriiq oo sallax ku wada yaal, isku xarriiqna ku qotomaa, waa //.

<u>ARAGTIIN:</u> Haddii laba xarriiq oo barbarro ah, u gudbane jaro xagla gudeedka talantaaliga ahi way isleeg yihiin.



<u>Siin</u> a // b, mid walba waxa jara gudbanaha t waxaanu ka jaraa C iyo D sida ay u kala horeeyaan. Waxaana sameeyaan xaglo gudeedka talantaaliga ah ∠1 , ∠2 iyo ∠3, ∠4.

<u>Caddee</u> in ∠1 = ∠2; ∠3 = ∠4

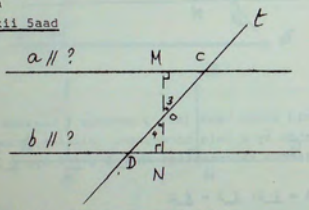
Saafid: si aad u heshid in ∠ luhu isleeg yihiin , isku day in aad heshid hal lammaane Δ oo ay ∠la hoodu yihiin
<u>Caddayn</u>) qaybaha isku aada ee Δ ≅

<u>Hawraar</u>	<u>Garaadayn</u>
1. O ahna bar badhtameedka CD, ku sawir qotomaha MN, ee ⊥ a, b-na ka jara barta N.	1. Xarriijin waa la kala dadbi karaa. Bar sallaxa debadda ka ahna xarriiq qoton waa looga soo sawiri karaa.
2. MN ⊥ b	2. Laba xarriiq oo // ah haddii xarriiq mid ⊥ u yahay, ka kalana waa u ⊥.
3. ∠ m iyo ∠ n waa ∠10 quman	3. Dhinacyo isku qotoma aya sameeyay.

4. $\angle 5 = \angle 6$	4. $\angle$Lo [illegible] foodsaar ah
5. CD = DO	5. O waxay kala badhaa CD
6. $\triangle$ ka quman ee OCM $\cong \triangle$ ka quman ee ah ODN	6. Shakaal iyo xagal (SH.X)
7. :. $\angle 1 = \angle 2$	7. Qisi =
8. $\angle 3 = \angle 4$	8. Xaglo isleegi waxay lee-yihiin xaglo buuxsha oo isleeg.

ARAGTIIN: Haddii laba xarriiq ay la sameeyaan gudbane xaglo gudeed talantaali ah, oo isleeg, markaa labada xarriiq waa barbarro.

Aragtiinkani
waa rogga
Araqtiinkii 5aad



Siin : a iyo b waxa jaray gudbanaha t oo jaray C iyo D sida ayu kala horeeyaan, waxaana sameysmay xaglo-gudeedka isleeg ee ah $\angle 1$ iyo $\angle 2$.

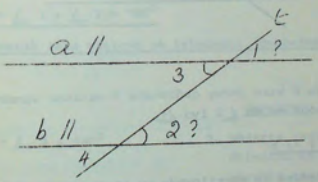
Caddee in a // b

Saafid a // b, haddii labaduba ay $\perp$ u yihiin, isku xarriiq.
Caddayn

Hawraar	Garaadayn
1. Barta O ahna bar badhtameedka CD, ka sawir MN oo barta M $\perp$ uga aha, kana jaraya b barta N.	1. Xariijin waa la kala badhi karaa. Bar sallaxa debedda ka ahna qotome xarriiq waa looga soo sawiri karaa.

2. $\angle 3 = \angle 4$	2. $\angle$o [illegible] foodsaar ah.
3. $\angle 1 = \angle 2$	3. Siin
4. OC = OD	4. O, waxay kala badhaa CD.
5. :. $\triangle$ ka OCM $\cong \triangle$ ka ODN	5. X.Dh. X.
6. $\angle$ M = $\angle$ N	6. Qaybaha isku aada ee seddex-xagallo isku sargo'ani way isleeg-yihiin(Qisi)
7. Mase-yeeshe $\angle$ M = $\angle$ quman	7. Qobomo waxay sameeyaan xaglio quman.
8. :. $\angle$ N = $\angle$ quman	8. Isku beddel
9. :. a, iyo b waa u qoton MN markaa a // b	9. Laba xarriiq oo isku xarriiq ku qotomaa waa //.

ARAGTIIN: Haddii laba xarriiq oo barbarro ah u gudbane jaro, xaglaha gudboone way isleeg yihiin.



Siin: a // b, mid walba waxa jaray gudbaraha t waxaana sameysmay xaglaha gudboone ee ah $\angle 1$ iyo $\angle 2$.

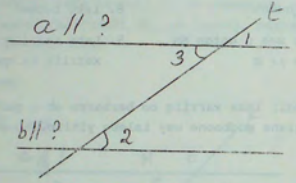
Caddee In $\angle 1 = \angle 2$?

Saafid: $\angle 1$ iyo $\angle 2$ mid walba tus in ay la mid tahay ama le'eg tahay $\angle 3$.

Caddayn

Hawraar	Garaadayn
1. $\angle 1 = \angle 3$	1. Xaqllo foodsaar ah
2. $\angle 3 = \angle 2$	2. Xaqllo qudeed talantaalli ah, ee xarriiqyo // ah.
3. :. $\angle 1 = \angle 2$	3. Astaanta dhexidda.

Xigasho: Haddii laba xarriiq ay la sameeyaan gudbane, xaqllo-gudboon oo isleeg, markaa labada xarriiq waa barbarro.



Siin: a iyo b waxa jeray gudbanaha t waxaana sameeysmay xaglaha gudboon ee ah $\angle 2$ iyo $\angle 1$.

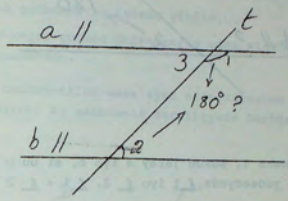
Caddee in a // b

Saafid: Caddee in xarriiqyadu ay la sameeyeen gudbanaha, xaqllo-gudeed talantaalli ah oo isleeg.

Caddayn

Hawraar	Garaadayn
1. $\angle 1 = \angle 3$	1. Xaqlo foodsaar ah.
2. $\angle 1 = \angle 2$	2. Siin
3. $\angle 2 = \angle 3$	3. Xaddiyo leeg. Xaddi isku mid ah, iyaguna way isleeg yihiin.
4. :. a // b	4. Haddii laba xarriiq, ay la sameeyaan gudbane xaqllo-gudeed talantaalli ah oo isleeg markaa xarriiqyadi waa //.

XIGASHO: Haddii laba xarriiq oo barbarro ah u gudbane jaro, labada xagal-gudeed ee dhinac ka wada xiga gudbanaha, way isbuuxshaan.



Siin: a // b, waxaana jaray gudbanaha t $\angle 1$ iyo $\angle 2$ waa laba xagal-gudeed.

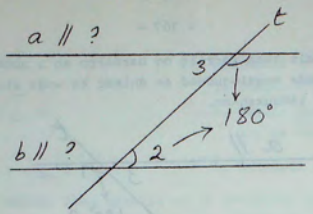
Caddee in $\angle 1 + \angle 2 = 180^\circ$

Saafid: Raadi laba xaglood oo isbuuxsha, debeedna isku beddel.

Caddayn

Hawraar	Garaadayn
1. $\angle 1 + \angle 3 = 180^\circ$	1. Wadarta xaglaha ee bari, dhinacna ka wada xiga xarriiq toosan = 180°.
2. $\angle 3 = \angle 2$	2. Xaglo-gudeedka xarriiqyo barbarro ahi way isleeg yihiin.
3. :. $\angle 1 + \angle 2 = 180^\circ$ markaa xagluhu way isbuuxshaan.	3. Astaanta isku-beddelidda xagla-isbuuxsha wadartoodu waa = 180°.

4. Xigasho Haddii laba xarriiq iyo gudbane ay sameeyaan, xaqllo-gudeed dhinac ka wada xiga gudbanaha; markaa labada xarriiq waa barbarro //.



Siin: Gudbanaha t, wuxuu jaray a iyo b, si uu u sameeyo xaglo gudeedyada, ∠1 iyo ∠ 2, ∠1 + ∠ 2 = 180°

Caddee in a // b

Saafid: Caddee in xarriiqyadu, la sameeyaan gudbanaha xaglo -gudeed isleeg .

Caddayn

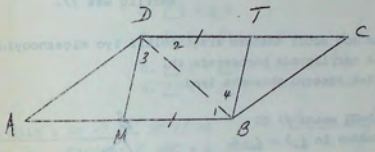
Hawraar	Garaadayn
1. ∠ 1 + ∠ 3 = 180°	1. Wadarta xaglaha ee bari, dhinacna ka wada xiga xarriiq toosani = 180°.
2. ∠1 ∠2 = 180°	2. Siin
3. :. ∠ 1 + ∠ 3 = ∠ 1+∠ 2	3. Dhadhaarka isku-beddelidda.
4. :. ∠ 3 = ∠ 2	4. Dhadhaarka kala go'ynta.
5. :. a// b	5. Haddii laba xarriiq ay la sameeyaan gudbane, xaglio gudeed isleeg, markaa labadd xarriiq waa // barbarro.

DARIIQOOYINKA GUUD

1. Si aad u caddaysid in laba xaglood ay isleeg yihiin, ka eeg shaxanka in:
   a) Xagluhu yihiin, xaglo-gudboon ee xarriiqyo barbarro ah.
   b) Xagluhu yihiin, xagllo-gudeed ee xarriiqyo barbarro ah.

2. Si aad u caddaysid in laba xarriiq ay barbarro yihiin, ka eeg shaxanka in:.
   a) Xaglo-gudeedka talantaaliga ahi ay isleeg yihiin.
   b) Xaglaha gudbooni ay isleeg yihiin.
   c) Labada xagal-gudeed dhinacna ka wada xiga gudbanaha ay isbuuxsaan.

Tusaalahani caddayntiisa waxa aynu adeegsanaynaa araqtimihii iyo xigashooyinki ku saabsanaa xarriiqyada barbarrada ah.



Siin: AB // CD,
AB = CD;
M waxay kala badhaa AB;
Tna waxay kala badhaa CD.

Caddee: In DM // TB

Caddayn

Hawraar	Garaadayn
1. Sawir DB	1. 2 laba barood waxa ay sameeyaan xarriiq.
2. ∠ 1 = ∠ 2	2. Xaglo-gudeedka ee xarriiqyo barbarro ahi way =/
3. AB = CD	3. Siin.
4. MB = ½ AB DT = ½ DC	4. Xarriiqyo la kala badhay.

5. :. MB = DT

5. Badhadhka laba xarriiq oo isleegi way isleeg yihiin.

6. BB = DB

6. Ka dhexeeye (Madoorshe)

7. :. △ ka MBD ≅ △ ka DBT

7. dh. x. dh.

8. :. ∠ 3 = ∠ 4

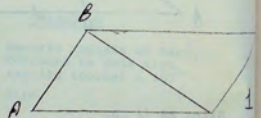
8. Qisi (qaybaha isku aada ee seddex-xagllo isku sargo'an way isleeg yihiin).

9. :. DM // TB

9. Haddii laba xarriiq ay la sameeyaan gudbane xagllo-gudeed isleeg; markaa labada xarriiq waa //.

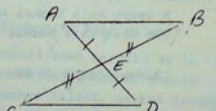
Weeydiimahan soo socda adeegso aragtiimihii iyo xigashooyinkii ku saabsanaa xarriiqyada barbarrada ah.
Weydiimaha 1-4 adeegso shaxanka 1aad.

1. Siin: AB=CD, waana // CD
Caddee in ∠ 3 = ∠ 4
iyo AD // BC



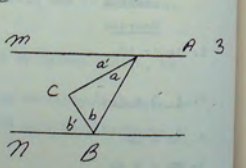
2. Siin : AB // DC, AD // BC
Caddee in ∠ A = ∠ C

3. Siin: AB // DC, AD // BC.
Caddee in AB = DC
iyo in AD = BC.



4. Siin : AB = DC, AD = BC
iyo in AD // BC
Caddee in: AB // DC
iyo in AD // BC

5. Siin: AE = ED, BE = EC
Caddee in : AB // CD

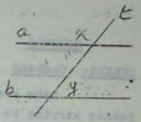


6. Siin Gudbanaha AB wuxuu jaray xarriiqyada m iyo n
∠a = ∠a'; ∠b = ∠b'
∠a + ∠b = 90°
Caddee in m // n

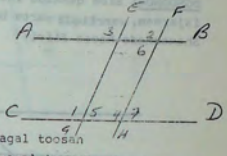
#

7. a) Siin : $y = \frac{2x}{3}$, ∠x = 108°, a iyo bi barbarro ma yihiin?

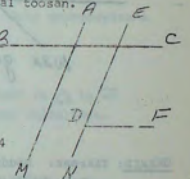
b) Siin: ∠ y = 73°, ∠x - ∠y = 32°
haddaba a iyo bi barbarro ma yihiin?



8. Siin: AB // CD, EG // PH
Caddee in a. ∠1 = ∠2
b. ∠3 = ∠4
c. ∠ 5 = ∠6
d. ∠ 5 + ∠2 = xagal toosan
e. ∠ 1 + ∠ 6 = xagal toosan.



9. Siin : AM // EN, BC // DE
Caddee: ∠ ABC = ∠ EDF;
∠ MBC = ∠ NDF



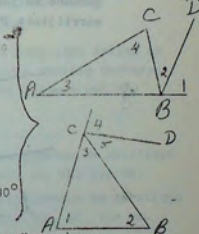
10. Siin BD // AC
Caddee : ∠ 1 + ∠ 2 = ∠ 3 + ∠ 4

11. Siin : ∠CA = CB; ∠4=∠2
Caddee : CD // AB

12. Siin : ∠ 4 + ∠5 = ∠1 + ∠2
∠5 = ∠ 4, CA = CB
Caddee : CD // AB

13. Siin CA = CB, ∠ 5 = ∠1
Caddee: CD // AB

14. Siin: CA = CB, ∠ 3 + ∠ 5 + ∠2 + ∠ = 180°
Caddee CD // AB



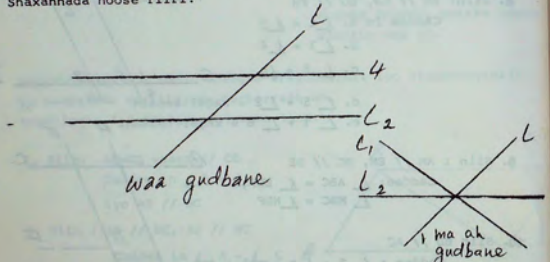
15. Haddii laba xarriiq oo barbarro ah uu gudbane jaro, caddee in xaglo dibadeedka talantaaliga ahi ay isleeg yihiin.

HORDHAC

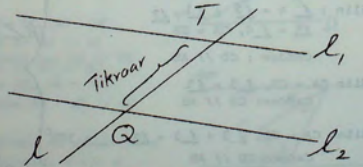
QEEXID: Gudbane

Laba xarriiq gudbanahoodu waa xarriiqa toosan ee labada xarriiq ka jara laba barood.

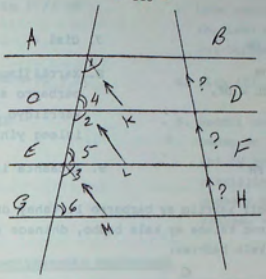
OGSOONOW: Sida qeexidu ina leedahay, haddii laba xarriiq ay isjaraan, xarriiqda marta barta ay iska jaraan ma aha gudbane. Shaxannada hoose fiiri:



QEEXID: Tikraar: Haddii l_1 iyo l_2 ay yihiin laba xarriiq, oo gudbane ka jarayo l_1 barta T, l_2-na barta Q, markaa xarriijinta TQ waxa la yidhaa tikraar.



ARAGTIIN: Haddii 3 ama in ka badan ee xarriiqyo barbarro ahi ay gudbane ka tikraaraan xarriijimo isleeg, markaa gudbane kasta waxa ay ka tikraaraan



Siin: // yaasha AB, CD, EF iyo GH Waxa jaray gudbanayaasha AG iyo BH, markaa AC = CE = EG.

Caddee: In BD = DF = FH.

Saafid: Marka hore raadi qaybaha isku aada ee $\triangle$ lo $\cong$ dabeed isticmaal dhardhaarka isku beddelidda.

Caddayn:

Hawraar

1. Sawirk AK, CL; EM // BH
2. AK, CL, iyo EM waa //

Garaadayn

1. Dhisme
2. Xarriiqyo isku sallax ah oo isku xarriiq barbarro u ahi iyaguna waa //.

Hawraar

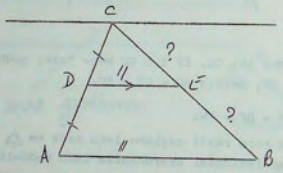
3. $\angle 1 = \angle 2 = \angle 3$
4. $\angle 4 = \angle 5 = \angle 6$
5. AC = CE = EG
6. $\triangle$ ka ACK $\cong$ $\triangle$ ka CEL $\cong$ $\triangle$ ka EGM

Garaadayn

3. Xaglo gudboon ee xarriiqyo // ah. AK, CL, iyo EM.
4. Xagllo-gudboon ee xarriiqyo // ah. CD, EF, iyo GH.
5. Siin.
6. x.dh. x

7. AK = CL = EM	7. Qisi =
8. Hase-yeeshe AK = BD, CL = DF, EM = FH.	8. Xarriijimo ee xarriiqyo barbarro ahi oo ay jareyn xarriiqyo barbarro ahi way isleeg yihiin.
9. BD = DF = FH	9. Astaanta isku beddelidda.

ARAGTIIN: (A) Haddii xarriiq ay barbarro la tahay dhinac seddexagal, dhinac kalana ay kala badho, dhinaca seddexaadna waa ay kala badhtaa.



Siin: △ ka ABC, DE // AB, AD = DC

Caddee in : CE = EB

Saafid: Raadi 3 xarriiqoo // h^1, oo gudbane ka jaraya xarriijimo isleeg.

Caddayn:

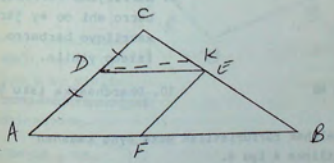
Hawraar	Garaadayn
1. C ka sawir xarriiqa l // AB	1. Bar debedeed xarriiq waa laga sawiri karaa xarriiq // u ah xarriiq lagu siiyay.
2. Hase-yeeshe DE // AB	2. Siin.

3. :. Mar kale l // DE	3. Laba xarriiq oo isku sallax ah, isku xarriiqna // u ahi waa barbarro.
4. Hase yeeshe AD = DC	4. Sida (2)
5. :. CE = EB	5. Haddii seddex ama in ka badan oo xarriiqyo barbarro ahi ay gudbane ka tikraaraan xarriijimo isleeg, markaa gudbane kasta waxay ka tikraaraan xarriijimo isleeg.

Xarriiq - badhtameedka seddexagal

Qeexid: Xarriiq-badhtameedka seddexagal waa xarriiqa isku xidhaya bar-badhtameedyada laba dhinac ee seddexagale.

ARAGTIIN (B) : Xarriiq-badhtameedka seddexagal waa u barbarro [illegible] dhinaca seddexaad, waxa uuna leegyahay dhinaca seddexaad badhkii.



Siin: △ka ABC, AD = DC, BE = EC

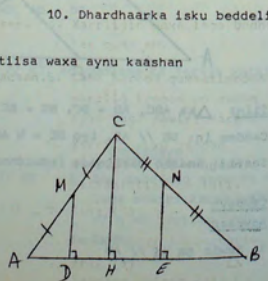
Caddee in: DE // AB, iyo DE = ½ AB.

Saafid: Kaasho dariiqada isduldhaca ee caddaynta dadban.

Caddeyn Hawraar	Garaadayn
1. Sawir DK ∋i // AB	1. Bar debedeed xarriiq waa laga sawiri karaa // u ah xarriiq lagu siiyay.

2. DK way badhaa BC	2. Haddii xarriiq barbarro la ∥ yahay dhinac △ u badhana dhinac kale ka seddexaadna wuu ka badhaa.
3. Hase-yeeshe DE wuu kala badhay BC	3. Siin.
4. :. K iyo E way isdul dhaceen.	4. Xarriijin waxa laga badhay bar qudh ah.
5. :. DK iyo DE way isdul dhacaan.	5. Laba barood qumaatigooda xarriiq toosan oo qudha ayaa laga sawiri karaa.
6. :. DE // AB	6. Xarriiq wuxu qaadan xarriiq isdul-dhacaan astaamihiisa oo idil.
7. Sawir si EF // AC	7. Sida (1)
8. EF waxay badhaa AB, ama AF = ½ AB	8. Sida (2)
9. AF = DE	9. Xarriijimo xarriiqyo barbarro ahi oo ay jareyn xarriiqyo barbarro ahi way isleeg yihiin.
10. :. DE = ½ AB	10. Dhardhaarka isku beddelidda.

TUSAALE: Tusaalahan furfuristiisa waxa aynu kaashan aragtiinka A iyo B.



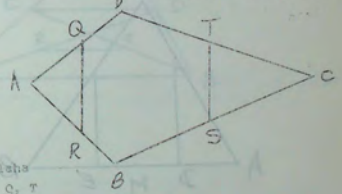
Siin: △ ka ABC
m iyo n waa baro-badhtameedyada AC iyo BC sida ay u kala horeeyaan. MD, CH, NE mid waliba waa ⊥ AB.

Caddee in: MD + NE = CH

Caddayn

Hawraar	Qaraadayn
1. MD // CH	1. Laba xarriiq oo xarriiq ku wada qotomaa waa //.
2. :. D waa bar-badhtameedka AH.	2. Haddii xarriiq u barbarro u yahay dhinac seddex-xagal, dhinac kalenina uu badho, markaa wuu badhaa dhinaca seddexaad.
3. MD = ½ CH	3. Xarriiq -cedhtameedka △ = ½ ka dhinaca seddexaad.
4. Sidii oo kale NE = ½ CH	4. Talleabooyinka 1-3, ee aynu ku isticmaaley △ ka CBH.
5. NE + MD = CH	5. Dhardhaarka isugaynta.

LAYLI



Siin: Afar geesoolaha ABCD, oo R, Q, T iyo S ay yihiin baro-badhtameedyada dhinacyada.

Caddee in QR = TS.

Biniix : Sawir BD adoo afar geesoolaha u qaybinaya laba △.
Dabeed kaasho xarriiq-badhtameedyo.

2. Caddee in xarriiqyada isku-xidhaya baro-badhtameedyada seddexagal in ay u qaybinayaan seddexagalka afar seddexagal oo isku sargo'an:
(Biniix: Kaasho dh.dh.dh. = dh.dh.dh.).

3. Siin: $\angle$ ka ABC oo baraha D, M, iyo E ay u qaybinayaan dhinaca AB afar qaybood oo isleeg.
T iyo Q waa baro-badhtameedyada AC / iyo BC.
Caddee in : DT // EQ.

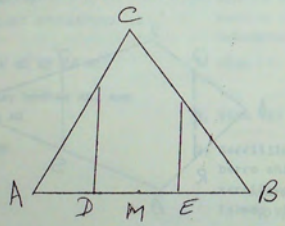
LAYLI

4. Siin : $\square$ABC D oo T ay tahay bar-badhtameedka DC,
Q waa bar kutaal AC, CQ = ½ AC;
TQ oo la fidiyayna waxa ay ka jaraysaa BC barta R.
Caddee in R ay tahay bar-badhtameedka BC.

Biniix: Sawir BD.

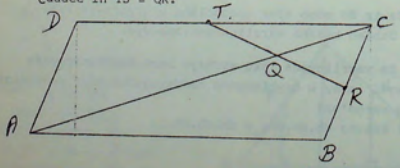
5. Siin : Laba jibaaranaha ABCD oo T, Q, R iyo S ay yihiin baro-badhtameedyada dhinacyada.

Caddee in TQRS u yahay laba jibaarane.



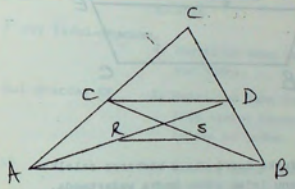
6. Siin: T, iyo Q waa bar-badhtameedyada AB iyo BC ee $\triangle$ka ABC; TS $\angle$ AC, QR $\angle$ AC

Caddee in TS = QR.



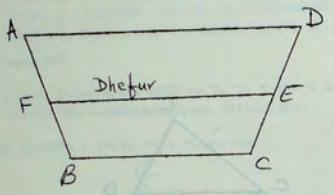
7. Siin : AD iyo BE waa dhexfurada seddexagalka ABC oo ku kulmaya barta O; R waa bar-badhtameedka OA, S waa bar-badhtameedka OB.

Caddee in : ED = RS, ED // RS

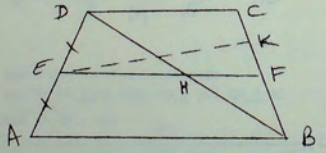


DHEXFURKA KOORTA

QEEXID: Dhexfurka koori waa xarriijinta isku xidha baro-badhtameedyada dhinacyada aan barbarrada ahayn ee koorta.



ARAGTIIN: Dhexfurka koori waa u barbarro salalka waxaanu le'eg yahay badka wadartooda.

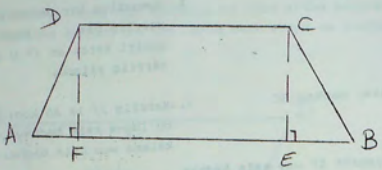


SIIN: Koorta ABCD dhexfurkeeda EF wuxuu badhayaa AD iyo BC
Caddee in : EF // AB iyo CD, EF = ½ (AB+CD)

Saafid: Kaasho caddaynta dadban, iyo aragtiinkii xarriiq-badhtameedka.

Caddayn	Garaadayn
1. SawirEK // AB	1. Qumaatiga bar debedeed xarriiq baa [illegible] laga sawiri karaa oo // u a xarriiq siismo.
2. EK waxay badhaa BC	2. Xarriiq // la ah koor salkeeda oo lugna kala badhaya, lugta kalena wuu kala badha.
3. Hase-yeeshe EF way kala badhaa BC	3. Siin
4. :. K iyo F way isdul-dhacaan.	4. Xarriijin waxa laga badhay bar qudha.
5. EK waxay dul dhacday EF	5. Qumaatiga laba barood waxa laga sawiri karaa xarriiq toosan oo qudha.
6. :. EF // AB	6. Xarriiq euxuu qaataa xarriiqu ku duldhoco astaamihiisa oo idil.
7. :. EF // CD	7. Laba xarriiq oo isku sallax ahi barbarrona u wada ah xarriiq 3aad waa barbarro iyaguna.
8. Sawir DB ha kana jaro EF barta H.	8. Dhisme.
9. H waa bar-badhtameedka DB	9. Haddii xarriiq u barbarro la yahay dhinac △, u dhinac kalena badho, dhinaca seddexaad-na wuu kala badhaa.
10. Sawir DB ha kana jaro EF barta H.	9. Dhisme.
10. :. EF = ½ AB	10. Xarriiq-badhtameedka △ = ½ dhinaca seddexaad.
11. :. EF = ½ (AB+CD)	11. Dhardhaarada isugaynta iyo isku beddelidda,

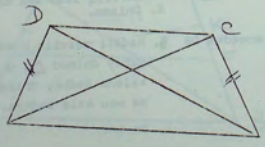
XIGASHO I: Xaglo-saleedka ee koor labaale ahi way isleeg yihiin.



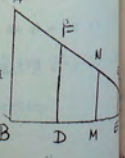
Biniix: Sawir CE iyo DF ⊥ AB. CE = DF?
△ ka quman ee BEC ≅ △ ka quman ee AFD? ∠A = ∠B?

XIGASHO II: Xaglagooyayaasha eekkoor labaale ahi way isleeg yihiin.

Biniix : Haaka tegin xagashadii kowaad.
∠ ABC = ∠ DAB . △ ka ABC ≅ △ ka DAB? AC = BD?



TUSAALE: Tusaalahan waxa aynu kaashan araqtiinkii hore.



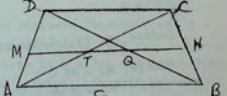
Siin: △ ka ABC oo baraha D iyo E ay ku yaalaan dhinaca BC. Markaa BD = EC. DF iyo EG // AB. Caddee in GE + DF = AB.

CADDAYN

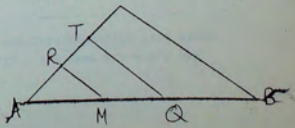
Hawraar	Qaraadayn
1. Sawir xarriiq-badhtameedka MN; MN // AB	1. Xarriiq-badhtameedka △ waa u // dhinaca 3xaad.
2. AN = NC, BM = MC	2. M iyo N waa baro-badhtameedyo
3. BD = EC	3. Siin
4. ∴ DM = ME	4. Dhar. kala goynta
5. FD // MN // GE	5. Xarriiqyada isku xarriiq barbarro u ahi, iyaguna waa //.
6. FN = NG	6. Xarriiqa salka koor barbarro u ahi lugaha badhaya lugta kalena wuu badhayna.
7. MN = ½ (GE + FD)	7. Dhexfurka koori wuxuu leegsha yahay badhka wadarta salalka.
8. MN = ½ AB	8. Xarriiq-badhtameedka △ = ½ dhinaca 3xaad.
9. ∴ ½ (GE + FD) = ½ AB	9. Isku beddelid
10. GE + FD = AB	10. Isku-dhufasho.

LAYLI: Layliyadan u kaasho araqtiinkii hore.

Siin: Koorta ABCD oo dhexfurka MN uu ka jarayo xagla gooyayaasha barta T iyo Q.

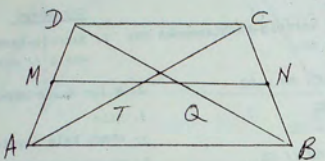


CADDEE IN: T ay tahay bar-badhtameedka AC iyo in Q ay tahay bar-badhtameedka BD.



2. Siin : △ ka ABC oo M iyo Q ay seddex goor badhayaan AB; MR iyo QT waa // BC.

Caddee in : MR + QT = BC



Biniix: Sawir XY, oo ah xarriiq ka yimaada bar-badhtameedka K ilaa iyo bar-badhtameedka MQ.

△ ka ABC, XY = ? shaxanka MQTR, XY = ? Isku beddel.

3. Caddee in shaxanka ka sameeysma marka laysku xidho baro-badhtameedyada lugaha iyo baro-badhtameedyada salalka koo u yahay qardhaas.

Siin : Koorta ABCD
oo dhexfurkeeda MN
uu ka jarayo xagla-
gooyaha AC barta T,
iyo xaglagooyaha BD
barta Q.

Caddee in: TQ = ½ (AB-CD).

Biniix: MQ - MT = ?
MQ = ? MT = ?

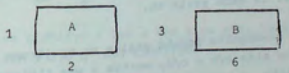
SAAMI IYO SAAMIGAL

Ka soo qaad in laba xarriijimood mid tahay 18", ta kalena tahay 24". Marka aynu labada xarriijimood is garab dhigno innaga oo kaashanayna qaybinta, waxay nu odhan karnaa ta qaabani waa 18/24 ama 3/4 marka loo eego tan dheer. is garab dhigaas oo lagu magacaabo saami waa laga dhaxaysiin karaa laba tiro oo kasta bishardi in aan hooseeyu eber ahayn. Haddii ay tirooyinku yihiin qaar cabbir waa inay isku hal-beeg yihiin. Tusaale ahaan saamiga ka dhexeeya 4" iyo 1 waar waa 4/36 ama 1/9.

QEEXID: Saamigal laba tiro waa qaybtooda.

Si loogu feejignaado is garabdhigga la saamiyaynaayo, saamigan 3/4 oo kale waxa loo qoraa, 3:4 waxaana loo akhriyaa 3 ilaa 4 ama seddex afraad.

Tirooyinka saamiga ku jira waxa lagu magacaabaa tibixaha saamiga. Haddiiba saamiga iyo jajabkuba ay qayb yihiin si isku mid ah ayaa loola macaamilaa.



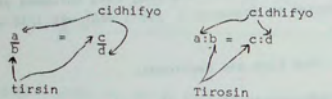
Saamiga ka dhexeeya balaadhka iyo dhererka laydiga A waa ½ . Kan B waa 3/6. Isleekaanshaha labadan saami waxa uu suura gelinayaa in aynu u qori karro in ½ = 3/6 ama 1:2 = 3:6. Jajabyada ½, 3/6, a/b iyo $\frac{x+y}{1+y}$ waa saamiyo: Addimada labadan laydi way saamigalsan yihiin.

QEEXID: Saamigal : Saamigal waa isleekaanshaha laba saami marka isleegyadan 9/15 = 3/5 ama a/b = c/d waa saamigalyo .

TIBIXO

Afarta xaddi ee saami-galku mid kastaaba waa tibix. Markaa saamigalka a/b= c/d, a waa tibixda kowaad, b waa tibixda labaad, c waa tibixda seddexaad, dna waa tibixda afraad.

Tibixda kowaad iyo ta seddexaad a,c, waa horrad ta labaad iyo ta afraad waxa weeye gadaaleeye tibixda kowaad iyo ta afraad a, d waxa la yidhaa cidhifyo, ta labaad iyo ta seddexaadna b,c waxa la yidhaa tiro sin.



Tibixda afraad ee d waxay saamigalka afraad u tahay seddexda tibixaad ee kale a,b,c, oo horsiimadaas u yaalla.

Saamigal isdaba yaal ah

Gadaaleeyaha saami kastaa waa antecedentka saamiga ku xiga, markaa, a/b = b/c = c/d = d/c ama ½ = 2/4 = 4/8 = 8/16 waa saamigal is daba yaala ah.

Haddii saamigalka isdaba yaalka ah ay jiraan laba saami oo qudha sida a/b = c/d, markaa a waa tibixda kowaad ee saamigalka, b waa tibixda labaad. cna waa tibixda seddexaad.

Haddaba b oo tibixda labaadi waa tiro-sinta saamigalka u dhexeeya labada tibixood ee kale ee ah a, iyo c , c oo tibixda seddexaad ahina waxay saamigalka seddexaad u tahay.
a, iyo b oo horsiimadaas ah.

Astaamaha saamigalka

1. Haddii afar xaddi ay saamigal yihiin, taranta tiro sin waxay le'eg tahay taranta cidhifyada, taasu waxay tahay haddii a/b = c/d markaa ad = bc.

Caddayn

a/b = c/d jajabka ay u dhigan tahay waxa aynu kaga beddeli innagoo labada dhinac ee isleegta ku dhufanna hooseeyaha ugu yar ee ay wadaagaan, oo ah bd :. bd a/b = c/d bd = ad = bc.

2. Haddii taranta laba xaddi leeg tahay taranta laba xaddi oo kale labada lammaane mid ahaan waxallaga dhigi karaa tiro sinta ka kalena cidhifyada saamigal, taasi waxay tahay, haddii xy = tq markaa y/t = q/x.

Caddeyn: xy = tq

Waxaynu kaashan dhardhaarka qaybta innagoo u qaybinayna labada dhinac ee isleegta taran seddexaad oo ka timi xaddi taranta kowaad ah iyo xaddi taranta labaad ah sida 1 xt.

Markaa $\frac{xy}{xt} = \frac{tq}{xt}$

:. $\frac{y}{t} = \frac{q}{x}$, haddaba tan waxa suura gala afar saamigal kuwaasoo ah:

1. $\frac{x}{t} = \frac{q}{y}$ ama $\frac{y}{t} = \frac{q}{x}$

2. $\frac{t}{x} = \frac{y}{q}$ ama $\frac{t}{y} = \frac{x}{q}$

Ta hore x iyo y waa cidhifyo t iyo q waa tiro sin, ta dambena x iyo y waa tiro sin t iyo q waa cidhifyo.

3. Haddii (horradka) saamigal ay isleeg yihiin (gadaaleeyuhuna) waa isleeg yihiin. Taasi waxay tahay, haddii a = c oo a/b = c/d markaa b = d.

Caddeyn: Astaantii (1aad ad = bc) u qaybi a = c:. d =b.

4. Haddii gadaaleeyaha saamigal ay isleeg yihiin horradyaduna way isleeg yihiin taasoo ah haddii y = w oo $\frac{x}{y} = \frac{r}{w}$,

markaa x = r. (Caddayntu waa tii seddexaad oo kale) X = R.

5. Haddii afar xaddi ay saamigal yihiin talantaalina waa ku saamigal. Taasu waxay tahay haddii a/b = c/d markaa a/c =b/d.

Caddeyn: Astaantii 1aad ad = bc. Labada dhinac ba u qaybi dc :. a/c = b/d, markan waxa aynu tallantaalli u qornay labada tiro sin, haddaba sidaa oo kale haddii aynu u talantaalli u qorno cidhifyada waxa aynu diiri saamigalkan (d/b=c/b

6. Haddii afar xaddi ay saamigal yihiin isweydaar ahaanna waa ku saami gal. Haddii a/b = c/d, markaa b/a = d/c.

Caddeyn: Astaantii 1aad bc = ad, labada dhinac ba u qaybi ac. Markaa waxa aynu helaynaa saamigalka b/a = d/c.

7. Haddii afar xaddi ay saamigalsan yihiin isugeynna waa ku saamigal. Taasi waxay tahay haddii a/b = c/d markaa

$$\frac{a+b}{b} = \frac{c+d}{d}.$$

Caddeyn: a/b = c/d labada dhinacba 1 u gee sidan a/b + 1 = c/d + 1. Marka aynnu fududaynona waxaynu heli sidan: $\frac{a+b}{b} = \frac{c+d}{d}$

8. Haddii afar xaddi ay saamigal yihiin qayb ama kala go'yna waa ku saamigal. Haddii a/b = c/d.
Markaa $\frac{a-b}{b} = \frac{c-d}{d}$

Caddeyn: a/b = c/d, labada dhinacba kagoo 1
Sidaan a/b -1 = c/d - 1, fududayntu waxay noqon sidan $\frac{a-b}{b} = \frac{c-d}{d}$

9. Haddii afar xaddi saamigal yihiin, isugayn iyo kala goyna waa ku saamigal.

Haddii a/b = c/d, markaa $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

Caddayn: (7) u qaybi (8)

10. Haddii a/b = c/d = e/f = g/h, marka $\frac{a+c+e+g}{b+d+f+f} = a/b$

Caddayn
a/b = c/d = e/f = g/h = r
as+(1) a = br, c = dr, e = fr, g = hr
Isugayn: a+c+e+g = br +dr+fr+hr

Isirayn : a + c+ e+g = r(b+d+f+h)
Isuqaybin: $\frac{a+c+e+g}{b+d+f+h} = r$

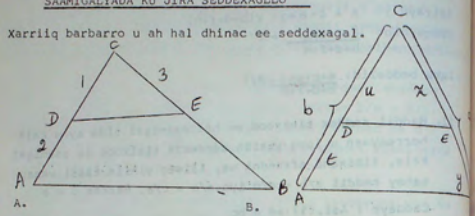
Isku beddelid: $\frac{a+c+e+g}{b+d+f+h} = a/b$

11. Haddii seddex tibixood ee hal saamigal sida ay u kala horreeyaan u leeg yihiin seddexda tibixood ee saamigal kale, tibixaha afraadna way isleeg yihiin taasi waxay tahay haddii a/b = c/d iyo a/b = c/e, markaa d = e

Caddeyn : Ast.(1) ad = bc
ae = bc
Markaa ad = ae
Qaybahaan d = e.

SAAMIGALYADA KU JIRA SEDDEXAGLLO

Xarriiq barbarro u ah hal dhinac ee seddexagal.



Shaxanadan haddii aynu dhinacyada siino dhereradaa waxa aynu samayn karnaa saamigalyadan.

A. $\frac{1}{2} = \frac{3}{6}$, $\frac{1}{3} = \frac{2}{6}$,

$\frac{1+2}{2} = \frac{3+6}{6}$,

$\frac{1+2}{1} = \frac{3+6}{3}$,

B. $\frac{u}{t} = \frac{x}{y}$,

$\frac{t}{u} = \frac{y}{x}$

$\frac{b}{t} = \frac{a}{y}$,

$\frac{b}{u} = \frac{a}{x}$,

$\frac{b}{a} = \frac{t}{y}$

$\frac{b}{a} = \frac{u}{x}$.

U qaadasho I: Xarriiqa barbarro la ah hal dhinac ee seddexagal jarayana labada dhinac ee kale wuxuu u qaybiyaa dhinacyadaa saamigal ahaan.

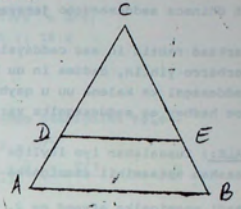
Odhaahda ah: Wuxuu u qaybiyaa dhinacyadaa saamigal ahaan, waxay tahay (in saamiyada (ratios) ee dhererada xarriijimaha isku aada ee labada dhinac ay isleeg yihiin.

TUSAALE I: $\triangle$ ka ABC, DE // AB.
Haddii CA = 10, CD = 7
CE = 9, Raadi CB

FURFURIS: U qaado dhererka CB X

$\frac{x}{9} = \frac{10}{7}$;

$\therefore x = \frac{90}{7}$ ama $12\frac{6}{7}$.



TUSAALE II. $\triangle$ ka ABC, DE // AB,
haddii CA = 8,
CD = 5,
CB =10,
Raadi CB.

FURFURIS: U qaado dhererka EB, X
Markaa $\frac{10-x}{10} = \frac{5}{8}$

$\therefore x = \frac{-30}{-8}$ ama $3\frac{3}{4}$

U qaadasho II: Haddii xarriiq u qaybiyo laba dhinac ee seddexagal saamigalsahaanwaa u barbarroadhin dhinaca seddexaad.

TUSAALE: $\triangle$ ka ABC, AC = 7, DC = 5, BC = 10.5
EC = 7.5; DE // AB?

FURFURIS: $\frac{7}{100.5} = \frac{5}{7.5}$, $\frac{1}{1.5} = \frac{1}{1.5}$;

$\therefore$ DE // AB (U qaadashadii labaad).

DARIIQOOYINKA GUUD

1. Markaad caddaynaysid in xarriijimo **xarriiq-ayl** saamigalsan yihiin, tus in ay yihiin xarriijimo **ku-yaal laba** dhinac ee seddexagal oo u sameeyay xarriiq barbarro la ah dhinaca seddexaadsoo **jarayana labada-dhinac**.

2. Markaad rabtid in aad caddaysid in laba xarriiq ay barbarro yihiin, caddee in uu hal xarriiq yahay dhinac seddexagal ka kalena uu u qaybinayo labada dhinac **oo** soo hadhay ee seddexagalka xarriijimo saamigal ah.

TUSAALE:: Tusaalahan iyo layliga soo socdaba waxa aynu kaashan astaamihii saamigalka .

1. Raadi saamigalka afraad ee 2, 6 iyo 3 oo loo qoray horsiimadaa.

Furfuris: Marka aynu u bixino ka afraad x, afarta tibixood ee saamigalku waxay yihiin 2,6, 3, iyo x sida ay u kala horeeyaan.

Markaa waxa aynu u qoray sidan $2/6 = 3/x$, u diid jajab markaa waxay noqon sidan:

$2x = 18$

$x = 9$ saamigalka afraad wuxuu yahay $\frac{9}{=}$.

2. Raadi tiro sinta saamigalka u dhexeeya 8 iyo 2.

FURFURIS: Weydiinta waxa aynu ka garan in ay cidhifyada saamigalku yihiin 8 iyo 2, markaa aan ku magacaawno **tiro** sida saamigalka x. Taas oo aynu u qori karno sidan:

$8/x = x/2 = x^2 = 16$, $x = \pm 4$.

LAYLI

Layliyada 1-6 sheeg astaamaha saamigalka aynu kaashanay.

1. Haddii $3x = 4y$, markaa $x/y = 4/3$.
2. Haddii $2/a = 5/b$, markaa $\frac{2+a}{a} = \frac{5+b}{b}$
3. Haddii $a/b = 3/4$, markaa $4a = 3b$.
4. Haddii $7/8 = t/q$, markaa $8/7 = q/t$.

5. Haddii $x/y = \frac{w}{t}$, , markaa, $\frac{x+y}{x-y} = \frac{w+t}{w-t}$

6. Haddii $a/b = x/y$, markaa $y/b = x/a$

7. Raadi qiimaha x ee ku jira saamigaladan soo socda:

a) $x/8 = \frac{1}{2}$, (b) $15/x = 3/4$,
c) $2:3 = x:9$, (d) $4:5 :: 16:x$
e) $a/b = c/x$

7. Layli

8. Talantaali ahaan ku samee saamigalka kale:.

a) $4/5 = 8/10$, b) $a/2 = b/3$
c) $\frac{x+2}{5} = \frac{y+5}{3}$, d) $\frac{a-3}{b+3} = 2/3$
e) $2/L = 8/4$.

9. Isugayn ku samee saamigalka kale:.

a) $a/4 = \frac{12}{24}$, b) $a/5 = 3/6$

c) $2/x = 4/3$.

10. Raadi saamiga x ilaa y

a) $3x = 4y$ b) $4x = y$
c) $\frac{ax}{b} = y/d$, d) $y/x = 4/5$
e) $x/2 = y/3$, f) $\frac{3y}{5x} = 2/3$

11. Raadi tiro sinta saamigalka ee u dhexaysa

a) 16 iyo 25, b) 9 iyo 4
c) 8 iyo 2 , d) 25 iyo 1.
e) $4a^2$ iyo 166^2, f) 3a iyo $27a^3$

12. Hawraarahan soo socda kuweebaa run ah, kuweebaase been ah.

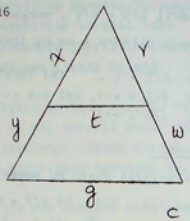
a) t/q waa saamigal
b) $a/b = b/c$; c waa saamigalka afraad ee ilaa a iyo b.
c) $b/a = c/b$, b waa tiro sinta saamigalka ilaa a iyo c.
d) $\frac{1}{3} = 5/7$ waa run.

13. Taranadan isleeg ee soo socda ka samee inta aad kari karto ee saamigalyo ah.

a) 8x3 = 2x12 , b) 8x6 = 3x16
c) tq = rs.

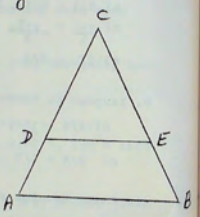
LAYLI

1. Kaasho shaxanka midigta ah t // q, sheeg isleegyadan lammaan kuwa runta ah.



a) Haddii x/y = ½ markaa r/w = 3/6
b) Haddii x/y = 6/7 , markaa r/w = 3/5
t) Haddii x/y = $\frac{t}{mt}$,markaa r/w = m/m^2
j) Haddii x/y = a/a^2 , markaa r/w = a^2/b^2

2. Layliyada b-x u tixraac shaxanka midigta kaas oo DE // AB.



b. Haddii EB = 2, EC = 5
AD = 3; Raadi CD.

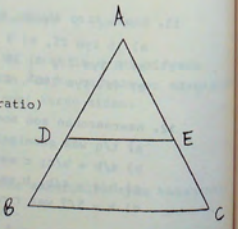
t. Haddii AC = 12, BC = 20, EB = 15,
Raadi AD, iyo DC.

j. Haddii AD = ⅓ CD, raadi saamida (ratio)
CE ilaa EB.

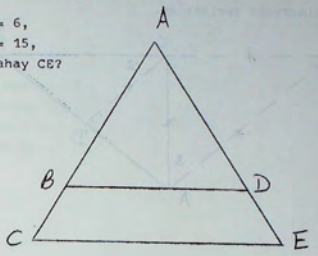
x. Haddii AC = 10, AD = 2, EB = 3,
Raadi BC iyo CE.

LAYLI

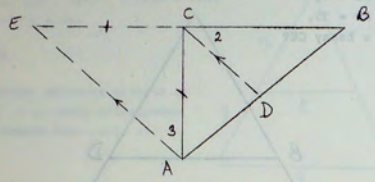
3. Shaxanka midigta ah,
haddii DE // BC.
Haddii AB = 6,
AD = 2, EC = 2½
Raadi dhererka AE.



4. Shaxankan kale ee midigta, haddii
AC = 15, BC = 6,
AE = 25, AD = 15,
BD // ma u tahay CE?



ARAGTIIN: Kala badhaha xagasha seddexagal wuxuu u qaybiyaa dhinaca ka soo horjeeda xarriijimo saamigal u ah dhinacyada deriska ah.



SIIN: △ ka ABC
oo CDeay kala
badhayso ∠ C,
kana jarayso AB barta D.

Caddee in $\frac{AD}{DB} = \frac{AC}{CB}$

Saafid: Si aynu u kaashano u qaadashooyinkii hore, salka △ ka kale barbarro uga dhig CD.

Caddayn

Hawraar

1. Sawir AE // CD fidina BC
2. △ ka ABE, $\frac{AD}{DB} = \frac{EC}{CB}$
3. ∠ 1 = ∠ 2

Garaadayn

1. Dhisme
2. Haddii xarriiq saamigal ahaan u qaybiyo laba dhinac ee △, waaau // dhinaca 3aad.
3. Siin, CD waxay kala badhaa ∠ ACB

Hawraar

4. ∠ 1 = ∠ 3
5. ∠ 2 = ∠ 4

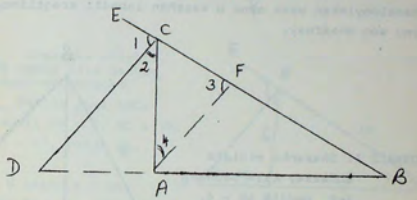
Garaadayn

4. Xagllo-gudeedyo talantaali ah ee xarriiqyo // ah.
5. Xagllo-gudboon ee xarriiqyo // ah.

6. ∠ 3 = ∠ 4
7. EC = AC
8. ∴ $\frac{AD}{DB} = \frac{AC}{CB}$

6. Dhardhaarka isku beddelidda.
7. △, dhinacyada ka soo horjeeda xagllo = way =
8. Sida (6)

ARAGTIIN: Kala badhaha xagal-debadeed ee seddexagal isma le'eka ahi, wuxuu debeddaa ka qaybiyaa dhinaca ka soo horjeeda, waxaanu u qaybiyaa xarriijimo saamigal u ah dhinacyada deriska ah.



Siin: △ ka ABC oo leh ∠ debadeedka ∠ ACE waxa kala badha CD, CD waxay ka jartaa BA oo la fidiyay barta D.

Caddee In $\frac{DB}{DA} = \frac{CB}{CA}$

Saafid: Sawir xarriiq // u ah salka CD ee △ ka BCD

Caddayn

Hawraar

1. Sawir AF // CD
2. $\frac{DB}{DA} = \frac{CB}{CF}$
3. ∠ 1 = ∠ 2

Garaadayn

1. Dhisme
2. Haddii xarriiq // u yahay hal dhinac ee △, jarayana labada dhinac ee kale wuxuu u qaybiyaa dhinacyada saamigal ahaan
3. Siin, CD waxay kala badhaa ∠ ACE.

4. $\angle 1 = \angle 3$ — 4. Xagllo-gudboon ee xarriiqyo // ah.

5. $\angle 2 = \angle 4$ — 5. Xagllo-gudeedyo ee xarriiqyo // ah.

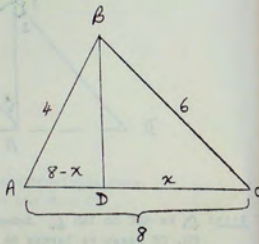
6. $\therefore \angle 3 = \angle 4$ — 6. Dhardhaarka isku beddelidda.

7. $\therefore CA = CF$ — 7. $\triangle$ dhinacyada ka soo horjeeda $\angle$ LO = way =.

8. $\therefore \frac{DB}{DA} = \frac{CB}{CA}$ — 8. Sida (6)

Tusaalooyinkan waxa aynu u kaashan labadii aragtiimoodeee aynu soo dhaafnay.

TUSAALE I. Shaxanka midigta BD waxay kala badhaa $\angle$ B, Haddii AB = 4, BC = 6, AC = 8, Raadi DC



FURFURIS: Weydiintaa waxa aynu ku furfuri laba dariiqo.

DARIIQADA HORE: Aan u qaadano dhererka DC in u yahay X
$\therefore$ AD = (8-x); markaa haddii aynu kaashan aragtidii hore waxa ay noqon sidan:

$\frac{x}{8-x} = \frac{6}{4}$: $4x = 48 - 6x$

$\therefore 10x = 48$

$\therefore x = \frac{48}{10}$

Markaa DC = $\frac{48}{10} = x$

DARIIQADA LABAAD

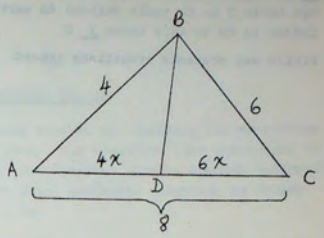
Waxa aynu u qaadan in AD = 4 x ay derisna u tahay AB. iyo DC = 6x, derisna u tahay BC.

$\therefore 4x + 6x = 8$

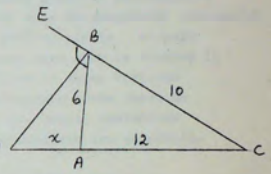
$10x = 8$

$\therefore x = 0.8$

$\therefore DC = 6x = 4.8$



TUSAALE II: Shaxanka midigta, DB waxay kala badhaa xagal-debadeedka $\angle$ EBA ee $\triangle$ka ABC. Haddii AB = 6, BC = 10, AC = 12, raadi DA.



FURFURIS: U qaado X = DA

Markaa DC = X + 12

Marka aynu kaashano aragtidii labaad waxa ay noqon sidan: $\frac{DA}{DC} = \frac{AB}{BC}$. Ku beddel: $\frac{X}{X+12} = \frac{6}{10}$

Markaa, $10x = 6x+72$; $4x = 72$

$DA = x = 18$

LAYLI

1. $\triangle$ ka ABC, BD waxay kala badhaa $\angle$ B. Haddii AB = 4, BC =8, AC = 7, raadi DC.

2. $\triangle$ ka ABC, AD waxay kala badhaa $\angle$ A. Haddii BC = 12 , BD = 4 4/5, AB = 6 , raadi AC?

3. Dhererka dhinacyada $\triangle$ ka ABC waa 12 hiish, 10 hiish iyo 8 hiish. Haddii dhinaca u dheer laga qaybiyo debedda, iyadoo la kala badhayo xagal-debedeedda ka soo horjeedda, raadi dhererka xarriijinteeda debedda xigta.

4. △ ka ABC, xariiqa CD [illegible] C ilaa
iyo barta D oo ku taala dhinaca AB markaa $\frac{AD}{BD} = \frac{AC}{BC}$.
Caddee in CD ay kala badho ∠ C

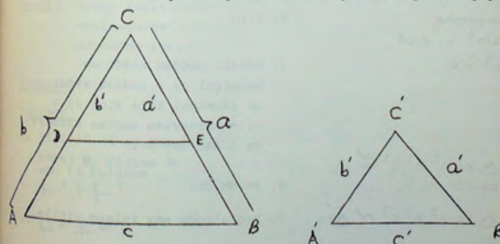
:. Biniix eeg shaxanka aragtiinka labaad.

SEDDEXAGALLO ISU EG

Buuggii kowaad waxa aynu ku soo qaadanay isu ekaanshaha laba seddexagal, wuxuu yahay, iyo aragtiimo laba seddexagal ka dhigi kara qaar isu eg. Hase-yeeshee midnaba maynaan caddeynin, markaa iminka waxa aynu unimi caddaynta aragtiimo ka dhigay laba seddexagal laba isu eg.

Haddaba haddii seddexagal noqday geesoole seddexagallo isu eg waxa aynu ogayn in ay ahaayeen kuwa dhinacyadooda isku aadani ay saamigal san yihiin xaglohooda - - - - - - - isku aadanina ay isle'eg yihiin. Arrinta guud ahaan ku saabsan geesoolayaashana waxa aynu ku aragnay in haddii labadaa xaladood mid qudhi jirto in aanay kufilayn isu ekaanshaha geesoolayaal. Hase-yeeshe arrin khaas ah ee seddexagallo waxa aynu caddeyn kari, haddii xaaladahaa midi jirto ama ay run tahay in seddexagalladu isu eg yihiin.

ARAGTIIN: Haddii dhinacyada laba seddex-xagal ay saamigalsan yihiin seddexagaladu way isu eg yihiin.



SIIN: △ lada ABC iyo $A^1 B^1 C^1$ oo $a/a^1 = b/b^1 = c/c^1$

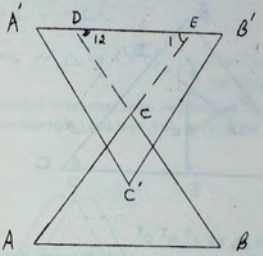
Caddee in △ ka ABC $\sim$ △ ka $A^1B^1C^1$

Saafid: △ ABC dushiisa ka dhis △,,dabeedna caddee in △ kaa dhismay uu u eg yahay △ ABC kuna sargo'an yahay △ $A^1B^1C^1$.

Caddayn

Hawraar	Garaadayn
1. b dusheeda ka samee CD = b^1 a dusheedana ka samee CE = a^1, sawir DE kuna magacaaw x	1. Dhisme
2. $a/a^1 = b/b^1$	2. Siin
3. $\angle c = \angle c$	3. Ka dhaxeeye.
4. $\triangle$ ka ABC $\triangle$ ka DEC	4. Haddii laba $\triangle$ $\angle$ sha midi ay = $\angle$ sha $\triangle$ ka kale dhinacyada xagaaha samaysaana ay saamigalsan yihiin $\triangle$ ladu way $\sim$.
Caddayn	Garaadayn
5. :. $a/a^1 = c/x$.	5. Dhinacyada isku aada ee $\triangle \sim$ way saamigalsan yihiin.
6. Hase-yeeshe $a/a^1 = c/c^1$	6. Siin
7. :. $c^1 = x$	7. Haddii seddex xaddi ee saamigal ay = seddex xaddi ee saamigal kale sida ay u kala horeeyaan marka xaddiyada 4 aad way =.
8. :. $\triangle$ ka $A^1 B^1 C^1 \cong \triangle$ ka CDE	8. dh.dh.dh.
9. :. $\triangle$ ka $A^1 B^1 C^1 \sim \triangle$ka BC .	9. Xaglahoodu way isleeg yihiin.

ARAGTIIN:
Laba seddexagal oo dhinacyadoodu sida ay u kala horeeyaan barbarro yihiin, way isu eg yihiin.



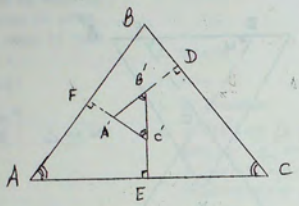
Siin: $\triangle$ ka ABC iyo $\triangle$ ka $A^1 B^1 C^1$ oo ay dhinacyadoodu //
Caddee in $\triangle$ ka ABC $\sim \triangle$ ka $A^1 B^1 C^1$

Saafid: Raadi xagla lamman oo gudboon ee xarriiqyo // ah iyo xaglo-gudeed talantaalli ah oo lamman ee xarriiqyo // ah dabeedna isku beddel.

Caddayn

Hawraar	Garaadayn
1. Fidi AC si ay ugu la kulanto $A^1 B^1$ barta B. Fidina BC si ay ugu la kulanto $A^1 B^1$ Barta D.	1. Dhsme.
2. $\angle A = \angle 1$	2. $\angle$ LLO gudeed talantaalli ah ee xarriiqyo // ah.
3. $\angle 1 = \angle B^1$	3. $\angle$ LLO-gudboon ee xarriiqyo // ah.
4. :. $\angle A = \angle B^1$	4. Isku beddel.
5. Sidaas oo kale $\angle B = \angle 2 = \angle A^1$	
6. :. $\triangle$ ka ABC $\sim \triangle$ ka $A^1 B^1 C^1$	6. 2 xaglood ee $\triangle$ = 2 xaglood ee $\triangle$ ka kale.

ARAGTIIN: Laba seddexagal oo dhinacyadoodu sida ay u kala horreeyaan isku qotomaan way isu eg yihiin.



SIIN: △ ka ABC iyo △ ka $A^1 B^1 C^1$
oo dhinacyadoodu sida ay u kalahoreeyaan way isku ∠ maan.

Caddee in △ ka ABC ~ △ ka $A^1 B^1 C$

Saafid: Raadi laba lammaane oo xaglo isleeg ah.

Caddayn:

Hawraar	Garaadayn
1. Fidi $A^1 B^1$ si ay kula kulanto CB barta D iyo $B^1 C^1$ oo kula kulmaysa AC barta E	1. Dhisme
2. ∠ 1 + ∠ D + ∠ C + ∠ E = 360°	2. ∠ laha afargeesoole = 4 xaglood oo quman.
3. Hase-yeeshe ∠ D + ∠ E = 180°	3. ∠ kastaa = ∠ quman, dabeedna isugayn.
4. :. ∠ 1 + ∠ c = 180	4. Kala gooyn.
5. Hase-yeeshe ∠ 1+ ∠ B^1 = 180°	5. Labadoodu waxay sameeyaan ∠ toosan.
6. :. ∠ B^1 = ∠ C	6. Isla xagashu waxay leedahay buuxis isleeg. (the same ∠ has = supplement).

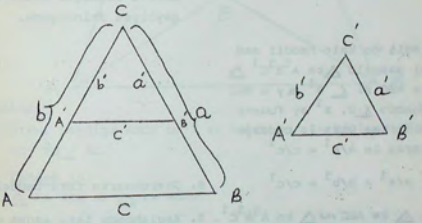
Hawraar | Garaadayn

7. Markaad $C^1 A^1$ u fidisid ilaa iyo AB waxa la caddayn karaa in ∠ C^1 = ∠ A.

8. :. △ ka ABC ~ △ ka $A^1 B^1 C^1$

8. Laba xaglood ee △ = 2 xaglood ee △ ka kale.

ARAGTIIN: Haddii xaglaha laba seddexagal ay isleeg yihiin seddexagalladu way isu eg yihiin.



SIIN △ ABC iyo $A^1 B^1 C^1$ oo ∠ A = ∠ A^1; ∠ B^1 = ∠ B ∠ C = ∠ C^1.

Caddee in : △ ka ABC ~ △ ka $A^1 B^1 C^1$

Saafid: Haddiiba ∠ lu = , dulsaar △
△ ka kale, dabeedna caddee in salka mid u // u yahay salka ∠ ka kale.

Caddayn

Hawraar	Garaadayn
1. Dulsaar △ ka $A^1 B^1 C^1$ △ ka ABC si ∠ C^1 u dul dacdo ∠ sha leeg ee ∠ C, b^1 ha dul fuusho b, a^1 ha dul fuusho a	1. Shaxan waa in la rari karaa.

2. $\angle A = \angle A^1$

3. $\therefore c^1 \parallel C$

Hawraar

4. $\therefore b/b^1 = a/a^1$

2. Siin

3. Haddii laba xarriiq ay la sameeyaan xaglo-gudboon oo = gudbane, xarriiqyadu waa //.

Qaraadaan

4. Haddii xarriiq laga sawiro qumaatiga laba dhinac oo $\triangle$ ta 3xaadna u // u yahay. Markaa saamigal ahaan buu u qaybiyaa dhinacyada.

5. Siddii oo kale haddii aad dul saartid $\triangle$ka $A^1B^1C^1$ $\triangle$ ka ABC si $\angle B^1$ ay u dul dhacdo $\angle B$, a^1 ay fuusho a markaa waxa la caddayn karaa in $a/a^1 = c/c^1$

6. $\therefore a/a^1 = b/b^1 = c/c^1$

6. Dhardhaarka isku beddelidda.

7. $\therefore \triangle$ ka ABC $\sim \triangle$ ka $A^1B^1C^1$.

7. Xaglahooda isku aadaa way = dhinacyadooda isku aadaana way saamigalsan yihiin.

Xigasho: Haddii laba xaglood ee $\triangle$ ay leeg yihiin laba xaglood ee $\triangle$ kale seddexagaladu way isu eg yihiin.

Xigasho: Xarriiqa qumaatiga u jara laba dhinac ee $\measuredangle$ dhinaca kalana u barbarro ahi wuxu ka sameeyaa seddexagalkii hore mid u eg.

LAYLI .

1. Caddee in laba seddexagale oo labaale ahi ay isu eg yihiin haddii xagal saleedka midi u leeg yahay xagal saleedka midka kale.

I.

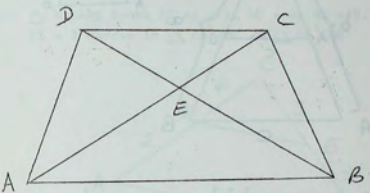
Siin:

Koorta ABCD,

oo xagla-gooyayaashu

ay iska jarayaan barta E.

Caddee in $\triangle$ka DCE $\sim \triangle$ka ABE.

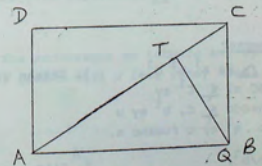


LAYLI:

3. Siin: Laydiga ABCD oo T ay tahay bar ku taal xaglagooyaha AC, TQ $\perp$ AC.

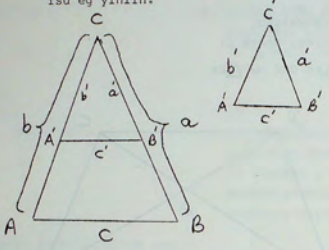
Caddee in:

$\triangle$ ka ATQ $\sim \triangle$ ka ACD.



4. AB waa shakaalka $\triangle$ ka quman ee ABC; xarriiq $\perp$ uga ah AB barta A wuxuu kula kulmay BC oo la fidiyay barta T, xarriiq $\perp$ uga ah AB barta B wuxuu kula kulmay isna AC oo la fidiyay barta Q. Caddee in $\triangle$ ka ABT $\sim \triangle$ ka BCQ.

ARAGTIIN: Haddii xagal △ L ay leeg tahay xagasha ku aada ee △ L kale oo dhinacyada xaglaha sameeyaana ay saamigalsan yihiin marka seddexagaladu way isu eg yihiin.



SIIN △ la ABC iyo $A^1B^1C^1$ oo ∠ C = ∠ C^1
$a/a^1 = b/b^1$

Caddee in △ ka ABC ~ △ka $A^1B^1C^1$

Saafid : Haddiiba 2 ∠ lo ═, dulsaar △ △ ka kale, debeed caddee in salka △ in u // u yahay salka △ ka kale.

Caddayn

Hawraar

1. Dulsaar △ ka $A^1B^1C^1$ △ ka ABC si ∠ C^1 ay u dul dhacdo ∠ C, b^1 ay u fuusho b a^1 ay u fuusho a.

2. $a/a^1 = b/b^1$.

Hawraar

3. :. c^1 // c

Garaadeyn

1. Shaxan waa la rari karaa.

2. Siin.

Garaadeyn

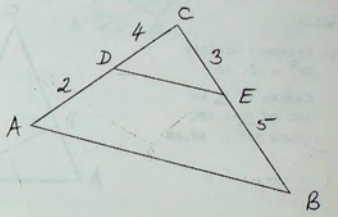
3. Haddii xarriiq saamigal ahaan uu u.. qaybiyo laba dhinac oo seddexagal waa u //ro dhinaca 3 xaadna.

4. :. ∠ A = ∠ A^1, ∠ B = ∠ B^1 — 4. Xagla-gudboon ee xarriiqyo // ah.

5. :. △ ka ABC ~ △ ka $A^1B^1C^1$ — 5. Xaglahooda oo idili way ═.

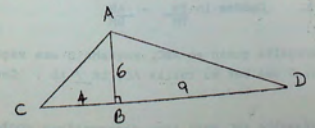
LAYLI

1. Siin: △ ka ABC lehna DE AD = 2, DC = 4, BE = 5, CE = 3 caddee in △ ka ABC ~△ ka CDE



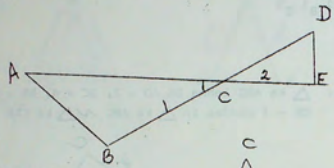
2. Caddee in Seddexagalka ka sameysma marka laysku xidho bar-badhtameedyada dhinacyada △ka ABC inu u eg yahay △ ka ABC.

3. Caddee in laba seddexagal oo labaale ahi ay isu eg yihiin haddii xagal geeska midi u leeg yahay xagal geeska ka kale.



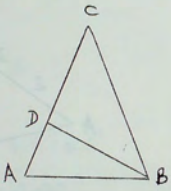
4. Siin:
AB // CD;
CB = 4,
AB = 6
BD = 9 Caddee in
△ ka ABC ~△ka ABD

5. Shaxanka midigta AC.EC = BC.CD
Caddee in $\triangle$ ka ABC ~ $\triangle$ka CDE.



LAHEL:

6. Shaxanka midigta
AB^2 = AC. AD
Caddee in $\triangle$ ka ABD ~ $\triangle$ka ABC
Ogoow AB^2 = AB.AB



7

7. Barta O kuna taala seddexagalka ABC waxa lagu xidhay geesaha seddexagalka. Waxa kaloo laysku xidhay bar-badhtameedyada A^1, B^1, iyo C^1 ee OA, OB iyo OC.
Caddee in $\triangle$ ka $A^1B^1C^1$ ~ $\triangle$ ka ABC.

8. Boqonada AB iyo CD ee goobo waxay iska jaraan barta E. Caddee in $\frac{CE}{EB} = \frac{AE}{ED}$

9. $\triangle$ ka ABC joogga AM iyo joogga BN waxay iska jarayaan barta T. Caddee in $\frac{TN}{TM} = \frac{AN}{BM}$

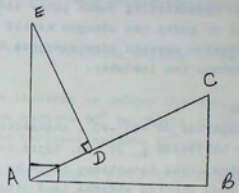
10. Seddexagalka quman ee ABC, xagasha C waa xagasha quman, D waa bar ku taalla AC, DE // AB . Caddee in $\frac{ED}{BC} = \frac{AD}{AB}$.

11. Seddexagalka ABC waxa lagu dhexmeeriyay goobo; AE waa dhexroorka goobada AD na waa joogga seddexagalka
Caddee in AB : AE = BD : CE.

12. AB waa dhexroorka goobo, BCna waa taanjentka, AC waxay kula kulantaa goobada barta D.
Caddee in $\overline{AB}^2$ = AC. AD.

13. Shaxanka midigta,
EA // AB; ED // AC
CB // AB.

Caddee in $\frac{CB}{AB} = \frac{AD}{ED}$



SEDDEXAGALLO GAAR AHAANEED

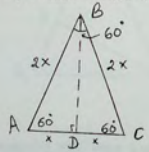
SEDDEXAGALKA $30^\circ - 60^\circ - 90^\circ$ ah.

Waxaynu soo ogaannay in dhinacyada seddexagalka quman la soo saari karo markaad haysatid laba lugood ama lug iyo shakaal, adigoo adeegsanaaya aragtiinka (Baytogaras) hasa yeeshee markuu seddexagalka quman yahay aad haysatidna hal lug ama shakaal oo qudha ama adeegan kartid aragtiinka (Bayt.). Hase-ahaatee waxaynu adeegsanaynaa taba kale oo ah tan xigashadanu ina leedahay.

XIGASHO:

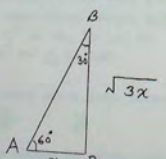
Seddexagalada 30°-60°-90°, shakaalku waa labanlaabka lugta ka soo horjeedda $\angle$ 30° ah, lugta ka soo horjeedda xagasha 60° ahina waxay le'egtahay lugta ka soo horjeedda xagasha 30° ah oo lagu dhuftay xiddika laba jibaar ee seddex $\sqrt{3}$.

Shaxanka midigtu waa seddex-xagalka ABC oo seddexle ah. Sidaa darteed dhinac walba waxaynu siin karnaa doorsome isku mid ah.



I

Sida shaxanku ku tusaayo waad raacin karta weheliye sida 2x, 3x iwm.



II

Ku noqo seddexagalka ABC, AB = 2 X = BC = AC ka soo jeex joogg geeska B, jooggaasuna ha badho xagasha $\angle B$ iyo AC, markaa goonni uu qaado seddexagalka ABD, ogowna seddexagalka ABD waa mid quman dhinac AD = X . Markaa inagoo adeegsanayna aragtiinka Baysoo-garaas $\overline{BD}^2 = \overline{AB}^2 - AD^2$ markaa $BD^2 = 4x^2 - x^2$

$BD^2 = 3x^2$ $BD = x\sqrt{3}$.

U fiirso xidhiidhka ka dhexeeya dhinacyada AB, BD AD, AB = 2x, AD = x markaa AB waa labanlaabka AD. BD waa AD oo lagu dhuftay $\sqrt{3}$.

Gebageba ahaan sidani waa sida lagu helo dhinacyada seddexagalka $30^\circ - 60^\circ - 90^\circ$.

1. Markaad haysatid dhinaca ka soo horjeeda xagasha 30°ka digirii ah.

   a) Ku dhufo xiddidka laba jibaaran ee seddex dhererka dhinaca ka soo horjeeda xagasha 30° ka digrii ah si aad u heshid dhinaca ka soo horjeeda xagasha 60° ah.

   b) Si aad u heshid shakaalka ku dhufo 2 dhererka dhinaca ka soo horjeeda xagasha 30° ka digirii ah.

2. Markaad haysatid dhinaca eegaya xagasha 60°ka digirii ah, si aad u heshid dhinaca ka soo horjeeda xagasha 30°ka digrii ah.

   a) U qaybi $\sqrt{3}$ dhinaca eegaya xagasha 60°ka digirii ah.

   b). Markaad shakaalka rabtid, ku dhufo 2 dhererka dhinaca ka soo horjeeda xagasha 30° ka digrii ah.

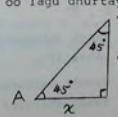
3. Marka lagu siiyo shakaalka.

   a) U qaybi 2 shakaalka si aad u heshid dhinaca ka soo horjeeda xagasha 30° ka digrii ah.

   b) Ku dhufo $\sqrt{3}$, dhinaca ka soo horjeeda xagasha 30°ka digrii ah, si aad u heshid dhinaca eegaya xagasha 60° ka digrii ah.

XIGASHO:

Seddexagalka quman ee labaale ahi ($\triangle$ka $45^\circ - 45^\circ - 90^\circ$). Shakaalku wuxuu leeg yahay labada lugood oo lagu dhuftay xiddidka laba jibaar ee laba $\sqrt{2}$.



Shaxanka midigta ka soo
qaad in labada lugood ee is leegi midiba
tahay x.

Markaa : $AB^2 = AC^2 + BC^2$

$AB^2 = x^2 + x^2$

$\sqrt{AB^2} \quad \sqrt{2x^2}$

$AB = x\sqrt{2}$

U fiirso xidhiidhka ka dhaxeeya
shakaalka iyo lugaha.

Ogsoonowna in x u taagan tahay tira kasta oo ay x tahay madoorsoome.

GEBGGEBA AHAAN

Sidani waa sida aad ku soo saari kartid dhinacyada seddexagalka ($45^\circ - 45^\circ - 90^\circ$) kuna ogaan kartid xidhiidhka ka dhexeeya dhinacyada.

1. Markaad og tahay laba lugood oo isleeg mid ahaan ku dhufo xidhiidka laba jibaaran ee laba si aad u heshid shakaalka.

2. Markaad ogtahay shakaalka u qaybi $\sqrt{2}$ shakaalka si aad u heshid labada lugood ee isleeg mid ahaan.

LAYLI:

1. Raadi xaglagooyaha laba jibaaraneh haddii dhiniciisu yahay 5 fuud.

2. Raadi lugaha seddexagalka 30°-60° iyo 90° haddii shakaalkiisu yahay 8".

3. Raadi xaglaqooyaha laydiga dhinacyadiisu yihiin 8', 8"

4. Raadi dhinacyada seddexagal seddexlanah haddu jooggiisu yahay 12'.

5. Haddii seddexagal quman oo labaala ah shakaalkiisu yahay 14', Raadi dhererka lugahiisa?

6. Soo saar wareegga seddexagalka 30°-60° iyo 90° haddii dhiniciisa ka soo horjeeda $\angle$ 30° ahi uu yahay 8'?

7. Boqonbaa dhererkiisu yahay 48 hiish, toban hiishna waxa u jiraa xuddunta goobada. Raadi gacanka goobada?

8. Lugaha seddexagal labaale ahi mid waliba 34" , salkuna waa 60". Raadi joogga salka ku taagan.

TIROOYINKA BAYSOGARAS

Sida badan ba, marka la raadinaayo dhinacyada seddexagalka quman waxa raacaa xidiidka laba jibaar. Hayeeshee mararbaa jira ay dhinacyadoo idil abyoonayaal yihiin, mararka khaaska ahi halkan ayay ku muuqanayaan, ee waxa fiican in la xusuusnaado, badanaaba waxa lala kulmaa seddexagalada leh dhinacyada 3,4,5, ama 5,12, 13 ama 8; 15, 17 iyo 7, 24, 25, ama dhufsanayaashooda ee kuwan raqayntooda uun ah.

Kooxahaa waxa loo yaqaanaa tirooyinka (Baysogaraa). Markan tirooyin badan oo la mid ah waxa lagu soo saari karaa jidkan,

(1) n, $\frac{n^2-4}{4}$, $\frac{n^2+4}{4}$ markay n tahay tiro dhaban ahi ama

(2) n, $\frac{n^2-1}{2}$, $\frac{n^2+1}{2}$ markay n tahay tiro kisi ah.

TUSAALE:

Bal eeg haddii 14= (dhaban)

n = 14

$\frac{n^2-4}{4} = \frac{14^2-4}{4} = 48$ Markay n tahay dhaban, dhinacyada seddexagalka quman waa 14, 48, 50.

$\frac{n^2+4}{4} = \frac{14^2+4}{4} = 50$

2. Ha yeeshee haddii n tahay kisi, ka soo qaad in ay tahay:11.

n = 11

$\frac{n^2-1}{2} = \frac{11^2+1}{2} = 60$ Haddana marka n tahay kisi, dhinacyada seddexagal qumani waa 11, 60, 61.

$\frac{n^2-1}{2} = \frac{11^2+1}{2} = 61$

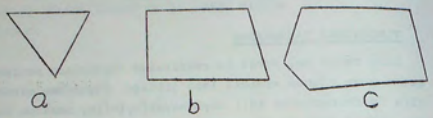
LAYLI

Raadi dhinacyada seddexagalka quman adoo mar walba n, ku beddelaya tirooyinkan, adeegsanayana labadii jed ee aad soo martay, kuna hubi jidka Baysogaras:

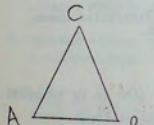
1, 2, 4, 3, 5, 13, 17, 7, 15;

GEESOOLAYAAL

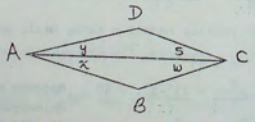
Waa maxay geesoole?



Shaxanadan oo idili waa geesoolayaal. Idilkoodna waa shaxanno oodan, shaxanno sallax ah, waxana ku wareegsan xarriiqyo toosan, waxaana lagu sawiri karaa dulfidsan. Geesoolayaasha xagal guudeedyadoodu ay ka yar yihiin 180° waxa la yidhaahdaa geesoolayaal tuur leh. Haddaba inkasta oo ay jiraan qaar kale waxa aynu ku koobnaan kuwa tuuraysan ama tuuraha leh. Geesoolaha ugu fudud waxa la yidhaahdaa seddexagal, kaas oo sidaad ogtahayba ay wadarta xagal gudeediisu tahay 180°.



$\angle A + \angle B + \angle C = 180^{\circ}$



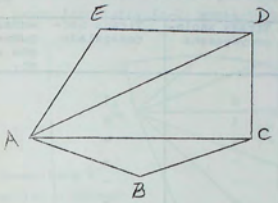
Afar geesoole isna waa geesoole afar dhinac leh. Marka aynnu sawirro xagllogooyaha AC shaxanka ABCD wuxuu u qaybsamay laba seddexagal.

Haddii $\angle D + \angle Y + \angle S = 180^{\circ}$. Waayo?
$\angle B + \angle X + \angle W = 180^{\circ}$ Waayo?

Haddaba waxa jirta haddii aynu xagllogudeedyada ABCD oo idil aynu isugayno waxa aynu heli sidan:

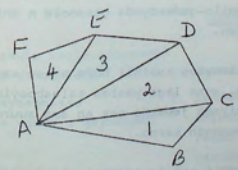
$\angle B + \angle D + (\angle Y + \angle X) + (\angle S + \angle W) = 360^{\circ}$

$\therefore \angle B + \angle D + \angle A \angle C = 360^{\circ}$



Geesoolaha shanta dhinac leh waxa la yidhaa "shandhinac leh". Marka aynu samaynay xaglagooyayaasha AC iyo AD ee shaxanka ABCDE, imisa seddexagal baa samaysmaya?

Ma sheegi kartaa wadarta xagalgudeedyada geesoole shan dhinac leh? Eeg uun inta seddexagal ee uu ka samaysan yahay.



Geesoolaha lixda dhinac leh isna waxa la yidhaahdaa lix dhinac leh. Xaglagooyayaasha AC, AD, AE ee shaxanka ABCDEF waxa ay inoo sameeyaan afar seddexagal. Waa imisa wadarta xagla gudeedyada lix dhinac leh.

Adigoo kaashanaaya falanqayntii hore iyo adoo samaysanaya washir ku habboon, dhamaystir tusahan.

Tirada dhinac-yada geesoola-ha	Tirada xagla-gooyayaasha	Tirada Sed-dexagalada	Wadarta xaglo-gudeedyada iya-goo xaglo quman ah.
3	0	1	2
4	1	2	4
5	-	-	-
6	-	-	-
7	-	-	-
8	-	-	-
9	-	-	-
10	-	-	-
11	-	-	-
12	-	-	-

Ma kuu muuqataa si gaar ah oo loo raaci karo tusaha sare?
Haddii geesooluhu uu leeyahay n dhinac, imisa xaglogooye ayaa ku sameysmi kara bal gees? Imisa seddexagal ayay u qaybinayaan xaglagooyayaashu? Adigoo waydiimanaan tixraacaya sheeg wadarta xagllo-gudeedyada geesoole n dhinac leh, kuna sheeg xagllo-quman.

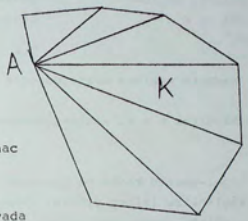
Markaad dhamaystiraysay tusihii hore amase aad ka jawaabaysay weydiimihii kale, waxa laga yaabaa talaabooyinkii aad qaaday in ay kuu hogaamiyeen jawaabo sax ah ama suurogal ah, hase-yeeshe kuma kalsoonaan karno.

Tusaala ahaan haddii aad is leedahay ka jawaab weydiintan soo socota maxaad odhan lahayd.
Ardayda Dugsiga ku jirta oo idili ma xidhan yihiin dhar isu eg?
In ay xidhan yihiin dhar isu eg waa wax suuragal ah amase dhici kara, hase-yeeshe markiiba ma gaadhi karno gebegaba deg-deg ah oo aynu leenahay, waxay xidhan yihiin dhar isu eg maxaa yeelay haddii xeer lagaga dhigana qaar uunbaa jebin.

Haddaba xeerarka xisaabtu waa qaar guud ahaan loo caddeeyay oo aan la jebin karin ama aan la beenayn karin, mana aha qaar marna jawaab sax ah ku siiya mar kalana jawaab qalad ah ku siiya.

Markaa tusihii hore iyo weydiimihiiba waxa aynu u kaashan aragtiimooyin soo socda oo la caddeyey.

ARAGTIIN: Wadarta xaglo-gudeedyada ee geesoole n dhinac lehi waxay leeg tahay (n-2) xagal-toosan ama (n-2) 180°.



SIIN: Geesoolaha K oo leh n dhinac

Caddee in: Wadarta xaglo-gudeedyada ee K = (n-2) xaglo-toosan ama (n-2) 180°

Saafid: Geesoolaha u qaybi seddexagallo.

Caddayn

Hawraar	Garaadayn
1. A ka soo sawir xagla gooyaasha suuragalka ah oo idil	1. Dhisme.
2. Xaglagooyayaashaasu geesoolaha waxay u qaybiyaan (n-2) $\triangle$	2. Seddexagal baa ka samaysma dhinac kasta, marka laga reebo labada deriska u ah geeska A.
3. Wadarta xagllo-gudeedyada $\triangle$ = xagal toosan	3. Aragtiin hore.

4. Wadarta xagllo-gueedyada ee seddexagalada oo idili = $(n-2)\ 180^{o}$

4. Dhardhaarka isku-dhufashada.

5. Wadarta xagllo-gudeedyada oo idili ee seddexagalada oo idili = wadarta xagllo-gudeedyada ee geesoole.

5. Wax dhami = wadarta qaybihiisa.

6. Markaa wadarta xaglo-gudeedyada ee K = $(n-2)\ 180^{o}$.

6. Dhardhaarka isku beddelidda.

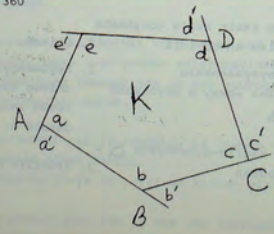
XIGASHO: Wadarta xaglaha afar geesoole waxay leeg tahay 360^{o}.

Biniix: Haddiiba n = 4, xaglo-gudeedyadu = $(4-2)\ 180^{o}$
$= 2 \times 180^{o} = 360^{o}$.

XIGASHO: Xagal-gudeed kasta ee geesoole xaglihiisu isleeg yihiin, lehna n dhinac waxay leeg tahay

$$\frac{(n-2)\ 180}{n}$$

ARAGTIIN: Haddii dhinacyada geesoole golxaysan loo fidiyo si qaabsan markaa wadarta xagllo-debadeedyada sameeysmay waxay leeg tahay laba xaglood oo toosan ama 360^{o}.



Siin: Geesoolaha K lehna n dhinac AB waxa loo fidiyay qumaatiga B, BC qumaatiga C, CD qumaatiga D iwm, si ay u sameeysmaan xagllo-debadeedyada $\angle a^1, \angle b^1, \angle c^1, \angle d^1, \angle e^1$

Caddee in: Wadarta xaglo-debedeedyada ee geesoolaha K = 360^{o}.

Saafid: Wadarta xagal-gudeeda iyo xagal-debadeeda ee gees kasta waxay tahay xagal toosan.

Caddayn

Hawraar	Garaadayn
1. $\angle a + \angle a^1$ = xagal toosan. $\angle b + \angle b^1$ = xagal-toosan.	1. Wadarta xaglaha ka sameeysmay bar dhinac uun kaga taal xarriiq toosan = 180^{o}.
2. Hase-yeeshe geesuhu waxay yihiin n gees	2. Siin
3. Xagllo-gudeedyo + xagllo-debadeedyo = n X xaglo-toosan.	3. Isku dhufasho.
4. Hase-yeeshe xagal-gudeed = n Xaglo-toosan - 2 xaglo-toosan.	4. Wadarta xagllo-gudeedyada geesoole leh n dhinac waxay tahay (n-2) xaglo toosan ah.
5. Markaa wadarta xagllo-debadeedyada K = laba xaglood oo toosan ama 360^{o}.	5. Kala goynta hawraarta seddex ka go'o hawraarta afraad.

XIGASHO: Xagal-debadeed kasta ee geesoole ay xaglihiisu isleeg yihiin dhinacyadiisuna ay n, yihiin waa $\frac{360^{o}}{n}$.

TUSAALE I: Raadi wadarta xagal-gudeedyada geesoole leh 10 dhinac leh.

FURFURIS: Kaasho jidkii ahaa $w = (n-2)\ 180^{\circ}$.

Haddiiba, $n = 10$ waxaynu ku beddeli n.

Markaa, $w = (10-2)\ 180^{\circ} = w = 8 \times 180^{\circ} = \underline{1440^{\circ}}$

TUSAALE II: Wadarta xagal-gudeedyada geesoole waa 1620°, imisa dhinac ayuu leeyahay geesoolahaasu?

FURFURIS: Kaasho jidkii hore.

$w = (n-2)\ 180$,

$1620^{\circ} = (n-2)\ 180^{\circ}$.

$1620 = 180\ n - 360$

$1620 + 360 = 180n$

$\frac{1620}{180} = \frac{180}{180} n$

$n = 11$ dhinac

TUSAALE III: Dhinacyada geesoole qaabsani waa 15 . Raadi qiimaha xagal-gudeed kasta.

FURFURIS: Kaasho jidkii ahaa xagal-gudeed = $\frac{(n-2)\ 180^{\circ}}{n}$,

Haddiiba $n = 15$

$\therefore \frac{(15-2)\ 180^{\circ}}{15} = \frac{13 \times 180^{\circ}}{15} = 156^{\circ}$

Xagal-gudeed kasta = $\underline{156^{\circ}}$

LAYLI :

1. Xagal-gudeed kasta ee geesoole qaabsan waa 160°, imisa dhinac ayuu leeyahay geesoolahaasu?
2. Raadi wadarta xagal-gudeedyada geesoole 6 dhinac leh
   " 8 dhinac leh
   iyo " 10 dhinac leh

3. Raadi tirada dhinacyada geesoole, wadarta xagal-gudeedyadiisu yihiin 1800°, (b) 1260°, (c) 540°.
4. Raadi qiimaha xagal-gudeed kasta ee geesoole 5 dhinac leh (b) 9 dhinac leh, (c) 12 dhinac leh.
5. Haddii seddexaglood ee afar geesoole yihiin, 75°, 85°, 100°. Raadi xagasha afraad?
6. Imisa dhinac ayuu leeyahay geesoole, haddii wadarta xagal-gudeedyadu ay tahay 5 , oo lagu dhuftay wadarta xagal-debadeedyada?

GOOBOOYIN, QAANSOOYIN, BOQONO IYO XAGLLO XUDUMEEDYO

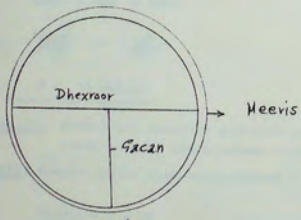
B. Qeexiddo hakhtiin ah oo ku saabsan goobooyinka.

1. Goobo waa xoodan oodan kuna jiifa ama ku lingaxan sallax barahoo idilina in u wada jiraan bar sallaxa ah oo la yidhaa xuddun.

2. Gacanka goobo waa xarriiq toosan oo ka yimaada xuddunta kuna dhammaada bar goobada ku taal.

3. Dhexroorka goobo waa xarriiq toosan oo mara xuddunta kuna dhammaada laba barood oo goobada ku yaal.

4. Meeriska goobo waa dhererka goobada. Goobo waxa inooga wakiil ahaan kara xuddunteeda, markaa goobada O waxay tahay goobada xuddunteedu tahay O.



Xigashooyin laga diiray geexo.

1. Gacanada isku goobada ah ama kuwa ka samaysama goobooyin is leegi way isleeg yihiin.

2. Dhexroorada isku goobada ahi way isleeg yihiin ama kuwa ka samaysama goobooyin isleeg way isleeg yihiin.

3. Dhexroorka goobo waa laba laabka gacanka goobada.

QAANSO

Qaanso waa qayb kasta oo ka mid ah goobada

1. Goobo badh: waa qaansada leeg goobo badhkeed.

2. Qaanso weyn: waa qaansada ka weyn goobo-badh .

3. Qaanso yar: Waa qaansada ka yar goobo-badh.
Haddaba marka qaanso lagu siiyo iyadoo aan laguu sheegin tay tahay waxa loo fahmaa in ay tahay qaansada yar.
Summadda laysku raacay ee qaansaduna waa xarriiq xoodan oo la dul dhigo xuruufta sheegaysa cidhifyada qaansad.
AB waxa loo akhriyaa "qaansada $\overset{\frown}{AB}$" waxaanay tahay qaansada u dhaxaysa baraha A iyo B.

4. Qaansooyinka isleegi waa qaansooyinka saani isugu dul dhaca.

Xagal xuddumeed: Waa xagasha geeskeedu yahay xuddunta goobada dhinacyadeeduna ay yihiin gacanada goobada.

1. Xagal xuddumeedku wuxuu tigraaraa qaansada taasoo ay jaraan dhinacyada xagashu.

2. Qaansada la tigraarayaana waxay laashaa xagal-xuddumeedkeeda.

T. Boqon: waa xarriiq toosan oo cidhifyadiisu ku yaallaan goobada shaxanka hoose xarriiqda toosan ee DE wa boqon.

1. Boqonka goobo wuxuu laala qaansooyinka uu isagu ka jaro goobada. Haddii aan la sheegin qaansada la'laalay waxa loo qaataa qaansada yar.

2. Qaansada goobo ee uu jaray boqon waxa laalay boqonka.

J. Siikant: waa xarriiq toosan kana jara goobada laba barood.

GOOBOOYIN, QAANSOOYIN, BOQONO IYO XAGLLO XUDUMEEDYO

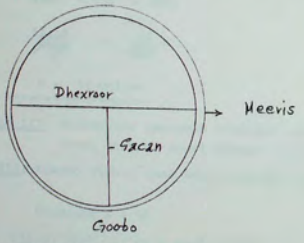
B. Qeexiddo nakhtiin ah oo ku saabsan goobooyinka.

1. Goobo waa xoodan oodan kuna jiifa ama ku lingaxan sallax barahoo idilina in u wada jiraan bar sallaxa ah oo la yidhaa xuddun.

2. Gacanka goobo waa xarriiq toosan oo ka yimaada xuddunta kuna dhammaada bar goobada ku taal.

3. Dhexroorka goobo waa xarriiq toosan oo mara xuddunta kuna dhammaada laba barood oo goobada ku yaal.

4. Meeriska goobo waa dhererka goobada. Goobo waxa inooga wakiil ahaan kara xuddunteeda, markaa goobada O waxay tahay goobada xuddunteedu tahay O.



Xigashooyin laga diiray qeexo:

1. Gacanada isku goobada ah ama kuwa ka samaysama goobooyin is leegi way isleeg yihiin.

2. Dhexroorada isku goobada ahi way isleeg yihiin ama kuwa ka samaysama goobooyin isleeg way isleeg yihiin.

3. Dhexroorka goobo waa laba laabka gacanka goobada.

QAANSO

Qaanso waa qayb kasta oo ka mid ah goobada

1. Goobo badh: waa qaansada leeg goobo badhkeed.

2. Qaanso weyn: waa qaansada ka weyn goobo-badh .

3. Qaanso yar: Waa qaansada ka yar goobo-badh.
Haddaba marka qaanso lagu siiyo iyadoo aan laguu sheegin tay tahay waxa loo fahmaa in ay tahay qaansada yar.
Summadda laysku raacay ee qaansaduna waa xarriiq xoodan oo la dul dhigo xuruufta sheegaysa cidhifyada qaansad.
AB waxa loo akhriyaa "qaansada $\overset{\frown}{AB}$" waxaanay tahay qaansada u dhaxaysa baraha A iyo B.

4. Qaansooyinka isleegi waa qaansooyinka saani isugu dul dhaca.

Xagal xuddumeed: Waa xagasha geeskeedu yahay xuddunta goobada dhinacyadeeduna ay yihiin gacanada goobada.

1. Xagal xuddumeedku wuxuu tigraaraa qaansada taasoo ay jaraan dhinacyada xagashu.

2. Qaansada la tigraarayaana waxay laashaa xagal-xuddumeedkeeda.

T. Boqon: waa xarriiq toosan oo cidhifyadiisu ku yaallaan goobada shaxanka hoose xarriiqda toosan ee DE wa boqon.

1. Boqonka goobo wuxuu laala qaansooyinka uu isagu ka jaro goobada. Haddii aan la sheegin qaansada la'laalay waxa loo qaataa qaansada yar.

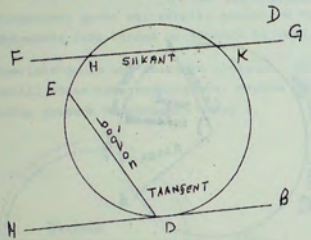
2. Qaansada goobo ee uu jaray boqon waxa laalay boqonka.

J. Siikant: waa xarriiq toosan kana jara goobada laba barood. Shaxanka hoose FG waa siikant.

1. Xubinta siikantka ee ku jirta goobada waxay tahay xarriijinta boqon ee siikantka. HK waa boqon xarriijimeedka siikantka FG.

X. Taanjent: waa xarriiq toosan oo dhererkiisu aanu xad lahayn, barqudhana ka taabanaaya goobada.

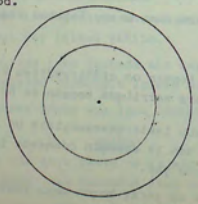
1. Barta ayay wadaagaan xarriiq iyo goobadu waxa la yidhaa "Barta taabashada " ama barta "Taanjetka". Shaxanka hoose AB waa Taanjent. D waa barta taabashada.



KH. Goobooyin Isku Xuddun ah

Waa goobooyinka xuddun wadaaga gacanadooduna aanay is leekayn. Haddii goobooyinku aanay isku xuddun ahayn, xuddimahooda waxa laysugu xidhi karaa xarriijin.

1. Xarriiq xuddumeedyada (line of centres) laba goobo oo aan isku xuddun ahayni waa xarriijinta isku xidha labadooda xudumood.



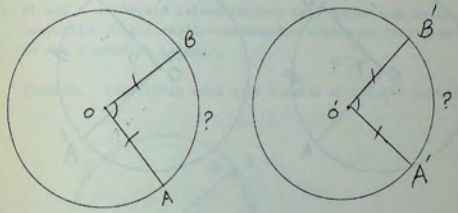
"Laba goobo oo isku xuddun ah"

XIGASHA 1: Laba goobo iskama jari karaan laba barood wax ka badan.

XIGASHO 2: Goobo lagama sameyn karo qumaatiga seddex barood oo si toos ah isu dabayaala ama isku toosan.

XIGASHO: Xarriiq toosani laba barood wax kabadan kama jari karo goobo.

ARAGTIIN: Isla goobada ama goobooyin isleeg, xagllo-xuddumeedyo isleegi waxay tigraaraan qaansooyin isleeg.



SIIN: Goobooyin isleeg O, iyo O^1 oo xagllo-xuddumeedyada $\angle$ AOB = xagal xuddumeedka $\angle A^1O^1B^1$.

Caddee in $\overset{\frown}{AB} = \overset{\frown}{A'B'}$

Saafid: Qaansooyin way isleeg yihiin marka laga dhigo qaar is duldhaca.

CADDEYN

Hawraar	Garaadayn
1. Goobada O^1 dulsaar goobada O si O^1 ay u fuusho O iyo si O^1A^1 ay ugu dul dhacdo OA.	1. Shaxan waa la rari karaa.
2. A^1 waxay fuushay A.	2. $OA = O^1A^1$
3. O^1B^1 waxay fuushay OB	3. Siin, $\angle$ AOB = $\angle A^1O^1B^1$.

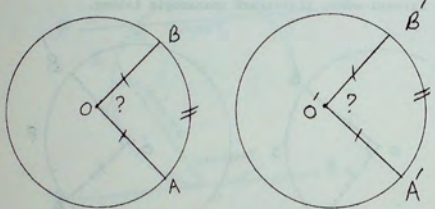
4. B^1 waxay fuushaa B — 4. $OB = O^1B^1$, gacanada goobooyin = way =.

5. $\overset{\frown}{AB}^1$ waxay dul dhacdaa $\overset{\frown}{AB}$ — 5. Qeexiddii goobo.

6. $\therefore AB = \overset{\frown}{AB}^1$ — 6. Way is dul dhacaan.

ARAGTIIN

Isla goobada ama goobooyinka isleeg, qaansooyinka isleegi waxay laalaan xagllo-xuddumeedyo isleeg.



SIIN: Goobooyin isleeg O iyo O^1 oo $AB = \overset{\frown}{AB}^1$
Caddee in $\angle AOB = \angle A^1O^1B^1$.

Saafid: Shaxannada waxay isku sargo'an yihiin marka laga dhigo qaar is dul dhaca.

CADDEYN:

Hawraar — Garaadayn

1. Goobada O^1 dulsaar goobada O si O^1 u fuusho O,OA^1 na u fuusho OA. — 1. Shaxan waa la rari karaa

2. Markaa A^1 waxay fuushay A — 2. $OA = O^1A^1$, gacanada goobooyin isleegi way is leeyihiin.

3. $\overset{\frown}{A^1B^1}$ waxay fuushay $\overset{\frown}{AB}^1$ — 3. Qeexidii goobo.

4. B^1 waxay fuushay B — 4. Siin, $\overset{\frown}{AB} = \overset{\frown}{A^1B^1}$

5. O^1B^1 waxay dul dhacday OB. — 5. Xarriiq toosan oo qudha ayaa laga sawiri karaa qumaatiga laba barood.

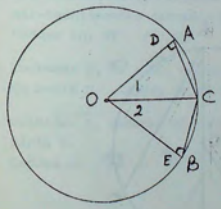
6. $\therefore \angle AOB = \angle A^1O^1B^1$. — 6. Giddigood way is dul dhaceen.

DARIIQOOYINKA GUUD

1. Si aad u caddaysid isleekaanshaha laba xaglood, tus in ay yihiin xagllo-xuddumeedyo ay laaleen qaansooyin isleeg oo isku goobo ah ama goobooyin isleeg.

2. Si aad u caddeysid isleekaanshaha laba qaanso tus in ay leeyihiin qaanso xagllo-xuddumeedyo isleeg oo isku goobo ah ama goobooyin isleeg.

Tusaale. Tusaalahan waxa aynu kaashan aragtiinka labaad.



SIIN: Goobadda O oo leh C ay tahayna bar-badhtameedka AB.
$\underline{CD} \perp$ gacanka OA, $\underline{CE} \perp$ gacanka OB.
Caddee : $CD = CE$

CADDEYN

Hawraar — Garaadayn

1. Sawir OC si aad u heshid $\angle 1$ aad iyo $\angle 2$aad ee barta O. — 1. Dhisme

2. $\overset{\frown}{AB} = \overset{\frown}{CD}$ — 2. Ognahay in C tahay bar-badhtanka $\overset{\frown}{AB}$.

3. $\angle 1 = \angle 2$

3. Goobo dhexdeed, qaansooyin isleegi waxay laalaan xagllo xuddumeedyo isleeg.

4. OC = OC

4. Ka dhexeeye.

5. Labada ODC iyo OEC waa xagllo-quman

5. Siin $\angle$s yaal waxay sameeyaan xagllo-quman.

6. $\triangle$ka quman ee ODC $\cong$ $\triangle$ka quman oo OEC .

6. Sh. x

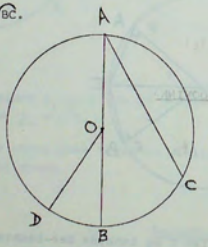
7. CD = CF

7. Qisi =

LAYLI: Layliyadan u kaasho labadii aragtiimo ee hore:

1. CD iyo EF waa dhexroorro isjaraya ee goobo.
   Caddee in $\overset{\frown}{CE} = \overset{\frown}{DF}$.

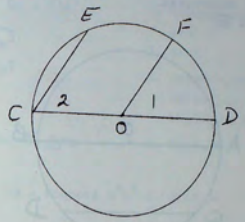
2. AOB waa dhexroorka goobada O; $\angle CAB = 25^{\circ}$, $\angle DOB = 50^{\circ}$.
   Caddee in $\overset{\frown}{BD} = \overset{\frown}{BC}$.



Biniix: Sawir OC. Raadi $\angle$ debadeedda $\triangle$

3. AOB waa dhexroorka goobadda O boqonada AC iyo AD waxa laga soo sawiri A, AB waxa ay kala sameeyeen xagllo isleeg barta A.
   Caddee in $\overset{\frown}{BD} = \overset{\frown}{BC}$.

4. COD waa dhexroorka goobadda O; gacanka OF wuxuu barbarro u yahay boqonka CE.
   Caddee in: $\overset{\frown}{EF} = \overset{\frown}{FD}$.

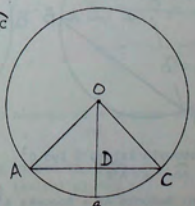


Biniix: Sawir EO.

5. COD waa dhexroorka goobada O CE waa boqon uun; Fwaa bar-badhtameedka qaansada DE.
   Caddee in: OF // CE.

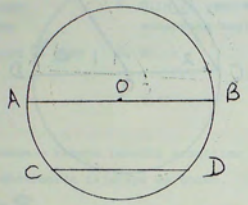
6. Goobadda O, $\overset{\frown}{AB} = \overset{\frown}{BC}$; boqonka AC wuxuu ka jaraa gacanka OB barta D. Caddee in OB badho AC.

7. Goobadda O, gacanka OB wuxuu kaga $\angle$ waa boqonka AC barta D.
   Caddee in : $\overset{\frown}{AB} = \overset{\frown}{BC}$

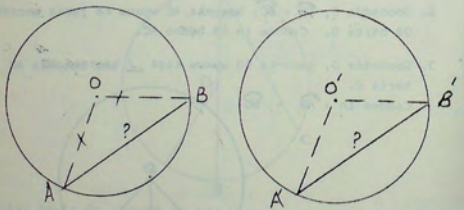


8. Siin: Goobada O, oo C ay tahay bar-badhtameedka qaansada $\overset{\frown}{AB}$. T waa bar-badhtameedka gacanka OA; Q waa bar-badhtameedka gacanka OB. Caddee in CT = CQ.

9. Goobada O, AOB
waa dhexroor CD waa boqon
barbarro u ah dhexroorka.
Caddee in $\widehat{AC} = \widehat{BD}$.



ARAGTIIN: Isla goobada, ama goobooyin isleeg qaansooyin isleegi waxay laalaan boqono isleeg.



SIIN: Goobooyin isleeg O iyo O^1
oo $\widehat{AB} = \widehat{A^1B^1}$

Caddee in: Boqonka AB = Boqonka A^1B^1

Saafid: Caddee in boqonadu yihiin qaybaha isku aada ee $\triangle$ $\cong$.

CADDAYN

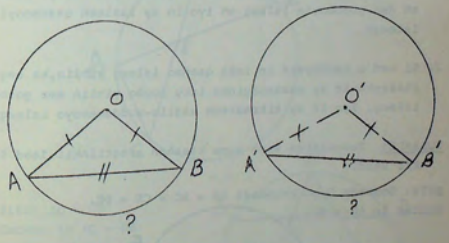
Hawraar

1. Sawir qacanada OB, OA, O^1B^1; O^1A^1.
2. OB = O^1B^1; OA = O^1A^1.
3. $\widehat{AB} = \widehat{A^1B^1}$
4. $\angle O = \angle O^1$
5. :. $\triangle$ka OAB $\cong$ $\triangle$ka $O^1A^1B^1$
6. :. Boqonka AB = boqonka A^1B^1

Garaadeyn

1. Dhisme
2. Qacanada goobooyin = way = yihiin.
3. Siin.
4. Qaansooyinka = ee goobooyinka = waxay laalaan xagllo-xuddumeedyo =
5. dh.x.dh.
6. Qisi = .

ARAGTIIN: Isla goobada, ama goobooyin isleeg boqono isleegi waxay laalaan qaansooyin isleeg.



SIIN: Goobooyinka O iyo O^1 oo boqonka AB = boqonka A^1B^1
Caddee in: $\widehat{AB} = \widehat{A^1B^1}$.

Saafid: Lammaane $\triangle$ $\cong$, ka caddee in xagllo-xuddumeedyada laalani ay =.

CADDAYN Hawraar	Garaadayn
1. Sawir gacanada OA, OB, O^1A^1, O^1B^1.	1. Dhisme.
2. OB = O^1B^1, OA = O^1A^1	2. Gacanada goobooyinka isleegi way = .
3. Boqonka AB = Boqonka A^1B^1	3. Siin.
4. $\triangle$ka OAB $\cong$ $\triangle$ ka $O^1A^1B^1$	4. dh. dh. dh.
5. :. $\angle$ O = $\angle$ O^1	5. Qisi =.
6. :. $\overset{\frown}{AB}$ = A^1B^1	6. Xagllo-xuddumeedyada =ee goobooyin = waxay tikraaraarn qaansooyin =.

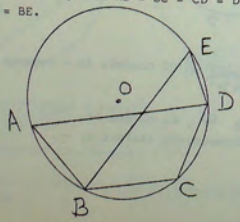
DARIIQOOYINKA GUUD

1. Si aad u caddaysid in laba xaglood isleeg yihiin, ka eeg shaxanka in ay xagluhu yihiin xagllo -xuddumeedyo isku goobo ah ama goobooyin isleeg ah iyo in ay laaleen qaansooyin isleegi.

2. Si aad u caddaysid in laba qaanso isleeg yihiin,ka eeg shaxanka in ay qaansooyinku isku goobo yihiin ama goobooyin isleeg, iyo in ay tikraareen xagllo-xuddumeedyo isleegi.

TUSAALE: Tusaalahan waxa aynu kaashan aragtiinkii 1aad iyo kii 2aad.

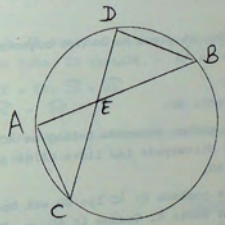
SIIN: Goobada O oo boqonada AB = BC = CD = DE.
Caddee in AO = BE.



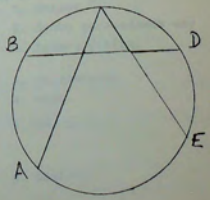
CADDEYN Hawraar	Garaadayn
1. AB = BC = CD = DE	1. Siin
2. $\overline{AB}$ = $\overline{BC}$ = $\overline{CD}$ = $\overline{DE}$	2. Boqono = ee goobo waxay laalaan qaansooyin =.
3. $\overline{AB}$ + $\overline{BC}$ + $\overline{CD}$ = $\overline{BC}$ + $\overline{CD}$ + $\overline{DE}$	3. Dhardhaarka isugaynta.
4. $\overline{AD}$ = $\overline{BE}$	4. Dhardhaarka isku beddelidda.
5. :. AD = BE	5. Qaansooyin = ee goobo waxay laalaan boqono =.

LAYLI Layliyadan u kaasho aragtiinka 1aad iyo ka labaad.

1. $\triangle$ ka TQR waxa lagu dhexmeeriyay goobo. $\angle$T = $\angle$Q.
Caddee in $\overline{TR}$ = $\overline{QR}$.



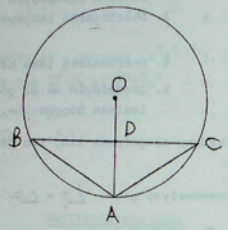
2. Siin: AB = CD
Caddee in AC = DB
Biniix: $\overset{\frown}{AB}$ = $\overset{\frown}{CD}$?



3. Siin: AC = BD
Caddee in AB = CD .

4. Siin: $\overset{\frown}{AB}$ = $\overset{\frown}{BC}$ = $\overset{\frown}{CD}$ = $\overset{\frown}{DE}$
Caddee in AC = BD = CE

5. Siin: ⊙O oo boqonka BC ka ⊥ gacanka OA barta D
   Caddee in: AB = AC



LAYLI

6. Siin : ⊙O , gacanka OA wuxuu ka badhaa boqonka BC barta D. Caddee in AB = AC.

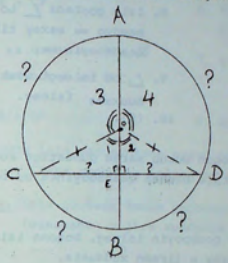
7. Siin: ⊙O AB = AC
   Caddee in OA ay badho BC.

8. AB waa dhexroorka goobo, boqonada isleeg ee AC iyo AD waxa laga sawiray dhinacyada isu lidka ah ee AB,
   Caddee in BC = BD.

9. AOB waa dhexroorka goobada O. AC iyo BD waa boqono isleeg iskana jara barta E. Caddee in AD = BC.

DHEXROOR, BOQON IYO FOGAANSHAHA ILAA IYO BOQONADA

ARAGTIIN: Xarriiqa mara qumaatiga xuddunta goobo, qotona u ah boqon wuxuu kala badhaa boqonka iyo qaansoo-yinka uu boqonku laalayba.



SIIN: Goobada O, AB waxay maraysaa qumaatiga xuddunta O qotona waxay uga tahay CD barta E.

Caddee In : CE = ED, $\overset{\frown}{CB} = \overset{\frown}{BD}$,
$\overset{\frown}{AC} = \overset{\frown}{AD}$.

Saafid: Caddee in xagllo-xuddumeedyada laalaysay isleeg yihiin.

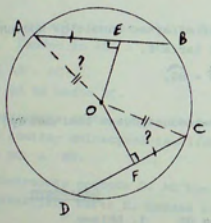
CADDAYN

Hawraar	Garaadeyn
1. Sawir gacanada OC, iyo OD	1. Dhisme
2. OC = OD	2. Goobo gacanadeedu way isleeg yihiin.
Ha...	
3. OE = OE	3. KA dhexeeye
4. OE ⊥ CD	4. Siin
5. △ ka quman ee OEC ≅ △ ka quman ee OED	5. Sh. L.
6. ∴ CE = ED	6. Qisi=

7. $\angle 1 = \angle 2$	7. (6)
8. $\overset{\frown}{CB} = \overset{\frown}{BD}$	8. Isla goobada $\angle$ Lo xudduu-meedyo = waxay tikraaraan qaansooyin =.
9. $\angle 3 = \angle 4$	9. $\angle$ Lo isleegi waxay leeyihiin buuxsha isleeg.
10. :. $\overset{\frown}{CA} = \overset{\frown}{AD}$	10. (8).

XIGASHO: Qotome badhaha boqon wuxuu maraa qumaatiga xuddunta goobada waxaanu kala badhaa qaansooyinka uu boqonku laalay.

ARAGTIIN: Isla goobo ama goobooyin isleeg, boqono isleegi isku fogaansho ayay u jiraan xuddunta.



SIIN: Goobada O leh boqonka AB = boqonka CD;
OE $\angle$ AB iyo OF $\angle$ CD.

Caddee in: Fogaanta OE = Fogaanta OF.

Saafid: Raadi qaybaha isku aada ee $\triangle$ =.

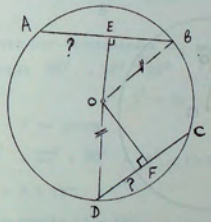
Caddayn

Hawraar	Garaadayn
1. Sawir gacanada OA iyo OC.	1. Dhisme.
2. OA = OC	2. Goobo gacanadeedu way =.
3. AB = CD	3. Siin

4. OE $\angle$ AB of ha $\angle$ CD	4. Siin
5. AE = ½ AB CF = ½ CD	5. Aragtiinkii hore
6. :. AE = CF	6. Badhadhka xaddiyo =way isleeg yihiin.
7. :. $\triangle$ ka quman ee OEA = $\triangle$ ka quman ee OFC.	7. Sh. L.
8. :. OE = OF	8. Qisi =

OGOOW isla caddayntaas ayaad kaashan kartaa haddii laba goobo oo isleeg aad qaadatid.

ARAGTIIN: Isla goobada amase goobooyin isleeg, boqonada isku fogaansho u jira xuddunta way isleeg yihiin.



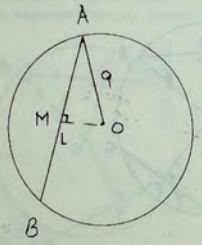
SIIN: Goobada O iyo boqonada AB iyo CD oo OE $\angle$ AB
OF $\angle$ CD;
Fogaanshaha OE = fogaanshaha OF
Caddee in AB = CD

CADDEYN

Hawraar	Garaadayn
1. Sawir gacanada OB iyo OD	1. Dhisme
2. OB = OD	2. Goobo gacanadeedu way =.
3. OE = OF	3. Siin
4. OE $\angle$ AB OF $\angle$ CD	4. Siin

5. $\triangle$ ka quman ee OEB $\cong$ $\triangle$ ka quman ee OFD	5. Sh. L = Sh. L
6. :. EB = FD	6. Qisi =
7. Hase-yeeshe EB = ½ AB FDna = ½ CD	7. Aragtiin hore
8. :. AB = CD	8. Laba laabyada xaddiyo isleegi way = .

TUSAALE I: Boqonbaa xuddunta goobo u jira 6 sm, haddii gacanka goobadu yahay 9 sm, raadi dhererka boqonka.



FURFURIS

Waxa aynu kaashan aragtiinkii Baysoogaras.

Markaa $\overline{OA}^2 - \overline{OM}^2 = \overline{AM}^2$

ama $\overline{AM}^2 = 9^2 - 6^2 = 81 - 36 = 45$

$AM = 3\sqrt{5}$

Hase-yeeshe AM = ½ AB

:. $AB = 6\sqrt{5}$

TUSAALE II: Boqonbaa 8 sm dhererkiisu yahay waxaanu u jiraa xuddunta 3sm. Raadi dhererka gacanka? Mar kale raadi dhererka boqon kale oo u jira xuddunta 2.5 sm.

FURFURIS: R waa bar-badhtameedka MN, amase 4 sm.

Markaa $\overline{OM}^2 = \overline{OR}^2 + \overline{MR}^2$

$= 4^2 + 3^2 = 25$

OM = 5 = gacan

$\overline{AQ}^2 = \overline{OA}^2 - \overline{OQ}^2 = 5^2 - (2.5)^2$

$AQ = 2.5\sqrt{3}$

Hase ahaate AB = 2AQ

Markaa $AB = 5\sqrt{3} \approx 8.66$ sm.

LAYLI

1. Boqon dhererkiisu yahay 15 sm wuxuu u jiraa xuddunta goobo 4 sm. raadi dhererka boqonka.

2. Boqonka AB iyo dhexroorka DD^1 barta e waxay iskaga jaraan xagllo-quman, haddii dhererka boqonku yahay 16 sm, DC = 4 sm raadi gacanka goobada.

3. Boqonka dhererkiisu yahay 12 sm. Imisa santimeter buu u jiraa xuddunta goobada gacankeedu yahay 18 sm.

4. AB waa dhexroor dhererkiisu yahay 34 sm, BC na waa boqon dhererkiisu yahay 8 sm. Inteebay BC u jirtaa xuddunta.

5. Laba boqon oo isleegi AB, iyo CD waxay iska jareen barta N. Caddee in AN = ND, BN = NC.

6. Dhererada laba boqon oo barbarro ahi waa 12 sm, iyo 8 sm. Haddii gacanku yahay 10 sm. Raadi fogaanshaha uu dhexeeya labada boqon:

   a) Marka ay dhinac kawada xiggaan xuddunta.

   b) Marka ay lid dhinacyada xuddunta kala yaallaan.

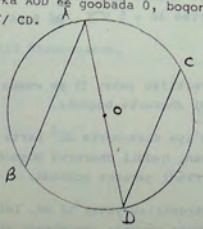
7. Raadi boqonka ay wadaagaan dhererkiisa marka laba goobo oo isleegi iska jaraan barta T, iyo Q, gacanadoodu waa 6 sm.

8. XYS waa seddexagal labaale ah, oo XS = YS.

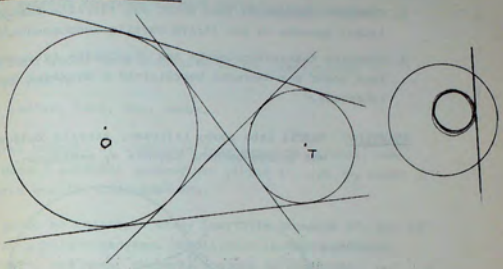
   Goobada xuddunteedu tahay S waxay ka jartay XY baraha A iyo B. Caddee in AX = BY.

9. Laba goobo oo leh xuddumaha O, O^1 waxay iska jareen baraha A iyo B, caddee in xarriiqa xuddumaha isku xidhaa yahay qotome-badhaha boqonka AB.

10. Siin: Dhexroorka AOD oo goobada O, boqonka AB = boqonka CD. Caddee in AB // CD.



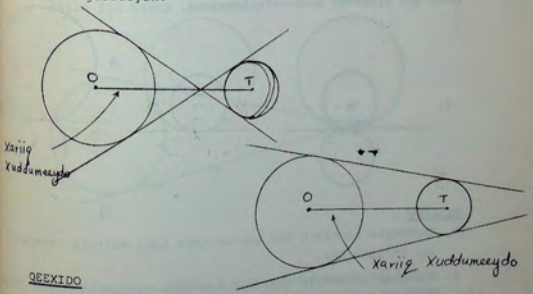
TAANJENTYO IYO GOOBOOYIN



Haddii la sawiro laba goobo oo aan isjarin, imisa xarriiq oo taanjent u ah labada goobaba ayaa la sawiri karaa?

Haddii goobo ku dhex jirto goobo kale barna aanay wadaagin, Imisa taanjent oo ay wadaagaanbaa jira?

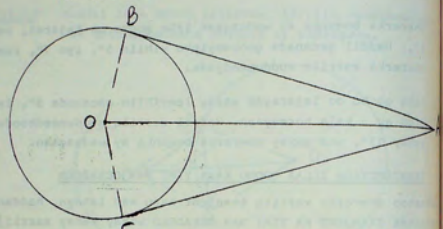
Qeexidaha taanjentyada ay wadaagaan laba goobo waxa lagu fududeeyay iyadoo marka hore la qeexo xarriijinta isku xidha xuddumaha goobooyin:



QEEXIDO

1. Xarriiq xuddumeedyada laba goobo waa xarriiqda isku xidha xuddumaha laba goobo.
2. Taanjent-gudeedka ay wadaagaan laba goobo waa xarriiqa taanjentka u ah labada goobaba, jarayana xarriiq xuddumeedyadooda.

ARAGTIIN: Taanjentyada illaa goobo kana yimid bar-debadeed way is dherer leeg yihiin, xagllo isleegna waxa ay la sameeyaan xarriiqa isku xidhaya bar debadeedka iyo xuddunta.



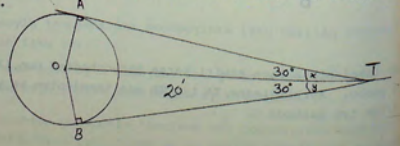
SIIN: Goobada O lehna taanjentyada AB iyo AC laga sawiray bar-debadeedka A, xarriiqa AO wuxuu ka yimaadaa A illaa xuddunta O.

Caddee in : AB = AC, iyo in
$\angle$ BAO = $\angle$ CAO .

Saafid: Raadi $\triangle$ llo $\cong$, oo AB iyo AC, $\angle$ BAO iyo $\angle$ CAO ay yihiin qaybaha isku aada.

TUSAALE

Bar 20^1 u jirta xuddunta goobo, ayaa taanjentyo laga soo sawiray illaa iyo goobada. Haddii xagasha taanjentyada u dhexaysa tahay 60^0, raadi dhererka gacanka iyo taanjent kastaba.



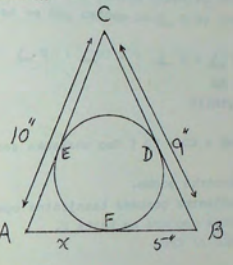
FURFURIS: $\angle$ x = $\angle$ y = 30^0,
$\angle$ OAT = 90^0

:. OA = $\frac{1}{2}$ OT = 10

TA = OA $\sqrt{3}$ = 10 $\sqrt{3}$

:. Gacan = 10, taanjent = 10 $\sqrt{3}$

TUSAALE II Shaxanka midigta goobo waxa lagu dhexmeershay seddexagal. Raadi qiimaha X.

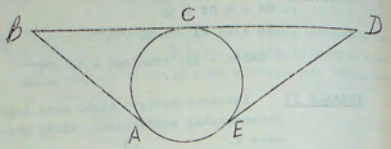


FURFURIS: Haddiiba
BF = 5, BD = 5
:. CD = 4, CEna = 4
:. AE = 6, AF ama X = 6

LAYLI

1. Caddee in dhererada taanjent-gudeedyada ay wadaagaan laba goobo oo aan isjarayni in ay isleeg yihiin?
2. Caddee in dhererada taanjent debadeedyada ay wadaagaan laba goobo oo aan isjarayni in ay isleeg yihiin.
3. Haddii xagasha u dhexaysa laba taanjent ay tahay 60^0, caddee in boqonka isku xidhaya baraha taabashada in u leeg yahay taanjentyada mid ahaan.
4. Gacanka goobo waa 6" taanjentyada laga soo sawiray bar-debadeedka T waxay sameeyaan xagasha 60^0. Inteebay T u jirtaa xuddunta.

LAYLI



5. Haddii AB, BD
iyo DE ay yihiin
taanjentyo
Caddee in AB + ED = BD (Eeg shaxanka sare)

6. EF waa dhexroorka goobo.
AEB waa taanjentka goobada taanjentna uga ah barta E, CFD waa taanjentka goobada ee barta F.
Caddee in AEB // CFD.

7. Haddii laba goobo oo aan isleegayni ay wadaagaan laba taan gudeed, caddee in xarriijimaha tanjentyada ee u dhexeeya baraha taabashadu in ay isleeg yihiin.

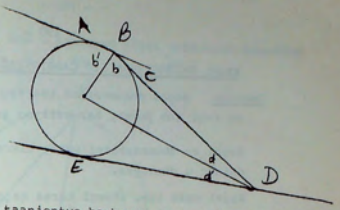
Biniix: Barta ay iska jaraan, raadi laba xaddi oo isleeg debeedna isugee.

LAYLI

8. Haddii laba goobo oo aan isleegayni ay wadaagaan laba taanjent-debadeed, caddee in xarriijimaha u dhex-xeeya, baraha taabashadu in ay isleeg yihiin.

Biniix: Fidi taanjentyada illaa ay kulmayaan.

9. AT waxay taanjent uga tahay goobada O barta A, TB waa xarri leeg TA, oo kula kulmaya goobada barta B.
Caddee in TB ay taanjent uga tahay goobada barta B.



10. AB iyo DE waa taanjentyo barbarro ah oo u jaray taanjentka seddexaad ee BD, caddee in ∠ 1 ay quman tahay.

Biniix: ∠ B + ∠ W = ? ∠ b = ∠ b^1?

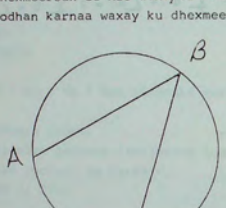
XAGAL DHEXMEERSAN IYO CABBIRKEEDA

Qeexid: Xagal dhexmeersan waa xagal ay sameeyaan laba boqon oo laga soo jeexay bar kali oo goobo dusheed ah.

Xagal ku dhexmeersani waxay tikraartaa qaansada labadeed dhinac u dhexaysa.

Xagal waxa lagu sheegi karaa xagal ku dhexmeersan qaanso dhexdeed haddii geeskeedu ku dul yaalo qaansada dhinacyadiisuna ay ku dhammaadaan qaansada dacaladeeda.

Xagasha ku dhexmeersan ee ABC waxay tikraartaa qaansada $\overset{\frown}{ADC}$, waxana aynu odhan karnaa waxay ku dhexmeersan tahay qaansada $\overset{\frown}{ABC}$.



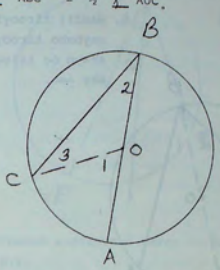
ARAGTIIN: Xagal ku dhexmeersan cabbirkeedu waa qaansada ay tikraarto badhkeed.

SIIN: $\angle$ ABC waxay ku dhexmeersan tahay goobada O.
Caddee in $\angle$ ABC $\overset{m}{=}$ ½ AC

Saafid: Waxa jirta seddex siyood oo lagama maarmaan ah in la tix geliyo.

(1) Xuddunta goobadu markay ku dultaal dhinaca xagasha.
(2) Xuddunta goobadu markay ku taal xagasha gudaheeda.
(3) Xuddunta goobadu markay ku taal xagasha debadeeda. Labada siyood ee 2 iyo 3 waxa lagu celin karaa sida kowaad marka la fidiyo dhexroorka mara geeska xagasha. Xagal xudduneedka oo lagu cabbiro qaansadiisa awgeed, marka aad doonayso in aad caddayso xaaladda kowaad ee aynu hore u soo sheegnay

waxaad jeexdaa CO si aad u sameeyso xagal xudduneed dabeedna caddee in $\angle$ ABC = ½ $\angle$ AOC.

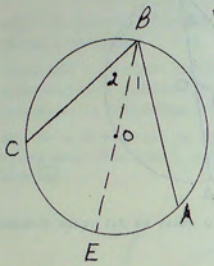


Xaaladda kowaad xuddunta O waxay ku dul taala dhinaca AB ee xagasha.

CADDEYN Hawraar	Garaadayn
1. $\angle$ ABC waxay ku dhexmeersan tahay goobada O dhexroorkeeduna yahay AB	1. Siin
2. Sawir CO	2. Qumaatiga laba barood waxa laga sawiri karaa **harxariiq** oo toosan oo qudha.
3. OC = OB	3. Gacanada goobo oo idili way isleeg-yihiin.
4. $\angle$ 2 = OB $\angle$ 3	4. Xagal-saleedyada $\triangle$ labaale ahi way =.
5. $\angle$ 1 = $\angle$ 2 + $\angle$ 3	5. Xagal-debadeedka $\triangle$ waxay leeg tahay wadarta labada xagal-gudeed ee fog-fog.
6. $\angle$ 1 = 2 $\angle$ 2	6. Tiro kasta waxa lagu beddeli karaa mid leeg tibaax kastaba.
7. $\angle$ 1 $\overset{m}{=}$ $\angle$ AC	7. Xagal-xudduneed waxa laga cabbiray qaansadiisa la tikraaray.

8. ∴ $\angle$ ABC = $\angle$ 2 $\overset{m}{=}$ $\frac{1}{2}\overset{\frown}{AC}$

8. Haddii tirooyin isleeg loo qaybsho tirooyin aan eber ahayn oo isleeg, qaybuhu way $\overset{m}{=}$.



Xaaladda labaad xuddunta 0 waxay ku taal gudaha xagasha.

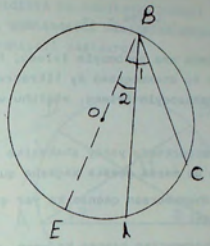
CADDAYN

Hawraar

1. Sawir ama jeex dhexroorka BE
2. $\angle$ 1 $\overset{m}{=}$ $\frac{1}{2}\overset{\frown}{EA}$
3. $\angle$ 2 $\overset{m}{=}$ $\frac{1}{2}\overset{\frown}{CE}$
4. $\angle$ 1 + $\overset{m}{=}\frac{1}{2}$ ($\overset{\frown}{EA}$ +$\overset{\frown}{CA}$)
5. ∴ $\angle$ CBA $\overset{m}{=}\frac{1}{2}$ CEA

Garaadayn

1. Qumaatiga laba barrood waxa laga sawiri karaa hal xarriq oo toosan oo qudha.
2. Xaaladda 1aad
3. Xaaladda 1aad
4. Haddii tirooyin isleeg loo geeyo tirooyin isleeg, wadarhu way isleeg yihiin.
5. Ururka baraha qaanso ku dul yaal waxay u qaybiyaan qaansada urur qaansooyin ah oo isku xuddun ah. Wadarta cabbiraadooduna ay la mid tahay cabbirka qaansada laysa siiyay.



Xaaladda seddexaad xuddunta 0 waxay ku dul taal debedda xagasha.

CADDEYN

Hawraar

1. Sawir dhexroorka BE
2. $\angle$ 1 $\overset{m}{=}$ $\frac{1}{2}\overset{\frown}{EAC}$
3. $\angle$ 2 $\overset{m}{=}$ $\frac{1}{2}\overset{\frown}{EA}$
4. $\angle$ 1 - $\angle$ 2 $\overset{m}{=}\frac{1}{2}$ ($\overset{\frown}{EAC}$ - $\overset{\frown}{CA}$)
5. ∴ $\angle$ ABC $\overset{m}{=}$ $\frac{1}{2}$ ($\overset{\frown}{AC}$

Garaadayn

1. Qumaatiga laba barrood waxa lagu sawiri karaa hal xarriiq oo toosan oo qudha.
2. Xaaladda 1aad
3. Xaaladda 1aad
4. Haddii tirooyin isleeg laga jaro tirooyin isleeg faraqyadu way isleeg yihiin.
5. La mid ah 5, ee xaaladda labaad.

1. XIGASHO: Xagal ku dhexmeersan goobo badh waa mid quman.

XIGASHO 2: Isla goobada ama goobooyin isleeg, haddii laba xaglood oo ku dhexmeersan ay tikraaraan isla qaansada ama qaansooyin isleeg, xagluhu way isleeg yihiin.

XIGASHO:3: Goobo dhexroorkeedu yahay shakaalka seddexagal quman waxa uu maraa geeska xagasha quman ee △ka.

XIGASHO:4: Xagal ku dhexmeersan qaanso ka yar goobo-badh waa xagal fiiqan.

XIGASHO 5:Xagal ku dhexmeersan qaanso ka weyn, goobo-badh waa xagal furan.

XIGASHO:6: Xaglaha iska soo horjeeda ee afargeesoole ku dhexmeersan goobo waa xagllo-isbuuxsha.

TUSAALE:

Dhinacyada △ lagu dhexmeershay goobo waxay sameeyaan qaansooyin saamigoodu yahay 1:3:5. Imisa digrii weeye xagal kastaa oo △ ku?

FURFURIS:

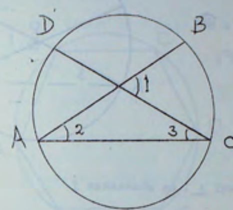
Ka soo qaad in qaansooyinku yihiin x, 3x, iyo 5x

$x+3x + 5x = 360^\circ$.

$x = 40$, $3x = 120$, $5x = 200$ sidaa awgeed xaglaha △ ku waa 20°, 60°, iyo 100°.

(5) Dhinacyada △ ee lagu meershay goobo waxay laalaan qaansooyin ah 120°, 130°, 110°. Imisa digrii weeye △ ka xagashiisi kastaaba?

6. Shaxanka midigta ku muujisan, haddii ∠B = 20°, qaansada $\widehat{AC}$ waa imisa digrii? △ ka xagashiisii kastaaba?



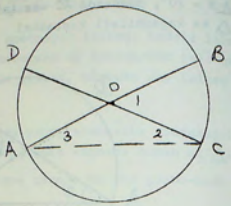
7. Xagal kasta oo shan geesle qaabsan ahi goobo lagu meershay waa imisa digrii?

8. Goobo ayaa loo qaybshay seddex qaanso oo saamigoodu yahay 2:3:7. Baraha qaybinta ayaa laysugu xidhay si is daba joog ah. Raadi tirada digriiyada ah ee xagal kasta oo marka goobada lagu meersho sidaa ku samaysanta.

Xagasha u dhaxaysa laba boqon oo goobo dhexdeed iska tikraara

ARAGTIIN: Xagasha ay sameeyaan laba boqon oo iska tikraaray goobo dhexdeed cabbirkeedu waa wadarta qaansooyinka ay tikraa- badhkbadhkood.

TUSAALE:
Raadi $\angle 1$ haddii $\overset{\frown}{AD} = 50^\circ$
$\overset{\frown}{BC} = 110^\circ$.



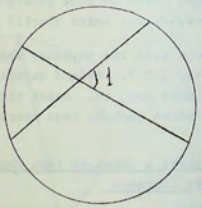
FURFURIS:
$\angle 1 \overset{m}{=} \frac{1}{2}(\overset{\frown}{AD} + \overset{\frown}{BC})$

$\overset{m}{=} \frac{1}{2}(50^\circ + 110^\circ)$

$= 80^\circ$

LAYLI
Suaalaha 1-3 raadi $\angle 1$ ee shaxankan I

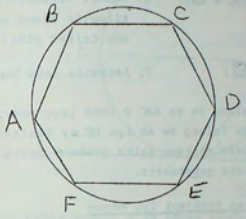
Shaxanka I



1. Siin: $\overset{\frown}{AD} = 20^\circ$, $\overset{\frown}{BC} = 60^\circ$
2. Siin : $\overset{\frown}{AD} = 70^\circ$, $\overset{\frown}{BC} = 40^\circ$
3. Siin: $\overset{\frown}{AD} = 90^\circ$; $\overset{\frown}{BC} = 90^\circ$

4. Haddii xagal saleedyada qardhaas goobo lagu meershay midkood yahay 80°, raadi xaglaha kale, caddeena in qardhaasku tahay qardhaas labaale ah.

5. Shaxanka II, $\overset{\frown}{AB} = 60^\circ$
$\overset{\frown}{BC} = 30^\circ$, $\overset{\frown}{CD} = 40^\circ$,
$\overset{\frown}{DE} = 50^\circ$, $\overset{\frown}{EF} = 70^\circ$.
Raadi cabbirka xagal kastaa ee geesoolaha.



Shaxanka II

Siin: Boqonada AB iyo CD waxay iska jaraan O
Caddee in $\angle 1 \overset{m}{=} \frac{1}{2}(\overset{\frown}{BC} + \overset{\frown}{AD})$

Saafid waxa aad raadisaa laba xaglood oo lagu meershay cabbirka midkiiba yahay wadarta qaansooyinka lagu meershay badhkeed. Dabeedna waxa aad muujisaa in $\angle 1$ le'eg tahay wadarta labadaa xaglood.

Hawraar	Garaadayn
1. Boqonada AB iyo CD waxay iska jaraan barta O.	1. O siin
2. Waxaad jeexdaa C	2. Laba barood waxa mari kara xariiq mid qudha ah oo keli ah oo toosan.
3. $\angle 1 = \angle 2 + \angle 3$	3. Xagal-debedeedka $\triangle$ waxa uu leeg yahay wadarta labada xagal-gudeed ee fog fog.

4. $\angle 2 \overset{m}{=} \frac{1}{2} \overset{\frown}{BC}$ — 4. Xagal qaanso lagu meershay cabbirkeed waa qaansada ay xagalshu tikraarto badhkeed.

5. $\angle 3 \overset{m}{=} \frac{1}{2} \overset{\frown}{AD}$ — 5. Waxay la mid tahay 4C.

6. $\angle 2 + \angle 3 = \frac{1}{2} (\overset{\frown}{BC} + \overset{\frown}{AD})$ — 6. Haddii tiroooin loo geeyo tirooyin isleeg wadarahoodu wey isleeg yihiin.

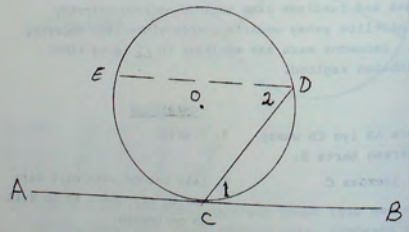
7. $\therefore \angle 1 = \frac{1}{2}(\overset{\frown}{BC} + \overset{\frown}{AD})$ — 7. Astaanta isku beddelka.

Qardhaasta labaalaha ah ee ABC D waxa lagu meeriyey goobo. Haddii dhinacyada isleeg ee AB iyo DC ay laalaan qaansooyin ah 60° midiba, isla markaan salka gaabani laalo qaanso ah 110°, raadi xaglaha qardhaasta.

Xagasha u dhaxaysa taanjent iyo boqon

Araqtiin:

Xagasha ka samaysanta meesha taanjanku ka taabto goobo ay sameeyaana taanjant iyo boqon cabbirkeedu wuxuu leeg yahay qaansada xagashu tikraarto badhkeed.



Siin: Goobada O lehna $\angle 1$ oo u sameeyay taanjentka ACB iyo boqonka CD.

Caddee in: $\angle 1 \overset{m}{=} \frac{1}{2} \overset{\frown}{CD}$

Caddayn aan dhamayn: Sawir DE // BA

$\angle 1 = \angle 2$

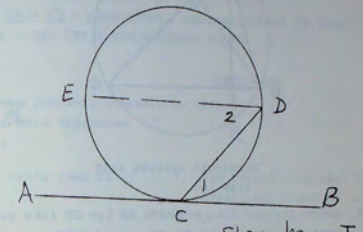
$\overset{\frown}{CD} = \overset{\frown}{EC}$

$\angle 2 \overset{m}{=} \frac{1}{2}$ EC

$\therefore \angle 1 = \frac{1}{2} \overset{\frown}{CD}$

LAYLI

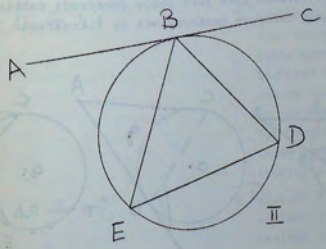
1. Shaxanka I, haddii $\overset{\frown}{EC} = 80^\circ$, raadi $\angle DCB$



Shaxanka : I

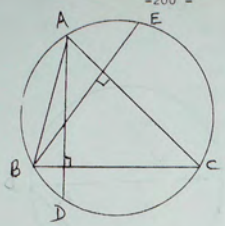
2. Haddii AB uu yahay boqon, c ay tahay badhtamaha qaansada AB, CD uu yahay tanjant, waxaad caddaysa in AB // CD.

3. Xarriiqda ABC waxay tanjant u tahay goobada waxayna ka taabataa barta B. $\overset{\frown}{DB} = 60^\circ$, $\angle C = 80^\circ$. Raadi $\angle DBE$.



Shaxankan sidaasha 3 aad ayaa leh.

4. $\triangle$ ABC waxa lagu meeriyey goobo. Boqonka AD $\perp$ BC, boqonka BE $\perp$ AC. Caddee in $\overset{\frown}{DC} = \overset{\frown}{CE}$.



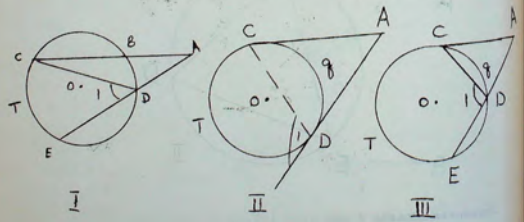
III

Shaxankan suaasha 4aad
ayaa leh.

5. Goobo dhexeed ayaa boqonada AB iyo CD iska gooyaan barta E. Caddee in △ AEC ∼ △ DEB.

Xaqasha ay sameeyaan laba siikant, laba tanjant, ama tanjant iyo siikant.

Aragtiin: Xagasha u dhaxaysa laba siikant, laba tanjant, ama tanjant iyo siikant iska jara goobo debedeeda cabbirkeed, waa faraqa u dhexeeya qaansooyinka ay tikraaraan.



I II III

Siin: Goobada O ee xagásha A ay sameeyaan siikanada AC iyo AE; labo tanjant AC iyo AD; tanjant iyo siikanta AC iyo AE oo goobada u qaybsha qaansooyinka T iyo Q .

Caddee: $\measuredangle A \overset{m}{=} \frac{1}{2}(\widehat{T} - \widehat{Q})$

Saafid: Xaalad kastaba jeex CD.

Caddee in $\angle A = \angle 1 - \angle DCA$ sidaa awgeedna dabeeto ay leeg tahay ½ (t - q)t iyo q waa tirooyin togan.

Hawraar	Garaadayn
1. $\angle A$ waxay leedahay qaansoo-yinka u kala tigraaran t iyo q	1. Siin
2. Xaalad kasta jeex CD	2. Laba barood waxa mari kara xarriiq mid qudha ah oo keli ah.
3. $\Delta^{ka}ADC$, $\angle 1 = \angle A + \angle C$	3. Xagal debedeedka △ waxay leeg tahay wadarta labada xagal-gudeed ee fog-fog.
4. $\angle A = \angle 1 - \angle C$	4. Haddii tirooyin isleeg laga jaro tirooyin is leeg farqiyadu way isleeg yihiin.
5. $\measuredangle 1 \overset{m}{=} \frac{1}{2}\widehat{t}$	5. Xaaladaha 1 iyo 3, xagal lagu meerishay waxay leeg tahay qaansada ay tigraarto badhkeed xaalada 2 na xagasha ay tanjant iyo boqon ka ka sameeyaan meesha tanjantka iyo goobadu iska taabtaan waxay leeg tahay qaansada ay tigraarto badhkeed.
6. $\measuredangle C \overset{m}{=} \frac{1}{2}\widehat{q}$	6. Xaalada 1 waxay la mid tahay 5; xaalada 2 waxay la mid tahay 5, xaalada 3 waxay la mid tahay xaalada 2.
7:. $\measuredangle A \overset{M}{=} \frac{1}{2}(\widehat{t} - \widehat{q})$	7. Astaanta isku beddelka.

LAYLI

1. Laba siikant ABC iyo ADE ayaa goobo ka jara baraha B,C iyo D,E sida ay u kala horeeyaan. Haddii qaansada $\widehat{BD} = 90^{o}$; qaansada $\widehat{CE} = 120$, raadi $\angle CAE$, $\angle ADC$, $\angle DCA$.

2. Haddii MN iyo MR ay tanjano ku yihiin goobo kana yimaadaan M, qaansada $\widehat{MB} = 100^{\circ}$ raadi /NMB, /MBN , iyo /MNB.

3. Taanjant QR iyo siikant QWX ayaa goobo ka jara baraha R,W, iyo X. Haddii qaahsada $\widehat{RX} = 200$, $\widehat{RW} = 100$, raadi /RQW, /RWX, /WRX.

Barbaroolaha ABCD waxa lagu meershay goobo. Xagashii la arkaaba waa imisa digrii?

Furfuris:

Ka soo qaad in $x = \widehat{AB} = \widehat{CD}$ isla marka $y = \widehat{BC} = \widehat{AD}$.

$2x + 2y = 360^{\circ}$

$x + y = 180^{\circ}$

Xagal kasta cabbirkeed waa $\frac{1}{2}(x+y) = \frac{1}{2}(180^{\circ})$

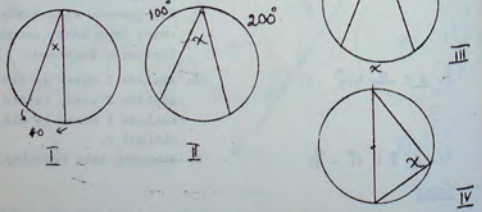
$= 90^{\circ}$

∴ Xagal kastaaba waa 90°.

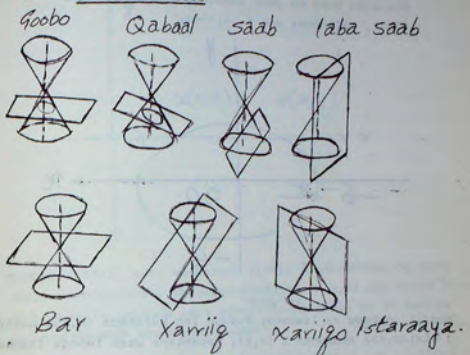
Dariiqo: Si aad u caddayso inilaba xaglood isleeg yihiin, waxaad tustaa in ay yihiin xaglo lagu meershay goobo geli ah ama goobooyin isleeg oo sameeya ama jara qaanso keli ah ama qaansooyin isleeg.

LAYLI:

Raadi qiimaha x ee shaxanada 1-4



JEEB TOOBINAYDYO

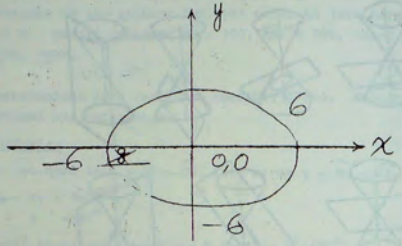


Jaantusyadu waxa ay tusayaan in qaabka xoodan la helayo ama la arki karaba marka sallax uu jaro toobin salkiisu yahay goobo, dhidibkiisuna ku qotomo sallaxa, iyada oo weliba la tixgelinayo xagasha u dhexaysa sallaxa iyo dhidibka toobinka. Haddaba, jarid kasta oo uu sallaxu sameeyaa waxa ay ku siinaysaa shaxan ka mid ah shaxanada magacyadoodu halkan ku taxan yihiin. Kuwaas oo kala ah, goobo, qabaal, saab, laba-saab, xarriiq, ama xarriiqo lammaan oo isjaraya. Ururadaa baraha ah ee samaynaya shaxanada aynu kor ku magacawnay ayaa guud ahaan loo yaqaan jeeb toobinaydyo.

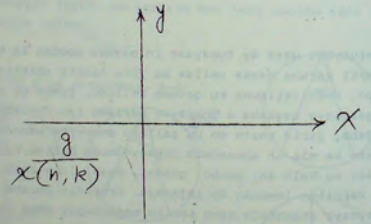
GOOBO

QEEXID: Goobo waa tub baro ah ee isku fogaansho u jira bar maguuraan ah.
Fogaanshaha waxa lagu magacaabaa gacanka goobada, barta maguuraanka ahna waxa lagu magacaabaa xuddunta goobada.

Shaxanka hoos ku yaal xudduntu waa unugga ama (0,0), gacankuna waa 6 halbeeg.

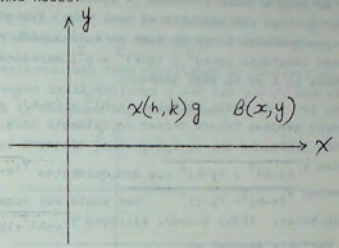


Haddii goobadu ay leedahay xuddun leh kulanada (h,k), waxaynu u bixinaynaa xuddunta x (h,k); gacankana waxa inooga taagaan kara g. Bal u fiirso shaxanka hoos ku yaal; waxa uu tusayaana marka aanay xudduntu ku oollin unugga.



Waxa jirta in xarriiq waliba ay leedahay isle'eg ama ha xoodnaado ama ha toosnaadee. Haddaba si aynu u raadino isle'egta goobo waa in aynu ogaano astaamaha gaarka ah ee ay leedahay bar kasta oo ku taalla goobada; astaamahaa gaarka ah oo aanay lahayn baraha aan ku oolini goobada.
Si aynu u helo isle'egta goobo bal aan qaadano marka ay goobadu leedahay $\overset{\text{xuddunta}}{x(h,k)}$, iyo bar ku taalla xudunta goobada dusheeda, lehna kulamada B (x,y); bal fiiro u yeelo

shaxanka hoose:



Markaa, haddii aynu adeegsano jidkii fogaanshaha, oo aynu raadino fogaansha u dhexeeya xuddunta x (h,k) iyo barta B(x,y) ee ku taalla goobada waxa aynu helaynaa fogaanshahaas oo la mid ah gacanka wuxuuna noqonayaa sidan: noocnayaa sidan.

$$\sqrt{(x-h)^2+(y-h)^2} = g$$

Bar kasta oo ku taalla goobada leh xuddunta x(h,k) iyo gacanka g, waxay leedahay kulanada (x,y); kuwaas oo raali gelinaya isle'egta $\sqrt{(x-h)^2+(y-k)^2} = g$

Bar kasta oo leh kulanada (x,y) raali gelinaysana $\sqrt{(x-h)^2+(y-k)^2} = g$, waxay ku taallaa goobada, xuddunteedu tahay x(h,k), gacankeeduna yahay g.
Markaa isle'egtan $\sqrt{(x-h)^2+(y-k)^2} = g$ waa isle'egta goobada leh xuddunta x (h,k) iyo gacanka, g.

Si aynu u soo saaro xidiska isle'egta kore, waxa habboon in aynu marka hore ka saaro isle'egta calaamadda xiddidle. Sidan oo kale $\left(\sqrt{(x-h)^2+(y-k)^2}\right)^2 = g^2$; kolkaa, waxa aynu helaynaa sidan $(x-h)^2 + (y-k)^2 = g^2$.

Inkasta oo aynu laba jibbaarnay labada dhinac ee isle'egta, waxa aynu soo gelinay laba barrood oo cusub oo ay kulanadoodu raali gelinayaan isle'egta $(x-h)^2 + (y-k) = g^2$, hase-yeeshe aan ku oollin goobada.

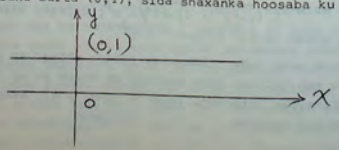
Haddaba si aynu u tusno in $(x-h)^2 + (y-k)^2 = g^2$ ay tahay isle'egta goobada leh xuddunta ku taal (h,k) iyo gacanka g, waa in aynu qaadanaa barta B, taas oo kulanadeedu raali gelinayaan isle'egta $(x-h)^2 + (y-k)^2 = g^2$; markaana waa in aynu tusnaa in B ay ku taal goobada.

Haddii x, iyo y ay raali geliyaan $(x-h)^2 + (y-k)^2 = g^2$, oo markaa aynu qaadano labada dhinac ee isleegta sare, xidid-kooda laba jibbaarka ah, x, iyo y waxay bilqasab ku raaligelin isle'egtan $\sqrt{(x-h)^2 + (y-k)^2} = -g$ ama isleegtan $\sqrt{(x-h)^2+(y-k)^2} = g$, hase-yeeshee $\sqrt{(x-h)^2+ (y-k)^2}$ mar kasta way togan tahay, -g, way ka-aban tahay. Sidaa awgeed, isleegta $\sqrt{(x-h)^2+(y-k)^2} = -g$, ma laha furfuris maangal ah.

Sidaa awgeed, kulanada x iyo y ee barta B way raali gelinayaan $(x-h)^2 + (y-k)^2 = g^2$, haddii iyo haddii oo keliya oo x, iyo y ay raali geliyaan isle'egtan $\sqrt{(x-h)^2 + (y-k)^2} = g$. Hase ahaatee barta B (x,y), way raali gelinaysaa isle'egta $\sqrt{(x-h)^2 + (y-k)^2} = g$ haddii iyo haddii oo keliya oo B ay ku taal goobada leh, xudunta (h,k) iyo gacanka g. Waxa aynu tusnay in isle'egta $(x-h)^2 + (y-k)^2 = g^2$ in ay tahay isleegta goobada, leh xuddunta (h,k), iyo gacanka g.

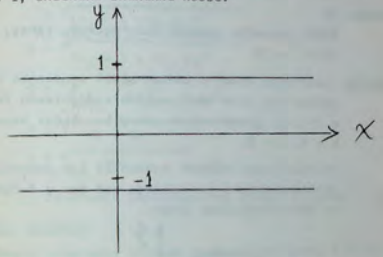
Marka aynu laba jibbaarno laba dhinac ee isle'eg, khatar weyn baa ku sugan, oo laga yaabaaba in aynu ku darro urur barro ah oo cusub, oo kulanadoodu ay aad u raali gelinayaan isleegta aynu helay, hase-yeeshee aan dul dhacaynin amase aan laga helayn xoodkii aynu ku bilowney. Markii aynu laba jibbaarnay labada dhinac ee isleegtii ahayd $\sqrt{(x-h)^2 + (y-k)^2} = g$ maynaan helin, wax urur barro ah oo cusub.

Bal hadda u fiirso isle'egta y= 1. Isle'egtaa iyada ah garaafkeedu waa xarriiq toosan, barbarrana la ah dhidibka -x, maraysana barta (0,1); sida shaxanka hoosaba ku tusayo:



Isle'egtii ahayd y = 1, haddii aynu labada dhinacba laba-jibbaarno, waxa aynu heli isleegtan, $y^2 = 1$; haddaba, waxa isweydiin leh, sida uu noqonayo garaafkeedu.

Garaafkeedu waa laba xarriiqood oo barbarro ah, maxaa yeelay kulanada raali gelinaya y =-1 iyo y = 1, way raali gelinayaan $y^2 = 1$; dheehana shaxanka hoose.



Haddaba, marka aynu laba jibbaarayno labada dhinac ee isleegta xoodka, si aynu u hello isle'eg cusub, waxa haboon in aynu hubinno, si aynaan u qaadanin, waxii barro cusub ah oo kulanadoodu raali gelinayaan isle'egta inoo soo baxday; hase yeeshe aan laga helayn xoodka ama xarriiqda isle'egta.

Bal aan u qaadano $(x-h)^2 + (y-k)^2 = g^2$ in ay tahay sansaan-keena beeggal ee isle'egta goobo, leh xuddunta x(h,k) iyo gacanka g. Markaa isle'egtan $(x-h)^2 + (y-k) = g^2$ garaaf-keedu waa goobo,leh xuddunta x(h,k) iyo gacanka g; haddaba, haddii goobada xuddunteedu tahay unugga (0,0) gacankeeduna yahay g, markaa isle'egta sansaankeeda beeggal wuxuu yahay ama noqonayaa sidan: $x^2 + y^2 = g^2$. Maxaa yeelay, h = 0, k = 0 oo halkoodii aynu ku beddelay eber, sidan oo kale.

$(x-h)^2 + (y-k)^2 = g^2$
$(x-0)^2 + (y - 0)^2 = x^2+y^2 = g^2$.

Taasuna waa marka ay xuddunta goobadu ku dhacdo unugga (0,0).

TUSAALE : Raadi isle'egta goobada, xuddunteedu tahay x(5,-3) gacankeeduna yahay 7.

Furfuris: h = 5, k = -3, g = 7

Waxa aynu adeegsanaynaa isle'egteenii ahayd $(x-h)^2 + (y-k)^2 = g^2$. Waxa ayna noqonaysaa sidan:

$(x-5)^2 + [y-(-3)]^2 = 7^2$

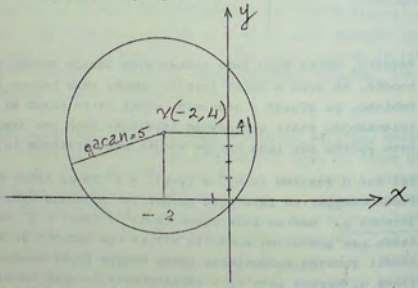
$(x-5)^2 + (y+3)^2 = 49$

TUSAALE: II

Sawir garaafka goobada isle'egteedu tahay, $(x+2)^2 + (y-4)^2 = 25$

Furfuris: Isle'egtu waxay u taalaa sidii sansaanka beeggal, markaa waa isle'egta goobada xuddunteedu tahay x (-2,4), gacankeeduna yahay 5. Sidaa awgeed, h = -2, k = 4, g = 5.

Haddiiba aynu haysano xudduntii iyo gacankii goobada sawirka garaafkeedu aad buu u fudud yahay oo wuxuu noqonayaa sidan:



Marka hore, la soo bax xuddunta goobada; xudduntaas oo ah, barta (-2,4). Marka xigana sawir goobo gacankeedu yahay 5 halbeeg, adoo ka bilaabaya xuddunta (-2,4), sida shaxanka sare uu kuu tilmaamayo.

TUSAALE:III

Raadi gacanka goobada xuddunteedu tahay (-3,1) maraysana barta (5,7).

Furfuris: Haddiiba goobadu ay leedahay xuddunta (-3,1), isla markaana ay marayso barta (5,7) gacanku wuxuu noqonayaa fogaanshaha u dhexeeya labada meelood, xuddunta (-3,1) iyo barta (5,7).

Wuu inoo cad yahay jidka ay tahay in aynu qaadno; waana jidkii fogaanshaha

$$= \sqrt{(x_2-x_1) + (x_2-y_1)^2}$$

$$g = \sqrt{(x_2-x_1) + (y_2-y_1)^2}$$

$$g = \sqrt{[5-(-3)]^2 + (7-1)^2}$$

$$g = \sqrt{64 + 36}$$

$$g = \sqrt{100} = 10$$

Isle'egta goobaduna waa $(x+3)^2 + (y-1)^2 = 100$

LAYLI Jawaabo

1. Raadi Isle'egta goobada xuddunteedu tahay x (0,0) gacankeeduna yahay 5.

   Jawaab $x^2 + y^2 = 25$

2. Raadi Isle'egta goobada xuddunteedu tahay x(4-2), gacankeeduna yahay 8.

   Jawaab: $(x-4)^2 + (y+2)^2 = 64$

3. Raadi isle'egta goobada xuddunteedu tahay x (-4,-2), maraysana barta (1,3)?

   Jawaab: $(x+4)^2 + (y+2)^2 = 50$

4. Raadi isle'egta goobada xuddunteedu tahay x(-5,6), taanjantna u ah dhidibka x (shaxan baa ku caawin kara)

   Jawaab: $(x+5)^2 + (y-6)^2 = 36$

5. Raadi isle'egta goobada xuddunteedu tahay x(2,-8) gacankeeduna yahay 5.

   Jawaab: $(x-2)^2 + (y+8)^2 = 5^2$

   ama $x^2 + y^2 - 4x + 16y + 43 = 0$

6. Soo saar isle'egta goobada taanjantka u ah dhidibka x, xuddunteeduna tahay x(-3,5/3)

   Jawaab: $3x^2 + 3y^2 + 18x - 10y + 27 = 0$

7. Raadi isle'egta goobada, dhexroorkeedu leeyahay kulanada (-3,12) iyo (7,16).

(Biniix: Marka, hore raadi kulanka xuddunta goobada taas oo ah bar-bartameedka dhexroorka goobada. Dabeedna raadi gacanka goobada.)

Jawaab: $(x-2)^2 + (y-14)^2 = 29$
ama $x+y^2 -4x-28y + 171 = 0$

Haddii aynu rabno in aynu raadino isle'egta goobada xuddunteedu tahay x (5,2), gacankeeduna yahay 6, waxa aynu odhan jirnay isle'egtu waa $(x-5)^2 + (y-2)^2 = 36$, ama $x^2+y^2 - 10x - 4y - 7 = 0$.

Ka soo qaad in aynu haysano isle'egtan $x^2+y^2 -8x+2y + 8 =0$. Haddaba, haddii ay taasu tahay, isle'egta goobo, waa in aynu ogaano xuddunta, iyo gacanka goobada labadaba. Ma malayn kartaa sidaynu ku heli karno xuddunta iyo gacanka goobada isleegtaa?

Innagoo raacayna ama kaashanayna darriiqadii DHAMAYSTIRKA LABAJIBBAAR, WAxa aynu ka dhigi karna isle'egteena sasaanka beeggal ee isle'egta goobo; sansaankaas oo ahaa sidan: $(x-h)^2+(y-k)^2 = g^2$.

Haddii tibxaha x, aynu ka soocno tibxaha y, oo aynu madoorsoomahana u rarno midigta isle'egta $x^2+y^2-8x+2y+8 = 0$ waxa aynu heli doonaa sidan
$x^2-8x + y^2+2y = - 8$.
Markaa, x^2-8x, waxa aynu u dhigi karnaa sansaanka ah $(x-h)^2$, h waxay innooga taagan tahay tiro.
y^2+2y, waxa aynu u dhigi karnaa sansaanka ah $(y-k)^2$, k waxay innooga taagan tahay tiro.
Taasaana lagu magacaabaa dhamaystirka laba jibbaar,

a. Haddaba si aynu u dhamaystirno laba jibbaarka x^2-8x, aan qaadano 4; afartaas oo ah badhka weheliyaha tibixda x, taas oo ah $(x-4)^2$.
Markaa, aynu isku dhufano $(x-4)(x-4) = x^2-8x+16$, waxa aynu helayna saddex tibixle laba ka mid ahi, x^2-8x, ay ku jiraan isleegteenii iyo tibix madoorsoome ah, 16 oo aynaan u baahnayn. Si aynu tibxihii isle'egteena ku jiray u hello, $(x-4)^2$ waxa aynu ka goynaynaa 16, sidan oo kale

$(x-4)^2 - 16$. Tan oo la mid ah $x^2 -8x +16 - 16$.

b. Sidii talaabadii hore oo kale, si aynu u dhamaystirno laba jibbaarka y^2+2y, waxa aynu qaadan badhka weheliyaha tibixda y; dabeedna waxa aynu ka dhigi sansaankan $(y+1)^2 = y^2+ 2y + 1$.

Markaa sida aad ku aragtidba waxa aynu helnay laba tibxood oo ah y^2 iyo 2y kuna jira isle'egteenii iyo tibix madoorsoome ah +1 , oo aynaan u baahnayn. Haddaba si aynu tibxaha isle'egteenii ku jiray ula hadno, y^2+2y+1 waxa ka gooynaynaa 1, sidan oo kale, $y^2+2y+1-1 = (y+1)^2-1$. Isle'egteenii hore waxay ahayd $x^2-8x + y^2+2y = -8$; markaa tibxaha x waxa aynu halkoodii dhigi qiimihii aynu tusnay in ay la mid yihiin; tibaxaha yha waxa iyana aynu halkoodii dhigi qiimihii aynu tusnay in uu la mid yahay sideeda tabanna waynu qaadan.
$(x-4)^2-16 + (y+1)^2 - 1 = -8$.

Madoorsoomayaasha oo idil marka aynu midigta isugu wareejino, waxa aynu helnay sidan $(x-4)^2 +(y+1)^2 = 17 -8=9$. Isle'egtani $(x-4)^2 + (y+1)^2 = 9$, waxay tahay sansaanka beeggal ee isle'egta goobo. Haddii aad qeexidihii baalaalka hore, ku qeexnaa la socotayna, waxa cad in aad sheegi kartid, xuddunta iyo gacanka goobada labadaba.

TUSAALE:

Ka dhig isle'egtan $x^2+y^2 - 6x+4y -3 = 0$
sansaanka beeggal ee isle'egta goobo, soona saar xuddunta iyo gacanka goobada.

Furfuris: Marka, aynu dhamaystirno labajibbaarka $x^2+y^2-6x+4y-3=0$ waxa aynu heli sidan:
$(x-3)^2 + (y + 2)^2 = 16$
Xudduntu waxay tahay (3,-2) gacankuna wuxuu yahay = 4.

LAYLI

Isle'egyadan soo socda mid walba ka dhig sansaanka beeggal; dabeed raadi xuddunta iyo gacankaba; sawirna garaafkooda haddii uu jiro.

1. $x^2 + y^2 - 6x - 4y - 3 = 0$
2. $x^2 + y^2 + 8x - 10y + 32 = 0$
3. $x^2 + y^2 + 18x - 2y + 82 = 0$
4. $x^2 + y^2 - 8x + 14y + 1 = 0$
5. $x^2 + y^2 - 12x + 6y + 70 = 0$
6. $x^2 + y^2 + Dx + Ey + F = 0$?

SAAB

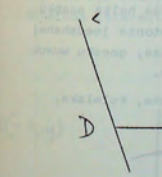
QEEXID:

Saab waa dhammaan ururka baraha ku yaalla sallax, kuwaas oo isku fogaansho u wada jira bar ma guuraan ah, iyo xarriiq ma guuraan ah, kuna yaalla sallaxa.

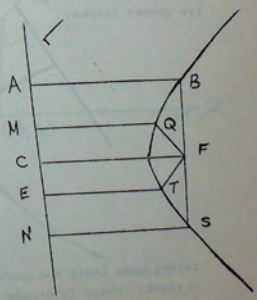
Barta ma guuraanka ah waxa la yidhaahdaa kulmiska saabka, xarriiqda ma guuraanka ahna waxa la yidhaahdaa jeedshe.

F = waxa ay innooga taagan tahay kulimiska
L = Waxa ay innooga taagan tahay jeedshe.
G = Waxa ay innooga taagan tahay geeska saabka.

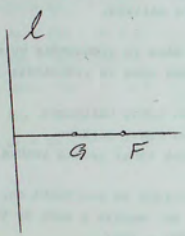
Haddii L ay tahay xarriiqda ma guuraanka ah, Fina tahay barta ma guuraanka ah, markaa B waxa ay ku taallaa saabka dushiisa haddii /BD/ = /BF/.



Bal hadda, u fiirso shaxankan hoose.

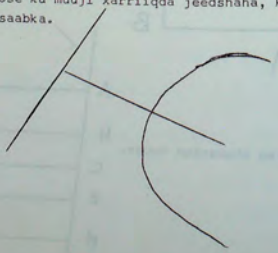


Haddii AB = BF, MQ = QF, CG = GF, ER = RF, NS = SF; markaa baraha B,Q,R,S, iyo G, waa baro raaligelinaya qeexiddii saabka sida jeedshaha iyo kulmiskuba ay u raaligeliyeen. Haddii aynu sawirno xarriiq ku qotonta jeedshaha, maraysana kulmiska, sida aad shaxanka hoose ku aragtid, markaas barta G ee ku taalla saabka ma tahay badhtamaha jeedshaha iyo kulmiska?



Jawaabtu waa haa, waxana innoo caddaynaya qeexiddii saabka. Barta G ee saabka ku taallaa, waxay dhacdaa halka saabku ka jaro xarriiqda marta kulmiska, kuna qotonta jeedshaha; waxana la yidhaahdaa geeska saabka. Markaa, geesku wuxuu kala badhaa jeedshaha iyo kulmiska saabka.

Shaxanka hoose ku muuji xarriiqda jeedshaha, kulmiska, iyo geeska saabka.

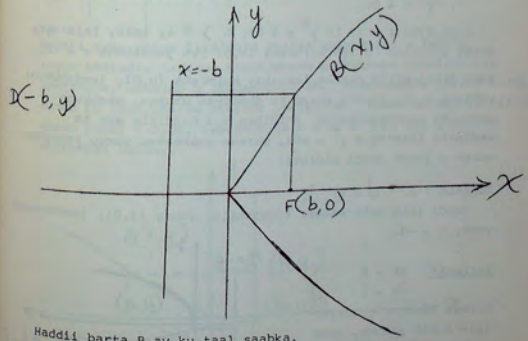


Iminka waxa inala soo gudboonaaday waqtigii iyo sidii aynu u raadin lahayn isle'egta saab; markaa, waa in aynu dhisnaa hab kulan, kaas oo jeedshaha, kulmiska iyo geeskaba sida ugu

hawl yar loogu muujiyo sallaxa; isla markaa waa in aynu arki karna astaamaha qaarka u ah ee ay leedahay bar.kasta oo ku taalla saabka, astaamahaas oo aanay lahayn baraha aan ku oollin saabka.

Waxa jirta inuu yahay ma guuraan fogaanshaha u dhexeeya jeedshaha iyo kulmiska, haddaba hawl yaraysi awgeed fogaanshahaa waxa aynu ka dhigan sidan $2b \geq 0$. Markaa, fogaanshaha u dhexeeya kulmiska iyo geesku waa b; fogaanshaha u dhexeeya geeska iyo jeedshuhuna waa b.

Haddaba inagoo maskaxda ku hayna qeexiddii saabka waxa aynu diiri isle'egta saab; saabkaas oo leh kulmiska (b,0), jeedshuhuna uu leeyahay isle'egta $x = -b, b = -b, b \geq 0$. Geeskuna uu ku dhacayo unugga dhidibada. Isle'egtana waxa aynu u diiri karnaa innagoo tusayna in bar kasta oo ku taalla saabka ay raaligelinayso isle'egta; bar kasta oo raaligelisa isle'egtana ay ku taallo saabka.
Bal aad ugu fiirso shaxankan hoose, iyo tallaabooyinka aynu diiridda isle'egta u qaadayno.



Haddii barta B ay ku taal saabka,
markaa, $/\overline{BF}/ = /\overline{BD}/$
$/\overline{BF}/^2 = /\overline{BD}/^2$

Hase-yeeshe haddii $/\overline{BF}/^2 = /\overline{BD}/^2$

Markaa, $/\overline{BF}/ = /\overline{BD}/$ ama $/\overline{BF}/ = -/\overline{BD}/$

Hase-ahaatee $/\overline{BF}/$ iyo $/\overline{BD}/$ labaduba way togan yihiin; sidaa awgeed ma jiraan baro raaligelinaya $/\overline{BF}/ = -/\overline{BD}/$.

Markaa roggaal ahaan baynu ku tusnay in bar kasta oo B ah oo raaligelisay $/\overline{BF}/^2 = /\overline{BD}/^2$ in ay haddana raaligelisay $/\overline{BF}/ = /\overline{BD}/$; sidaana ay ku noqotay mid ku taal saabka. Sidaa daraadeed isle'egta saabku waxay tahay $/\overline{BF}/^2 = /\overline{BD}/^2$ markaa, innagoo qaadanayna kulanadii aynu ku muujinay shaxanka, adeegsanaynana jidkii fogaanshaha waxa aynu helaynaa sidan:

$$\left[\sqrt{(x-b)^2 + y^2}\right]^2 = \left[\sqrt{(x+b)^2}\right]^2$$

ama $(x-b)^2 + y^2 = (x+b)^2$

Marka, aynu isku dhufano isirada waxa aynu heli doonaa sidan:

$$x^2 - 2bx + b^2 + y^2 = x^2 + 2bx + b^2$$

Tanna waxa loo sii fududayn karaa sidan:

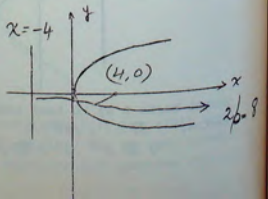
$$y^2 = 4bx.$$

Waxa aynu tusnay in $y^2 = 4bx$, $b > 0$ ay tahay isle'egta saab; saabkaas oo leh kulmiska (b,0) iyo jeedshaha $x = -b$.

Waxa jirta marka saabku leeyahay kulmiska (b,0), jeedshuhuna yahay $x = -b$, geesku wuxuu ku dhacayaa unugga, saabkuna wuxuu ku wanqaranayahay dhidibka x) haddiiba aan la beddelin isle'egta $y^2 = 4bx$, markaa saabkaasu wuxuu leeyahay qolxo u jeeda xagga midigta.

TUSAALE : I

Raadi isle'egta saabka kulmiskiisu yahay (4,0), jeedshuhuna yahay $x = -4$.



Furfuris: $2b = 8$

$b = 4$

Markaa innagoo raacaynn isle'egtii saabka, waxa aynu helaynaa $y^2 = 4bx$

$y^2 = 4.4x$

$y^2 = 16x$

==========

TUSAALE :II

Raadi kulmiska iyo jeedshaha ee saabkan $y^2 = 8x$; garaafkana sawir.

Furfuris: Isle'egtan $y^2 = 8x$, waxay u taallaa sidii sansaanka beeggal ee ahaa $y^2 = 4bx$, markaa, $8x = 4bx$

Kulmisku waa (+2,0) $2 = b = 2$

Jeedshuna waa $x = -2$

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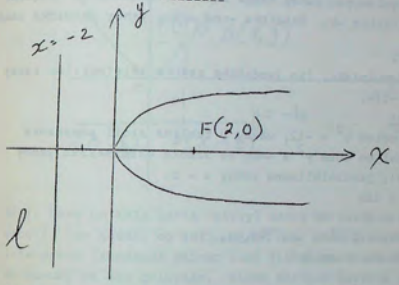
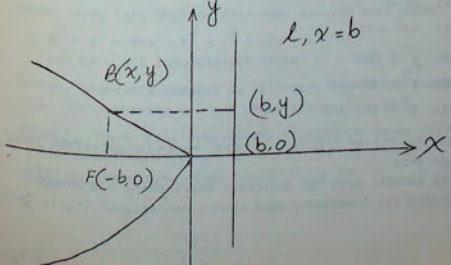


Haddii aynu rabno isle'egta saabka qolxadiisu u jeeddo xagga bidixda, haddana fogaanshaha u dhexeeya kulmiska iyo jeedshaha, waxaynu u qaadanaynaa 2b. Hase-yeeshe iminka kulmisku wuxuu ku samaysmayaa bidixda jeedshaha. Sidaa awgeed kulmisku wuxuu yahay (-b,0), jeedshuhuna $x = b$, $b > 0$. Bal u fiirso shaxankan hoose.



Marka aynu diirayno isle'egta saabka u golxaysan xagga bidixda waxa aynu raaci isla dariiqadii hore ee saabku u golxaysnaa xagga midigta. Hase-yeeshe kulanada ayuunbaa isbeddel yari ku dhacay sida aad ku aragtayba shaxankan ku qoran hoosta bogga 217. Isle'egtuna waxay noqonaysaa sidan: $y^2 = -4bx$, oo sansaanka beeggal ah. Bal isku day in aad diirtid isle'egta. Saabka isle'egtiisu tahay $y^2 = -4bx$, kulmiskiisu waa $(-b,0)$; jeedshihiisuna wuxuu leeyahay isle'egta $x = b$. Sidaa awgeed wuxuu u golxaysan yahay xagga bidixda; waxaanu ku wanqaran yahay dhidibka -x. Dhidibka -xna wuxuu yahay dhidibka saabka.

TUSAALE:I

Raadi kulmiska, iyo jeedshaha saabka isle'egtiisu tahay $y^2 = -12x$.

Furfuris:

Isle'egtan $y^2 = -12$, waxay u taallaa sidii sansaanka beeggal ee ahaa $y^2 = -4bx$ ee saabka kulmiskiisu yahay $(-b,0)$, jeedshihiisuna yahay $x = b$.

$\therefore -4bx = 12x$

$b = 3$

Markaa, kulmisku waa $(-3,0)$.

Jeedshuhuna waa 3

LAYLI :

1. Saabakkan kuweebaa midigta u golxaysan, kuweebaase bidixda u golxaysan. Mid walbana sheeg jeedshaha iyo kulmiskeeda.

b. $y^2 = -4x$

t. $y^2 = 7x$

j. $-y^2 = -14x$

s. $y^2 = 6x$

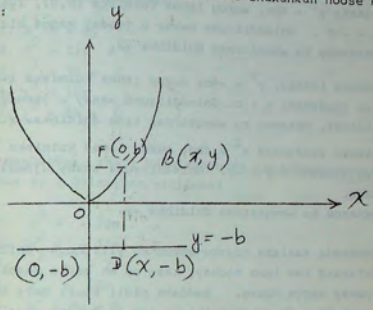
kh. $y^2 + 16x = 0$

d. $y^2 - 10x = 0$

Hadda [illegible] saabka marka uu golxaysan yahay xagga sare.

Sidii labadii aynu soo dhaafnay, fogaanshaha u dhexeeya kulmiska iyo jeedshaha, waxa aynu u qaadanaynaa 2b, $(b > 0)$.

Hase-yeeshe iminka kulmisku wuxuu ku yaallaa dhidibka -y, waxana uu yeelanayaa kulanada $(0,b)$, jeedshuhuna wuxuu yeelanayaa isle'egta $y = -b$, sida aad shaxankan hoose ku aragtid:



Sidii hore oo kale barta B(x,y) waxay ku taallaa saabka haddii iyo haddii oo keliya oo $/\overline{BF}/ = /\overline{BD}/$ marka aynu isle'egtan labadeeda dhinac laba jibbaarnana wax baro ah, oo cusubi ma soo gelayaan. Sidaa darteed barta B waxay ku taallaa saabka haddii iyo haddii oo keliya oo $/\overline{BF}/^2 = /\overline{BD}/^2$.

Haddaba, innagoo adeegsanayna jidkii fogaanshaha iyo shaxankeena kore, waxaynu heli sidan:

$$\sqrt{x^2 + (y-b)^2} = \sqrt{(y+b)^2}$$

ama $x^2 + (y-b)^2 = (y+b)^2$

Marka, aynu isku dhufano isiradana waxaynu heli sidan:

$x^2 + y^2 - 2by + b^2 = y^2 + 2by + b^2$

tan oo u sii fududaanaysa sidan: $x^2 = 4by$, $b > 0$

Markaa, sansaanka beeggal ee isle'egta saabka leh kulmiska $(0,b)$ iyo jeedshaha $y = -b$ wuxuu yahay $x^2 = 4by$.

TUSAALE: Raadi Isle'egta saabka kulmiskiisu yahay $(0,4)$ jeedshihiisuna yahay $y = -4$.

Furfuris: Waxa aynu ogaan karnaa in ay $2b = 8$

$\therefore b = 4$

$\therefore x^2 = 4.4y$

$x^2 = 16y.$

Ilaa iyo hadda waxa aynu falanqaynay saddex saab oo kala duwan.

1. Saabka $y^2 = 4bx$, wuxuu lahaa kulmiska (b,0), iyo jeedshaha $x = -b$. Golxadiisuna waxay u jeeday xagga midigta, waxaanuu ku wanqaarnaa dhidibka -x.

2. Saabka labaad, $y^2 = -4bx$ wuxuu lahaa kulmiska (-b,0), iyo jeedshaha $x = b$. Golxadiisuna waxay u jeeday xagga bidixda, waxaanu ku wanqaarnaa isna dhidibka- x.

3. Saabka saddexaad $x^2 = 4by$, wuxuu lahaa kulmiska (0,b), iyo jeedshaha $y = -b$. Golxadiisuna waxay ujeeday xagga sare.
waxaanu ku wanqaarnaa dhidibka -y.

Saddexda xaalaba geesku-wuxuu ku yiilunuga dhidibada mid afaraad baa inoo hadhay, kaas oo ah marka saabku u golxaysan yahay xagga hoose. Haddaba sidii kuwii hore oo kale, fogaanshaha u dhexeeya kulmiska iyo jeedshaha waxa aynu u qaadanaynaa 2b, $b > 0$. Sidaa awgeedna kulmisku wuxuu noqonayaa (0,-b), jeedshununa waxa uu noqonayaa $y = b$. Bal adigu diir isle'egta saabka marka uu hoos u jeedo ilaa aad ka gaadhaysid isle'egtan $x^2 = -4by$, oo ah sansaanka beeggal marka saabku hoos u golxaysan yahay.

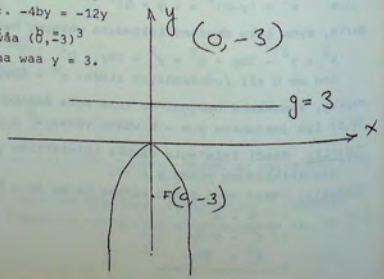
TUSAALE Raadi kulmiska iyo jeedshaha ee saabka $x^2 = -12y$, garaafkana sawir.

Furfuris: Isle'egteenii sansaanka beeggal ahayd, waxa ay ahayd $x^2 = -4by$

$\therefore -4by = -12y$

$\therefore$ Kulmisku waa $(0,-3)^3$

jeedshuhuna waa $y = 3$.



Layliga sheeg saabab kan golxadoodu xagga ay u jeeddo. (Midig, bidix, hoos, ama sare)

1. $x^2 = -9y$
2. $x^2 - 12y = 0$
3. $y^2 - 7x = 0$
4. $y^2 = 16x$
5. $y^2 + 8x = 0$

Layliga saabab kan, raadi kulmisyadooda jeedshayaashooda, iyo jahada ay u golxaysan yihiinba?

1. $y^2 = -20x$
2. $y^2 = -11x = 0$
3. $x^2 = -7y$
4. $y^2 = 10x$
5. $x^2 - 8y = 0$

Layli:

1. Raadi isle'egta saab kulmiskiisu yahay (3,0), jeedshuhuna yahay $x = -3$.

2. Raadi isle'egta saab, haddii kulmiskiisu yahay (-6,0) geeskiisuna yahay (0,0)?

3. Raadi isle'egta saab, haddii geeskiisu yahay (0,0) jeedshuhuna yahay $y = -8$.

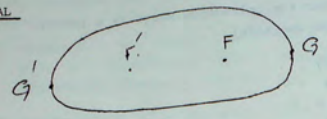
4. Raadi isle'egta saabka kulmiskiisu yahay (0,-2) geeskiisuna yahay (0,0)?

5. Raadi isle'egta saabka kulmiskiisu yahay (0,8), jeedshuhuna yahay $y = -8$.

6. Raadi isle'egta saabka u golxaysan xagga midigta, geeskiisuna yahay (0,0), marayana barta (8,8).

7. Haddii kulmisku yahay (-6,0), geeskuna yahay (0,0), saabku wuxuu u golxaysan yahay xagga_____. isle'egtuna waa sansaanka_____ bna waxay = _____.

QABAAL



/kuna wanqarma labada

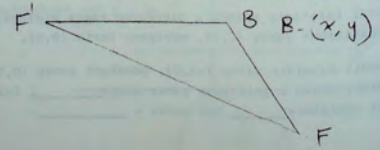
Waxa aynu soo baranay saababka maraya unugga/dhidib mid uun. Baabkan waxa aynu ku baranaynaa shaxanada toobineysan mid ka mid ah oo la yidhaaho qabaal; waxase aynu ku koobnaan qabaal-ka xudduntiisa maraysa unugga, kuna wanqaran labada dhidib.

F = waxa ay inooga taagan tahay kulmi.
F^1 = Waxa ay inooga taagan tahay kulmis kale
G. = Waxa ay inooga taagan tahay geeska.

QEEXID:

Qabaal waa dhammaan ururka baraha ku jira sallaxa, wadarta fogaanshahooda ay laba barrood, oo ma guuraan ahi ay u jiraanna tahay madoorsoome.
Baraha maguuraanka ah waxa la yidhaa kulmisyada qabaalka.

Bal u fiirso shaxankan hoose. Ka soo qaad in ay F iyo F^1 ay yihiin kulmisyada qabaal. Haddaba qeexidcahaan barta B (x,y) waxay ku taallaa qabaalka haddii iyo haddii oo kaliya, oo /FB/ + /F^1B/ ay le'egtahay madoorsoome lagu siiyay.



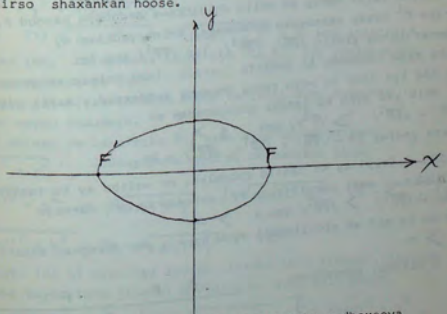
(Sawir)

Sida ugu dhib yar ee dhisi kara qabaal waxa aynu u raaci karnaa tallaabooyinka soo socda:

1. Soo qaado miiq dun ah.
2. Cidhifyada miiqa isku gunud
3. Waxa kale oo aad soo qaadataa laba musmaar.
4. Dul dhig miiqa aad isku gunuday xaashi cad
5. Gudaha miiqa guntan ee xaashida dul saaran ka taag labada musmaar oo aad ku kala taagto laba meelood oo kala duwan miiquna waa in u giigtiraadaa.
6. Soo qaado qalin kale gudaha miiq saar hana taabto miiqa iyo xaashidaba.
7. Dhinaca aad rabtid u wareeji qalinka isagoo caaradiisa marna aanaad ka qaadin xaashida.
   Shaxanka aad heshay waa qabaal.

Si aynu u raadino isle'egta qabaal waxa aynu dhisi hab kulanno ah. Kulmisyadana waxa aynu u dhigi sida inoogu hawl yar.

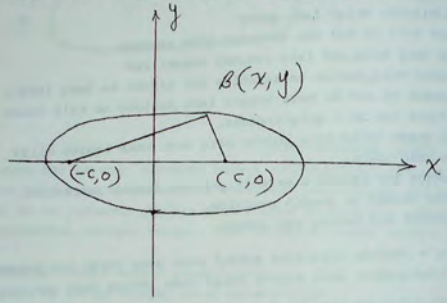
Sida ugu fudud ee aynu ku dooranayno dhidibka - x waa isaga oo mara kulmisyada F iyo F^1, dhidibka-yna wuxuu inoogu qo-toma dhidibka -x .
Bal u fiirso shaxankan hoose.



Sida aad shaxankan sare ku aragtidba fogaanshaha u dhexeeya labada kulmis waa ma guuraan, waxaynuna u qaadan karnaa, 2c.

Mar haddii aynu dooranay dhidibadeenii, markaa kulmisyada kulanadoodu waxay noqonayaan sidan :

(a) F (c,0) (b) F^1 (-c,0), u fiirso shaxankan hoose.



Qeexidii qabaal inagoo la kaashanayna wadarta $/\overline{FB}/ + /\overline{BF^1}/$ waxay ahayd madoorsoome, waxaynuna u qaadanaa in ay le'eg tahay 2a, ama sidan oo kale $/\overline{FB}/ + /\overline{BF^1}/ = 2a$.

Haddii B (x,y) aanay ku oolin dhidibka-x saddexda barood F, B iyo F^1 waxay sameeyaan seddexagal dhinacyadiisu ay dherer le'eg yihiin $/\overline{FB}/$, $/\overline{BF^1}/$, iyo $/\overline{FF^1}/$ ama 2c.

Waxa aynu naqaanay in wadarta dherada laba dhinac ee seddexagal ay had iyo jeer ka weyn tahay dhinaca seddexaad ; marka waxa si dhib yar aynu ku gaadhi gebagebadan ah

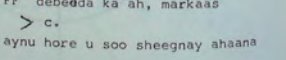
$/\overline{FB}/ + /\overline{BF^1}/ > /\overline{F^1F}/$ ama $a > c$

maxaa yeelay $2a = /\overline{FB}/ + /\overline{BF^1}/$, $/\overline{FF^1}/ = 2c$.

Haddii B (x,y) ay ku taallo qabaalka, oo waliba ay ku taallo dhidibka -x meel xarriijinta FF^1 debedda ka ah, markaas $/\overline{FB}/ + /\overline{BF^1}/ > /\overline{FF^1}/$ ama $a > c$.

Tan oo la mid ah xidhiidhkii aynu hore u soo sheegnay ahaana $a > c$.

Bal u fiirso, shaxankan:



(shaxan)

Hase-yeeshe haddii B ay ku taallo qabaalka ay kuna taallo xarriijinta FF^1 markaa $/\overline{FB}/ + /\overline{BF^1}/ = /\overline{FF^1}/$, ama $a = c$

Tan waxa aad fahmi markaad qabaal qabatid geesaha kulmisyada ee aad kala jiidid ilaa barta B ay fuusho xarriijinta FF^1, qabaalkuna u noqdo xarriiq toosan; taas waxa aad sameeyn karta markaad isticmaashid dun ama wax kala jiidma.

Rogaal ahaan haddii B ay ku taallo qabaalka oo, $a = c$, marka, waxa lama huraan ah in B kutaallo xarriijinta FF^1.

Waxa aynu tusnay marka aanay barta B ku oollin xarriijinta FF^1 in $a > c$; marka $a = c$ qabaalka garaafkiisu waa dhammaan ururka baraha ku yaalla xarriijinta FF^1.

Haddaba haddii $a < c$, ma jiraan wax baro ah oo raaligelinaya qeexiddii qabaalka . Sidaa awgeed had iyo jeer waxa aynu u qaadan in $a > c$.

Barta B (x,y), waxay ku taallaa qabaalka haddii iyo haddii oo keliya, oo $/FB/ + /BF^1/ = 2a$.

Innagoo la kaashanayna jidkii fogaanshaha, barta B (x,y) waxay ku taallaa qabaalka haddii iyo haddii oo keliya, oo

$\sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2+y^2} = 2a$

Markaa (1) $\sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2 + y^2} = 2a$ waa isle'egta qabaalka leh. Kulmisyada (c,0) iyo (-c,0), iyo fogaanshaha madoorsoomaha ah ee 2a.

Isle'egta (1) Sidaa ku dayn mayno ee intii aynu fududeen karnaba waynu fududeyn.

Labada dhinac ee isle'egta (1) waxa aynu ka go'ynaynaa $\sqrt{(x-c)^2 + y^2}$. Jadeeyaduna waxa ay noqonaysaa sidan:

$$\sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = 2a - \sqrt{(x-c)^2+y^2}$$

$$= \sqrt{(x-c)^2 + y^2} = 2a - \sqrt{(x-c)^2+y^2}. \quad (2)$$

Isle'egta (2) si aynu uga saarno xididka laba jibbaar, labada dhinacba waynu laba jibaari markaa waxa aynu heli sidan.

(3) $\left(\sqrt{(x+c)^2+y^2}\right)^2 = \left[2a - \sqrt{(x-c)^2+y^2}\right]^2 = (x+c)^2 + y^2 =$

$$\left[2a - \sqrt{(x-c)^2 + y^2} =\right]^2 = x^2+2cx+c^2+y^2 = 4a^2 - 4a\sqrt{(x-c)^2+y^2+x^2} -2cx+c^2+y^2$$

Isle'egta (4) aad marka aynu sii fududayno waxa aynu helaynaa sidan (5) $2cx = 4a^2 - 4a\sqrt{(x-c)^2+y^2} - 2cx$ oo la mid ah tan (6) $4a\sqrt{(x-c)^2+y^2} = 4a^2 - 4ac$.

Isle'egta 6aad marka aynu dhinac walba u qaybino 4 waxa ay noqon sidan $\frac{4a}{4}\sqrt{(x-c)^2+y^2} = \frac{4}{4}(a^2-cx)$.

Isle'egyadan ta ugu hoosaysa haddii aynu labada dhinacba laba jibbaarno waxay noqonaysaa sidan:

$$\left[a\sqrt{(x-c)^2+y^2}\right]^2 = (a^2-cx)^2$$

$$a^2x^2 - 2a^2cx + a^2c^2 + a^2y^2 = a^4 - 2a^2cx + c^2x^2$$

Tan wixii isugo'aya marka aynu isugoyno waxa aynu heli sidan: $a^2x^2 + a^2c^2 + a^2y^2 = a^4 + c^2x^2$. Tan markaynu tibxaha x iyo y marka aynu bidixda marino, madoorsoomayaashana midigta isugu wareejino waxa ay noqonaysaa sidan:
$(a^2-c^2)x^2 + a^2y^2 = a^4 - a^2c^2$ ama $(a^2-c^2)x^2 + a^2(y^2) = a^2(a^2-c^2)$.
isleegtan haddii aynu dhinac walba u qaybino $a^2(a^2-c^2)$ waxa aynu heli sidan:

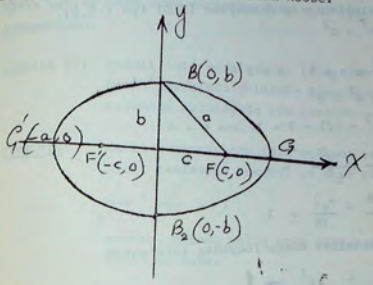
$$\frac{x^2}{a^2} + \frac{y^2}{a^2-c^2} = 1$$

Waxa aynu diirnay in isle'egta $\frac{x^2}{a^2} + \frac{y^2}{a^2-c^2} = 1$ ay tahay isle'egta qabaal: hqrana waxa aynu u tusnay in $a > c$. Sidaa awgeed $a^2 - c^2 > 0$; sidaa darteedna waxa aynu meesha soo gelin xaddi cusub oo ah $b = \sqrt{a^2-c^2}$ ama $b^2 = a^2 - c^2$, ama $b^2 + c^2 = a^2$ markaa hadii $a^2 - c^2 = a^2$ waxa aynu dhigi b^2, oo waxa ay noqon isle'egta ugu dambaysaa sidan: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

==========

Waana sansaanka beegga ee isle'egta qabaal.

Bal fiiro gaar ah u yeelo shaxankan hoose.



Si loo helo geeska G^1 iyo geeska G waa in aynu raadinaa tikraarada -x ee isle'egta qabaal.

Markaa $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; hase yeeshee marka y = 0, waxa aynu helaynaa $\frac{x^2}{a^2} = 1$ ama $x^2 = a^2$

$x = \pm a$

$G^1(-a,0)$, G (a,0) . Sida aad shaxanka sare ku aragtid.

Sidaas oo kale kulanada B iyo B^1 waa in aynu raadinaa tikraarada -y ee isla isle'egteenii markaa $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; hase yeeshee x = 0. marka waxa aynu helaynaa $\frac{y^2}{b^2} = 1$ ama $y^2 = b^2$

$y = \pm b$

B $(+\overline{b},0)$, B^1 (-b,0)

==========

OGSOONOW:

1. Xarriiqda maraysa kulmisyada F iyo F^1 waxa la yidhaahdaa dhidibka weyn.
2. Ka ku qotomana waxa la yidhaa dhidibka yar.
3. c = fogaanshaha u dhexeeya kulmiska iyo xuddunta .
4. b = fogaanshaha u dhexeeya xuddunta iyo geeska yar.
5. a = fogaanshaha u dhexeeya xuddunta iyo geeska weyn.
6. Dhererka dhidibka yari = 2b
7. Dhererka dhidibka weyni = 2a.

TUSAALE. Raadi isle'egta qabaalka kulmisyadiisu yihiin (3,0) iyo (3,0), geesihiisuna ay dhacayaan (5,0) iyo (-5,0): Ogoow in $b^2 = a^2 - c^2$

FURFURIS: Markaa a = 5, c = 3

$b^2 = a^2 - c^2$

$\therefore b^2 = 25 - 9 = 16$ ama b = 4

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\therefore \frac{x^2}{25} + \frac{y^2}{16} = 1$$

Tusaale II Qabaalbaa wuxuu leeyahay isle'egtan

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Raadi kulmisyada iyo geesaha qabaalka; garaafkiisna sawir.

FURFURIS: $a^2 = 25$, $b^2 = 9$

$\therefore a = \pm 5$, $b = \pm 3$

Waxa aynu naqaanay in

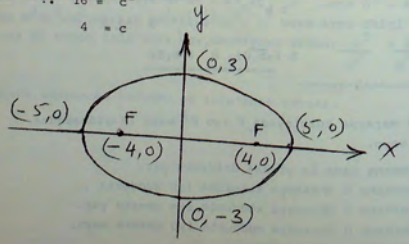
$a^2 - c^2 = b^2$

$\therefore 25 - c^2 = 9$

$\therefore 25 - 9 = c^2$

$\therefore 16 = c^2$

$4 = c$



(Shaxan)

Maxaa yeelay waxa aynu naqaanay waxa a,b, iyo c ay inooga taagnaayeen.

TUSAALE III: Haddii isle'egta qabaal tahay $36x^2 + 100y^2 = 3,600$. Raadi dhererada dhidibka weyn, iyo ka yar, iyo kulanada kulmisyada iyo geesaha.

FURFURIS: Marka u horraysaba isle'egteena waxa aynu u dhigi sidii sansaanka beeggal ee ahaa sidan:-

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Markaa, dhinac walba waxa aynu u qaybin 3,600. Sidan oo kale.

$$\frac{36x^2}{3600} + \frac{100y^2}{3,600} = \frac{3,600}{3,600}$$

$$\frac{x^2}{100} + \frac{y^2}{36} = 1$$

$\therefore a^2 = 100$, $b^2 = 36$ waxa kaloo jirtay in

$a = \pm 10$ $b = \pm 6$ $\therefore 100 - c^2 = 36$

$100 - 36 = c^2$

$\therefore$ Dhidibka weyni = 2a = 20 $64 = c^2$

Dhidibka yari = 2b = 12 $\pm 8 = c$

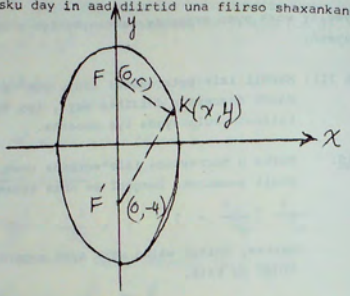
Markaa geesuhu = G (10,0), G^1 (-10,0)

Kulmisyadu = F(8,0), F^1 (-8,0)

Ilaa haatan waxa aynu falanqaynay marka kulmisyada qabaalku yihiin (c,0) iyo (-c,0), kuna yaallaan dhidibka -x. Dhidibka -xna uu ahaa dhidibka weyn ee qabaalka, dhidibka yarina ahaa dhidibka-y. Hase-yeeshe haddii aynu rabno in uu dhidibka-y noqdo dhidibka weyn ee qabaal, waa in aynu ka dhignaa kulmisyadeena dhidibka -y, oo ay yeeshaan kulanada F(0,c), iyo (0,-c), fogaanshahii madoorsoomahuna uu isla kii yahay 2a (a > c). Waxa aynu diiri karnaa isle'egta qabaalka jaadkaas ah go'aankina waxa uu yeeshay sansaankan:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad (a > b)$$

Bal hadda isku day in aad diirtid una fiirso shaxankan.



$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad (a > b)$$

TUSAALE I Haddii isle'egta qabaal ay tahay $\frac{x^2}{9} + \frac{y^2}{25} = 1$

Raadi dhererada dhidibada (ka yar iyo ka weyn) iyo kulamada kulmisyada iyo geesaha.

FURFURIS: $a^2 = b^2 + c^2$ $\quad a^2 = 25$

$a = \sqrt{b^2 + c^2}$ $\quad b^2 = 9$

∴ Kulmisyadu
= (0,4) iyo (0,-4). $25 = 9 + c^2$ $\quad a = 5$
Geesuhu = (0,5) iyo(0,-5) $25-9 = c^2$ $\quad b = 3$
dhererada dhidibada $16 = c^2$
ka yar iyo ka weyn waa (0,-5) $\quad 4 = c$
(6 iyo 10)

LAYLI

1. Qor qeexida qabaal
2. Adoo kaashanaya qeexiddii qabaal diir isle'egta qabaalka kulmisyadiisu yihiin F(c,0) iyo F^1 (-c,0) geesihiisuna yihiin G (a, 0) iyo G (-a,0).

3. Raadi isle'egta qabaal, haddii dhidibkiisa weyni leeyahay kulanada (7,0) iyo (-7,0); dhidibkiisa yarina leeyahay kulanada (0,5) iyo (0,-5).

4. Raadi isle'egta qabaalka kulanada kulmisyadiisu yihiin (0,5) iyo (0.-5); dhidibka yarina leeyahay kulanada (7,0) iyo (-7,0)

5. Raadi isleegta qabaalka geesihiisu leeyihiin kulanada (9,0) iyo (-9,0) marayana barta $(\sqrt{\frac{81}{26}}, 5)$

6. Haddii isle'egta qabaal tahay $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

Raadi (a) Kulmisyada
(b) Geesaha
(c) iyo dhererada dhidibka yar iyo ka weyn.

LAYLI

7. Haddii isle'egta qabaal tahay $\frac{x^2}{4} + \frac{y^2}{3} = 1$
(a) Raadi dhererka dhidibka yar
(b) Kulanada kulmisyada iyo geesahaba, garaafkana sawir.

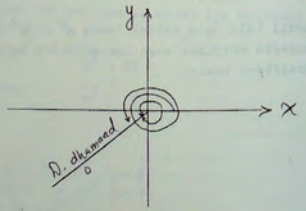
8. Haddii isle'egta qabaal tahay $x^2 + 7y^2 = 7$. Raadi dhererka dhidibka weyn iyo kulanada kulmisyada iyo geesahaba, garaafkana sawir.

TIRIGNOOMETERI

"Tirignoometeri" waa eray giriig ah kana kooban (tirigoon oo ah saddexagal iyo meteri oo cabbir ah). Waayadii hore waxa loogu dhaqmi jiray cabbiraadda xaglaha iyo fogaanta xiddigaha. Maantase waxay door weyn ka ciyaarta, baarista atoomigga, aragtida elegtrigga, gariirada kala duwan. Intaa waxaa dheer bedadkana idil waxay leeyihiin astaan soo noqnoqosho (periodic characteristics).

1-1. Kulanno iyo tirignoometeri

Xagalo isku dhinac billow iyo isku dhinac dhamaad ah waxa la yira xaglo dhamaad wadaag ah. Bar kasta, oo aan ahayn unuga, kuna taal sallaxi kulan (x iyo y), waxay sugta xaglo aan kobnayn oo dhamaad wadaag ah; oo mid waliba geeskeedu yahay unugga, dhinac billowgeeduna yahay dhanka togan ee dhidib-x; Fallaarta OD-na waa dhinac dhamaadka xagasha xagal kasta oo kuwaas ka mid ah waxa la yiraaha xagal-rug-beeggal-ah. Fiirso, cabbirada xaglo dhamaad wadaag ahi waxay is dheer : yihiin dhufsane abyoone ah oo 360, macnee haddii x ay tahay xagal rug beeggal ah, x waxa la dhamaad wadaag ah dhamaan xaglaha ka mid ah ururka $x + 360\ n\ n \in \{0, \pm 1, \pm 2, \ldots\}$.



Xaglo dhamaad wadaag ah oo rug beeggal ah.

(Shaxan) 1

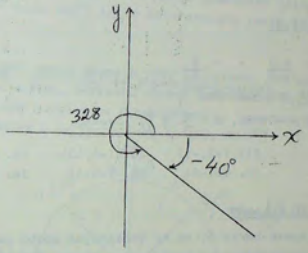
Markan ognahay cabbirka xagal ruggeedda barta D iyo fogaant OD ee u dhaxaysa D iyo unugga, waan garan karnaa meesha D ay kaga taal sallaxa.

TUSAALE:

Meele barta D, haddii OD = 3, cabbirka xagal ka mid ah xaglo rugeed keeduna uu yahay -40. Tus, sheegna cabbirka xagal rugeed togan ee D.

FURFURIS:

1. Sawir fallaarta q ee la samaysa dhidib -x togan xagal cabbirkeedu yahay -40°.
2. q ka cabbir 3 halbeeg oo laga billaabo O. Barta la gaaray waa D .
3. Xagal rugeed togan ee D cabbirkeedu waa $(-40+360^\circ)$, ama 320°.



(Shaxan) 2

LAYLI

Barta inta ay ka fog tahay unugga iyo cabbirka xagal rugeedkeeda lagu siiyay, muuji. Waxaa kale oo aad muujisaa, ka sheegtaana cabbirrada laba xaglood oo kale, mid togan iyo mid taban, oo ayaguna ka mid ah ururka xaglo rugeedyada barta.

1. 2 (fogaanta laga billaabo unugga), 180° (cabbir xagal)
2. 3; 210° 5. 5; -45° 8. $2\frac{1}{2}$, -225° 11. 0; 150°
3. 4, 60° 6. 7,240° 9. $3\frac{1}{2}$; 720° 12. 0, 330°
4. 1, -30° 7. 3/2; -360° 10. $\frac{1}{4}$; -540°

Sawir fallaarta ah garaafka xiriir kasta ee soo socda, muujina xagal rugeed togan iyo mid taban oo ay fallaartu u tahay dhinac dhamaad.

13. $\{(x.y) / y = \frac{1}{2}x, x \geq 0\}$ 16. $\{(x,y) / y = -5x, x \geq 0\}$
14. $\{(x,y) / y = \frac{1}{3}x, x \leq 0\}$ 17. $\{(x,y) / x = 0, y \leq 0\}$
15. $\{(x,y) / y = -4x, x \leq 0\}$ 18. $\{(x,y) / y = 0, x \leq 0\}$

Haddii barta D ay leedahay kulannada la isasiiyey, sawir falaarta OD, sheegna xiriirka ay u tahay garaaf; muujina xagal rugeed togan oo ay u tahay dhinac dhammaad.

TUSAALE D (-3,4)

Furfuris

Tirada OD = $\frac{4-0}{-3-0} = -\frac{4}{3}$

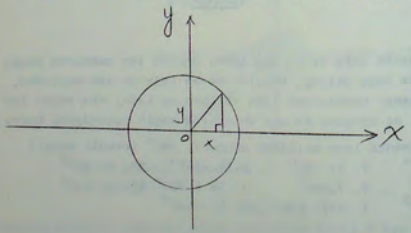
:.Fallaarta OD waa garaafka

$\{(x,y) : y = -4/3x, x \leq 0\}$, Jawaab.

19. (6,8) 21. (-2,-1) 23. (-5,10) 25. (4,0)
20. (12,5) 22. (3,-6) 24. (-9,3) 26. (0,-2)

1-2 GOOBO IYO KULANNO

Tixgali barta D, oo ku waniineysa goobo gacankeedu yahay 1, xuddunteeduna ku taal unugga (goobo halbeeg).



(Shaxan 3).

Haddaan isticmaalno jidka fogaanta ama aragtiinka pythegorus

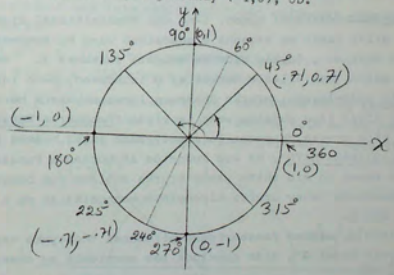
OD = $\sqrt{(x-0)^2+(y-0)^2}$ = 1 (oo waa gacanka goobada)

:. $\boxed{x^2 + y^2 = 1}$; kani waa isle'egga goobo xuddunteedu tahay unugga, gacankeeduna le'eg yahay 1. (Xusuusnow!).

1-3. FANSAARADA SAYN IYO KOSAYN

Tixgeli barta D oo ku waniinaysa goobo halbeeg. Haddii D ka dhaqaaqdo barta (1,0) ayadoo u socota lid sacad wareeg, marka cabbirka xagal rugeedadu wuxuu qaadanaya qiimayaasha ka billaabma 0^o ilaa 360^o, kolba inta xagashu le'eg tahay. (eg shax. 3).

Haddii aan u dayno D inay socodkeeda lid saacad-wareeg ugu noqnoqoto, iyo haddii ay jiho saacad wareeg socodkeeda u wadaba, markasta waxaad heli karta xagal rug beeggal ah, cabbirkeeda darajeedna ku aaddismaayo lammaanayaal horsan ee (a,b). Marka D ay tagantahay (1,0), OD.



OD waxay dul fuusha dhidib-x togan waxayna la samaysa xagal 0^o ah. D marka ay marayso barta (0.87, 0.5), OD waxay la samaynaysa dhinac billawga (x togan) xagal 30^o ah.
Sida oo kale marka D marayso (0,1) xagashu waa 90^o; barta (-1,0) xagashu waa 180^o; barta (D,-1) xagashu waa 270^o. Barta (1,0) xagashu waa 360^o.

Jibeyhcoyinkan waxa aynu isugu soo urumrin karnaa tuse

Cabbirka θ D	a	b
0^o	1	0
30^o	0.87	0.50
45^o	0.71	0.71
90^o	0	1
225	-0.71	-0.71
270^o	0	-1
360^o	1	0

Tuse -1

Guud ahaan, ka soo qaad in D tahay doorsoome urur horaadkiisu yahay xagio rugeedka sallaxa ku yaal oo dhan. Marka D ku wareegayso goobada, xagal rugeedkeeda θ ah oo keliyi isbeddeli maayo, laakiin kulannadeeduna (a,b) waa ay isbeddelayaan , bishardi in fogaanta u dhaxaysa θ iyo unugu mar kasta ay noqoto 1, oo macnaheedu yahay, $\sqrt{a^2 + b^2} = 1$, (ogow: a, iyo b waxa weeye lugaha saddexagal quman, 1-na waa shakaalkiisa) sida kuu muuqata qiimi kasta oo xagasha θ qaadataba waxa ku beegmaaya lammaane madi ah , (a,b), sida ka muuqata shaxanka 3. Haddaan u dhabo galno, mar allaale markii ay θ isbadasho, a-na (qiimiga x). waa ay is baddeleysaa; ururka dhammaan lammaanayaasha horsan (θ,a) ee sidaa lagu sugaana waxa la yiraa "Fansaarka Kosayn"; Sida oo kale ururka dhammaan la maanayaasha (θ,b), waxaa la yiraa "FANSAARKA SAYN". Ma kuu muuqataa in horaadka Fansaar kasta ee kuwaa ka mid ahiba yahay ururka xaglaha rug beegaalka ah, dambeedkuna yahay ururka tirooyinka maangalka ah ee u dheexeya 1 iyo 0.

Xagal kasta θ, waxaan Fansaarkeeda niraahnaa "Kosaynka xagal D" iyo "Saynka xagal θ", sida qeexidda soo socotaaba ay sheegeyso.

QEEX (1)

Ka dhig in D u taagan tahay xagal kasta oo rug beeggal ah. Haddii (a,b) u taagan yihiin kulannada bar unugga u jirta hal halbeeg, kuna taal dhinac dhammaadka θ, markaa:

Kosaynka xagasha θ = a,

Saynka xagasha θ = b.

U fiirso in (Cosθ, Sinθ) yihiin kulannada barta ay isku gooyaan goobada halbeeg iyo dhinac dhammaadka θ.

Haddii, laysa siiyo bar kasta T, aan ahayn unugga, kuna taal dhinac dhammaadka xagasha θ oo rug beeggal ah, waad sugi karta kulannada barta D ee fallaarta OT ay ka gooysa goobada halbeeg, ⁂ $x^2 + y^2 = 1$.

Taa micnaheedu waxa weeye waad heli karta Cosθ iyo Sinθ.

TUSAALE I:

T(-4,3) waa bar xagal rugeedkeedu tahay θ. Raadi Cosθ iyo Sinθ.

FURFURIS:

Ka soo qaad in D tanay barta ay isku gooyaan fallaarta OT iyo goobada $x^2+y^2 = 1$.

1. Fallarta OT waa garaafka

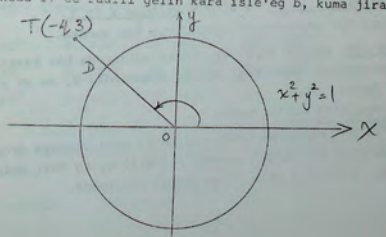
$\{(x,y) : y = -3/4x, \; x \quad 0\}$.

2. Kulannada D waa inay raalli geliyaan saddexdan weer-xisaabeed ee furan:

a. $x^2 + y^2 = 1$ (isle'egga goobada halbeeg)

b. $y = \frac{-3}{4} x$ (isle'egga fallaarta)

t. $x \leq 0$ (macnii waxa weeye, maaddaama OT ay tahay fallaarta unugga iyo T(-4,3). Baraha kale ee ku yaal waxaad IV ee raalli gelin kara isle'eg b, kuma jiraan).



(Shaxan 4).

3. Fur fur isle'egyada wada jira $x^2 + y^2 = 1$, iyo $y = -3/4\ x$

$x^2 + y^2 = 1$, $y = -3/4x$

$y^2 = 1-x^2$

$y = \sqrt{1- x^2}$

$\therefore -3/4\ x = \sqrt{1- x^2}$

$\frac{9}{16} x^2 = 1 - x^2$

$\frac{25}{16} x^2 = 1$

$x = 4/5$ ama $- 4/5$

Maaddaama $4/5 > 0$, qiimigaas ka tag.
Marka $x = -4/5$, $y = 3/5$.

Lammaanaha $(-4/5, 3/5)$ ayaa raalli gelinaaya saddexdii xiriirba.

$\therefore$ Kulannada D waa $(-4/5, 3/5)$, taasoo inna gaarsiineysa in $Cos\theta = -4/5$, $Sin\theta = 3/5$ Jawaab.

Haddaad u fiirsatid hooseeyaha jajabyada $-4/5$ iyo $3/5$, oo ah 5 waxaad arki doontaa in uu yahay fogaanta laga billaabo unugga ilaa T, $(5 = \sqrt{(-4)^2 + 3^2}$, halka $g = \sqrt{a^2 + b^2})$; natijada tusaalaha aan soo dhaafnay waxa loo qori kara sidan:

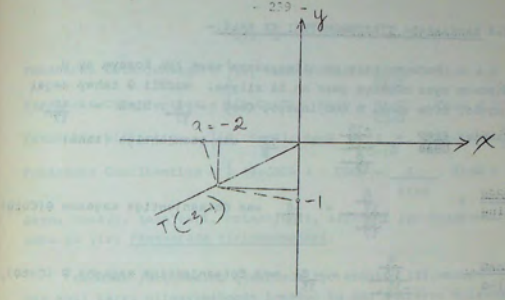
$Cos\theta = \frac{qiimiga\ x\ ee\ T}{g}$, $Sin\theta = \frac{qiimiga\ y\ ee\ T}{g}$.

Tani waxay noo garqaadaysa aragtiinta soo socota.

ARAGTIIN (1) Haddii (a,b) ay yihiin kulannada bar kasta oo aan ahayn unugga, kuna taal dhinac dhammaadka θ, oo ah xagal rugeed, marka

$Cos\theta = a/g$, $Sin\theta = b/g$, marka

$g = \sqrt{a^2 + b^2}$ (xuuuso iidka fogaanta) adoo raacaya aragtiin (1) raadi $Cos\theta$ iyo $Sin\theta$, haddii T $(-2,1)$ ay ku taal dhinac dhamaadka θ (fiiri shax.5) ee bogga soo socda.



FURFURIS:

$a = -2$, $b = -1$; $g = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$

$\therefore Cos\theta = \frac{-2}{\sqrt{5}} = \frac{-2.\sqrt{5}}{\sqrt{5}.\sqrt{5}} = -2/5\ \sqrt{5}$

$Sin\theta = \frac{-1}{\sqrt{5}} = \frac{-1.\sqrt{5}}{\sqrt{5}.\sqrt{5}} = -\frac{1}{5}\sqrt{5}$

LAYLI

(B) Xagal kasta ee soo socota θ ku washir sallaxa kulan, adoo isticmaalaaya xagal-beegyo goobo gacankeedu yahay 1; dabadeedna qiyaas $Cos\theta$ iyo $Sin\theta$.

1. 120^0 4. 315^0 7. -60^0 10. 1080^0
2. 210^0 5. 330^0 8. -150^0 11. -810^0
3. 225^0 6. -385^0 9. -225^0 12. 720^0

Raadi $Cos\theta$ iyo $Sin\theta$, haddii θ tahay xagal rugeedka barta kulannadeeda la isa siiyay. Ku tibaaxi xididlayaalka si fudud.

13. $(-8,-15)$ 15. $(-3,0)$ 17. $(-2,2)$ 19. $(4,2)$
14. $(30,16)$ 16. $(0,-4)$ 18. $(5,-5)$ 20. $(-6,-3)$.

1-4 FANSAARADA TIRIGNOOMETERI EE KALE:-

Racaymo kale oo qiimayaasha sayn iyo kosayn ay u dhacaan ayaa magacyo gaar ah la siiyaa. Haddii θ tahay xagal rugeed, sida shax. 6 muujinaayo, $\text{Cos}\theta = \frac{8}{17}$, $\text{Sin}\theta = -\frac{15}{17}$.

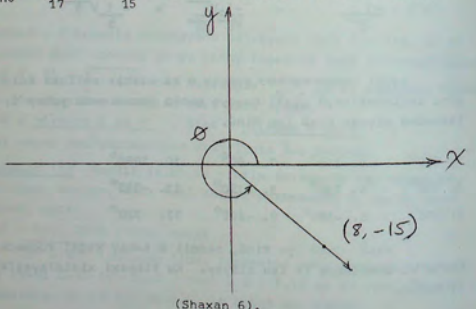
Saamiga $\frac{\text{Sin}\theta}{\text{Cos}\theta} = \frac{\frac{-15}{17}}{\frac{8}{17}} = -\frac{15}{8}$ waa taanjentiga (tanθ)

$\frac{\text{Cos}\theta}{\text{Sin}\theta} = \frac{\frac{8}{17}}{\frac{-15}{17}} = \frac{-8}{15}$ waa Cotaanjentiga xagasha θ(Cotθ)

$\frac{\text{Cos}\theta}{\text{Sin}\theta} = \frac{\frac{8}{17}}{\frac{-15}{17}} = -\frac{8}{15}$ waa Cotaanjentiga xagasha θ (Cotθ),

$\frac{1}{\text{Cos}\theta} = \frac{1}{\frac{8}{17}} = \frac{17}{8}$ waa siikantiga xagasha θ (sec θ)

$\frac{1}{\text{Sin}\theta} = \frac{1}{\frac{-15}{17}} = \frac{17}{15}$ waa Kosiikantiga xagasha θ (CSCθ)



(Shaxan 6).

Markan waxa aan qiri karna qeexida afar fansaar oo horaadkoodu yahay hormo urur ee ururka xaglaha θ ee rug beeggal ka ah;

Fansaarka Cotanjentiga = $\left\{(\theta, \tan\theta): \tan\theta = \frac{\text{Cos}\theta}{\text{Sin}\theta}, \text{Sin}\theta \neq 0\right\}$

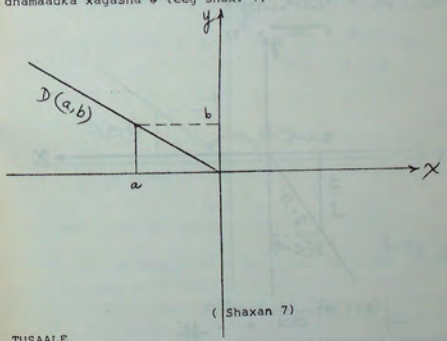
Fansaarka Cotanjentiga = $\left\{(\theta, \text{Cot}\theta): \text{Cot}\theta = \frac{\text{Sin}\theta}{\text{Cos}\theta}, \text{Cos}\theta \neq 0\right\}$

Fansaarka siikantiga = $\left\{(\theta, \text{Sec}\theta): \text{Sec}\theta = \frac{1}{\text{Cos}\theta}, \text{Cos}\theta \neq 0\right\}$

Fansaarka Cosiikantiga = $\left\{(\theta, \text{CSC}\theta): \text{CSC}\theta = \frac{1}{\text{Sin}\theta}, \text{Sin}\theta \neq 0\right\}$

Sayn, Kosayn, taanjenti, kotaanjenti, siikanti iyo kosiikanti waxa la yira Fansaarada tirignoometeri.

Haddaan isticmaalno qeexidada iyo aragtin (1), markaa waa beli karra qiimayaashooda innakoo ku soo saarayna kulannada (a,b) ee bar kasta D, oon ahayn unugga, kuna taal dhinac dhamaadka xagasha θ (eeg shax. 7)



(Shaxan 7)

TUSAALE

Haddii $\text{Cos}\theta \neq 0$

$\tan\theta = \frac{\text{Sin}\theta}{\text{Cos}\theta} = \frac{\frac{b}{\sqrt{a^2+b^2}}}{\frac{a}{\sqrt{a^2+b^2}}}$ $\therefore \tan\theta = \frac{b}{a}$, $a \neq 0$

garaadeyn-taa la mid ah waxay keenaysa tibaaxaha Cotθ, Secθ, iyo CSCθ ee aragtiinka soo socda:

Aragtiin (2)

Haddii D(a,b) ay tahay bar ka gedisan unuga kuna taal dhinac dhammaadka xagal rugeedka θ markaa

$\sin\theta = \frac{b}{\sqrt{a^2+b^2}}$ $\csc\theta = \frac{\sqrt{a^2+b^2}}{b}$, $b \neq 0$

$\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$ $\sec\theta = \frac{\sqrt{a^2+b^2}}{a}$, $a \neq 0$

$\tan\theta = \frac{b}{a}$, $a \neq 0$ $\cot\theta = \frac{a}{b}$

TUSAALE

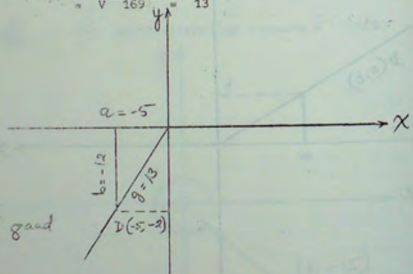
Raadi qiimayaasha fansaarada tirignometeri ee xagal rugeedka θ, haddii D (-5,-12) ay ku taal dhinac dhammaadka θ.

FURFURIS:

$a = -5$, $b = -12$

$\therefore \sqrt{a^2 + b^2} = \sqrt{(-5)^2+(-12)^2}$

$= \sqrt{169} = 13$



F.G shaxa 8 gaad

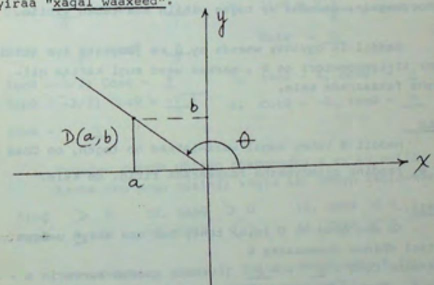
$\sin\theta = -\frac{12}{13}$ $\csc\theta = -\frac{13}{12}$

$\cos\theta = -\frac{5}{13}$ $\sec\theta = -\frac{13}{5}$

$\tan\theta = \frac{12}{5}$ $\cot\theta = \frac{5}{12}$

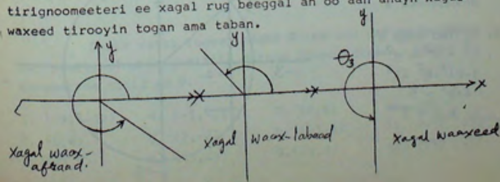
U fiirso, qiimayaasha fansaarada tirignoometeri waxay ku xiran yihiin rugta dhinac dhammaadka xagasha oo keli ah. Sababteeo ah cabbirka xagal rug beeggal ah wuxuu suga rugta dhinac dhammaadka; si aad u sheegtid xagashaas oo kale waxaa kugu filan inaad sheegtid cabbirkeeda. Bil matal, waxad qori karta "Sin 30°" intaad qori lahayd "Saynka xagal rug beeggal ah oo cabbirkeedu yahay 30°".

Xagallo rug beeggal ah waxaa badanaaba lagu kala hufaa hadba waaxda dhinac dhammaadkoodu ku yaal. Sidaa darteed xagasha θ_1 ee Shax.8 waxa la yira "<u>xagal waax-afraad</u>", ta θ_2 waxa la yiraa "<u>xagal waax-labaad</u>". Marka dhinac dhammaadka dul fuulo dhidibka -x ama -y, sida xagasha θ_3, xagashaa waxaa la yiraa "<u>xagal waaxeed</u>".



F.Gaara: (Shax.7)(242)

Tusaha soo socda wuxu u kala dhigayaa qiimayaasha fansaarada tirignoomeeteri ee xagal rug beeggal ah oo aan ahayn xagal waxeed tirooyin togan ama taban.



Qiimi	W a a x			
	I	II	III	IV
SinΘ iyo CSCΘ	Togan	Togan	Taban	Taban
CosΘ iyo SecΘ	Togan	taban	taban	togan
TanΘ iyo CotΘ	Togan	taban	togan	taban

Tuse 2

Tusahan waxad si fiican u garanaysaa haddaad xusuusatid waaxaha kulannada a iyo b oo kala qiimi x iyo y sey u kala horreeyaan, waaxaha ay togan yihiin ama taban yihiin.

Haddii la ogyahay waaxda ay Θ ku joogsato iyo qiimi fansaar tirignoometeri ee Θ , markaa waad sugi kartaa qiimayaasha fansaarada kale.

TUSAALE

Haddii Θ tahay xagal waax-labaad oo togan, oo $\cos\theta = \frac{-2}{3}$, sawir Θ, raadina qiimayaasha fansaarada tirig. ee kale.

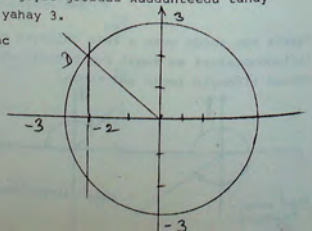
FURFURIS:-

Ka soo qaad in D (a,b) tahay bar aan ahayn unugga kuna taal dhinac dhammaadka Θ

1. Maadaama $\cos\theta = \frac{a}{g} = \frac{-2}{3}$, waxaad qaadan karta in a = -2, g = 3. Markaa D waa barta waaxda labaad ku taal, halka xarriiqda x = -2 ay ka goyso goobada xuddunteedu tahay unugga, gacankeeduna yahay 3.

2. Muujin OD tahay dhinac dhammaadka Θ

3. Si loo sugo b:
$a^2 + b^2 = g^2$
$(-2)^2 + b^2 = 3^2$
$4 + b^2 = 9$
$\therefore b = \sqrt{5}$



4. Isticmaal aragtiin 2 si aad u heshid

$\sin\theta = \frac{\sqrt{5}}{3}$ $\quad \csc\theta = \frac{3}{\sqrt{5}} = \frac{3.\sqrt{5}}{\sqrt{5}.\sqrt{5}} = \frac{3\sqrt{5}}{5}$

$\cos\theta = \frac{-2}{3}$ $\quad \sec\theta = -\frac{3}{2}$

$\tan\theta = -\frac{\sqrt{5}}{2}$ $\quad \cot\theta = \frac{-2}{\sqrt{5}} = \frac{-2.\sqrt{5}}{\sqrt{5}.\sqrt{5}} = \frac{-2\sqrt{5}}{\sqrt{5^2}} = \frac{-2\sqrt{5}}{5}$

LAYLI

(B) Sheeg qiimiyaasha fansaarada xagasha Θ ee lagu weeydiyey.

Tusaale $\cos\theta = \frac{-8}{17}$, $\sin\theta = \frac{15}{17}$; waxaad tiraa $\tan\theta = \frac{-8}{15}$

1. $\sin\theta = \frac{3}{8}$; $\csc\theta = \frac{?}{?}$
2. $\cos\theta = -\frac{1}{5}$; $\sec\theta = \frac{?}{?}$
3. $\sec\theta = -3/2$; $\cos\theta = ?$
4. $\sin\theta = -2/3$; $\cos\theta = \frac{-\sqrt{5}}{3}$
   $\tan\theta = ?$
6. $\sin\theta = \frac{\sqrt{3}}{2}$, $\cos\theta = ?$, $\cot\theta = ?$
7. $\tan\theta = 4$, $\cot\theta = ?$
8. $\cot\theta = -5$, $\tan\theta = ?$

Magacaw waaxda dhinac dhammaadka Θ ay ku oolli karto (ku noqo tusihii xagla aan aheyn xaglo-waaxeed)

9. $\sin\theta > 0$ 11. $\tan\theta > 0$ 13. $\sec\theta < 0$
10. $\cos\theta > 0$ 12 $\csc\theta < 0$ 14. $\cot\theta < 0$

Waaxdeebaa ay tahay in dhinac dhammaadka Θ inu yaal, si qiimayaasha lagu siiyey ay dhab u noqdaan?

15. $\sin\theta > 0$, $\cos\theta < 0$ 18. $\csc\theta > 0$, $\cos\theta > 0$
16. $\sin\theta < 0$, $\cos\theta < 0$ 19. $\sec\theta < 0$
17. $\sin\theta < 0$ $\tan\theta > 0$

Sawir xagal rugeed togan Θ ee ugu yar oo baraha la isasiiyey ay ku yaalliin dhinac dhammaadkeeda, qiime fansaarada tirig ee Θ.

1. (-9, -12)
2. (8,6)
3. $(\sqrt{3}, 1)$
4. $(-1, \sqrt{-3})$
5. (2,-2)
6. (-3,-3)
7. (0,4)
8. (-3,0)
9. (-1,7)
10. (-5,3)
11. $(-\sqrt{2}, \sqrt{6})$
12. $(\sqrt{3}, \sqrt{15})$

Ku sawir xagasha rug beeggalka ah ee taban θ ee cabbirka astirada ugu yar leh kuna dhamaata waaxda la isasiiyey; sheeg qiimiyada fansaarada tirig ee θ.

J) 13. $\sin\theta = -2/5$; III
14. $\cos\theta = 3/7$; IV
15. $\tan\theta = 5/4$; I
16. $\cot\theta = \frac{1}{3}$; III
17. $\cot\theta = -2$; IV
18. $\tan\theta = 3$; I
19. $\sec\theta = 1.5$; IV
20. $\csc\theta = 1.25$; III

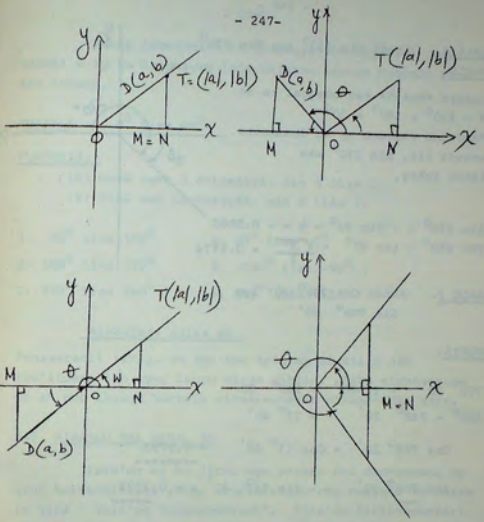
Qiimayaasha fansaarada tirig ee xagal waaxeedo. Xarfiiq (-), micnaheedu waa ma qeexna, ama qiimi sugan ma leh. Tixraac: Shax. 3.

θ	Sinθ	Cosθ	Tanθ	Cscθ	Secθ	Cotθ
0^c	0	1	0	-	1	-
90^c	1	0	-	1	-	0
180^c	0	-1	0	-	-1	-
270^c	-1	0	-	-1	-	0

XAGAL TIXRAAC

Waxaad buuggi kowaad ku soo baratay sida fansaarada xagal fiiqan oo togan looga raadiyo tusaha fansaarada tirig.

Si ay u suurowdo in isla tusahaas laga helo cabbirka iyo fansaarada xagal kasta, xagal kasta θ oo waax-kowaad ah waxan u bixinayna "xagal tixraaca θ". Ka dhig D(a,b) in ay tahay bar kasta ee ka mid ah dhinac dhammaadka θ, oo aan ahayn unugga, kana dhig in T ku taal waaxda kowaad kulanadeeduna yihiin (/a/, /b/). Xagasha fiiqan (ama quman) ee rug beeggalka ah, fallaarta OT-na tahay dhinac dhammaadkeeda waa xagal tixraaca θ waxana lagu magac dara W. (fiiri Shax.9)



(Shax. 9)

Haddaad dheehatid Shax.9 waxaad arki karaysaa in /DOM = W. Sababto ah dherarada dhinacyadooda isu dhigma ee saddexagalada quman DOM iyo TON waa isle'eg yihiin, maadaama MD = NT = /b/, OM = ON = /a/, OT = OD = $\sqrt{a^2 + b^2}$.

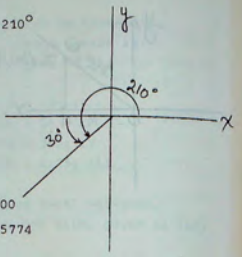
Haddaba, cabbirada xaglaha isku dhigma waa isle'eg yihiin. Taa micnaheedu waa inaad sugi kartid cabbirka w adoo helahaya cabbirka xagasha fiiqan ee dhinac dhammaadku θ la sameeyo dhidib-x.

Summadda qiimayaasha fansaarada θ ay qaadanaayan (togan ama taban) waxaad ka helaysa Tuse 2.

TUSAALE 1: Raadi Sin 210° iyo Tan 210°

FURFURIS:

1. Washir xagasha rug beeggal ka ah
2. W = 210° - 180° = 30°
3. Maadaama xagashu ay ku dhammaato waaxda III, Sin 210° waa taban yahay,



∴ Sin 210° = - Sin 30° = ½ = - 0.5000

Tan 210° = tan 30° = $\frac{\sqrt{3}}{3}$ = 0.5774

TUSAALE 2. Raadi Cos 708° 20' iyo Sin 708° 20'

FURFURIS:

1. 708° 20' - 1.360° = 348° 20'
2. 360° - 348° 20' = 11° 40'

Cos 708° 20' = Cos 11° 40' = 0.9793

Sin 708° 20' = - Sin 11° 40' = - 0.2022

LAYLI

Sawir xaglaha leh cabbirada la isasiiyey.
Raadi xagal tixraacooda:

(B) 1. 120° 5. -150° 9. -20°
2. 225° 6. - 94° 10. -315°
3. 330° 7. 760° 11. -240°
4. -60° 8. 1040° 12. 540°

Ku tibaax fansaar xagal fiiqan oo togan.

13. Cos 160° 17. Cot 440° 20. Cos (-1045°)
14. Sin 130° 18. Sec 195° 21. CSC (-39° 20')
15. Tan 200° 19. Sin (-365°) 22. Cot (-54° 40')
16. Cos 310°

Sheeg isbeddelka ku dhacaaya (b) Cosθ (t) Sinθ haddii θ ay ka kordhayso laga bilaabo waaxda koowaad ilaa tan labaad.

TUSAALE : 0° ilaa 90°

FURFURIS:

(b) Cosθ wuxu u dhinmaaya: min 1 ilaa 0.
(t) Sinθ wuu kordhaayaa: min 0 ilaa 1.

1. 90° ilaa 180° 4. 90° ilaa 0°
2. 180° ilaa 270° 5. -180° ilaa -90°
3. 270° ilaa 360° 6. 360° ilaa 450

Midaalada salka ah

Fansaaradii tirig. ee aad soo aragtay si xiisa leh faa'iidana leh ayey isugu xiran yihiin. Isku xirnaantaas oo ah hub lagaga hortago xiraaleyaasha xisaabaadka sare.

1-5 MIDAALO HAL XAGAL AH

Isle'eg ay ku jirto ugu yaraan hal doorsoome, oo urur horaadkiisuna yahay ururka xaglo rug beeggal ah waxaa la yira " isle'eg tirgnoometeri". Isle'eg tirignoometeri sida $(2\text{Sin}\theta + 1)(2\text{ Sin}\theta - 1) = 4(\text{Sin}\theta)^2 - 1$ ee qiima kasta oo θ ay qaadataba dhab ka dhigaaya isle'egga, labadiisa dhinacna ay qeexan yihiin, waxaa la yira "MIDAALO TIRIGNOOMETERI".

Midaalada tirig. waxay ku xiran yihiin qeexihi fansaarada tirig iyo aljebrada tirooyinka maangalka ah.
Ma sharxi karta sababta weer kasta ee soo socota ay run ugu tahay xagal walba θ ee fansaarku qeexan yahay?

1. $\text{Tan}\theta = \frac{\text{Sin}\theta}{\text{Cos}\theta}$ 3. $\text{Sec}\theta = \frac{1}{\text{Cos}\theta}$ 5. $\text{Cot}\theta = \frac{1}{\text{Tan}\theta}$

2. $\text{Cot}\theta = \frac{\text{Cos}\theta}{\text{Sin}\theta}$ 4. $\text{Csc}\theta = \frac{1}{\text{Sin}\theta}$

Midaalayaalka 1-4 waxay si toos ah uga yimaadeen qeexihi fansaarada tirig. Madaalka 5-na waxay ka dhex dhalatay 1 iyo 2, u fiirso haddii $\sin\theta \neq 0$, $\cos\theta \neq 0$, waxad heleysaa:

$$\frac{1}{\tan\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta}} \quad \text{(isticmaal midaalo 1)}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta \quad \text{(isticmaal mid. 2)}$$

$$\therefore \cot\theta = \frac{1}{\tan\theta}$$

Haddaad xusuusatid, xagal kasta θ ee rug beeggal ah, ($\cos\theta$, $\sin\theta$) waa kulannada bar ku taal goobo halbeeg, isle'egeduna yahay $x^2 + y^2 = 1$, waxaad soo bandhigi karta midaalaha $(\cos\theta)^2 + (\sin\theta)^2 = 1$ ama

6. $\sin^2\theta + \cos^2\theta = 1$

Haddii dhinac kasta ee midaal 6 aad u qaybisid $\cos^2\theta$, waxad dhiraandhirin kartaa midaal kale,

$$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}, \text{ ama } 1 + \left(\frac{\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta}\right)^2,$$

$\cos\theta \neq 0$

Markaad ku isticmaashid midaalada 1 iyo 2 waxaad helaysaa:

7. $1 + \tan^2\theta = \sec^2\theta$

$(\cos\theta)^2 = \cos^2\theta$, $(\sin\theta)^2 = \sin^2\theta$ ma

Ma sheegi karta sida loo dhiraandhiriyay midaalkan soo socda?

8. $1 + \cot^2\theta = \csc^2\theta$

Midaalada 1-8 waxaa la yira "MIDAALADA SALKA AH EE TIRIG". Adoo ku isticmaalaya iyaka iyo astaamaha tirooyinka maangalka ah, waxaad qori kartaa tibaax kasta oo ay ku jiraan qiimeyaasha fansaarada tirig ee xagal θ adoo ku soo saaraaya qiimaha $\sin\theta$ ama fansaar tirig oo xagal θ ee kale.

Tusaale 1: $\cos\theta$ ku tibaaxi $\sin\theta$

Furfuris: $\cos^2\theta + \sin^2\theta = 1$ (Midaal 6)

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\therefore \cos\theta = + \sqrt{1-\sin^2\theta}, \text{ haddii } \theta \text{ ay ku taal waax I ama IV}$$

ama $\cos\theta = - \sqrt{1-\sin^2\theta}$, haddii θ ay ku taal waax II ama III

Janaabta tusaale 1, waxay innoo sheegaysa habka soo socda ee loo helo $\cos\theta$, haddii la isasiiyo in θ ay ku taal waaxda labaad oo $\sin\theta = 3/5$.

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\left(\frac{3}{5}\right)^2} = -\sqrt{\frac{16}{25}} = -4/5$$

Tusaale 2. Ku tibaaxi $\cos\theta$, raadina tibaax u dhiganta $(1+\sin\theta)(\sec\theta - \tan\theta)$.

Furfuris:- Tibaaxda la isasiiyey waxay tilmaamaysa inay maangal ah, bisharti $\cos\theta \neq 0$. Isticmaal midaalada 1 iyo 3.

$$(1+\sin\theta)(\sec\theta-\tan\theta) = (1+\sin\theta)\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)$$

$$= (1+\sin\theta)\left(\frac{1-\sin\theta}{\cos\theta}\right) = \frac{1-\sin^2\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta} = \cos\theta \quad \text{Jawaab}$$

(B) LAYLI

Ku tibaaxi tibaaxaha soo socda hal fansaar oo tirig.

1. $1 + \tan^2 B$
2. $1 - \cos^2\theta$
3. $\tan\theta \sec\theta \cos\theta$
4. $\csc\theta \sin\theta \cot\theta$
5. $\sin^2 A + \cos^2 A + \tan^2\theta$
6. $\csc^2 r - \cot r + \tan^2 r$
7. $\dfrac{(\sin^2 d + \cos^2 d)(\sec^2 d - \tan d)}{\tan r}$
8. $\sqrt{\dfrac{\sec^2 - 1}{\csc^2\theta - 1}}$
9. $\sqrt{\dfrac{1-\sin^2\theta}{1 + \tan^2\theta}}$

(T) θ waxay ku dhammaataa waaxda la isasiiyey; raadi qiimayaasha fansaaradeda tirig.

10. IV; $\cos\theta = 4/5$
11. III; $\tan\theta = 8/15$
12. II; $\csc\theta = 13/12$
13. III; $\sin\theta = -\dfrac{7}{25}$
14. SinA ku tibaaxi: $\tan^2 A (\csc^2 A - 1) + \tan A \cos A$.
15. $\sec\theta$ ku tibaaxi: $\sin\theta \csc\theta + \dfrac{\sin\theta}{\cos\theta \cot\theta}$

(J) 16. $\tan\theta$ ku tibaaxi : $\csc^2\theta (\sec^2\theta - 1)(\sin\theta \cos\theta)$

17. $\cos\theta$ ku tibaaxi : $1 + \tan^2\theta - \dfrac{\sin^2\theta}{\csc^2\theta - 1}$

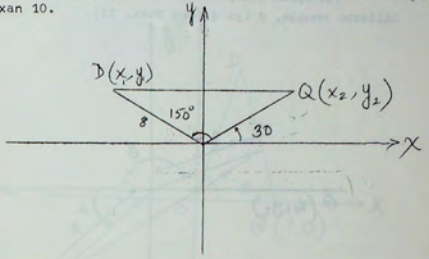
Sayn iyo Kosayn oo kali ah ku tibaax, fududeena.

18. $\dfrac{(\cos r - \sec r}{\sec r} + \cos^2 r \tan^2 r) \quad \dfrac{(\tan r - \sin r)}{\tan r}$
19. $(\tan u + \sin u)(1 - \cos u) + \dfrac{\cos u}{\csc u}$
20. $\sin A \sec A (\cos A + \dfrac{\csc A}{\sec^2 A}) + (\csc A + \sec A)$

Midaallo ay ku jiraan labo xaglood

1-6 Jidka Fogaanta

Ka soo qaad in D iyo Q ay yihiin baraha ku muujisan shaxan 10.



(Shax.10)

Si loo raadiyo DQ, waxaad qaadi karta tillaabooyinka hoos ku xusan.

1. Isticmaal araqtiin 1, si aad u sugtid kulannada (x_1,y_1) ee D iyo (x_2,y_2) ee Q.

D : $\cos 150^\circ = \dfrac{x_1}{8} \longrightarrow x_1 = 8 \cos 150^\circ = (-\frac{1}{2}\sqrt{3}) = -4\sqrt{3};$

$\sin 150^\circ = \dfrac{y_1}{8} \longrightarrow y_1 = 8 \sin 150^\circ = 8 (\frac{1}{2}) = 4$

Q: $\cos 30^\circ = \dfrac{x_2}{6} \longrightarrow x_2 = 6 \cos 150^\circ = 6 (\frac{1}{2}\sqrt{3}) = 3\sqrt{3};$

$\sin 30^\circ = \dfrac{y_2}{6} \longrightarrow y_2 = 6 \sin 30^\circ = (\frac{1}{2}) = 3.$

2. Isticmaal jidka fogaanta:

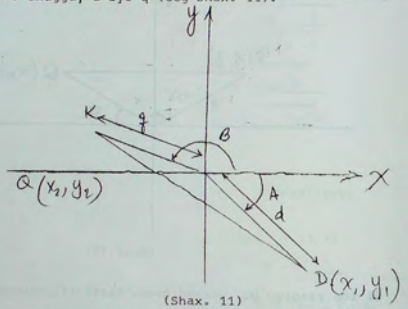
$(DQ)^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$(DQ)^2 = (-4\sqrt{3} - 3\sqrt{3})^2 + (4-3)^2 = 49 (3) + 1 = 148$

$\therefore DQ = \sqrt{148} = 2\sqrt{37}$

Haddaad raacdid tallaabooyinkaa kore, waxaad dhiran-dhirin kartaa jidka fogaanta u dhaxaysa laba barood ee kasta D iyo Q adoo ku soo saaraaya xaglo rugeedkooda.

A iyo B sidey u kalahorreeyaan iyo fogaanta laga billaabo unugga, d iyo q (eeg Shax. 11).



(Shax. 11)

1. D : $x_1 = d \text{ Cos } A$; $y_1 = d \text{ Sin } A$

   Q: $x_2 = q \text{ Cos } B$; $y_2 = q \text{ Sin } B$

2. $(DQ)^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$= (d \text{ Cs } A - q \text{ Cos } B)^2 + (d \text{ Sin } A - q \text{ Sin } B)^2$ (Astaanta isku beddelka)

$= d^2 \underline{\text{Cos}^2 A} - 2 dq \text{ Cos } A \text{ Cos } B + q^2 \text{ Cos}^2 B$

$+ \underline{d^2 \text{ Sin}^2 A} - 2 dq \text{ Sin } A \text{ Sin } B + q^2 \text{ Sin}^2 B$

$= d^2 (\text{Cos}^2 A + \text{Sin}^2 A) + q^2 (\text{Cos}^2 B + \text{Sin } B)$

$- 2 dq (\text{cos } A \text{ Cos } B + \text{Sin } A \text{ Sin } B)$

(Isku dhufashadu way ku kala dhiganta isugeynta)

$= d^2 (1) + q^2 (1) - 2 dq (\text{Cos } A \text{ Cos } B + \text{Sin } A \text{ Sin } B)$

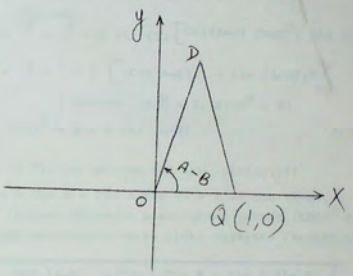
(Midaal 6)

/:. $(DQ)^2 = d^2 + q^2 - 2 dq (\text{Cos } A \text{ Cos } B + \text{Sin } A \text{ Sin } B$ /

.....jidka fogaanta

1-7 Kosaynka Farqiga laba xaglood:

Ka soo qaad in A iyo B ay u taagan yihiin xaglo kasta oo rug beeggal ah; ka dhigna in barta D I oo ku taal dhinac dhammaadka A) iyo Q (oo ku taal dhinac dhammadka B) ay unugga u jiraan 1 halbeeg (fiiri shax. 12)



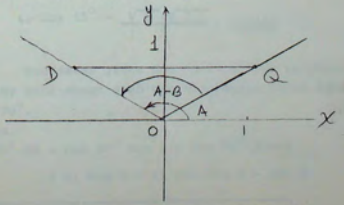
(Shax. 13)

Isticmaal jidka fogaanta, kuna beddel 1 d iyo q,

$(DQ)^2 = 1^2 + 1^2 - 2 (1) (1) (\text{Cos } A \text{ Cos } B + \text{Sin } A \text{ Sin } B)$

:. $(DQ)^2 = \underline{2 - 2 (\text{Cos } A \text{ Cos } B + \text{Sin } A \text{ Sin } B)}$ (b)

Markan, waxaad doorataa habdhis kulanno cusub oo Q tahay barta (1,0); fallaarta OQ, dhinac dhammaadka B, waa dhidibka -x ee togan; xagasha A-B ayaduna waa rug beeggal (fiiri shaxan 13, ee loo sawiray si loo muujiyo in dhidibka cusub ee x u yahay xarriiq jiif).



(Shax. 14)

Si loogu xisaabiyo $(DQ)^2$ habdhiskan kulanno aad ugu fiirso waxa soo socda:

Q 1 halbeeg ayey u jirta unugga, xagal rugeedeeduna waa 0°.

D 1 halbeeg ayey u jirta unugga, xagal rugeedeeduna waa A-B.

Haddaba,

$$(DQ)^2 = 1^2 + 1^2 - 2\ (1)\ (1) \left[\text{Cos}(A-B)\ \text{Cos}0^\circ + \text{Sin}\ (A-B)\ \text{Sin}0^\circ\right]$$

$$= 1 + 1 - 2 \left[(\text{Cos}\ A-B).1 + \text{Sin}\ (A-B).0\right]$$

(Xusuuso $\text{Cos}0^\circ = 1$, $\text{Sin}0^\circ = 0$)

$$\therefore (DQ)^2 = 2 - 2\ \text{Cos}\ (A-B) \qquad (t)$$

U fiirso isle'eg (b) iyo isle'eg (t)

$2-2\ (\text{Cos}\ A\ \text{Cos}\ B + \text{Sin}\ A\ \text{Sin}\ B) = 2 - 2\ \text{Cos}\ (A-B)$.

(Labada dhinacba waxay le'eg yihiin $(DQ)^2$) taniwaxay noo hogaaminaysa jidka Kosaynka farqiga laba xaglood:

9. $\text{Cos}\ (A-B) = \text{Cos}\ A\ \text{Cos}\ B + \text{Sin}\ A\ \text{Sin}\ B$

Maaddaama A iyo B ay yihiin xaglo rugeed, jidka Cos (A-B) waa midaal aad A iyo B ku beddeli kartid xagal kasta.

Tusaale 1. Sug qiimaha $\text{Cos}\ 15^\circ$

Furfuris: $15^\circ = 45^\circ - 30^\circ$

$$\therefore \text{Cos}\ 15^\circ = \text{Cos}\ (45^\circ - 30^\circ)$$

$$= \text{Cos}\ 45^\circ\ \text{Cos}\ 30^\circ + \text{Sin}\ 45^\circ\ \text{Sin}\ 30^\circ$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \tfrac{1}{2}$$

$$\therefore \text{Cos}\ 15^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}\text{, JAWAAB}$$

Maaddaama jidka Cos (A-B) u dhab ku yahay xagal kasta A, waxay gaar ahaan dhab ugu noqonaysaa marka cabbirka A uu yahay 90°.

Haddaba,

$$\text{Cos}\ (90^\circ-B) = \text{Cos}\ 90^\circ\ \text{Cos}\ B + \text{Sin}\ 90^\circ\ \text{Sin}\ B$$

$$= 0.\ \text{Cos}\ B + 1.\ \text{Sin}\ B = 0 + \text{Sin}\ B.$$

$\therefore$ 10. $\text{Cos}\ (90^\circ-B) = \text{Sin}\ B$

Maaddaama isle'ega ugu dambeeyaay ay tahay midaal ay ku jirto B, waligeed dhab ayey ahaaneysaa markii B lagu baddelo $90^\circ - B$.

Sidaas ayaad ku helaysa

$\text{Cos}\ [90^\circ - (90^\circ - B)] = \text{Sin}\ (90^\circ - B)$

$\text{Cos}\ B = \text{Sin}\ (90^\circ - B)$, ama

11. $\text{Sin}\ (90^\circ - B) = \text{Cos}\ B$

Intaa waxaa dheer, maadaama

$$\text{Tan}\ (90^\circ - B) = \frac{\text{Sin}\ (90^\circ - B)}{\text{Cos}\ (90^\circ - B)} = \frac{\text{Cos}\ B}{\text{Sin}\ B} = \text{Cot}\ B$$

Haddaba,

12. $\text{Tan}\ (90^\circ - B) = \text{Cot}\ B$

, bishardi in qiimayaasha fansaaradu ay qeexan yihiin, caddaynta midaalada soo socda adaa layli ahaan lagugu daayey :

$\text{Cot}\ (90^\circ - B) = \text{Tan}\ B$, $\text{Sec}\ (90^\circ - B) = \text{Csc}\ B$,
$\text{Csc}\ (90^\circ - B) = \text{Sec}\ B$.

LAYLI

Ku tibaaxi layli kasta ee soo socda qaabka $\text{Cos}\theta$, ee θ ku habboon tahay.

(B) 1. $\text{Cos}\ 260^\circ\ \text{Cos}\ 190^\circ + \text{Sin}\ 260^\circ\ \text{Sin}\ 90^\circ$

2. $\text{Cos}\ 310^\circ\ \text{Cos}\ 50^\circ + \text{Sin}\ 310^\circ\ \text{Sin}\ 50^\circ$

3. $\frac{1}{2}\ \text{Cos}\ 40^\circ - \frac{\sqrt{3}}{2}\ \text{Sin}\ 40^\circ$

Ku tibaaxi kosaynka farqiga laba xaglood, qiimeena.

4. $\text{Cos}\ 75^\circ$ 5. $\text{Cos}\ 195^\circ$ 6. $\text{Cos}\ 225^\circ$ 7. $\text{Cos}\ 105^\circ$

Jubi midaalada soo socda:

8. $\text{Cos}\ (45^\circ - B) = \frac{\sqrt{2}}{2}\ (\text{Cos}\ B + \text{Sin}\ B)$

9. $\text{Cos}\ (150^\circ - B) = -\frac{1}{2}\ (\sqrt{3}\ \text{Cos}\ B - \text{Sin}\ B)$

Caddee in kuwa soo socda ay yihiin midaallo (ku noqo tusaale 1):

10. Cot $(90^\circ - B)$ = Tan B
11. Csc B = Sec $(90^\circ - B)$
12. Csc $(9^\circ - B)$ = Sec B
13. Cos $(180^\circ - B)$ = - Cos B
14. Sin $(180^\circ - B)$ = Sin B
15. Sin $(270^\circ - B)$ = - Sin B

Fudude.

(T) 16. Cos $(90^\circ - A)$ Sin $(180^\circ - B)$ + Cos $(360^\circ - A)$ Sin $(90^\circ - B)$

17. Cos $(A - 90^\circ)$ Sin $(90^\circ - B)$ + Sin $(B - 270^\circ)$ Cos $(90^\circ - A)$

18. Tan $(90^\circ - B)$ Tan $(180^\circ - B)$ Sec B + Csc A Sin $(90^\circ - A)$ Csc $(90^\circ - A)$

19. Csc $(90^\circ - \theta)$ Sec $(360^\circ - \theta)$ - Tan $(720^\circ + \theta)$ Cot $(450^\circ - \theta)$

1-8 Fansaarada Wadarta iyo farqiyada xaglaha

Midaallo badan ayaa ka dhasha jidka Cos (A-B) = Cos A Cos B + Sin A Sin B.

Tusaale ahaan, haddii A cabbirkeedu yahay 0° waxaad helaysaa in: Cos $(0^\circ - B)$ = Cos0° Cos B + Sin0° Sin B

Cos (-B) = 1. Cos B + 0. Sin B

:. Cos (-B) = Cos B.

Si aad Sin B ugu tibaaxdid Sin (-B), B ku beddel -B midaalaha Sin B = Cos $(90^\circ - B)$:

Sin (-B) = Cos $\left[90^\circ - (-B)\right]$

:. Sin (-B) = Cos 190° B)
= Cos $\left[B - (-90^\circ)\right]$
= Cos B Cos(-90°) + Sin B Sin (-90°)
= Cos B.0 + Sin B. (-1)

:. Sin (-B) = - Sin B

Adoo isticmaalaaya in A + B = A - (-B) ay run tahay, waxaad markan dhiraandhirin karta jidka kosaynka oo wadarta laba xaglood.

Cos (A +B) = Cos $\left[A - (-B)\right]$
= Cos A Cos (-B) + Sin A Sin (-B)
= Cos A Cos B + Sin A (-Sin B)

:.(13) Cos (A + B) = Cos A Cos B - Sin A Sin B

Maaddaama saynka xagali uu le'eg yahay kosaynka xagasha ku sidkan, jid waad u heli karta Sin (A + B):

Sin (A + B) = Cos $\left[90^\circ - (A + B)\right]$
= Cos $\left[(90 - A) - B\right]$
= Cos $(90^\circ - A)$ Cos B + Sin $(90^\circ - A)$ SinB

:. (14) Sin (A + B) = Sin A Cos B + Cos A Sin B

Haddaad B ku beddeshid -B waxaad helaysa

(15) Sin (A - B) = Sin A Cos B - Cos A Sin B

Tusaale 1: Fudude: Sin 160° Cos 20° + Cos 160° Sin 20°
Furfuris: Sin 160° Cos 20° + Cos 160° Sin 20° = Sin $(160^\circ + 20^\circ)$
= Sin (180°) = 0
:. Sin 160° Cos 20° + Cos 160° Sin 20° = 0, JAWAAB.

Tusaale 2: Sug Sin (A - B), haddii A ay tahay waax-saddexaadda oo Cos A = - 3/5,
B-na tahay waax-labaadda oo waliba Sin B = 8/17.

FURFURIS:-
Cos A = - 3/5; Sin B = 8/17 Ogaal
:. Sin A = - 4/5; Cos B = - 15/17 Isticmaal kabka qaybta 1-4.
Sin (A - B) = Sin A Cos B - Cos A Sin B
= $-\frac{4}{5}$ (-15/17) - (-3/5) (8/17)
= 60/85 + 24/85 = 84/85 , JAWAAB

Jid ma u dhiraandhirin karta Tan (A + B) ? Haddii
Cos (A + B) $\neq$ 0,
Tan (A + B) = $\frac{\text{Sin (A+B)}}{\text{Cos (A+B)}}$ = Sin A Cos B + Cos A Sin B / Cos A Cos B - Sin A Sin B

Haddaad u qaadatid in Cos A $\neq$ 0, Cos B $\neq$ 0, waxaad jajabki midaalka dhinaciisa midig u beddeli karta jajab u dhigma. Markaad u qaybisid sareeyaha iyo hooseeyahaba Cos A Cos B.

$$\text{Tan } (A+B) = \frac{\text{Sin A Cos B}}{\text{Cos A Cos B}} + \frac{\text{Cos A Sin B}}{\text{Cos A Sin B}}$$

$$\frac{\text{Cos A Cos B}}{\text{Cos A Cos B}} - \frac{\text{Sin A Sin B}}{\text{Cos A Cos B}}$$

16:	Tan (A + B)	$= \frac{\text{Tan A} + \text{Tan B}}{1 - \text{Tan A Tan B}}$

Naqtiin midaaladii iyo ka shaqayn layliyadan:

/LAYLI/

Fududee:

(B) 1. Cos (-B) Sec (-B) - Cos B Sin (-B)
2. Tan B Cos B - Cot (-B) Sec (-B) - Csc B - Sin (-B)
3. Cos 137° Cos 47° + Sin 137° Sin 470°
4. Sin 26° Cos 94° + Cos 26 Sin 94°
5. Cos 708° Sin 753° - Sin 708° Cos 753°
6. Cos 157° Cos 173° - Sin 157° Sin 173°

Ku dhagan jidadka wadarta iyo farqiyada xaqlaha si aad u heshid qiimiga mid kasta ee soo socda:

7. Tan 75°
8. Sin 75
9. Sin 15°
10. Tan 15°
11. Cos 285°
12. Sin 285°
13. Sin 285°
14. Cos 165°
15. Sin 195°
16. Cot 165°

17. Haddii A tahay xagal waax-koowaad oo Sin A = 4/5, B-na tahay xagal waax-labaad oo Cos B = - 51/149, raadi (b) Sin (A +B); (t) Cos (A +B) (j) Sin(A-B); (x) Cos (A- B); (kh) Tan (A +B).
18. Haddii A tahay xagal waax-saddexaad oo Csc A = -13/5, B-na tahay xagal waax-afraad oo Sec B = 25/7, raadi (b) Sin(A+B); (t) Cos (A+B); (j) Sin (A-B); (x) Cos(A-B); (kh) Tan (A+B)

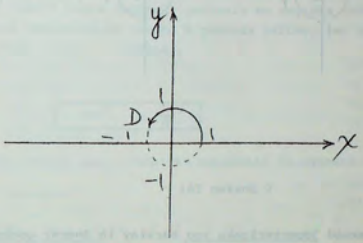
Qiime:

19. Csc 60° $\quad \frac{\text{Tan } 47° + \text{Tan } 13°}{1 - \text{Tan } 13° \text{ Tan } 47°}$

20. $\frac{\text{Tan } 279° + \text{Tan } 144°}{\text{Tan } 144° \text{ Tan } 279° + 1}$ + Sec 139° Cos 139°

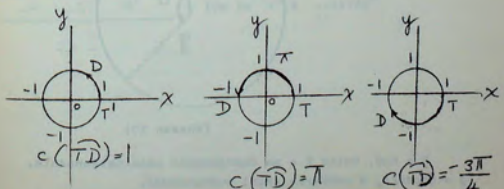
1-9 CABBIRKA QAANSO IYO XAGAL

Shaxan 14 wuxuu muujinaya barta T(1,0) ee kutaal goobda halbeeg $\{ (x,y) : x^2 + y^2 = 1 \}$, ee meriskeedu yahay 2 π (1) = 2π. Barta goobada ku wareegaysa ee ka dhaqaaqda T tagtana D, waxay sameysey qaanso goobeed, $\widehat{TD}$.
Haddii aad ogtahay dherarka qaansadaa $\{ C(\widehat{TD}) \text{ ama cabbirka } \widehat{TD} \}$, iyo jihada wareegga, markaa waad meelayn kartaa D.



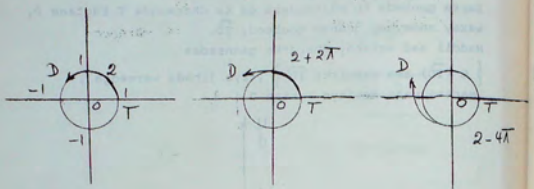
(Shaxan 14.)

Shaxan 15 wuxuu muujinayaa rugaha kala duwan ee D kuna beegan dheerarka qaansooyinka kala jadka ah. U fiirso in cabbir togan la siiyay qaansooyinka ka dhashay sodcaalka lid saacad wareeg, cabbir taban-na la siiyey kuwa ka dhasha sodcaalka saacad wareeg.



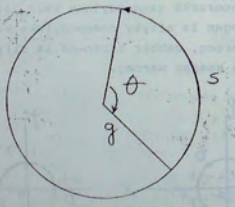
(Shaxan 15)

Ma sharxi kartaa sababta ay xaglaha ku kala duwan dhufsane abyoone ee 2π ugu wada dhammaadaan isku bar? Shaxan 16 wuxuu muujinaaayaa saddex qaanso oo dhammaad wadaag ah.



(Shaxan 16)

Waxaad joometerigaku soo baratay in dherar goobeedka qaanso, ee lagu aqoonsado S, ay u saami galsan tahay dhererka gacanka goobada g iyo cabbirka xagal xudduneedda θ ee ay qaansadu leesho (substended) sida shax. 17 uu muujinaayo, taas mirneheedu waa:



(Shaxan 17)

S = Kgθ, halka K = ma doorsoomaha saamigalsanaanta, g = gacanka, θ cabbirka xagal xuddumeedda.

Haddaan halbeeg ku habboon u doornno cabbirka xagasha, waxaan ka dhigi karnaa madoorsoomaha saamigalsanaanta K inuu noqdo 1. Marka S = $\frac{1}{1}$.gθ = gθ; taasoo haddaan tixgelinno goobo gacankeedu yahay 1, aan helayno in

S = $\frac{1}{1}$. θ ama S = θ , θ = S.

Haddaba, qaanso dhererkeedu yahay 1 oo ku taal goobo gacankeedu yahay 1 waxay leesha xagal cabbirkeedu yahay 1. Halbeggan cabbir xagleed waxaan nira, 1 gacansin oo loo qoro 1^G.

Ma raadin karta cabbirka gacansin ee xagasha 180° ah? Maaddaama xagashani ay leesho ½ goobada halbeeg ama qaanso dhererkeedu yahay halbeeg.

$$180^\circ = \pi^G$$

Taasi macneheedu waa: Cabbirka gacansiin ee xagasha 180° waa ^{G}. Haddaba

$$1^G = \frac{180^\circ}{\pi} \quad \text{ama} \quad 1^\circ = \frac{\pi^G}{180^\circ}$$

Haddaad isticmaashid saddexdan xiriir, waxaad cabbirka gacansiin u rogi kartaa darajooyin (digirii) darajocyinkana gacansiin .

Maaddaama π ≈ 3.1416, waxaad helaysaa in

$1^G = 57^\circ\ 18'$ iyo in $1^\circ \approx 0.01745^G$.

Tusaale 1: $\frac{1}{2}^G$ u rog cabbir darajo .

Furfuris:

$\frac{1}{2}^G = (\frac{1}{2} \cdot \frac{180^\circ}{\pi}) = \frac{90^\circ}{\pi} \doteq \frac{90^\circ}{3.1416} \doteq 28.65^\circ$, Jawaab

Tusaale 2 . 30° u rog cabbir gacansiin

Furfuris:

$30^\circ = (30 \cdot \frac{\pi}{180})^G = \frac{\pi}{6}^G \doteq \frac{3.1416}{6} = 0.5236^G$, JAWAAB

LAYLI

Keen cabbirka gacansiin ee mid kasta oo soo socda:

(B) 1. 45° 5. -330° 9. -75°
2. 90° 6. 450° 10. 270°
3. 300° 7. 450° 11. -360°
4. 180° 8. -15° 12. 210°

Adoo isticmaalaya $\pi \doteq 3.1416$, u rog cabbir kasta gacansiin boqoleed ee ugu dhow.

13. $11\frac{1}{2}^\circ$ 14. 90° 15. -160° 16. 430°

LEEB JOOMETERI - BUUG II -

LEEBAB

KALA GOYNTA LEEBABKA

Buuggii hore ayaan ku soo aragnay in labadii leeb (ama in ka badan) ee kasta wadartoodu ay^{u}dhiganto leeb kali ah; sida leeb wadareedka aan ku heli jirnayna waxay ahayd isugeynta xubnaha isku beegan ee biirooyinka. Waxa kale oo aan xusuusa-nahay in leeb kastaa uu leeyahay weydaar. Ka soo qaad in $\overrightarrow{A}(a,b)$ uu yahay leeb; $-\overrightarrow{A}$ waa weydaarka $\overrightarrow{A}$ xubnihiisuna waa -a iyo -b.

$\overrightarrow{A} + (-\overrightarrow{B})$ waa isugeynta leeb $\overrightarrow{A}$ iyo weydaarka B, waxana niraaha kala goynta leeb $\overrightarrow{A}$ iyo leeb $\overrightarrow{B}$ sida loo helo leeb u dhigmana waxa inuu sharxaaya qeexda soo socota:

QEEX: Faraqa laba leeb, $\overrightarrow{A}$ iyo $\overrightarrow{B}$ waxa lagu helaa faraqyada xubnahooda isu dhigma.

TUSAALE: Raadi faraqa $\overrightarrow{A} = (3,4)$ iyo $\overrightarrow{B} = (2,-3)$

FURFURIS:

$$\overrightarrow{A} - \overrightarrow{B} = (3,4) - (2,-3) = \big((3-2), 4 - (-31)\big)$$
$$= (1, 7)$$

LAYLI :

(1) Raadi leeb faraqa $\overrightarrow{A} - \overrightarrow{B}$, haddii

(a) $\overrightarrow{A} = (3,3)$, $\overrightarrow{B} = (-1,-2)$; (b) $\overrightarrow{A} = (0,-3)$, $\overrightarrow{B} = (4,-6)$

(t) $\overrightarrow{A} = (6,3)$, $\overrightarrow{B} = (-3,0)$; (j) $\overrightarrow{A} = (4,-1)$, $\overrightarrow{B} = (1,1)$

(x) $\overrightarrow{A} = (-2,-5)$, $\overrightarrow{B} = (8,1)$; (kh) $\overrightarrow{A} = (13,9)$, $\overrightarrow{B} = (20,-5)$

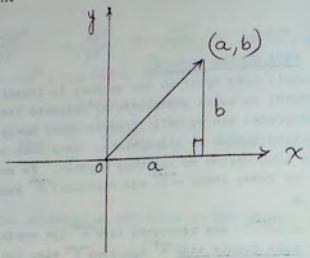
(d) $\overrightarrow{A} = (-4,0)$, $\overrightarrow{B} = (-6,-8)$; (r) $\overrightarrow{A} = (-20,-30)$, $\overrightarrow{B} = (-14,-8)$

(2) Raadi leeb faraqa $\overrightarrow{B} - \overrightarrow{A}$ ee layli 1

BAAXADDA LEEB

Joometeri ahaan leeb wax kale mahee, waa uun xarriijin jiho leh. Madaxa fallaartu wuxuu tilmaamayaa jihada, dheerarka xarriijintana waxa muujiya baaxadda leebka.

Bal aan qaadanno leeb $\overrightarrow{OD}$ oo bar billaawgiisu tahay unugga sallax kulan.



(Shaxan 1)

Leebka $\overrightarrow{OD}$ xubintiisa -x waaa a - 0 = a, xubintiisa -yna waa b-0 = b (Waxana samaysmaaya saddexagal quman oo lugihiisu yihiin a iyo b, OD-na tahay shakal).

Haddaba, maaddaama aan helnay saddexagal quman, aan isticmaalno aragtiinka "Betagooras" si aan ku helno dhererka $\overrightarrow{OD}$.

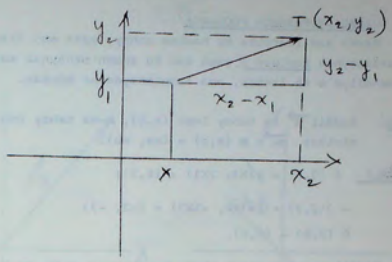
$$OD = \sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$$

Summadda, fogaanshaha ama dhererka leeb $\overrightarrow{OD}$ waa $/\overrightarrow{OD}/$ oo kali ah, maaddaama uu yahay leeb rug-beegal ah.

Guud ahaan, haddii bar billawgu ka duwan tahay unugga, lammaanayaasha horsan ee bar-billowga iyo bardhammaadkuna ay kala yihiin B (x_1, y_1), iyo T (x_2, y_2) sida ay u kala horreeyaan. Lammaanayaasha horsan ee $\overrightarrow{BT}$ waa $x_2 - x_1$, $y_2 - y_1$ (dheeho shaxan 2).

$$\therefore\ /\ BT\ / = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

.....jidka fogaanta



(Shaxan 2)

QEEX : Leebka dhererkiisu yahay 1, waxa la yira "leeb halbeeg ah".

TUSAALE I: Raadi baaxadda leeb rugeed .

FURFURIS: $\overrightarrow{A}(3,4)$..

$$/\overrightarrow{A}/ = \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{9 + 16}$$

$$= \sqrt{25} = 5$$

TUSAALE II: Raadi dhererka leeb $\overrightarrow{LM}$
haddii L = (5,2), M = (-3,8)

FURFURIS: $/\ LM\ / = \sqrt{(-3-5)^2 + (8-2)^2} = \sqrt{(-8)^2 + (6)^2}$

$$= \sqrt{64 + 36} = \sqrt{100} = 10$$

LAYLI :

1) Raadi dhererka leeb rugeedada soo socda:

$\overrightarrow{B}$: (1,1); $\overrightarrow{T}$:(4,0), $\overrightarrow{J}$ (3,-3), $\overrightarrow{K}$(-3, -2)

D : (4_1 - 4), $\overrightarrow{R}$: (0, -3), $\overrightarrow{W}$(12, 5), $\overrightarrow{M}$: (5,4)

2) Raadi dhererka leebabka soo socda, haddii bar billawyadoodu iyo bar dhammaadyadoodu ay yihiin sida ay u kala horreeyaan:

(b) (3,4) iyo (-3,-1); (t) (4,-1) iyo (0,3); (j) (1,-3) iyo (5,1); (x) (-3,-2) iyo (0,2) (kh) (30, 10) iyo (18,15); (d) (0,5) iyo (4,9) (r) (18,15) iyo (30,10); (s) (9,4) iyo (5,0).

ISKU DHUFASHADA FOOL-WAA :-

Marka aan leebabka ku hawlan nahay, waxa aan tirooyinka u qaadanaynaa foolwaa ; waxa aan ku aqoon sanaynaa xaraf yar oo weheliye u ah leebka, ama lammaanayaasha horsan.

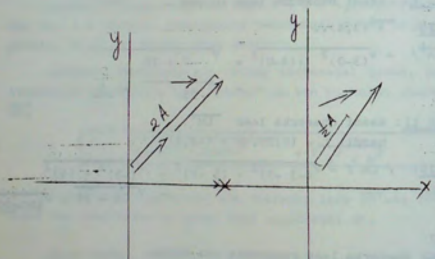
QEEX: Haddii $\vec{A}$ ay tahay leeb (a,b), m-na tahay foolwaa, markaas $\vec{mA}$ = m (a,b) = (ma, mb).

TUSAALE 2 (3,1) = (2X3, 2X1) = (6,2);

- 1(2,3) = (-1x2, -1X3) = (-2, -3)

0 (a,b) = (0,0).

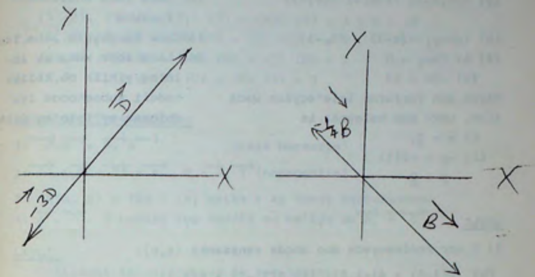
Haddii aan joometeri ahaan ku sharaxno isku dhufashada foolwaa markaxay m togan tahay, jihada leebku ma doorsoonto, hase yeeshee baaxaddiisu (dhererkiisu) waa ay isbeddeshaa, asaga oo fidaaya ama gaabanaaya.



(Shaxan 3)

Haddii ay m taban tahay, jihada leebka cusubi waxayay noqonaysaa lidka jihadii leebkii hore; baaxaddiisuna waa ay fidaysaa, ama yaraanaysaa.

(Fiiri shaxan 4)



(Shaxan 4)

Astaamaha isku dhufashada foolwaa

1. 1.$\vec{A}$ = $\vec{A}$.1 = $\vec{A}$
2. c (d $\vec{A}$) = (cd) $\vec{A}$, taasoo c iyo d ay yihiin foolwaa
3. c $(\vec{A}+\vec{B})$ = c $\vec{A}$ + c $\vec{A}$ + c $\vec{B}$
4. (c+d) $\vec{A}$ = c $\vec{A}$ + d $\vec{A}$
5. 0. $\vec{A}$ = 0
6. (-c) $\vec{A}$ = -c $\vec{A}$

TUSAALE I:

U qor foolwaayada soo socda qaabka (a,b), iyagaoo a iyo b yihiin tirooyin maangal ah.

a) 5(0.1) + (-2) (6,-3) (b) 2(-1,-2) + 6 (-3,0) + 0.(7,1)

FURFURIS

5(0,1) + (-2) (6, -3)
= (0,5) + (-12,+6) =(0-12,5+6)
= (-12,11), = JAWAAB

FURFURIS

2 (-1,-2) + 6(-3,0) + 0.(7,1)
= (-2,-4) + (-18,0) + (0,0)
= (-2-18, 0, -4 +0+0)
= (-20, - 4) JAWAAB

TUSAALE II: Raadi x iyo y

x(2,-3) +y(-1,0) = (0, -3)

FURFURIS

(1) X (2, -3) + y (-1,0) = (0,-3)
(2) (2x,1-3) +(-y,0) =(0,-3)

(3) (2x-y, -3x+0) = (0,-3)
(4) i) 2x-y = 0
ii) -3x = -3
Marka aan furfurno isle'egyada wada jira, waxa aan heleynaa in
i) x = 1,
ii) -y = -2(1)
y = 2

(1) Siin
(2) Qeex isku dhufasho foolwaa
(3)Qeex isugeynta laba leeb.
(4) Laba leeb waxa ay isle'eg yihiin oo keliya haddii xubnahooda isu dhigmaa ay isle'eg yiihin.

LAYLI

1) U qor foolwaayada soo socda sansaanka (a,b):

(b) 6(1,0) + 4(-2,5); (t) 8(1,-1) + 6(4,3)
(j) -2(7,11) +5(-3,6); (x) 4(-3,1) - 5(6,0)
(kh) -3(9,-2) +2(5,6); (d) 6 (1,8) - 3(12,0)

2) Sheeg qiimaha x iyo kan y ee weeraha soo socda run ka dhigaya:

b) x (-4, -8) + y (3,6) = (1,5)
t) -10(0,0) + 2 (x+y, x-y) = (10,6)
j) 3(x-1,½y)- (2x,4y) = (20,15)
x) 2(x, ½y) - 3(2x, ½ y) = (8,-10)

TARAN DHEXE

Taranta dhexe ama taranta bar waa "xisaab falka ku aaddiya labadii leeb ee kastaba foolwaa.

QEEX : Taranta dhexe ee labaleeb
$\overrightarrow{B}$: (b_1,b_2) iyo $\overrightarrow{T}$: (t_1, t_2) waxa lagu qeexaa inay tahay foolwaa $b_1t_1 + b_2t_2$.

Summadda tarantan waa bar u dhexaysa labada leeb, sida

$$B.\ T = (b_1,b_2) \cdot (t_1,t_2) = b_1t_1 + b_2t_2$$

TUSAALE

1. (3,-2). (1,4) = (3,) (1) + (-2) (4) = 3 - 8 = -5
2. (5,2) . (1,1) = (5) (1) + (2) (1) = 7
3. (-4,1) . (0,0) = (-4) (0) + (1) (0) = 0
4. (1,0) (0,1) = (1) (0) + (0) (1) = 0

Astaamaha TarantaDhexe

1) $\overrightarrow{A}.\overrightarrow{B} = \overrightarrow{B}.\overrightarrow{A}$ (kala hormarin)
2) $\overrightarrow{A}.(\overrightarrow{B} + \overrightarrow{T}) = \overrightarrow{A}.\ B + \overrightarrow{A}.\overrightarrow{T}$ (hormogalin)
3) k(A. B) = (kA) .B, halka k ay tahay madoorsoome.
4) $\overrightarrow{A}.\overrightarrow{A} = 0$ haddii iyo haddii oo keliya oo $\overrightarrow{A} = \overrightarrow{0}$

LAYLI

Xisaabi tarantadhexe ee soo socta:

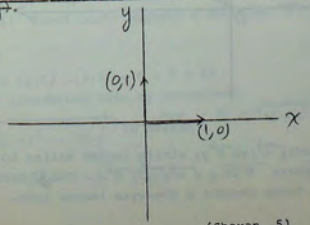
b) (2,1). (1,2); (t) (6,-2). (-2,0); (j) (1,3).(2,3);
x) (4,-1).(-2,-1); (kh) (3,0) .(4,1): (r) (4,-2).(3.-5)
s) (6,1).(0,0); (sh) (3,1).(-1,-1) (dh) (3,4).(0,0)
c) [3(4,1). (2,2)]; (g) (3,2).(4,1) + (1,1)

GUNDHIG BEEGGALKA LEEBABKA

Ka soo qaad in $\overrightarrow{B}$ = (1,0) , $\overrightarrow{T}$ = (0,1), markaas, $/\overrightarrow{B}/$ = $\sqrt{(1-0)^2 + (0-0)^2}$ = 1, sidoo kale $/\overrightarrow{T}/$ = 1

Hore ayeynu u soo aragnay in ay leebabka jaadkaas ahi yihiinohalbeegyo, waxase soo korodhay in leeb kastaa (a,b) = a(1,0) + b(0,1) maaddaama a (1,0) + b(0,1) = (a,0)+(0,b) = (a,b)

Leebabka (1,0) iyo (0,1) waxa la yiraa "Gundhig beeggalka ururka leebabka laba addimaale" summaddiisuna waa: (1,0) = $\overrightarrow{i}$, (0,1) = $\overrightarrow{j}$.



Haddaba, leeb kasta waxa aan ku tibaaxi karna leebabkan gundhigga ah.

TUSAALE: - $\overrightarrow{A}$: (a,b) = $a\overrightarrow{i} + b\overrightarrow{j}$

$\overrightarrow{B}$: (c,d) = $c\overrightarrow{i} + d\overrightarrow{j}$

Bal hadda aan fiirino sida ay noqonayso taranta dhexe ee leebab gundhig ah.

1) $\overrightarrow{i}.\overrightarrow{j} = \overrightarrow{j}.\overrightarrow{i} = (1,0) . (0,1) = 0 + 0 = 0$

2) $\overrightarrow{i}.\overrightarrow{i} = (1,0) . (1,0) = 1 + 0 = 1$

3) $\overrightarrow{j} . \overrightarrow{j} = (0,1) . (0,1) = 0 + 1 = 1$

Haddaba, $\overrightarrow{i}.\overrightarrow{i} = 1$

$\overrightarrow{j}.\overrightarrow{j} = 1$

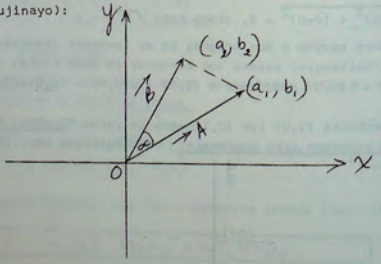
$\overrightarrow{i}.\overrightarrow{j} = 0$

Taranta Dhexe ee Sallax ku taal:-

Ka soo qaad in leebabka A : (a_1, b_1) iyo $\overrightarrow{B}$: (a_2, b_2) ay yihiin rug beeggal, markaas taran dhexeda

A . B = $(a_1 m\ b_1) . (a_2, b_2) = a_1a_2 + b_1b_2$

Bal aan dhisno saddexagalka OAB(Sida Shaxan 6 uu muujinayo):



(Shaxan 6)

ARAGTIIN: Haddii $\overrightarrow{A}$ iyo $\overrightarrow{B}$ ay yihiin leebab sallax ku yaal, markaas A .B = $/\overrightarrow{A}/./\overrightarrow{B}/$ Cos α, ayada oo α ay tahay xagasha u dhexaysa labada leeb.

CADDAYN

1. $/\overrightarrow{AB}/^2 = /\overrightarrow{OA}/^2 + /\overrightarrow{OB}/^2 - 2/\overrightarrow{OA}$. /OB/ Cosd (xeerka Kosayn)

2. $/AB/^2 = (a_2 - a_1)^2 + (b_2 - b_1)^2$ (jidka fogaanta)

$= a_2^2 - 2a_2 a_1 + a_1^2 + b^2 - 2b_2b_1 + b_1^2$

$= (a_1^2 + a_2^2) + (b_1^2 + b_2^2) - 2(a_1b_2 + a_1b_2)$

3. $/OA/^2 = \left(\sqrt{(a_1 - 0)^2 + (b_1 - 0)^2}\right)^2 = a_1^2 + b_1^2$

$/OB/^2 = \left(\sqrt{(a_2 - 0)^2 + (b_2 - 0)^2}\right) = a_2^2 + b_2^2$

4. $/\overrightarrow{OA}/^2 + /OB/^2 - 2.\ \overrightarrow{OA} . \overrightarrow{OB} = /\overrightarrow{OA}/^2 + /OB/^2 - 2\ /\overrightarrow{OA}/./\overrightarrow{OB}/\text{Cos}\alpha$

(Ku beddel talaabada (2) $a_1^2 + b_1^2$, $/OA/^2$; $(a_2^2 + b_2^2)$ ku beddel $/OB/^2$; $\overrightarrow{OA}.\overrightarrow{OB}$ waa la mid $\overrightarrow{A}.\overrightarrow{B}$)

5. $-2.\overrightarrow{OA}.\overrightarrow{OB} = -2\ /\overrightarrow{OA}/\ .\ /\overrightarrow{OB}/\text{Cos}\alpha$ (isu bixi tibxaha isle'eg ee laba dhinac ku jira)

6. $\overrightarrow{OA}.\overrightarrow{OB} = /\overrightarrow{OA}/\ .\ /\overrightarrow{OB}/\ \text{Cos}\alpha$ (isu bixi tibxaha isle'eg ee laba dhinac ku jira.

ama

:. $\boxed{A . B = /A/ . /B/ \text{Cos}\alpha}$

W.D.M.

Jidkan wuxu furayaa si fudud oo loo helo xaglaha u dhexeeya leebab, maadaama

$$\boxed{\text{Cos} = \frac{\overrightarrow{A}.\overrightarrow{B}}{/\overrightarrow{A}./\overrightarrow{B}/}}$$

TUSAALE I: Raadi koska xagasha u dhexaysa $\overrightarrow{A}$: (2,1) iyo $\overrightarrow{B}$:(3,6).

FURFURIS:

$\overrightarrow{A}.\overrightarrow{B} = (2,1) .(3,6) = 6 + 6 = 12$

Dhererka leebabkuna waxa ay noqonayaan

$/A/ = \sqrt{(2-0)^2 + (1-0)^2} = \sqrt{5}$; $/B/ = \sqrt{(3-0)^2 + (6.0)^2} = \sqrt{45}$

:. $\text{Cos} = \frac{12}{\sqrt{5}.\sqrt{45}} = \frac{12}{\sqrt{255}} = \frac{12}{15} = \frac{4}{5}$

TUSAALE II. Raadi xagasha u dhexaysa $\overrightarrow{A}$: (1,1) iyo $\overrightarrow{B}$: (0,1)

FURFURIS:

$\overrightarrow{A} . \overrightarrow{B} = (1,1) . (0,1) = 0 + 1 = 1$

FURFURIS:

$\overrightarrow{A} . \overrightarrow{B} = (1,1) . (0,1) = 0 + 1 = 1$

$/ A / = \sqrt{1^2 + 1^2} = \sqrt{2}$

$/ B / = \sqrt{0^2 + 1^2} = \sqrt{1} = 1$

$\text{Cos } \alpha = \frac{1}{\sqrt{2} \cdot 1} = \frac{1}{\sqrt{2}} \approx \frac{1}{1.414} \approx 0.707$

Tusaha trig. waxan ka heleynaa in 0.707 ay tahay Cos 45°. Haddaba $\alpha = 45^\circ$.

Xigasho: Haddii $/ \overrightarrow{A} / / \overrightarrow{B} / \text{ Cos } \alpha = 0$, markaas ugu yaraan weeraha soo socda mid baa run ah: $/ \overrightarrow{A} / = 0$, $/ \overrightarrow{B} / = 0$ ama Cos $\alpha = 0$. Maaddaama A iyo B aanay ahayn leeb-ebero, waxa markaas cad in Cos $\alpha = 0$. Haddaba $\alpha = 90^\circ$.

$\overrightarrow{A}$ iyo $\overrightarrow{B}$ waa ay isku qotomaan.

Summad ahaan : $\overrightarrow{A} \perp \overrightarrow{B}$, haddii iyo haddii oo keliya oo ay $\overrightarrow{A} . \overrightarrow{B} = 0$.

TUSAALE II:- Kala sheeg in leebabkan, $\overrightarrow{A}$: (.1,2) iyo $\overrightarrow{B}$: (2,1) iyo in kale.

FURFURIS: Waxa aan raadinayna Coska xagasha u dhexaysa labada leeb. Haddii uu yahay eber, markaas waa ay isku qotomaan.

$\text{Cos } \alpha = \frac{A \cdot B}{/ \overrightarrow{A} / . / \overrightarrow{B} /}$; $\overrightarrow{A} . \overrightarrow{B} = (-1,2) . (2,1) = -2+2 = 0$

$\text{Cos } \alpha = \frac{0}{/ \overrightarrow{A} / . / \overrightarrow{B} /} = 0 \quad \therefore \alpha = 90^\circ, \quad \overrightarrow{A} \perp \overrightarrow{B}$

TUSAALE IV. Raadi taranta dhexe ee leebabka $-3 \overrightarrow{v} - 4 \overrightarrow{j}$ iyo $3 \overrightarrow{n} + 4 \overrightarrow{j}$.

FURFURIS:

Waxa aan u dhigi karna leebabkan sansaan lammaanayaal horsan.

$-3\overrightarrow{i} - 4\overrightarrow{j} = (-3,-4)$; $3\overrightarrow{i} + 4\overrightarrow{j} = (3,4)$

Tarantooda dhexena waa

$(-3, -4) . (3, 4) = -9 - 16 = -25$

LAYLI

1. Raadi taranta dhexe ee lammaanayaalka leeb ee soo socda:

a) $5\overrightarrow{i} - 5\overrightarrow{j}$ iyo $0\overrightarrow{i} + 3\overrightarrow{j}$; (d) 2i- 6j iyo 5i + 7j

b) $3\overrightarrow{i} + \overrightarrow{j}$ iyo $-\overrightarrow{i} + 3\overrightarrow{j}$; (e) 10i + 4j iyo 12i - 13j

c) $-2\overrightarrow{i} + 0\overrightarrow{j}$ iyo $4\overrightarrow{i} + 3\overrightarrow{j}$. (f) 11i + 10j iyo 20j + 2j

2. Raadi kosaynka xagasha u dhexaysa labadii leeb ee kasta.

b) $\overrightarrow{B}$: (1,0); $\overrightarrow{T}$: (4,3); (t) $\overrightarrow{B}$: (-1, -2); $\overrightarrow{T}$: (3,6);

j) $\overrightarrow{B}$: (4,0), $\overrightarrow{T}$: (-4,4); (x) $\overrightarrow{B}$: (3,3), T: (2,-2)

3. Labadeebaa isku qotoma leebabkan?

b) (3,1) iyo (1,3); (t) (4,0) iyo (0,2)

j) (0,0) iyo (6,3); (x) (-5,-2) iyo (4,10)

(kh) (12, 5) iyo (0,3), (d) (-13, -5) iyo (17,6)

ASTAAMAHA GUNDHIGGA U AH TIRADA

Haddii A iyo B ay yihiin ururo kooban, markaa, n(AXB) = n(A). n(B), macnee tirada kutirsanayaasha taranta Kaartis ee A iyo B waa taranta tirada kutirsanayaasha A iyo B kuwa B.

Matalan 5 wado-baabuur (urur A) ayaa isku xira magaalada R iyo ta M, weliba 2 wado-baabuur ayaa isku xira magaalada M iyo ta S (urur B). Markaa ku tirsanayaasha A **mid** kastaa waxa uu leeyahay 2 ah kutirsanayaasha B. Wadarta wadooyinka suuragalka ah ee qof mari karaana waa n(AXB) =

$$n(A).n(B) = 5.2 = 10$$

QEEX: Raabaqaadka urur A waa horsiimeynta (Kowaad, Labaad, Saddexaad, ...) kutirsanayaasha A.

ARAGTIIN: ka dhig Bn,n tirada raabaqaadyada kala gedisan ee ururka A, oo n(A) = n markaa Bn, n = n!

sumadda Bn,n waxa loo akhriyaa, "tirada raabqaadyada n walaxood oo marba la isku qaadey n".

CADDEYN : Ka dhig A_1, ururka xulashooyinka koowaad ee suuragalka ah. Kolkaa A_1 = A, dabadeedna $n(A_1) = n(A)$. ka dhig A_2 ururka xulashooyinka Labaad ee suurogalka ah. Markaa, A_2 CA dabeedna $n(A_2) = n(A_1) - 1 = n - 1$, Habkaas oo la sii wado oo weliba lala kaalmeysto astaanta tirsiimo n(AXB) = n(A).n(B) waxa ay inoo horseedeysaa in

$$Bn,n = n(A_1) .n(A_2).n(A_3)...n(A_n)$$
$$= n(n-1) (n-2) \dots 1 = n!$$

TUSAALE: Imisa siyood ayaa 5 ciyaaryahan loo kala siin karaa rugo (positions), si ay u sameeyaan koox ciyaarta kubadda koleyga?

FURFURIS: Ka dhig ururka ciyaaryahanada A, markaa n(A) = 5. Wadarta siyaabaha 5 nin loo kala siin karo 5 ruggood waa,

$$B_{5,5} = 5! = 5.4.3.2.1 = 120$$

ARAGTIIN: Ka dhig $B_{n,r}$ tirada raabaqaadyada ururka A $\left[n(A) = n\right]$ oo marba la isku qaadey r; macnee, $B_{n,r}$ waa tirada horsiimooyinka kala gedisan ee r kutirsane marka aynu keysano ururka A oo n kutirsane leh.

Markaa $B_{n,r} = n(n-1) (n-2) \dots [n-(r-1)]$

$$= n(n-1) (n-2) \dots (n-r+1).$$

TUSAALE: Imisa siyood ayaa loo sameyn karaa koox ciyaartooy ah oo tiradoodu dhan tahay 5 haddii ururka aan kala baxeynaa uu ka kooban yahay 10 ciyaar-yahan?

FURFURIS: Ka dhig A ururka ciyaaryahanada, markaa n (A) = 10

$$B_{10,5} = 10.9.8 \dots (10 - 5 + 1) =$$
$$10.9.8.7.6 = 30246$$

$B_{n,r}$ waxa kale oo loo qori karaa sidan:

$$B_{n,r} = n(n-1) (n-2)\dots(n-r+1)$$
$$= n(n-1) \dots(n-r+1) (n-r)!$$

$$B_{n,r} = \frac{n!}{(n-r)!}$$

Weydiinta ah "doon tirada raabaqaadyada kala gedisan ee n walaxood oo marba la isku qaadey n, haddii walaxaa qaarkood ay midaalsn yihiin", waxa ay u baahan tahay saafid. Tusaale ahaan, tixgeli tirada raabaqaadyada xarfaha ereyga KACAAMEYN saddexda "A" aan kala siino hoosgalayaal si aynu u heysano 9 xaraf oo kala gedisan.

K, A_1, C, A_2, A_3, M, E, Y, N

Tirada raabaqaadyada 9kaa xaraf waa 9! . Haddii xarfaha aan ahayn A_1, A_2 iyo A_3 lagu ilaaliyo meesha ay joogaan, A_1, A_2 iyo A_3 waxa dhexdooda lagu sameyn karaa 3! raabaqada. Haddii B ay tahay tirada raabaqaadyada kala gedisan ee xarfaha K, A, C, A, A, M, B, Y, N, oo isla markaa raabaqaad kastaa uu leeyahay 3! siyood oo I-yada loo horsiimeyn karo, markaa:

$$3 ! B = 9 !$$
$$B = \frac{9 !}{3 !}$$

TUSAALE: Tixgeli xarfaha ereyga "MAMMAL" waxa jiri lahaa 6! raabaqaad oo kala gedisan haddii xaraf kastaa uu ka gedisan yahay midka kale, laakiin xarfaha M iyo A midina 3 jeer ayey

ereyga ku jirtaa, midina 2 jeer.

:. 3! 2! B = 6 !

$$B = \frac{6\ !}{3!2\ !} = \frac{6.5.\overset{2}{\cancel{4}}.\overset{1}{\cancel{3}}.2!}{2.1.\underset{1}{\cancel{3}}.\underset{1}{\cancel{2}}!} = 60$$

LAYLI:

TUSAALE: Haddii A = $\{a,b,c\}$, B = $\{c,d\}$,

doon n (AUB), n(A∩B), n (AXB)

FURFURIS:: AUB = $\{a,b,c,d\}$. :. n(AUB) = 4

A B = $\{ c \}$. :. n (A∩B) = 1

A X B = $\{ (a,c), (a,d), (b,c),(b,d),(c,c),(c,d)\}$

:. n (AXB) = 6

1. (b) A = $\{ d, e \}$, B = $\{ e, f, g, h \}$
   (t) A = $\{ e \}$, B $\{ a, b, c, d \}$
   (j) A = $\{ 1, 2, 3 \}$, B = $\{ 3, 4, 5, 6 \}$
   (x) A = $\{ 1, 2 \}$, B = $\{ 3, 4, 5 \}$
   (kh) A = $\{ 1, 2 \}$, B = $\{ 1, 2 \}$
   (d) A = Ø , B = $\{ 2, 3, 4 \}$

2. Imisa astiro oo kala gedisan oo midiba tahay laba god ayaa laga sameyn karaa astirooyinka 5 iyo 6?

3. Imisa astiro oo kala gedisan oo midiba tahay laba god ayaa laga sameyn karaa astirooyinka 7, 8, 9?

4. Doon tirada raabaqaadyada kala gedisan ee xarfaha ereyga(i) LIMIT (ii) Soomaaliya (iii) Jabuuti.

RACAYMO

Inta aynaan u tegin "Itimaal" waxa aynu u baahan nahay xeer tiro oo kale oo la yiraahdo, doonidda tirada hormooyinka r-kutirsane leh ee kala geidsan ee urur n-kutirsane leh.

QEEX: Hormada r-kutirsane leh ee ururka n-kutirsane leh ayaa la yiraahaa racayn.

Markaa racayni waa urur walaxo ah oo horsiimadu aanay muhiim ahayn, Matalan, haddii afarta xaraf ee a,b,c,d, aynu ka doorano kooxo midiba saddex xaraf tahay, waxa aynu heli 4 kooxood oo kala ah abc, acd, abd, bcd.

Tirada racaymuhu waxa ay ku xiran tahay tirada raabaqaadyada . Waxa aynu haqaan in tirada raabaqaadyada ururka A $\lceil n(A) = n \rceil$, marka marba la isku qaado r ay tahay:

$B_{n,r} = \frac{n!}{(n-r)!}$. Adiga oo taas madaxa ku haya, tixgeli aragtiinkan soo socda.

ARAGTIIN: Ka dhig $\binom{n}{r}$ tirada racaymaha kala gedisan ee kutirsanayaasha ururka A oo ka kooban n walaxood $\lceil n(A) = n \rceil$, oo hadba la isku qaaday r, markaa

$$\binom{n}{r} = \frac{B_{n,r}}{r\ !} = \frac{n\ !}{r!(n-r)!}$$

TUSAALE: Imisa siyood ayaa guddi 5 qof ah looga dooran karaa urur 12 qof ah?

FURFURIS:

Waxa aan rabnaa waa tirada hormooyinka midiba 5 kutirsane leedahay ee ururka 12ka kutirsane leh; kolkaa

$$\binom{12}{5} = \frac{12\ !}{5!7!} = \frac{12.\ 11.\ 10.\ 9.\ 8.\ 7\ !}{5!}$$

$$= \frac{12.\ 11.\ 10.\ 9.\ 8}{5.\ 4.\ 3.\ 2.\ 1} = 792$$

Maadaam tirooyinka $\binom{n}{r}$ ay yihiin weheliyayaasha fidinta $(a+b)^n$ oo weliba weheliyayaashaasi wanqaaran yihiin (Symmetric), waxa aynu dheegi karaa aragtiinkan soo socda:

ARAGTIIN: $\binom{n}{r} = \binom{n}{n-r}$

CADDEYN: Waxa aynu ognahay in $\binom{n}{r} = \frac{n\ !}{r\ !\ (n-r)\ !}$

iyo in $\binom{n}{n-r} = \frac{n\,!}{(n-r)\,!\,[n-(n-r)]\,!} = \frac{n\,!}{(n-r)!r!}$

$\therefore \binom{n}{r} = \binom{n}{n-r}$.

LAYLI:

Qiimee:

1. (i) 5 ! (ii) $\frac{6\,!}{3\,!}$ (iii) $\binom{5}{3}$ (iv) $\binom{5}{2}$ (v) $\binom{8}{4}$
(vi) $\frac{8\,!}{5\,!}$ (vii) $B_{4,4}$ (viii) $B_{7,4}$ (ix) $\binom{n}{3}$.

2. Doon qiimaha x: (i) $\binom{x}{1} = 3$ (ii) $\binom{x}{2} = 1$

(iii) $\binom{2x}{2} = 3$ (iv) $B_{x,2} = 3$

MUUNAD DULALAATI IYO WAQDHACYO

Marka la sameeyo tijaabo, tijaabadaa waxa la xiriira urur ah natiijooyinka suuragalka ah. Matalan marka laadhuu la tuuro, waxa ay istaagi doontaa iyadoo astirooyinka 1,2,3,4,5,6 midi uun ay sarreyso.

Qeex: Ururka ka kooban dhamaan natiijooyinka suuragalka ah ee tijaabo ayaa la yiraahaa Muunad Dulalaatiga tijaabo.

QEEX: Kutirsane kasta oo ka mid ah muunad dulalaatiga waxa la yiraahaa natiijo ama bar-muunadeed.
Waxa suuroobi karta in tijaabo ay yeelato muunad dulalaatiyo fara badan. Tixgeli sanduuq ay ku jiraan kubbado yaryari; kubbadahaa qaar ka mid ahi waxa ay ka sameysan yihiin quraarad, inta kalena waxa ay ka sameysan yihiin caag. Nooc kasta, qaarna waa gaduud, qaarna waa cagaar. Haddii aynu haddaba sameyno tijaabo ah "kubbad ka soo saar sanduuqa" waxa laga yaabaa in aynu u jeedno waxyaabahan soo socda miduun:

b) Sheyga ay kubbadu ka sameysan tahay:
Markaa haddii q ay ka taagan tahay quraarad, c-na ay ka taagantey caag. Muunad dulalaatigeenu waxa weeye $\{q,c\}$.

(t) Midabka kubbadda:

Marka muunad dulalaatigeenu waa

$\{q,c'\}$ oo q ay ka taagan tahay gaduud, c'-na cagaar.

(j) Midabka iyo sheyga ay ka sameysan tahay, Labadaba.

Markaa, haddii q,c,g,c' ay yihiin waxa aynu ku soo sheegnay, muunad dulalaatigeenu waa;

$\{(q,g), (q,c'), (c,g), (c,c')\}$

QEEX: Hormooyinka muunad dulalaatiga mid kasta waxa la yiraahaa WAQDHAC, waxana badanaaba lagu tilmaansadaa xarfka W.

Marka aynu tijaabo sameyno, waxa laga yaabaa in aynu madaxa ku heyno urur natiijooyin ah oo aynaan rabin natiijooyin keli keli ah (individual outcomes).
Matalan marka la tuuro laadhuu, haddii muunad dulalaatiga loo qaato $\{1,2,3,4,5,6\}$, kolkaa waqdhaca tilmaamaya abyoone kisi ah waa ururka $\{1,3,5\}$.
Labadan waqdhac oo kale waxa la yiraahaa WAQDHACYO DULEEDINSAN.
Tirada waqdhacyada suuragalka ah ee ku jira muunad dulalaati n kutirsane lihi waa tirada hormooyinka suuragalka ah ee urur n kutirsane leh ama 2^n; macnee

$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n} = 2^n$
oo $\binom{n}{o}$ ay tahay waqdhaca $\emptyset$.

LAYLI:

(1) Laadhuu baa la tuuray. Tax muunad dulalaatiga.
Tax waqdhaca ah "astirada ay u dhacday waa ay ka weyntey 2 ".

(2) Kuumi baa la tuuray. Tax muunad-dulalaatiga. Tax waqdhaca ah daabac ayuu u dhacay.

(3) Laba kuumi ayaa la tuuray. Tax muunad dulalaatiga. Tax waqdhaca ah in ay u dhacayaan laba daabac.

(4) Laba kuumi baa la tuuray. Tax muunad dulalaatiga.
Tax waqdhaca ah in ay u dhacayaan ama laba dur ama laba daabac.

(5) Imisa waqdhac ayaa ku jira muunad-dulalaatiga $\{1,2,3\}$?

(6) Imisa waqdhac oo midkiiba ugu yaraan leeyahay hal kutirsane ayaa ku jira muunad dulalaatiga $\{1,2,3,4,5,6\}$?

FANSAARADA ITIMAAL

Marka aynu tijaabo sameyno, natiijooyinka tijaabadaas waxa aynu niraahaa waa natiijooyin siman haddii itimaalkoodu is wada le'eg yahay. Matalan marka aan tuuro laadhuu, natiijooyinka 1,2,3,4,5,6 haddii ay siman yihiin mid kasta itimaalkiisu waa $\frac{1}{6}$.

QEEX: Ka dhig M muunad dulalaati ka kooban natiijooyin siman, H-na ka dhig fansaar maangal ah oo horaadkiisu yahay dhammaan waqdhacyada $W \subseteq M$, danbeedkiisuna yahay $\{ y \in M \ / 0 \leq y \leq 1 \}$. Markaa H waa fansaar itimaal haddii iyo haddii oo keliya oo xaaladahan soo socdaa ay rumoobaan.

(1) $H(W) \geq 0$, waqdhac kasta $W \subseteq M$

(2) $H(M) = 1$

(3) Haddii $W_1 \cap W_2 = \emptyset$, markaa,

$$H(W_1 \cup W_2) = H(W_1) + H(W_2).$$

QEEX: Itimaal Waqdhac: Ka dhig M muunad dulalaati kooban oo kutirsanayaashiisu ay yihiin natiijooyinka siman ee tijaabo. Ka dhig W waqdhac ku jira M, markaa itimaalka W,

$$H(W) = \frac{n(W)}{n(M)} .$$

TUSAALE (1): Haddii laadhuu la tuuro, waa maxay itimaalka ay u dhici karto tiro dhaban ahi?

FURFURIS: $H(W) = \frac{n(W)}{n(M)}$;

$W = \{2, 4, 6\}$

$\therefore n(W) = 3$

$M = \{1, 2, 3, 4, 5, 6\}$

$\therefore n(M) = 6$

Markaa $H(W) = 3/6 = \frac{1}{2}$.

TUSAALE (2) 9 xaashadood oo yaryar ayaa lagu kala qoray astirooyinka 1 ilaa 9, markaasaa la baandheeyey; kadibna waxa laga saarey mid. Waa maxay itimaalka ay xabada la saarey ku noqon karto mid ay tiro dhabani ku qoran tahay?

FURFURIS: $H(W) = \frac{n(W)}{n(M)}$

$W = \{2, 4, 6, 8\}$

$\therefore n(W) = 4$

$M = \{1, 2, 3 \ldots 9\}$

$\therefore n(M) = 9$

Markaa $H(W) = 4/9$.

Haddii ashuun ay ka buuxaan kubbado cagaarani, itimaalka lagaga soo saarayo kubbad cagaarani waa 1. Maxaa yeelay $W = M$, $\therefore n(W) = n(M)$,
dabeedna,

$$H(W) = \frac{n(W)}{n(M)} = 1.$$

Markaas oo kale waxa aynu niraahnaa dhicitaanka waqdhaca W waa mid la hubaa (certain). Haddii se aan damacno in aan kubbad cad ka soo saaro ashuun ay ka buuxaan kubbado cagaarani, itimaalka aynu kaga soo saareynaa waa 0. Maxaa yeelay $W \neq \emptyset$, $\therefore n(W) = 0$,

Kolkaa $H(W) = \frac{n(W)}{n(M)} = \frac{0}{n(M)} = 0$

Markaas oo kalena waxa aynu niraahnaa dhicitaanka waqdhaca W waa mid aan suuragal ahayn (impossible).

ARAGTIIN:- Haddii W ay tahay waqdhac kasta $0 \leq H(W) \leq 1$.

ARAGTIIN: Haddii $\overline{W}$ ay tahay duleedka W, markaa $H(\overline{W}) = 1 - H(W)$.

TUSAALE 1) Haddii itimaalka uu dagaal uga dhici karo Bariga Dhexe uu yahay 3/7. Waa maxay itimaalka aanu dagaal uga dhici karini?

FURFURIS: Itimaalka aanu dagaal uga dhici karin Bariga Dhexe waa $1 - 3/7 = 4/7$.

TUSAALE 2: Haddii itimaalka uu roob ku da'ayo 1-da Maajo uu yahay 1/7, waa maxay itimaalka aanu roob ku dii' doonini?

FURFURIS: Itimaalka aanu roob ku dii' doonini waa
1 - 1/7 = 6/7.

WAQDHACYO MASIYAAB

QEEX:- Labada waqdhac W_1 iyo W_2 oo ku jira muunad dulalaati waxa ay yihiin waqdhacyo masiyaab, haddii dhicitaanka mid aanu wax raad ah ku lahayn dhicitaanka ka kale. Haddii laba waqdhac aanay masiyaab ahayn, waxa la yiraahaa waqdhacyo siyaab.

ITIMAALKA WAQDHACYO MASIYAAB

Itimaalka ay laba waqdhac oo masiyaab ahi iskaga daba dhici karaan waa taranta itimaalada ay mid kastaa ku dhici karto. Haddii $H(W_1)$ ay tahay itmaalka waqdhaca W_1 ay ku dhici karto, isla markaasna

$H(W_2)$ ay tahay itimaalka waqdhaca W_2 ay ku dhici karto, markaas;

$H(W_1$ iyo $W_2) = H(W_1).H(W_2)$, bishardi in W_1 iyo W_2 ay yihiin waqdhacyo masiyaab. Matalan itimaalka, H(b) ee Cali uu imtixaanka xisaabta ku liibaani karaa waa 3/10. Itimaalka H(s) ee cnasha ay imtixaanka af Soomaaliga ku liibaani kartaana waa $\frac{8}{10}$. Markaa maadaam

$$H(b) = 0.3,\ H(s) = 0.8$$

Itimaalka, H(b+s) ee labada waqdhacba ku dhici karaan waa:

$$H(b+s) = H(b).H(s) = (0.3)\ (0.8) = 0.24$$

Ama, haddii laba kuumi la tuuro (iskumar ama la iska daba tuuro) itimaalka lagu heli karo daabac iyo dur waa 1/4. Itimaalka kuumiga hore uu ugu dhici karo daabac, H(d) waa ½. Itimaalka kuumiga danbe ugu dhici karo dur H(t) waa ½.

$\therefore H(d).H(t) = ½.½ = 1/4.$

TUSAALE 1

Sanduuq ayey ku jiraan 4 kubbadood oo cagaaran iyo 2 cadcadi. Haddii kubbad laga soo saaro oo lagu celiyo, oo haddana kubbad labaad laga soo saaro, waa maxay itimaalka ay kubbadii hore ku noqon karto cagaar tii danbena caddaan?

FURFURIS: Itimaalka lagu soo saari karo kubbad cagaarani, H(c), waa 2/3. Itimaalka lagu soo saari karo kubbad caddina, H(c'), waa ⅓. Itimaalka lagu soo saari karo kubbad cagaaran iyo kubbad caddi waa

$$H(c).\ H(c') = 2/3.\ ⅓ = 2/9.$$

TUSAALE 2:

Itimaalka nin 40 jir ku gaadhi karo 96 waa 0.005. Itimaalka naagtiisu oo 36 jiri ay ku gaadhi karto 96-na waa 0.005. Waa maxay itimaalka ay ninka iyo naagtuba ku gaadhi karaan 96?

FURUFIRIS:

Itimaalka uu ninku ku gaadhi karo 96 waa H(M) = 0.005. Itimaalka ay naagtu ku gaadhi karto 96 waa H(n) = 0.005
$H(m+n) = H(m).H(n) = (0.005)\ (0.005) = 0.000025.$

LAYLI

1. Haddii laba kuumi la tuuro, waa maxay itimaalka lagu heli karo 2 daabac?
   Laba dur? daabac iyo dur(labada horsiimada)?
2. Haddii kuumi la tuuro 3 jeer, waa maxay itimaalka lagu heli karo daabac labada jeer ee hore.
3. Haddii 3 kuumi la tuuro, waa maxay itimaalka lagu heli karo 2 daabac iyo hal dur (ha tixgelin horsiimada)?
4. Haddii laadhuu la tuuro 2 jeer, waa maxay itimaalka ay marka hore ugu dhacayso 3, marka danbena 6?
5. Haddii laba laadhuu la tuuro laba jeer, waa maxay itimaalka lagu heli karo 7 iyo 11?

WAQDHACYO KALA EDEG AH

QEEX: Haddii laba (ama in ka badan) waqdhac aanay wada dhici karin mar keliya waxa la yiraahaa waqdhacyo kala edeg ah. Waqdhacyada W_1 iyo W_2 waa kala edeg haddii

$$W_1 \cap W_2 = \emptyset .$$

ARAGTIIN: Haddii waqdhacyada W_1 iyo W_2 ay kala edeg yihiin, markaa $H(W_1 \cap W_2) = H(W_1) + H(W_2)$.

TUSAALE: Ashuun ay ku jiraan 8 kubbadood oo cas, 4 cagaaran iyo 3 madoobe ayaa laga soo saarey kubbad. Waa maxay itimaalka kubbaddaas la soo saaray ay ku noqon karto madow ama casaan?

FURFURID:

Ka dhig W_1 waqdhaca ah "soo saaridda kubbad cas", W_2-na "soo saaridda kubbad madow".

Maadaam aynaan soo saari karin kubbad casaan iyo madow wada ah, W_1 iyo W_2 waa waqdhacyo kala edeg ah.

Muunad dulaalaatiga waxa ku jira

4 + 8 + 5 = 17 natiijo. Markaas
$H(W_1) = 8/17$, $H(W_2) = 5/17$.

Waqdhaca aynu rabnaa waa $W_1 \cup W_2$ oo ah wadarta kutirsanayaasha W_1 iyo W_2.
Markaa $H(W_1 \cup W_2) = \frac{8+5}{17} = \frac{13}{17}$

F.G (fiiro gaar ah) $H(W_1 \cup W_2) = \frac{8+5}{17} = \frac{8}{17} + \frac{5}{17} = H(W_1) + H(W_2)$

LAYLI:

Ashuun ayaa waxa ku jira 16 kubbadood, 9 cas, 5 madow iyo 2 cagaaran. Haddii kubbad laga soo saaro, waa maxay itimaalka ay ku noqoneyso.

(1) Casaan
(2) Madow
(3) Casaan ama madow
(4) Cagaar
(5) Madow ama cagaar
(6) Casaan ama cagaar

(7) Haddii la tuuro laadhuu, waa maxay itimaalka lagu heli karo 4 ama 5?

(8) Haddii sanduuq ay ku jiraan 19 qalin, 6 cagaar ah, 3 cas, 6 madow, 4 cad, lagana soo saaro sanduuqa qalin, waa maxay itimaalka uu ku noqon karo cagaar ama caddaan?

(9) Haddii fasal ay ku jiraan 20 wiil iyo 15 gabdhood, oo macallinku uu arday ka doorto (doorashada wa ay u siman yihiin), waa maxay itimaalka uu ku noqon karo
(i) Wiil ama gabadh
(ii) Wiil
(iii) Gabadh

(10) Haddii tirsiimooyinka 1 ilaa 20 lagu qoro xaashiyo yar yar oo dabeedna mid laga saaro (saaridda wa ay u siman yihiin), waa maxay itimaalka ay tirada xaashidaa ku qorani ku noqon karto :

(i) Mid ka weyn 10
(ii) Mid 9 ka weyn ama ka yar 4.
(iii) Ama kutirsane u ah $\{5, 7, 11\}$ ama ka weyn 15.

Waxa aynu aragnay in haddii W_1 iyo W_2 ay kala edeg yihiin, markaa

$$H(W_1 \cup W_2) = H(W_1) + H(W_2).$$

Matalan waqdhacyada W_1 iyo W_2 maaha kala edeg, macnee $W_1 \cap W_2 \neq \emptyset$. Haddaba si aan u helo dariiqada loo raadiyo $H(W_1 \cup W_2)$ marka $W_1 \cap W_2 \neq \emptyset$, aan tixgelino tijaabo la tuuray laadhuu. Haddii W_1 ay tahay waqdhaca ah "waxa soo sareeya tiro 3 ka yar" W_2-na ay tahay waqdhaca "dhinaca sare waxa ku yaal tiro kisi ah" markaa $W_1 = \{1,2\}$, $W_2 = \{1,3,5\}$.

$W_1 \cup W_2 = \{1, 2, 3, 5\}$, $W_1 \cap W_2 = \{1\}$

Maadaam W_1 iyo W_2 ay dhextaal leeyihiin, maaha kala edeg. Waqdhaca $W_1 \cup W_2$ kutirsanayaashiisu waa 4. Kolkaa $H(W_1 \cup W_2) = \frac{4}{6} = \frac{2}{3}$.

Haddii aynu isticmaali lahayn jidka isutagga waqdhacyo kala edeg ah, waxa aynu heli lahayn $H(W_1 \cup W_2) = H(W_1) + H(W_2) = 2/6 + 3/6 = 5/6$.

Sababta ay labadaa qiime u kala gedisan yihiin waa: Marka aynu $H(W_1)$ iyo $H(W_2)$ u kala xisaabino gooni, natiijada "1" ayeynu tiriney 2 jeer. Marna W_1 ayeynu ku tiriney, marna W_2. Si aan u raadino itimaalka dhicitaanka W_1 iyo W_2 oo aan ahayn kala edeg waxa aynu isticmaali jidkan:

$$H(W_1 \cup W_2) = H(W_1) + H(W_2) - H(W_1 \cap W_2)$$
$$= 2/6 + 3/6 - 1/6 = 2/3.$$

TUSAALE 1:

Matalan 2 astiro oo ku jira $\{1, 2, 3, 5, 7\}$ ayaa la bixiyey (bixinta wey u siman yihiin) si loo sameeyo astiro 2 god ah. Waa maxay itimaalka 5 ama 7 ay ku jiri karaan astirada sameysantay?

FURFURIS:

Ka dhig W_1 waqdhaca "astiro 2 god ah, oo labada god mid yahay 5" W_2-na waqdhaca "astiro 2 god ah, oo labada god mid yahay 7". Waxa innoo fiican in aan taxno natiijooyinka suuragalka ah dhammaantood.

12	13	15	17	tirada natiijooyinka suuragalka ah
21	23	25	27	(tirada kutirsanayaasha muunad-dulalaatiga)
31	32	35	37	waa 20.
51	52	53	57	
71	72	73	75	

Markaa waxaad aragtaa in

$W_1 = \{15, 25, 35, 75, 51, 52, 53, 57\}$

W_2-na $= \{17, 27, 37, 57, 71, 72, 73, 75\}$

$W_1 \quad W_2 = \{57, 75\}$

Maadaam W_1 iyo W_2 aanay kala edeg ahayn waxa aynu isticmaali jidka

$$H(W_1 \cup W_2) = H(W_1) + H(W_2) - H(W_1 \cap W_2)$$
$$= 8/20 + 8/20 - 2/20 = 14/20 = 7/10.$$

TUSAALE 2:

1000 qof oo Soomaali ah ayey 420 ka midi cunaan qaadka, 105 kalena cabbaan sigaarka. 45 ayaa qaadkana cuna sigaarkana cabba. Waa maxay itimaalka ay ku dhici karto in qof dadkaas ka midi ama uu qaadka cuno ama uu sigaarka cabbo?

FURFURIS:

Muunad-dulalaatigu wuxuu ka kooban yahay 1000 natiijo oo laga yaabo in uu mid waliba yahay qof ama qaadka cuna ama sigaarka cabba. Ka dhig Q ururka qaad cunada, S-na ururka sigaar cabyada.

Markaa $n(Q) = 420$, $n(S) = 105$, $n(Q \cap S) = 45$;

dabeedna $H(Q \cup S) = \frac{480}{1000} = 0.48$.

ama maadaam $H(QUS) = H(Q) + H(S) - H(Q \cap S)$,

$H(Q \cup S) = \frac{420}{1000} + \frac{105}{1000} - \frac{45}{1000} = 0.48.$

LAYLI:

Sharrax sida loo raadiyo itimaalka dhicitaanka ama waqdhaca W_1 ama W_2 marka:

(1) W_1 iyo W_2 ay kala edeg yihiin.

(2) W_1 iyo W_2 aanay kala edeg ahayn.

Weydiimaha 3-8 waxa lagu siiyey $n(M)$, $n(W_1)$, $n(W_2)$ iyo $n(W_1 \cap W_2)$. Sheeg

(b) in W_1 iyo W_2 ay kala edeg yihiin

(t) $H(W_1 \cup W_2)$

TUSAALE:

$n(M) = 20$, $n(W_1) = 10$, $n(W_2) = 8$, $n(W_1 \cap W_2) = 3$.

FURFURIS:

(b) W_1 iyo W_2 maaha kala edeg maxaa yeelay
$W_1 \quad W_2 \neq \emptyset$.

(t) Isticmaal $H(W_1 \cup W_2) = H(W_1) + H(W_2) - H(W_1 \cap W_2)$
$H(W_1 \cup W_2) = 10/20 + 8/20 - 3/20 = 15/20$ ama $3/4$.

(3) $n(M) = 20, n(W_1) = 12, n(W_2) = 5, n(W_1 \cap W_2) = 0$
(4) $n(M) = 20, n(W_1) = 8, n(W_2) = 8, n(W_1 \cap W_2) = 6$
(5) $n(M) = 15, n(W_1) = 8, n(W_2) = 5, n(W_1 \cap W_2) = 0$
(6) $n(M) = 15, n(W_1) = 7, n(W_2) = 3, n(W_1 \cap W_2) = 2$
(7) $n(M) = 25, n(W_1) = 10, n(W_2) = 10, n(W_1 \cap W_2) = 5$
(8) $n(M) = 100, n(W_1) = 40, n(W_2) = 20, n(W_1 \cap W_2) = 10$

Ururka $\{1,2,3,4,5,6,7,8,9,10\}$ ayaa laga saarey hal tiro (Saaridda waa ay u siman yihiin). Haddii W_1 ay tahay waqdhaca "tirada la saarey waa ay ka yar tahay 4 " P-na tahay waqdhaca "tirada la saarey waa dhaban".

(9) W_1 iyo W_2 ma yihiin kala edeg?

(10) Imisa natiijo ayaa ku jira
(b) W_1 (t) W_2 (j) $W_1 \cap W_2$?

(11) Waa maxay $H(W_1 \cup W_2)$?

12) Qor tusaha hoos ku yaal; kadibna buuxi meelaha maran:

Tirada ku tirsanayaasha ku jira				$H(W_1)$	$H(W_2)$	$H(W_1 \cap W_2)$	$H(W_1 \cup W_2)$
Muunad dulalaati	W_1	W_2	$W_1 \cap W_2$				
15	5	7	2	5/15	7/15	2/15	2/3
15	6	3	1				
15	11	7	3				
25	15	7	5				
25	18	10	5				
30	5	25	3				
30	17	6	0				

13. Ururka $\{ 11, 12, 13, \ldots. 20 \}$ ayaa laga saarey hal tiro. Haddii W_1 ay tahay waqdhaca "tirada la saaray waa dhufsane 3" W_2-na ay tahay waqdhaca "tirada la saarey waa mid ka weyn 16?

(i) W_1 iyo W_2 ma yihiin kala edeg ?

(ii) Imisa natiijo (outcome) ayaa ku jira W_1,
W_2, $W_1 \cap W_2$?

(iii) Waa maxay $H(W_1 \cup W_2)$.

14. Fasal 35 arday ah ayaa sidan u qaybsan:
20 arday af sawaaxiliga ayey taqaan laakiin af Soomaaliga ma taqaan. Tobanna af Soomaaliga ayey taqaan laakiin af Sawaaxiliga ma taqaan; shanina labadaba waa ay taqaan. Haddii arday laga doorto fasalkaa, waa maxay itimaalka uu ku noqonaayo mid ama af Soomaaliga yaqaan ama af Sawaaxiliga yaqaan.

ITIMAAL ODOROSAN (EMPERICAL PROBABILITY)

QEEX: Itimaal edorosan waa itimaal ku xiran jibeyto, tirokoob (Statistical data). Matalan macalin Cabdi ayaa fasal u dhigaayey Kimistriga 20 sanadood, fasalkaa waxa soo maray 2000 oo arday. Labaatankaa sanadood wuxuu bixiyey 250A,550 B, 800 C, iyo 100E. Waxa haddaba laga yaabaa in aan niraahno itimaalka uu arday fasalkaa ku jiraa ku heli karo A waa $\frac{250}{2000} = 1/8$, ama itimaalka uu arday fasalkaa ku jiraa ku heli karo C waa $\frac{800}{2000} = 2/5$.

Markaas oo kale ayaa itimaalka la oran karaa waa itimaal odorosan.
Helitaanka uu arday helayo A ama B ama C ama E waxa uu ku xiran yahay isiro fara badan oo ay ka mid yihiin caafimaadkiisa, xisaab yaqaan-nimadiisa, dedaalkiisa iyo qaar kale oo badan.

Haddaba derejada uu arday helay waxa aynu war rasmi ah ka bixin karnaa marka aynu isfiradaa dhammaantood warbixin sugan ka heynano.

Sidee aysad u qiyaasi kartaa in uu roob di'i doono bisha Maajo kowdeeda 1977?

Haddii aad u qaadato in labada waqdhac "roob" iyo "roob la'aan " ay innoo siman yihiin maalin kasta, markaa H(roob ku da'ayo 1da Maajo) waa ½, Laakiin itimaalka uu roob ku di'i karo maalin ogaali (given day) waxa ay ku xiran tahay xilliga.

Haddii aad xafiiska war ururinta saadaasha hawada taqto oo aad ogaato in kontonkii sanadood ee inna dhaafay uu 3 jeer roob da'ay 1da Maajo, markaa waxa aad oran kartaa

$$H(\text{roob ku da'ayo 1da Maajo}) = \frac{3}{50}$$

LAYLI:

Ashuun ayaa waxa ku jira kubbado aanad tiradooda iyo midabadooda midna war ka hayn. 26 jeer ayaa waxa laga soo saarey kubbadaha midabkooda iyo tiradoodu ay ku muujisan yihiin tusahan:.

Cagaar	////		/
Caddaan	////	////	//
Gaduud	////		///

Waa maxay itimaalka ay soo saaridda 27aad ku noqon karto.

(1) Kubbad cagaaran

(2) Kubbad cad

(3) Kubbad aan gaduudneyn.

FILASHO XISAABEED (MATHEMATICAL EXPECTATION)

Taranta H.L oo H ay tahay itimaalka lagu heli karo hanti lacag ah L ayaa la yiraahaa filasho xisaabeed F.

:. F = L.H

Bakhtiyaa-nasiib ayaa itimaalka uu ku guuleysan karo ninka boqolkii bilyeyti mid haystaa uu yahay 0.01. Waxa aynu u qaadaneynaa in dadka bilyeytiyada haystaa ay u siman yihiin guusha. Haddii ninka guuleystaa uu helayo 25 gini, filasho xisaabeedka ninka haysta hal bilyeyti waa

F = (0.01) (25 G) = 0.25 G.

TUSAALE 1:

Bakhtiyaa-nasiib lagu helayo baabuur qiimahiisu yahay 30,000 Sh. ayaa bilyeytiyada la iibshay tiradoodu tahay 1000. Waa maxay filasho xisaabeedka ninka haysta 2 bilyeyti?

FURFURIS:

Itimaalka uu ninka 2 biliyeyti haystaa wax ku heli karaa waa $\frac{2}{1000}$ = 0.002 qiimaha baabuurkana waa 30,000 Sh.

:. F = (0.002) (30,000 Sh.) = 60 Sh.

TUSAALE 2:

Itimaalka waqdhac uu ku dhici karaa waa 0.23. Haddii waqdhacaasi dhaco, faarax wuxuu helayaa 500 Sh. Waa maxay filasho xisaabeedkiisu?

FURFURIS: F = HL = (0.23) (500) = 115 Sh.

LAYLI 2:

1. Waa maxay filasho xisaabeedka aad ku heli kartid 35,75 gini haddii itimaalka aad hantidaa ku heli kartaa yahay 1/25?

2. Macallin ayaa ardaydiisii ku yiri, buug ayaan siinayaa ardayga 100% hela imtixaanka soo socda. 21 imtixaan oo sanadkan la qaaday ayuu Xasan 5 ka mid ah helay 100%. Haddii la tixraaco imtixaanadiisii hore, waa maxay filasho xisaabeedka haddii buugga qiimihiisu yahay 10 sh.?

3. Bakhtiyaa-nasiib ayaa lagu helayaa 50 gini, bilyeytiyada la gadayaana waa 70. Waa maxay filasho xisaabeedka qofka haysta 2 bilyeyti? 3 Bilyeyti? 4 bilyeyti? 5 bilyeyti?

4. Baabuur qiimahiisu yahay 25,000 Sh. ayaa la soo dhigay bakhtiyaa-nasiib. Imisa bilyeyti oo midkiiba yahay 1.00 Sh. ayaa la gadi doonaa haddii baabuurka laga rabo macaash ah 40%?

ARAGTIINKA LABA TIBIXLE (THE BINOMIAL THEOREM)

waa ay fududahay in isku dhufasho lagu sugo (determine) tarahana

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3 b + 6a^2b^2 + 4ab^3 + b^4$

Haddii aynu taranaha sare u fiirsano waxa si toos ah inoogu muuqan kara hubaalahan:

(1) Fidinta $(a+b)^n$, $n \in \{1,2,3,\ldots\}$ waxa ay leedahay (n+1) tibxood.

(2) b uma aha isir tibixda koowaad, a-na uma aha isir tibixda ugu danbeysa fidinta.

(3) Jibbaarka doorsoomaha tibixda koowaad iyo tibixda ugu danbeysaa waa n.

(4) Marka tibxaha fidinta aad deristid, jibbaarada a mid ba kan ka horeeya ayuu 1 ka yar yahay, jibaaradda b-na midba kan ka horeeya ayuu 1 ka weyn yahay.

(5) Tibix kasta, haddii aad jibaarka a ku dhufato weheliyaha oo aad dabeedna tarantaas u qaybisid tirada tibixda, jadeeyada aad heshaa waa weheliyaha tibixda ku xigta.

(6) Tibix kasta, wadarta jibbaarada a iyo b waa n.

Taranta $(a+b)^4$ waxa ku jira $\binom{4}{3} = \frac{4.3.2}{1.2.3} = 4$ ama $\binom{4}{1} = 4/1 = 4$ tibxood oo ah a^3b. Sidoo kale, waxa aynu arki karaa in tirada tibxaha a^2b^2 ay noqonayaan $\binom{4}{2} = \frac{4.3}{1.2} = 6$. Kolkaa $(a+b)^4$ waxa loo qori karaa sidan:

$(a+b)^4 = a^4 + \binom{4}{1} a^3b + \binom{4}{2} a^2 b^2 + \binom{4}{3} ab^3 + b^4$

Isla sidaas ayeynu ku tusi karaa in

$(a+b)^5 = a^5 + \binom{5}{1} a^4b + \binom{5}{2} a^3 b^2 + \binom{5}{3} a^2b^3 + \binom{5}{4} + b^5$

waxa markaa suurogal ah in aynu dheegno go'aanka ah, haddii $n \in \{1,2,3,\ldots\}$, markaa $(a+b)^n = a^n + \binom{n}{1} a^{n-2}b^2 + \ldots + \binom{n}{n-1} ab^{n-1} + b^n$ natiijadaa ugu danbeysa ayaa la yiraahaa Aragtiinka laba tibixle.

TUSAALE 1:

Ku isticmaal aragtiinka laba tibixle si aad u hesho $(x-2)^6$

$$(x-2)^6 = x^6 + 6/1x^5 (-2)^1 + \frac{6.5}{1.2} (x^4)(-2)^2 + \frac{6.5.4}{1.2.3} (x^3)(-2)^3 + \frac{6.5.4.3}{1.2.3.4} (x^2)(-2)^4 + \frac{6.5.4.3.2}{1.2.3.4.5} (x)^1(-2)^5 + (-2)^6$$

$$= x^6 + 6x^5 (-2) + 15x^4(4) + 20x^3 (-8) + 15x^2(16) + 6x(-32) + 64$$

$$= x^6 + 12x^5 + 60x^4 - 160x^3 + 240x^2 - 182x + 64$$

Aan xasuusano in $\binom{6}{5} = \binom{6}{1}$ iyo in $\binom{6}{4} = \binom{6}{2}$ markaas lagama maarmaan maaha in la wada qoro isirada ku jira "weheliyayaasha laba tibixle ee tibxaha 5aad iyo 6aad.

HUBSIIMO: Ka dhig x = 1

$$(x-2)^6 = x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$$

$$(1-2)^6 \quad 1^6 - 12(1)^5 \; 60(1)^4 - 160(1)^3 + 240(1)^2 - 192(1) + 64$$

$$+1 \quad 1-12 + 60 - 160 + 240 - 192 + 64$$

$$1 = 1$$

Marka aad isticmaaleyso aragtiinka laba tibixle, waa in aad ogaataa in $\binom{n}{r} = \binom{n}{n-r}$.

Taasi waxa ay gaabin doontaa qoritaanka "Weheliyayaasha laba tibixle " Sida

$\frac{6.5.4.3}{1.2.3.4}$ oo le'eg $\frac{6.5}{1.2}$

TUSAALE 2: Qor tibxaha 5aad iyo 6aad ee $(2W + 3)^7$

FURFURIS: Aragtiinka laba tibixle ayaa inna tusaya in tibixda seddexaad ay tahay $\binom{n}{2} a^{n-2} b^2$, tibixda toddobaadna ay tahay $\binom{n}{6} a^{n-6} b^6$, tibixda r aad-na $\binom{n}{r} a^{n-r} b^r$

∴ Tibixda 5aad ee $(2 W + 3)^7$ waa

$$\binom{7}{4} (2w)^3 (3)^4 = \frac{7.6.5}{1.2.3} \; 8 W^3 (81) = 22,680 W^3$$

Sidoo kale tibixda 6aad ee $(2w+3)^7$ waa

$$\binom{7}{5} (2w)^2 (3)^5 = \frac{7.6}{1.2} (4w^2)(243)$$

$$= 20,412 W^3$$

LAYLI :

Raadi taran kasta adiga oo isticmaalaya araqtiinka laba tibixle.

1. $(x+y)^4$
2. $(d+1)^7$
3. $(1+2x)^6$
4. $(2x-w)^5$
5. $(3a+\frac{1}{3}b)^5$
6. $(x-y)^6$
7. $(m+2)^5$
8. $(1-3y)^4$
9. $(1.01)^8$
10. $(2a-\frac{1}{2}b)^7$

Doon tibixda 3aad iyo ta 4aad ee taran kasta:

11. $(x+y)^{10}$
12. $(c-1)^{12}$
13. $(1.02)^8$.
14. $(1+d)^{15}$
15. $(1.03)^6$

Itimaal iyo Araqtiinka laba tibixle

Haddii kuumi la tuuro 5 jeer, itimaalka lagu heli karo daabac saddexda jeer ee hore iyo dur labada jeer ee danbe waa $(\frac{1}{2})^3 (\frac{1}{2})^2 = 1/32$. Itimaalka lagu heli karo saddex daabac iyo laba dur marka 5 jeer la tuuro waa:

$\binom{5}{2} (\frac{1}{2})^3 (\frac{1}{2})^2 = 10\ (1/32) = 5/16$.

Waxa aynu aragnaa in ay taasi run tahay, maxaa yeelay saddexda daabac waxa laga yaabaa in lagu helo 3 racaymood oo kasta oo ka mid ah 5ta jeer ee la tuuray:
Matalan tuurmooyinka koowaad, labaad iyo afraad; ama tuurmooyinka labaad, afraad iyo shanaad.

Waxa jira $\binom{5}{2} = \binom{5}{3}$ racaymood oo tuurmooyin ah. Taasi waxa ay inoo sheegeysaa in tibaaxaha araqtiinka laba tibixle ee $(\frac{1}{2}+\frac{1}{2})^5$ ay inna siinayaan itimaalada racaymaha daabac-dur ee kala gedisan.

$(\frac{1}{2}+\frac{1}{2})^5 = (\frac{1}{2})^5 + \binom{5}{4} (\frac{1}{2})^4 (\frac{1}{2})^1 + \binom{5}{3} (\frac{1}{2})^3 (\frac{1}{2})^2$

Itimaal 5daabac 4daabac, 1 dur 3 daabac, 2 dur

$+ \binom{5}{2} (\frac{1}{2})^2 (\frac{1}{2})^3 + \binom{5}{1} (\frac{1}{2})^1 (\frac{1}{2})^4 + (\frac{1}{2})^5$

2 daabac, 3 dur 1 daabac, 4 dur 5 dur.

Araqtiinka laba tibixle wuxuu aad u anfacaa sugidda itimaalka tijaabo la celceliyey (repeated trials).

TUSAALE: Laadhuu ayaa la tuurey 6 jeer, waa maxay itimaalka lagu heli karo ugu yaraan 3 afaraad.

FURFURIS:

Itimaalka 4 lagu heli karo tuurmo keliya waa 1/6, ta 4 aan lagu heleynina waa 5/6.

$$(1/6+5/6)^6 = (1/6)^6 + \binom{6}{1}(1/6)^5(5/6) + \binom{6}{2}(1/6)^4(5/6)^2 + \binom{6}{3}(1/6)^3(5/6)^3$$

+...+ tibxaha kale loogama baahna weydiintan

$$= (1/6)^6 + 6(1/6)^5(5/6)^1 + 15(1/6)^4(5/6)^2 + 20(1/6)^3(5/6)^3 + \ldots$$

$$= \frac{1 + 30 + 375 + 2500}{6^6}$$

$$= \frac{2906}{6^6} = \frac{2906}{46,656} = \frac{1453}{23,328}$$

Matalan tibixda $\binom{6}{4}(1/6)^2(5/6)^4$ waxa ay u taagan tahay itimaalka lagu heli karo afar 2da tuurmo ee hore, laakiin maaha itimaalka lagu heli karo afar 4ta tuurmo ee kale. Sidaas awgeed tibixdu kuma jirto tusaalahan.

LAYLI:

1. Kuumi ayaa la tuurey 3 jeer. Waa maxay itimaalka lagu heli karo 2 daabac iyo 1 dur? Lagu heli karo ugu yaraan 1 dur?

2. Kuumi ayaa la tuurey 7 jeer. Waa maxay itimaalka lagu heli karo 4 daabac iyo 3 dur? Lagu heli karo ugu yaraan 4 daabac?

3. Sanduuq ayaa waxa ku jira 3 kubbadood oo cas, 3 cad, iyo 3 cagaaran. Haddii 3 kubbadood laga saaro, waa maxay itimaalka ay ku dhici kartaa in ay 3da midabba ka koobnaadaan?

TIROKOOB

Tirooyinka siyaabo badan ayeynu u isticmaalnaa si ay war tafatiran inooga siiyaan mawaadiic fara badan oo aad u kala gedisan ayna matalan ka mid yihiin tirada dadka, wax soo saarka wershad, ciyaaraha ama isboortiga, cimilo-gooreedka, shilalka wadooyinka, iwm. Astirada warka ee jaadkan ah iyo habayntoodaba waxa saldhig u ah laanta xisaabta ah ee loo yaqaan tirokoob.

Farsamada ururinta, isku dubaridida, saafidda iyo keenidda (presentation) jibayto ayaa loo yaqaan tirokoobka sifeynta (descriptive statistics). Haddii tarjumidda jibayto ay soo marto heerar ama marxalado kala duwan, oo ay ka mid tahay iska qaadashada hawraareed ilaa go'aamo iyo saadaalo cad oo sugan la gaaro, waxaynu ku magacawnaa tirokoob-dhuu-xideed (inferential statistics). Hase yeeshee halkan waxaynu kula jeex-jeexi doonaa sifeynta tirokoobka oo keliya.

Had iyo jeerba waxa aynu ka dhadhansanaa ama kuba jirta tirokoobka macno leebid. Cabbiraadda jibeytooyinka la soo ururinayaa waa leebidda tirooyin. Waxaba iska dhici karta in muunadaha la qaataa aanay si fiican u matalin ama ugu taagnaanin jibeytada guud. Waxa had iyo jeerba kaaliya barashada tirokoobka laanta xisaabta ah ee loo yaqaan itimaal sababtoo ah labada cutub si weyn ayey isugu xir-xiran yihiin oo runtii aanay midina midda kale ka maarmi karin.

CABBIRIDDA DHEXAADYADA - TIROSINKA ARITMATIG

Celcelisyada oo dhammi waxa weeye cabbir dhexaadyo. Celcelis waxa weeye tiro u taagan ama metisha urur tirooyin ah. Celceliska caadiga ah waxa lagu helaa isugeynta tirooyinka ururka ku jira oo loo qaybiyey tirada kutirsanayaasha ururka. Celceliska jaadkan ah ayaa la tirosinka aritmatig. Tusaale ahaan tirosinka aritmatig ee 89,73, iyo 92 waxa weeye $\frac{89+73+92}{3}$ = 84 2/3. Sida runtu tahay waxaynu hawl yaraan isaga isticmaalnaa marka aynu u jeedno tirosinka aritmatig magaca tirosin oo keliya.

Haddii X ay tahay doorsoome u taagan ku-tirsane kasta oo ku jira urur jibeyto, markaa tirosinka aritmatig $\overline{X}$ (loona akhriyo X - jiitin) ee tirooyinka n waxa inna siiya jidka ah

$$\overline{X} = \frac{x_1 + x_2 + x_3 + \text{----} + x_n}{n} \qquad (I)$$

Ogow in $x_1, x_2, x_3 \cdots x_n$ ay u taagan yihiin kutirsanayaasha ururka jibeytada. Waxa jirta summad fududaysa, tustana wadarta urur tirooyin ah. Summaddan aynu ka hadlaynaa waxa weeye xaraf ka mid xarfaha waaweyn ee afka Giriiga. Xarafkaa isaga ah waxa loogu dhawaaqaa sigma, summad ahaanna waxa loo qoraa (Σ). Summaddan iyada ah waxa la yiraahdaa summad wadareed. Wadarta tibxo la caddeeyey waxa loo soo gaabin karaa sida soo socota.

$$\sum_{i=1}^{n} X_i = X_1 + X_2 + X_3 + \text{.....} + X_n$$

Dhinaca bidix waxa loo akhriyaa "wadaraynta X-i ku hoos dhaban iyada oo i = 1 ilaa n". Summadda X_i waxay u taagan tahay ku-tirsanayaasha isxigga ee urur jibeyto marka i ay qaadato qiimayaal abyan oo isxigga oo ka bilaabma 1 kuna dhammaada n. Haddaba jidka tirosin aritmatig waxa uu dabeed noqonayaa:

$$X = \frac{1}{n} \sum_{i=1}^{n} X_i \qquad (II)$$

TUSAALE: Raadi tirosinka aritmatig ee : 17, 18,19,20, 21.

FURFURIS: N = 5

$$\overline{X} = \frac{1}{n} \sum_{i-1} X_i = 1/5\ (17+18+19+20+21) = 19\ 1/5$$

Tirosinka aritmatig waxa looga fekeri karaa xuddunta miisaanka ee jibeyto haddii ku tirsanayaasha ay yihiin culaysyo (weights).

LAYLI

1. Raadi tirosinka 87-2, 68.5, 74.8, 94.0, 82.2, 96.1
2. Haddii nin beeraley ahi uu iibiyo jawaamo galley ah oo culayskoodu kala yahay 241 Kg. 305 Kg. 289 Kg. 262 Kg. 300 Kg, 267 Kg. waa maxay tirosinka culaysada?

3. Xaas dhaqaalihiisu iska ladan yahay ayaa temeshle maalin Sabti ah ku tegey tuulo magaala-madaxda u jirta 329 Km, Axadiina tegey tuulo u jirta 401 Km, Isniintiina tegey tuulo u jirta 105 Km, Talaadadiina tegey 306 Km, Khamiistiina tegey tuulo u jirta 211 Km, Jimcihiina tegey tuulo u jirta 511 Km. Haddaba raadi tirosinka fogaanshaha uu xaaskaasi socday?

4. Axmed shan imtixaan oo isxiga calaamaadkiisii waxa ay kala noqdeen sida soo socota, 84, 72, 91, 64, 83, Raadi tirosinka?

5. Tirosinka joog ee 5 wiil waa 67 inches. Haddii joogga hal will uu yahay 5 fdh, joogga mid kalena uu yahay 6 fdh bal keen ama sheeg joogagga suurtagalka ah ee 3da wiil ee haray?

DHEXFUR

Mar marka qaarkood dhexfurka ayaa ah cabbir si aad ah uga fiican tirosinka aritmatig marka aad haysato koox jibeyto ah. Dhexfurka oo summad ahaan loo qoro M_r waxa weeye qiimo-badhtameedka urur jibeyto.

Inta aanad dhexfurka soo saarin waa in aad jibeytada ceerin teed ahaan u qortaa. Tirooyinka aan sigaarjah u horsaneyn ayaa loo yaqaan jibeytada ceerin. Haddaba si aynu teed ahaan ugu qoro tirooyinkaas waa in aynu u ratibnaa suntooda si horsan iyaga oo u kala horsan sida ay u kala baxd weyn yihiin. Cigaalku waxa weeye faraqa u dhexeeya tirada ugu weyn iyo ta ugu yar ee tirooyin teedsan.

TUSAALE 1: Samee tirooyinka teedsan ee ah calaamadaha imtixaan ee arday. Waa maxay cigaalku?

(82, 79, 91, 57, 71, 87, 64, 95, 73, 75, 62).

FURFURIS:

Teedku waa: 57, 62, 64, 71, 73, 75, 79, 82, 87, 91, 95, cigaalkuna waa 95 - 57 = 38.

Dhexfurka tiro kisi ah ee tirooyin waxa weeye tirada badhtamaha ee teedkooda. Dhexfurka tiro dhaban ah ee tirooyin waxa weeye tirosinka aritmatig ee labada tiro ee teedkooda badhtamaha dhaca.

TUSAALE 2:

Waa maxay dhexfurka tirooyinkan 17, 31, 15, 28, 35, 30, 29, 19, 19? Waa maxay cigaalku?

FURFURIS:

Kolka hore samee teedka: 15, 17, 19, 19, 28, 29, 30, 31, 35. Waxa aynu haysanaa 9 tiro, kolkaa tirada badhtamaha ama tirada shanaad marka dhinac kasta laga soo tiriyo waxa weeye 28. Haddaba dhexfurku waa 28. Cigaalkuna waxa weeye 35-15 = 20.

TUSAALE 3:

Waa maxay dhexfurka tirooyinkan .014, .019, .010, .023, .045, .009? Waa maxay cigaalku?

FURFURIS:

Teedku waxa weeye: .009, .010, .014, .019, .023, .045. Mar haddii tirooyinku yihiin lix oo ah tiro dhaban markaa dhexfurku waa tirosinka labada tiro ee badhtamaha dhacaya.

Haddaba $M_d = \frac{.014 + .019}{2} = .0165$

Cigaalkuna waa .045 - .009 = .036

LAYLI

1. - Raadi dhexfurka 64sh, 82sh, 51sh, 90sh, 67sh, 71sh, 58sh, 94sh, 63sh? Waa maxay cigaalku?
2. - Waa maxay dhexfurka 5' 7'', 4' 8'', 6' 1'', 5' 5'', 8' 0'', 9' 1'', 6' 7'', 5' 4''? Waa maxay cigaalku?
3. - Raadi faraqa u dhexeeya tirosinka aritmatig iyo dhexfurka tirooyinkan soo socda. 144, 175, 192, 138, 166, 159, 171, 180, 162?
   Haddii tirada 127 lagu daro jibeytada, sidee bay raad ugu yeelanaysa tirosinka iyo dhexfurka?
4. - Maxaa ku dhacaya dhexfurka haddii cidhifyada teedka la beddelo? Muxuuse isbeddelkani u keenayaa tirosinka? Marna ma is dul dhici karaan tirosinka iyo dhexfurku?
5. - Bal hadda dhugo tijaabadan soo socota. Ul bir ah oo dhexerkeedu yahay hal mitir ayaa meel laga soo lulay iyada oo badhtamaha xarig lagaga xiray si ay u dheelitiranto. U qaado in culaysyo hal-garaam ah laga lulay gogaanshooyinka soo socda dacal ka mid ah dacalada

usha: 5sm, 20sm, 37sm, 44 sm, 52 sm, 68sm, 71sm, iyo 85sm, waxa la arkay inaanay ushu ka dheelitirmayn calaamada ah 50 sm. Haddaba xagee baa laga lulaa hal-garaam oo kale si ay ushu u dheelitiranto? (sarid: mar haddii culaysyadu ay is le'eg yihiin 50 waa in uu noqdaa tirosinka sagaalka fogaanshooyin) Waa maxay dhexfurka fogaanshooyinka?

BADIDHACE

Badidhacaha urur tirooyin waxa weeye tirada inta ugu badan laga helo ururka tirooyinka marka loo eego tirooyinka kale ee ururka ku jira. Waxa si hawl yar loo helaa marka tirooyinka loo qoro teed ahaan. Badidhacaha tirooyinkan 1.29, 1.37, 1.29, 1.25, 1.37, iyo 1.29 waxa weeye 1.29. Tiradan iyada ah saddex jeer ayaa laga helayaa ururka; haddii se aad fiirisid tirooyinka kale waxa aad arkaysaa in tirona aanay dhacayn ururkaas isaga ah wax laba jeer ka badan. Waxa mar marka qaarkood dhacda inaan la soo saari karin badidhacaha, gaar ahaan marka ururka tirooyinku uu yar yahay. Waxa kale oo dhici karta in hal badidhace in ka badan, marar laga helo ururka tirooyinka ah. Jibeytada leh laba badidhace waxa la yiraahdaa laba badidhacaale (bimodal). Badidhacuhu waxa uu muhiim yahay marka jibeytadeenu ku saabsan tahay baaxadaha kabaha iyo dharka. Waa maxay sababtu?

LAYLI:

1. Raadi badidhacaha 36.1, 42.4, 62.5, 51.7, 60.8, 63.5, 42.4, 56.0, 63.5, 55.1
2. Raadi badidhacaha .412, .408, .410, .408, .401, .401,.401, .420, .408.
3. Haddii imtixaan aritmatig ah oo aad u fudud la siiyo tiro aad u badan oo arday ah, badidhacaha calaamaduhu ma laga yaabaa inuu ka weynaado ama ka yaraado tirosinka? Haddiise imtixaanku uu aad u adag yahay, badidhacaha calaamaduhu ma laga yaabaa inuu ka weynaado ama ka yaraado tirosinka?

FIRIDHSANAANTA CABBIRYADA — TIROSINKA WEECSANAAN

Tirosinka aritmatig iyo dhexfurku waxa weeye cabbir dhexaadyo; haddaba waxa la oran karaa waa cabbirro wax ka sheegaya sifooyinka gaarka ah ee urur jibeyto. Hase-yeeshee marnaba waxba kama sheegaan firidhsanaanta jibeytada. Tusaale ahaan tirosinka 35, 40, iyo 45 waa 40. Sidoo kale tirosinka 10, 40, iyo 70 isna waa 40. Waxa halkan ka caddaan ah in firidhsanaanta tusaalaha danbe ay ka weyn tahay firidhsanaanta tusaalaha hore. Hase yeeshee tirosinku innooma sheegayo aida urur jibeyto uu u firidhsan yahay.

Haddaba cabbirka firidhsanaantu waxa weeye tirosinka weecsanaanta urur jibeyto oo kasta oo la qaato, wadarta ka weecsanaanada tirosinka waxa ay le'eg tahay eber. Summad ahaanna waxa lagu soo qaabin karaa jidka ah

$$\sum_{i=1}^{n} (x_i - \bar{x}) = 0$$

Weecsanaanada tirosinku way togan yihiin, qaarna way taban yihiin. Kolka waxa si hawl yar kuugu muuqan karta in wadarta aljebra ee tirooyinka togan iyo kuwa tabani ay tahay eber.

Hase yeeshee haddii calaamadaha la iska dhaafo, taas oo macneheedu yahay in qiimaha sugan ee tirooyinka la qaato, waxa suurtagal ah in la helo celceliska weecsanaanada. Tirosinka aritmatig ee qiimayaasha sugan ee ka weecsanayaasha tirosinka urur jibeyto ayaa loo yaqaan tirosinka weecsanaanta. Tusaaleheenii ahaa 35,40, iyo 45 ka weecsanayaasha tirosinku waa -5, 0, +5 sida ay u kala horreeyaan. Tirosinka aritmatig ee qiimayaasha sugan ee weecsanayaashani waxa weeye $\frac{5 + 0 + 5}{3}$ = $\frac{10}{3}$ = 3 ⅓. Haddaba tirosinka weecsanaantu waxa weeye 3.3 ugu dhawaan ama marka la seebo. Tusaalaha dambe ka weecsanayaasha tirosinku waa - 30, 0, + 30 sida ay u kala horreeyaan.

Kolka tirosinka weecsane waa $\frac{30 + 0 + 30}{3} = \frac{60}{3} = 20$

Hadda si gaaban oo fudud ayaa loo qori karaa jidka tirosinka weecsanaaan, waxana weeye jidka leh sansaankan soo socda:

$$T.W = \frac{1}{n} \sum_{i=1}^{n} / x_i - \bar{x} / \qquad (III)$$

LAYLI

1. Waa maxay tirosinka weecsanaaan ee: 87.2, 68.5, 74.8, 94.0, 82.2, 96.1?
2. Raadi tirosinka weecsanaaan ee : 84, 72, 91, 64, 83.
3. Raadi tirosinka weecsanaaan ee: 53,43,56,34, 33,50, 46,35, 38,57,44, 63,37,31,47,28, 34, 46, 60, 41,40, 39,40,34,42, 68,36, 40,37, 38.
4. Raadi tirosinka weecsanaaan ee: 48,46,44,50,47,45,49,48,52, 46, 43,48,45,48,47,45,50,46,45,48.
5. Waa maxay tirosinka weecsanaaan ee: 56, 48, 59, 53, 46,50, 51,56, 45, 49, 58, 50, 61, 48, 42, 55, 50, 62, 56, 45.

Markii aynu bilaabaynay cutubkan tirokoobka ah waxa aynu tilmaanay in aynu u baahan nahay tiro u taagnaan karta ama matilli karta koox jibeyto ah. Ka bacdina waxa aynu qeexnay dhawr cabbir dhexaad oo kii la rabo loo qaadan karo in uu u taagnaado kooxda tirooyinka ah. Haddana waxa aynu ka hadalay cabbir sheega firidh sanaanta tirooyin urur ku jira. Bal iminkana aan qeexno cabbir isna ku saabsan firidhsanaanta jibeyto, kaas oo intabadanba ku xiran tirosinka aritmatig si tilmaan fiican looga helo urur jibeyto. Cabbirka jaadkan ah ayaa loo yaqaan weecsanaanta beeggal. Sidii weecsanaanta tirosinka, weecsanaanta beeggal waa cabbirka celceliska xadiyada ay kutirsanayaasha ururka jibeytadu ka weecsan yihiin tirosinka aritmatig.

Weecsanaanta beeggal ee urur tirooyin ahi waa tirosinka wadarta laba jibbaarada weecsanayaasha.
Weecsanaan keli ahaaneed waa jadeeyada u dhexeeya tirosinka aritmatig iyo tiro keli ahaaneed oo ka mid ah jibeytada. Summad ahaanna waa $x_i - \bar{x}$. Sidii aynu horeba u araqnay, faraqyadaa qaarkood waa ay taban yihiin, marba haddii la labajibbaarayo, jadeeyooyinku waa ay tognaanayaan. Haddii

$\bar{x}$ ay u taagan tahay tirosinka, isla markaas $x_i (i = 1.2...n)$ ay iyana u taagan tahay tirooyinka keli ahaaneed ee ay jibeytadu ka kooban tahay, markaa weecsanaanta beeggal σ (sigma) ee jibeytadu waa:

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

TUSAALEheenii ahaa 35, 40, 45, waxa uu lahaa weecsanaanada kala ah -5, 0, +5,. Kolka laba jibbaarada weecsanaandan waa 25, 0, 25, sida ay u kala horreeyaan. Haddaba weecsanaanta beeggalku $\sigma \sqrt{\frac{25+0+25}{3}}$

$= \sqrt{\frac{50}{3}} = 4.1$ ugu dhawaan

Weecsanaanta beeggal ee 10, 40, 70 waa

$$\sigma = \sqrt{\frac{(-30)^2+0^2+(+30)^2}{3}}$$

$= 24.5$ ugu dhawaan.

Weecsanaanta beeggal ayaa dh ka ugu muhiimsan cabbirka firidhsanaanta.

TUSAALE 1:

Xisaabi tirosinka aritmatig iyo weecsanaanta beeggal ee tirooyinkan soo socda: 54, 57, 59, 59, 60, 60, 61, 61, 62, 62, 62, 63, 63, 63, 64, 65,65, 66, 66,67,68,68,68,68,69,69,69,70, 71,71,72,72,73,75,76,77,79,81,83,90.

FURFURIS: $\bar{x} = 1/41 (54+57+....+90) = 68$. Dabeedna weecsanaanadu waa: 54-68,57-68,59-68,....90-68, ama -14,-11,-9,....+22. Laba jibbaarada weecsanaanaduna waa 196,121,81,....,484. Kolka tirosinka laba jibbaaradu waa 54.39. Haddaba weecsanaanta beeggal waa $\sigma \sqrt{54.39} = 7.3$

LAYLI

1. Xisaabi weecsanaanta beeggal ee .6, .6, .7, .8, 1.0, 1.2, 1.4,
2. Xidigiye ayaa sameeyey 10 cabbir oo ku saabsan fogaansho xagleedka u dhexeeya laba xidigood. Cabbiraadii uu sameeyey xidigiyuhu waatan hoos ku qoran iyada oo halbeegga cabbirku uu yahay digrii.

   Xisaabi weecsanaanta beeggal ee : 11.21°, 11.17°, 10.93°, 11.06°, 11.20°, 10.97°, 11.10°, 11.05°, 11.23°, 11.01°.

3. Xisaabi weecsanaanta beeggaliyo weecsanaanta tirosin ee tirooyinkan: 46, 83, 74, 49, 58, 65, 72, 41, 75, 63, 66, 57, 68, 53, 61. Labada cabbir ee firidhsanaanta keebaa weyn?
4. Xisaabi weecsanaanta beeggal ee 40, 54, 32, 30, 45, 35, 72, 48, 65, 23, 36, 10, 58, 43, 16, 50, 40, 45, 38, 60.
5. Xisaabi weecsanaanta beeggal iyo weecsanaanta tirosin ee 58, 59, 55, 61, 60, 57, 60, 62, 56, 54, 57, 55, 54, 56, 56, 60, 57, 62, 58, 60, 56, 59, 63, 61, 54, 56, 58, 57, 55, 60, 57, 61, 63, 58, 57, 61, 55, 60, 55, 57.

FILIQSANAANTA RAKAADKA

Marka tirada ku-tirsanayaasha urur ee jibeyto ay bataan (qiyaas ahaanna noqdaan 50 iyo in ka badan) waxa loo baahan yahay in jibeytada loo kooxeeyo habdhiska loo yaqaan filiqsanaanta rakaadka . Kooxaynta waxa aynu uga jeednaa samaynta aynu samaynayno dhawr goosan si ku-tirsanayaasha goosanka loogu taxi karo sarab ahaan la isuguna kooxayn karo. Bal aan tusaale ku muujino sida kooxaynta jibeyto loogu muujo filiqsanaanta rakaadka.

TUSAALE 1:

Bal ka soo qaad in ay 200 oo wiil dugsi ku jiraan; waxana loo dhabogalay culaysyada kala duwan ee wiilashaas. Jibeytada markiiba waxa laga helayaa xaashiyaha diiwaan gelinta caafimaadka dugsiga. Haddaba sideebbaa loo diyaarin karayaa filiqsanaanta rakaadka?

FURFURIS:

Ugu horaynba jibeytada ceerin waa in laga sooguuriyaa xaashiyaha diiwaan gelinta caafimaadka dugsiga, laguna qoraa xaashiyaha jibeytada. Ka dibna waa in teed ahaan loo taxaa jibeytada ceerin, dabeedna la soo saaraa cigaalka. Bal ka soo qaad in culayska ugu yari uu yahay 99 Kg. ka ugu weynina uu yahay 203 Kg. Marka cigaalku waxa weeye 203-99 = 104 Kg.

Hadda waa in jibeytada loo kooxeeyo goosanno. Runtii waxa aynu imminka samayn karaa kow iyo toban goosan oo mid waliba uu ku fidsan yahay ilaa 10 Kg. Bal ka soo qaad in goosanka ugu hooseeya uu ka bilaabmo 95 Kg. ilaa 105 Kg. Goosanka xigaana waxa uu ka bilaabmayaa 105Kg ilaa 115Kg. Sidaas ayaa hanaanku u soconayaa ilaa tobanka goosanba la suubiyo. Tirooyinka ah 95, 105, 115, ... waxa loo yaqaan xadadka goosanada.

Haddii culays uu dhaco mid ka mid ah xadadka goosannada sida 115Kg, markaa waa in aynu isku raacnaa in aynu culeyskaa u sarabayno goosanka sare ee ah 115-125 oo aan loo sarabaynin goosanka hoose ee ah 105-115. Si hawl yaraan ah waxa aynu u soo saari karaa badhtamaha goosannada, waxana loo yaqaan calaamadaha goosannada oo loo taago xarafka x. Calaamadaha goosannadu waxa weeye susuntan ah 100, 110, 120, ...,200.

Sansaanka filiqsanaanta rakaadku waa sida soo socota:

Xadadka goosannada	Calaamadaha goosannada (x)	sarab	rakaad F(x)
95-105	100	///	3
105-115	110	~~////~~ //	7
115-125	120	~~////~~ ~~////~~ ~~////~~	15
125-135	130	~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ////	39
135-145	140	~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ //	42
145-155	150	~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ///	38
155-165	160	~~////~~ ~~////~~ ~~////~~ ~~////~~ ////	24
165-175	170	~~////~~ ~~////~~ ~~////~~	15
175-185	180	~~////~~ ~~////~~ /	11
185-195	190	~~////~~ //	7
195-205	200	////	4
		Wadar	200

Gaaliska goosan waxa weeye hadbaiinta goosankaa isaga ahi uu ku fidsan yahay. Marka aynu ka fekero tusaabeheenii hore, gaaliska goosanku waxa uu ahaa 10. Sida badanba waxa habboon in la qaato gaaliyo goosaneedyo iswada le'eg marka la haysto filiqsanaan gaar ah.

Calaamadda goosanku waxa weeye celceliska xadka hoose iyo ka sare ee goosannada. Tirada goosannada filiqsanaani waxa ay noqon kartaa inta u dhexeysa 5 ilaa 20 iyada oo tiradanu ay ku xiran tahay qodobo badan sida cigaalka, tirada ku-tirsanayaasha jibeytada ku jira, danta laga leeyahay kooxaynta, iwm. Filiqsanayaasha rakaadka qaarkood waxay yeelan karaan 5 goosan ama wax ka yar, ama 20 goosan iyo wax ka badan. Hase yeeshee wax 20 ka badan sideedaba lama jecla. Summadda rakaadka ee F(x) waxa weeye wadarta sarabka ee goosan.

Waa in madaxa lagu hayaa in ku-tirsanihii kasta ee ku jira jibeytada uu luminayo midaclnimaditsii marka la suubinayo filiqsanaanta rakaad. Tan macneheedu waxa weeye marka ku-tirsanayaasha goosan la saxmeeyo kutirsane kasta oo ka mid ah ku-tirsanayaasha goosanka waxa lala kooxeeyey ku-tirsanayaasha kalebaankujiraagoosanka. Waxa aynu iska qaadanay in jibeytada ku jirta goosan ay xi qaabsan ugu qaybsan tahay ama ugu filiqsan tahay goosankaa isaga ah laftiisa. Iska qaadashadan jadeeyadeedu waxa weeye calaamaddii kasta ee goosan in ay tahay tirsinka jibeytada goosankaa isku jirta.

TUSAALE 2:

U samee filiqsanaanta rakaadka imtixaan xisaab ah oo laga qaaday arday ku jirta fasalka shanaad kana kooban 20 su'aalood; calaamadkiina sidan ayey u kala baleen: 13,19,17,15,20,9,16,15, 17,14,10,16,19,20,13,17,15,18,12,16,14,18,16,7,17,19,15.

FURFURIS: Hor iyo abaataba samee teedkan oo ah 7,9,10,12,13, 13,14,15,15,15,15,16,16,16,16,17,17,17,17,18,18,19,19,29,20, 20.

Cigaalku waa 20-7=13. Dal hadde aan isku dayno in aynu samayno 7 goosan-oo mid waliba ay leedahay gaalis ah 2.

Si aynu u soo gelino tirada ugu yar ee jibeytada waa in xadka goosankee goosankeena ugu hoosaysa ka bilowdaa 6.5.

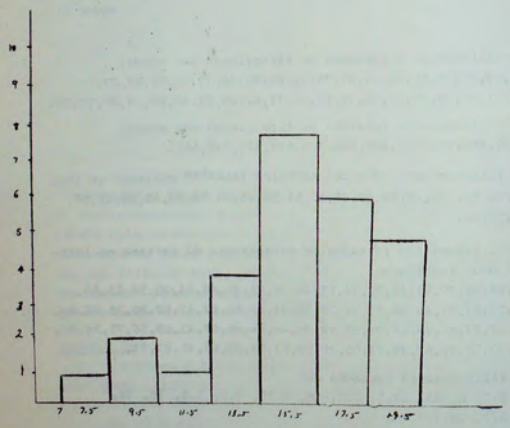
Xadadka goosanada	Calaamadaha goosannada (x)	Sarab	Rakaad F(x)
6.5 - 8.5	7.5	/	1
8.5 -10.5	9.5	//	2
10.5 -12.5	11.5	/	1
12.5 -14.5	13.5	////	4
14.5 -16.5	15.5	~~////~~ ///	8
16.5 -18.5	17.5	~~////~~ /	6
18.5 - 20.5	19.5	~~////~~	5
		Wadar	27

LAYLI

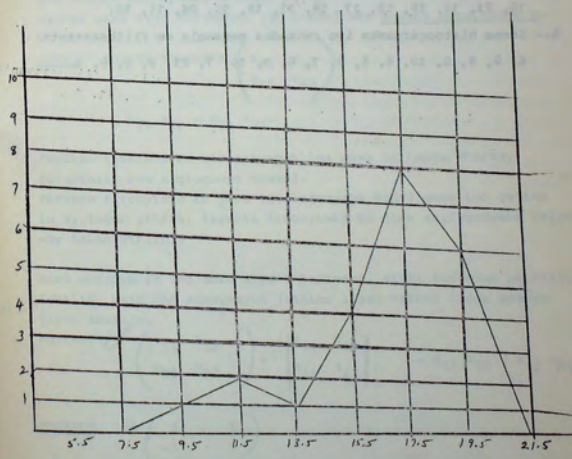
1. Samee filiqsanaanta rakaadka ee tirooyinkan soo socda:
   64,71,57,67,74,65,59,62,67,75,72,84,60,68,72,91,55,69,71, 93,69,71,69,75,59,60,70,76,62,66,77,62,68,81,68,63,79,88,57,78.
2. Samee filiqsanaanta rakaadka ee tirooyinkan soo socda:
   445,460,460,475,475,500,500,500,520,525,530,550.
3. Samee filiqsanaanta rakaadka ee cufka labaatan cabirood oo lagu cabbiray Kg. 56,48,59,53,46,50,51,56,45,49,58,50,61,48,42,55, 50,62,56,45.
4. Samee filiqsanaanta rakaadka ee calaamaadka 80 tartame oo imtixaan u wada fadhiistay:
   60,56,59,58,79,91,46,50,54,51,76,52,76,70,46,64,78,58,53,50, 38,83,55,53,41,59,48,61,94,74,48,82,32,65,52,37,67,80,38,61,64, 77,57,88,57,85,29,66,72,59,42,34,44,54,88,97,43,69,56,75,56,60, 62,69,40,69,92,67,79,72,55,26,52,83,83,75,67,45,87,51,
5. Samee filiqsanaanta rakaadka ee:
   12.5, 6.7, 8, 15, 10.5, 9.5, 16, 6.75, 7.5, 7.5, 16, 16, 12.5, 6.7, 10.5.

KU MUUJINTA JIBEYTO GARAAFYO

Mar haddii la sameeyo ama la suubiyo filiqsanaanta rakaadka waabay iska hawl yar tahay sida jibeyto loogu muujiyo garaaf. Calaamadaha dabaqadaha x waxa lagu cabbiraa dhidibka jiifa, rakaadkana F(x) waxa lagu cabbiraa dhidibka taagan. Dabeedna baar garaaf taagan oo loo yaqaan histoogaraam ayaa la suubin karaa. Hoos waxa ku muujisan histoogaraamka filiqsanaanta rakaadka ee tusaalaha labaad ee ku saabsan 27 arday oo fasalka shanaad dhigta, imtixaan xisaab ahna wada galay.



Nooc kale oo ka mid ah garaafyada lagu muujo jibeytada tirokoobka waxa loo yaqaan rakaad geesoole (frequency polygon). Mar marna waxaaba lagu magacaabaa garaaf xariiqeedka jajaban. Salka geesooluhu waxa weeye dhidibka jiifa. Hoos waxa ku muujisan rakaadka geesoole ee tusaalihii labaad ee histoogaraamkiisa aynu hadda dhow suubinay:



Garaafyada lagu muujo jibeytada tirokoobka runtii waa ay badan yihiin. Hase yeeshee qaybtii hore ee tirokoobka, kuna jirtay buugga kowaad ayaaba si tafatiran uga hadashay noocyada kala duwan ee ah garaafyada lagu isticmaalo tirokoobka.

LAYLI:

1.- Samee histoogaraamka filiqsanaanta rakaadka tirooyinka soo socda: 28,31,35,35,37,39,40,43,44,46,47,50,51,52.

2.- Samee rakaadka geesoole ee: 134, 137, 138, 141, 143, 146, 147, 148,150,153,157.

3.- Samee histoogaraamka filiqsanaanta rakaadka tirooyinkan: 29,40,44,45,49,51,52,56,56,59,63,65,67,71.

4.- Samee rakaadka geesoole ee: 12.9, 13.0, 13.3, 13.6, 13.7, 13.9, 14.2.

5.- Samee histoogaraamka iyo rakaadka geesoole ee filiqsanaanta: 16, 21, 14, 19, 13, 27, 18, 31, 19, 15, 24, 11, 18.

6.- Samee histoogaraamka iyo rakaadka geesoole ee filiqsanaanta: 6, 9, 8, 5, 10, 6, 5, 9, 7, 6, 7, 10, 7, 11, 9, 5, 8, 8.

SUGAHA TAXANE

Taxane, kasta A oo laba jibbaarane ahba, lehna ku-tirsanayaal tirooyin maangal, waxaynu la xiriirinaa tiro maangal oo la yidhaa sugaha A; waxana lagu gartaa ama la siiyay summad $\mathcal{S}_A$ (waxana loo akhriyaa sugaha A).

Sidaa darteed, $\mathcal{S}$ (delta) waa fansaar.
Horaadkeeduna waa taxanayaasha laba jibbaarane oo dhan, lehna ku-tirsanayaal ah tirooyin maangal. Dambeedka fansaarkuna waa ururka tirooyinka maangal oo dhan.
Markaa waxa aynu niraahnaa $\mathcal{S}$ (Anxn) waa sugaha horsiimada n.

QEEX: Sugaha taxanaha $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$$a_{11}\,a_{22} - a_{12}\,a_{21}.$$

(Halkan taxanaha ku-tirsanayaashiisa waxa la isugu dhuftay talantaali ama xaglogooye ahaan).
Taranta tirooyinka ku jira (xaglogooyaha door) waxa loo qaataa in ay togan yihiin, taranta tirooyinka ku jira xaglagooyaha kalena way taban yihiin).

Waxa caadiya in loo qoro sugaha taxanaha, sidii taxanaha oo kale, laakiin, waxa loo samaynayaa jiitimo ligan halkii looga samayn jiray sakallo.

Markaa, $\mathcal{S}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\,a_{22} - a_{12}\,a_{21}$

TUSAALE $A\begin{pmatrix} 3 & 1 \\ -2 & 3 \end{pmatrix}$

Marka $\mathcal{S}\begin{pmatrix} 3 & 1 \\ -2 & 3 \end{pmatrix} = \begin{vmatrix} 3 & 1 \\ -2 & 3 \end{vmatrix} = 3.3 - (-2).1 = 9+2 = 11$

QEEXID: Sugaha taxanaha $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ waxa la

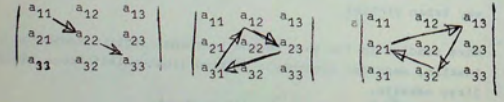
siiyay ama lagu magacaabay

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$; waxana uu le'eg yahay

$a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{13}a_{22}a_{31} -$

$a_{12}a_{21}a_{33} - a_{23}a_{32}a_{11}$.

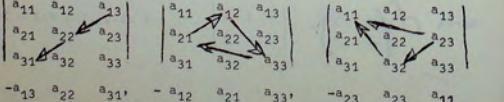
Tibaaxdan waxaynu ku haysanaa lix taramood oo mid waliba ay leedahay saddex kutirsane. Taramaha saddex ka midi way togan yihiin. Kuwaa waxa lagu helaa iyadoo la isku dhufto ku-tirsanayaasha ku jira xaglagooyaha door, iyo iyadoo la isku dhufto ku-tirsanayaasha dhaca ama yaala geesaha labada saddexagal ee dhinacyadoodu ay barbarro la yihiin xaglagooyaha door. Sida ka muuqata labada shaxan ee midigta. Bal u fiirso habkan soo socda.



$a_{11}a_{22}a_{33}$, $a_{12}a_{23}a_{31}$, $a_{21}a_{32}a_{13}$

Kuwaasi waa taramaha togan.

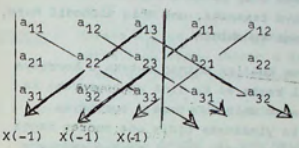
Taramaha tabanina waxay ka samaysmaan xaglogooyaha kale, iyo saddexagalada leh dhinacyada la barbarro ah xaglogooyahan, sidan oo kale.



$-a_{13}a_{22}a_{31}$, $-a_{12}a_{21}a_{33}$, $-a_{23}a_{23}a_{11}$

Kuwaasina waa taramaha taban.

Haddana waxa aad isticmaali kartaa dariiqadii xaglogooyaha oo kale, iyada oo uu kaa caawinayo habkan kale ee hoos ku yaal:



X(-1) X(-1) X(-1)

Halkan labada joog u tax ee hore ee sugaha ayaa mar labaad la dhigay ama lagu celiyay debedda jiitimaha ligan (verticalbar)

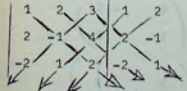
Sida aad ku aragtana waxa la dhigay midigta sugaha. Markaa waxa la qaadanayaa taramaha laga helay xaglogooyayaasha u jeeda xagga midigta; lana beddeli maayo calaamadaha (signs).

Taramaha laga helay xaglogooyayaasha u jeeda xagga bidixdana waxa lagu dhuftaa (-1)

TUSAALE:

Raadi sugaha $\begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ -2 & 1 & 2 \end{vmatrix}$

FURFURID:



markaa, $\begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ -2 & 1 & 2 \end{vmatrix}$ = 1.1.2+2.4.(-2)+3.2.1-3.(-1)(-2)-1.4.1-2.2.2= 2-16+6-6-4-8 = -10.

Waxa cad in taxanaha leh ama wata ku-tirsanayaal maangal ah, sugihiisuna yahay tiro maangal. Waxaynu ku tibaaxi doonaa tiradan maangal in ay tahay qiimaha sugaha.
Sida loo soo saaro tirada waxa la yidhaa "FIDINTA SUGAHA"
Gaar ahaan, qiimaha sugaha taxanaha (a_{11}) waa a_{11}.

TUSAALE: Taxanahan (3) sugihiisu waa $|3| = 3$.

Dariiqada xaglogooyaha ee fidinta sugaha waxa la yidhaa dariiqada saarus (Sarrus); waxana loogu magac daray xisaabyankii soo saaray ama sahamiyay dariiqada iyada ah.

DARIIQADA SAARUS (Sarrus' Method)

(1) Guuri taxanaha lagu siiyay markaa joog-u-taxa u dambeeya midigtiisa ku celi oo marlabaad dhig ku-tirsanayaasha ku jira labada joog-u-tax ee u horreeya taxanaha, una dhig sidoodii hore, iyada oo aanay wax isbeddel ahi ku dhicin.

(2) Ku-tirsanihii kasta ee ku-jira dhinac u-taxa u horreeya waxaad ku-dhufataa ku-tirihii kasta ee ku jira xaglogooyaha ka soo fiday ku-tirsanahii kasta ee dhinac u-taxa u horreeya. Xagla gooyayaashu waa in ay ka yimaadaan bidix una socdan xagga midigta; markaa taramaha la helay waxay ina siinayaan saddexda tibixoole hore ee sugaha. Tibixahaasuna idilkood way togan yihiin.

(3) Sidaas oo kale ku-tirsanihii kasta ee ku jira dhinac u-taxa u horreeya waxaad ku-dhufataa ku jirihii kasta ee ku jira xaglogooyaha ka soo fiday ku-tirsanihii kasta ee dhinac-u-taxa u horreeya. Xaglagooyayaashu waa in ay ka yimaadaan xagga midigta una socdaan xagga bidixda. Saddexda tibixood eetabani waxay ina siiyaan saddexda tibixood ee dambe ee sugaha.

LAYLI

Raadi sugayaashataxanayaashan soo socda:

1. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ 2. $\begin{pmatrix} -2 & 4 \\ -3 & 6 \end{pmatrix}$ 3. $\begin{pmatrix} -1 & 2 \\ 4 & -8 \end{pmatrix}$

4. $\begin{pmatrix} 6 & -2 \\ -1 & 1 \end{pmatrix}$ 5. $\begin{pmatrix} 0 & 1 \\ 6 & -2 \end{pmatrix}$ 6. $\begin{pmatrix} 4 & -5 \\ 3 & 2 \end{pmatrix}$

7. $\begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix}$ 8. $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ 9. $\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$

10. $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 4 \\ 3 & 0 & -2 \end{pmatrix}$ 11. $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 4 & 1 \\ 0 & -8 & 5 \end{pmatrix}$ 12. $\begin{pmatrix} 5 & 0 & -6 \\ 0 & 8 & -2 \\ 5 & 1 & 0 \end{pmatrix}$

13. $\begin{pmatrix} 1 & 3 & 2 \\ 5 & 6 & 7 \\ 2 & 3 & 4 \end{pmatrix}$ 14. $\begin{pmatrix} 8 & -2 & 4 \\ 3 & -1 & 4 \\ 6 & -3 & 5 \end{pmatrix}$ 15. $\begin{pmatrix} 3 & 6 & 5 \\ 2 & 1 & 3 \\ 4 & -5 & 6 \end{pmatrix}$

QEEXID : Yaraha M_{15} ee ku-tirsanaha a_{15} waa sugaha ku soo hadha marka laga reebo dhinac u taxa i aad iyo joog u taxa j aad ee sugaha lagu siiyay.

M = waxay inooga taagan tahay yaraha.

Taasu waxay tahay, si loo helo yaraha kutirsane kasta ee sugaha, waxaynu iska dhaafaynaa ama ka tegaynaa dhinac utax iyo joog u tax wadaag kutirsanahaa.

TUSAALE Haddii aynu haysano sugaha $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

1) Yaraha ku-tirsanaha a_{11} waa M_{11}:

$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$ taas oo aynu ka tagnay dhinac u taxa iyo joog u tax wadaaga ku-tirsanaha a_{11}.

Bal eeg hoos:

$\begin{vmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

2) Yaraha-kutirsanaha a_{23} waa M_{23}:

$M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$ tanna waxaynu ka tagnay dhinac u tax iyo joog u tax wadaaga ku-tirsanaha a_{23}; bal hoos eeg. (fiiri bogga 318)

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

3) Yaraha ku-tirsanaha a_{31} waa M_{31}:

$$M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

Tan oo aynu ku helay sidan:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Bal haddaba adigu raadi yaraha kutirsanaha a_{12}.

Isticmaalidda fikrada yare waxaynu ku raadin karnaa sugaha horsiimo kasta leh kana weyn horsiimada 2aad. Tusaale ahaan hadda aan fiirsano sugaha leh horsiimada saddex. Waxa jirtay in aynu sidan ku raadin jirnay:

$$\delta(A) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33}-a_{11}a_{23}a_{32}+a_{12}a_{23}a_{31} - a_{12}a_{21}a_{33}+a_{13}a_{21}a_{32}-a_{13}a_{32}a_{31}$$

Haddii aynu isirayno tibxaha lammaan ee isir wadaaga ah, waxanu helaynaa sidan:

$$\delta(A) = a_{11}\ (a_{22}a_{33}-a_{23}a_{32})+a_{12}\big(a_{23}a_{31}-a_{21}a_{33}\big)+a_{13}(a_{21}a_{32}-a_{22}a_{31})$$

Iminka haddii isirka laba tibixle ku jira tibixda dhexe uu yahay sidan:

$-(a_{21}\ a_{33} - a_{23}\ a_{31})$, waxaynu haysana

$$\delta(A) = a_{11}\ (a_{22}a_{33}-a_{23}a_{32}) - a_{12}\ (a_{21}a_{33}-a_{23}a_{31}) + a_{13}\ (a_{21}a_{32} - a_{22}\ a_{31})$$

oo la mid ah

$$\delta(A) = a_{11}. \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}. \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

11) Markaa, waxaynu haysanaa ama ay la mid tahayba

$$\delta(A) = a_{11}.y_{11} - a_{12}y_{12} + a_{13}y_{13}$$

Tani waa fidinta sugaha inaga oo isticmaalay yarayaasha ku saabsan <u>dhinac utaxa ugu horreeya</u>.

Guud ahaan waa aad fidin karta sugaha adoo isticmaalaya yarayaasha, taas oo ku saabsan dhinac-utax ama joog utax.

<u>Gebagebo</u>:

1. Ku dhufo ku-tirsane kasta ee ku jira dhinac utax ama joog u tax aad dooratay yarahiisa.
2. Taran kasta ku dhufo 1 ama -1, taas oo aad ku kala dooranayso wadarta hoosdhawgay wataan [illegible] tax [illegible] ahaasu ku [illegible] taxu u [illegible]
   Haddii ay kisi tahay qaado -1, haddii ay dhaban tahayse qaado +1.
3. Isugee taramaha soo baxa.

<u>OGSOONOW</u> Tibaaxdan $a_{11}\cdot y_{11} - a_{12} + a_{13}\cdot y_{13}$,

Ku-tirsanaha a_{11} wuxu ku jiraa dhinac u taxa ugu horreeya iyo joog u taxa ugu horreeya, markaa, haddiiba 1+1 =2 (abyoone dhaban ah), taranta ugu horreeysa waxa lagu dhufanayaa 1(ama sida ay tahay u daa).
Ku-tirsanaha a_{12} wuxu ku jiraa dhinac u taxa ugu horreeya iyo joog utaxa labana; markaa, haddiiba 1+2 = 3 (abyoone kisi ah), taranta labaad waxa lagu dhufanayaa -1;
Wadarta tirada(hoos dhawyada) ee ku-tirsanaha a_{13} waa 1+3 =4 (abyoone dhaban ah), markaa taranta saddexaad waxa loo deynayaa sideeda ama +1 baa lagu dhufanayaa.

TUSAALE

Haddii $A = \begin{pmatrix} 3 & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 3 & 2 \end{pmatrix}$, Raadi $\delta(A)$ adoo ku fidinaya joog u taxa ugu horreeya.

FURFURID:

Waxa aad ogsoon tahay in $a_{11} = 3$, $a_{21}=0$, $a_{31}=1$,

Markaa 1+1 = 2, 2+1=3, 3+1=4
(dhaban) (Kisi) (dhaban)

Markaa, waxaynu helaynaa

In $\delta(A) = 3.\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} - 0.\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} + 1.\begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} =$

$\delta(A) = 3\,(4+6) - 0 + 1\,[(-4) - 1] = 30 + (-5) = 25$

TUSAALE Ku fidi yarayaal sugaha adoo isticmaalaya ama adeegsanaya dhinac u taxa ugu horreeya.

$$\begin{vmatrix} 2 & 1 & 0 & 3 \\ 4 & 2 & 5 & 1 \\ 6 & 3 & 4 & 5 \\ 1 & 0 & 0 & 2 \end{vmatrix}$$

FURFURID

Marka u horayaa ka dhig sugaha leh horsiimada afaraad, mid leh horsiimada saddexaad, kaas oo weliba laga dhigi karo suge leh horsiimada labaad.

Markaa $\begin{vmatrix} 2 & 1 & 0 & 3 \\ 4 & 2 & 5 & 1 \\ 6 & 3 & 4 & 5 \\ 1 & 0 & 0 & 2 \end{vmatrix} = +2.\begin{vmatrix} 2 & 5 & 1 \\ 3 & 4 & 5 \\ 3 & 4 & 5 \\ 0 & 0 & 2 \end{vmatrix} -1.\begin{vmatrix} 4 & 5 & 1 \\ 6 & 4 & 5 \\ 1 & 0 & 2 \end{vmatrix}$

$+ 0 - 3.\begin{vmatrix} 4 & 2 & 5 \\ 6 & 3 & 4 \\ 1 & 0 & 0 \end{vmatrix} =$

Marka aynu fidino suge walba inagoo isticmaalayna dhinac u taxa ugu horreeya waxaynu helaynaa sidan.

$2.\left[2.\begin{vmatrix} 4 & 4 \\ 0 & 2 \end{vmatrix} - 5.\begin{vmatrix} 3 & 5 \\ 0 & 2 \end{vmatrix} + 1.\begin{vmatrix} 3 & 4 \\ 0 & 0 \end{vmatrix}\right] -$

$- \left[4.\begin{vmatrix} 4 & 5 \\ 0 & 2 \end{vmatrix} - 5.\begin{vmatrix} 6 & 5 \\ 1 & 2 \end{vmatrix} + 1.\begin{vmatrix} 6 & 4 \\ 1 & 0 \end{vmatrix}\right]$

$- 3.\left[4.\begin{vmatrix} 3 & 4 \\ 0 & 0 \end{vmatrix} - 2.\begin{vmatrix} 6 & 4 \\ 1 & 0 \end{vmatrix} + 5.\begin{vmatrix} 6 & 3 \\ 1 & 0 \end{vmatrix}\right] =$

$2\,(2.8 - 5.6 + 1.0) - (4.8-5.(12-5)\ 4.(-4)\ ) +$

$-3.(4.0-2.(-4)+4.(-3)) = 2\,(-14)-(-7)\ -3(-7) =$

$-28 + 7 - 21 = 0$

LAYLI

I.- Ka shaqee waydiimahan

(1) $\begin{vmatrix} -1 & 8 & 2 \\ -2 & 1 & 0 \\ 0 & 1 & -3 \end{vmatrix}$ (2) $\begin{vmatrix} 2 & 1 & 4 \\ 3 & 2 & 6 \\ 5 & -3 & 10 \end{vmatrix}$ (3) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 3 & 4 \end{vmatrix}$

(4) $\begin{vmatrix} 14 & 7 & 4 \\ 5 & 2 & 3 \\ 6 & 3 & 3 \end{vmatrix}$ (5) $\begin{vmatrix} a & b & 1 \\ a & b & 1 \\ 1 & 1 & 1 \end{vmatrix}$ (6) $\begin{vmatrix} 0 & 0 & x \\ 0 & x & 0 \\ x & 0 & 0 \end{vmatrix}$

(7) $\begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 3 & 2 \\ 5 & -1 & 2 & 1 \\ 1 & 0 & 1 & 1 \end{vmatrix}$ (8) $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix}$

II.- Fidi sugayaashan lagu siiyay adoo mid walba adeegsanaya dhinac u taxa ama joog u taxalagu siiyay:

(9) $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 1 & 0 & 2 \end{vmatrix}$ (10) $\begin{vmatrix} -1 & 2 & -3 \\ 4 & 5 & 1 \\ 6 & 7 & 0 \end{vmatrix}$ (11) $\begin{vmatrix} 0 & 4 & -1 \\ 2 & 0 & 3 \\ 4 & 5 & 6 \end{vmatrix}$

Dhinac u taxa labaad , Dhinac u taxa seddexaad , Joog u taxa ugu horreeya.

OGSOONOW: Kuwan iyo tusaalooyinka soo socdaba sugayaasha waxa lagu fidiyay iyadoo la isticmaalayo dhinac u taxa ugu horreeya.

ASTAANTA 2 Haddii laba dhinac u tax ama laba joog u tax ee suge ay isku mid yihiin, markaa, suguhu waa eber.

TUSAALE $\begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix} = 1 - 6 + 5 = 0$

ASTAANTA 3: Haddii dhinac u taxyadan iyo joog u taxyadan sugaha oo idil si habsami ah (in order) la isku beddelo, markaa sugahasoo baxaa wuxuu la mid yahay sugihii hore.

TUSAALE:

$S_1 = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 2 \\ 3 & 1 & 2 \end{vmatrix} = 6, \quad S_2 = \begin{vmatrix} 1 & 4 & 3 \\ 2 & 0 & 1 \\ 3 & 2 & 2 \end{vmatrix} = 6$

:. 6 = 6

Ma arki kartaa kuwa la isku beddelay:

1 2 3 oo ah dhinac u tax ugu horreeya ee S_1 wax lagu beddelay 1 4 oo ah joog u taxa ugu horreeya ee S_1, waaana dhinac u taxa ugu horreeya ee S_2.

Sidaas oo kale 4 0 2 oo ah dhinac u taxa labaad ee S_1 waxa lagu beddelay 201 oo ah joog u taxa labaad ee S_1, waana dhinac u taxa labaad ee S_2.

ASTAANTA 4 Haddii ku-tirsane kasta oo ku jira hal dhinac u tax ama joog u tax ee suge lagu dhufto tiro maangal ah K, markaa sugaha soo baxaa waa K oo lagu dhuftay sugihii hore .

TUSAALE $S_1 = \begin{vmatrix} 4 & 5 & 4 \\ 1 & 1 & 1 \\ 3 & 1 & 4 \end{vmatrix} = 12 - 5 - 8 = -1$

$S_2 = \begin{vmatrix} 4 & 5 & 4.2 \\ 1 & 1 & 1.2 \\ 3 & 1 & 4.2 \end{vmatrix} = \begin{vmatrix} 4 & 5 & 8 \\ 1 & 1 & 2 \\ 3 & 1 & 8 \end{vmatrix} = 24-10-16 = -2$

OGOOW

:. $S_2 = 2\,S_1$

ASTAANTA 5: Haddii hal dhinac u tax (ama hal joog u tax)uu qidigii wato ku-tirsanayaal eber ah, markaa suguhuna waa eber.

TUSAALE

$\begin{vmatrix} 1 & 1 & 0 \\ 3 & 5 & 0 \\ 2 & 7 & 0 \end{vmatrix} = 1.\begin{vmatrix} 5 & 0 \\ 7 & 0 \end{vmatrix} - 1.\begin{vmatrix} 3 & 0 \\ 2 & 0 \end{vmatrix} + 0.\begin{vmatrix} 3 & 5 \\ 2 & 7 \end{vmatrix}$

$= 0.0+0 = 0$

Bal adigu isku day marka hal dhinac u tax gidigii yahay eber dabeed eeg waxa soo baxa.

ASTAANTA 6: Haddii hal dhinac u tax (ama joog u tax) ee suge uu yahay dhufsanaha dhinac-u-tax (ama joog u tax) kale, markaa qiimaha suguhu wuxuu noqon eber.

TUSAALE $\begin{vmatrix} 3 & 2 & 1 \\ -1 & -2 & 4 \\ 6 & 4 & 2 \end{vmatrix} = 3.\begin{vmatrix} -2 & 4 \\ 4 & 2 \end{vmatrix} - 2.\begin{vmatrix} -1 & 4 \\ 6 & 2 \end{vmatrix} +$

$1.\begin{vmatrix} -1 & -2 \\ 6 & 4 \end{vmatrix} = -60 + 52 + 8 = 0$

Halkan dhinac u taxa saddexaad waa dhufsanaha dhinac u taxa ugu horreeya.

ASTAANT 7: Haddii ku-tirsane kasta ee dhinac u-tax (ama joog u tax) ee suge lagu dhufto tiro maangal ah K, oo markaa taramaha soo baxa loo geeyo ku-tirsanayaasha ku beegan ee dhinac u tax (ama joog u tax) kale, sida ay u kala horreeyaan, markaa sugaha la helaa wuxuu le'eg yahay ama la mid yahay sugihii hore.

$$\text{TUSAALE } S_1 = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 2 \\ 3 & 1 & 2 \end{vmatrix} = 6,$$

K = 2

$$S_2 = \begin{vmatrix} 1+2.3 & 2 & 3 \\ 4+2.2 & 0 & 2 \\ 3+2.2 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 7 & 2 & 3 \\ 8 & 0 & 2 \\ 7 & 1 & 2 \end{vmatrix} = 6$$

Markaa 6 = 6

ASTAANTA 8: Haddii ku-tirsane kasta oo ku jira dhinac u tax (ama joog u tax) ee suge loo qoro wadarta laba tibxood, markaa sugaha waxa loo qori karaa wadarta laba suge. Sida soo socota:

TUSAALE:

$$\begin{vmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 1 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0+1 & 0+1 & 4+1 \end{vmatrix} = \begin{vmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{vmatrix} + \begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix}$$

$$4 \begin{vmatrix} 4 & 0 \\ 1 & 5 \end{vmatrix}$$

$$4\,(20-0) = \begin{vmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0+1 & 0+1 & 4+1 \end{vmatrix} = 4 \begin{vmatrix} 4 & 0 \\ 0 & 4 \end{vmatrix} + 4 \begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix}$$

80 = 4 (16-0) + 4(4-0)

64 +16 = 80

Ka shaqee

TUSAALE $\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 6 \\ -3 & 5 & 6 \end{vmatrix}$

FURFURID

Fidinta sugaha waxaynu isticmaalaynaa astaanta 7aad, markaa si loo helo suge la mid ah; hase-yeeshe leh dhinac u tax ama joog u tax ku-tirsanayaashiisu ay yihiin eber mid mooyaane. Hadda taa aan u doorano joog u taxa ugu horreeya.

Talaabada I ku dhufo laba taban (-2) dhinac u tax ugu horreeya dabeedna u gee waxa soo baxa dhinac u tax labaad.

$$\begin{vmatrix} 1 & 3 & 3 & 4 \\ 2+(-2) & 1+(-6) & 6+(-8) \\ -3 & 5 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 4 \\ 0 & -5 & -2 \\ -3 & 5 & 6 \end{vmatrix}$$

TALAABADA II: Ku dhufo saddex dhinac u taxa ugu horreeya dabeedna waxa soo baxa una gee, dhinac u taxa saddexaad.

$$\begin{vmatrix} 1 & 3 & 3 & 4 \\ 0 & -5 & -2 \\ -3+3 & 5+9 & 6+12 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 4 \\ 0 & -5 & -2 \\ 0 & 14 & 18 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 3 & 4 \\ 0 & -5 & -2 \\ 0 & 14 & 18 \end{vmatrix}$$

TALAABADA SADDEXAAD:

Ku fidi yareyaal, qaadana joog u taxa ugu horreeya

$$1. \begin{vmatrix} -5 & -2 \\ 14 & 18 \end{vmatrix} - 0. \begin{vmatrix} 3 & 4 \\ 14 & 18 \end{vmatrix} + 0. \begin{vmatrix} 3 & 4 \\ -5 & -2 \end{vmatrix} =$$

$$1. \left[(-90) - (-28) \right] = -62$$

LAYLI

Ka shaqee sugayaashan lagu siiyay; adeegsana astaamo, si ay shaqadu u fududaato.

1) $\begin{vmatrix} 1 & 2 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 2 \end{vmatrix}$ (2) $\begin{vmatrix} 1 & 0 & 6 \\ 1 & 0 & 3 \\ 1 & 0 & 4 \end{vmatrix}$ (3) $\begin{vmatrix} 1 & 2 & 3 \\ -1 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix}$

4) $\begin{vmatrix} 60 & 30 & 20 \\ 30 & 15 & 10 \\ 70 & 80 & 93 \end{vmatrix}$ (5) $\begin{vmatrix} -2 & 1 & 0 \\ 3 & 4 & 1 \\ -4 & 2 & 0 \end{vmatrix}$ (6) $\begin{vmatrix} 2 & 3 & 1 \\ 0 & 0 & 0 \\ -1 & 2 & 0 \end{vmatrix}$

7)3 $\begin{vmatrix} 3 & 1 & 3 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix}$ (8) $\begin{vmatrix} 0 & 1 & 0 & 2 \\ 0 & 2 & 0 & 3 \\ 2 & -1 & 1 & 0 \\ 0 & 0 & 8 & 8 \end{vmatrix}$ (9) $\begin{vmatrix} 2 & 3 & 1 & 1 \\ 2 & 0 & 1 & 2 \\ 2 & 3 & 1 & 1 \\ 0 & 1 & 2 & 0 \end{vmatrix}$

10) $\begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 4 & 8 & 4 \\ -2 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 \end{vmatrix}$

ISWEYDAARKA TAXANE

QEEX: Taxanaha A^{-1}waa isweydaarka taxanaha A, haddii tarantooda

$$A.A^{-1} = A^{-1}.A = 1.$$

Markaa taxanaha A mar kale waa isweydaarka taxanaha A^{-1}. (A iyo A^{-1}, waxay yihiin taxanayaal labajibbaarane ah oo isku addimo ah))

OGSOONOW: A^{-1} la mid ma aha $\frac{1}{A}$.

TUSAALE fududdbaynu u fiirsan doonaa oo ah taxanayaasha ah 2 X 2.

HADDii aynu u qaadano in

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad A^{-1} \text{ na } = \begin{pmatrix} b & c \\ d & e \end{pmatrix}$$

Markaa tarantoodu waxay noqon sidan:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b & c \\ d & e \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Tani waxay inoo hoggaaminaysaa

$$\begin{pmatrix} a_{11}\ b + a_{12}\ d & a_{11}\ c + a_{11}\ e \\ a_{21}\ b + a_{22}\ d & a_{21}\ c + a_{22}\ e \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

oo run ah, haddii iyo haddii oo keliya oo

$a_{11}\ b + a_{12}\ d = 1 \qquad a_{11}\ c + a_{12}\ e = 0$

$a_{21}\ b + a_{22}\ d = 0 \qquad a_{21}\ c + a_{22}\ e = 1$

Marka isle'egyadan wada jir ahaan loogu furfuro b,c,d,e waxaynu helaynaa sidan:

1. $(a_{11}\ a_{22} - a_{12}\ a_{21})\ b = a_{22}$
2. $(a_{11}\ a_{22} - a_{12}\ a_{21})\ c = a_{12}$

3. $(a_{11}\,a_{22} - a_{12}\,a_{21})\,d = -a_{12}$

4. $(a_{11}\,a_{22} - a_{12}\,a_{21})\,e = a_{11}$

(isle'egta 1aad waxa lagu helay sidan soo socota)

(1) $a_{11}\,b + a_{12}\,d = 1$

(2) $a_{21}\,b + a_{22}\,d = 0$ Markaa $a_{22}d = -a_{21}\,b$,

(3) $$d = \frac{-a_{21}\,b}{a_{22}}$$

Markaa haddii aynu d ku beddello isle'egta kowaad (1) oo aynu ku beddello inta d ay la mid tahay isle'egta saddexaad (3), waxaynu helaynaa sidan:

$$a_{11}\,b + a_{12}\left(\frac{-a_{21}}{a_{22}}\right) b = 1$$

Tan waxaynu ka helaynaa isle'egtan $a_{22}\,a_{11}\,b - a_{12}\,a_{21}\,b = a_{22}$

$$(a_{22}\,a_{11} - a_{12}\,a_{21})\,b = a_{22}.$$

Markaa $$b = \frac{a_{22}}{a_{22}a_{11} - a_{12}a_{21}}$$

(Sidaas oo kale ayeynu ku heli karnaa isle'egyada kale)

Markaa qiimayaasha b, c, d, iyo e waxa ku siiya tibaaxahan

$$b = \frac{a_{22}}{a_{11}\,a_{22} - a_{12}a_{21}}\;;\quad c = \frac{-a_{12}}{a_{11}\,a_{22} - a_{12}\,a_{21}}\;;$$

$$d = \frac{-a_{21}}{a_{11}\,a_{22} - a_{12}\,a_{21}};\qquad e = \frac{a_{11}}{a_{11}\,a_{22} - a_{12}\,a_{21}};$$

Shardi waxa ah in $a_{11}\,a_{22} - a_{12}\,a_{21} \neq 0$

Waad arki kartaa, in hooseeyayaasha jajabkanu uu la mid yahay qma le'eg yahay sugaha δ_A,

Markaa, $$A^{-1} = \begin{pmatrix} \frac{a_{22}}{\delta(A)} & \frac{-a_{12}}{\delta(A)} \\ \frac{-a_{21}}{\delta(A)} & \frac{a_{11}}{\delta(A)} \end{pmatrix} = \frac{1}{\delta(A)} \begin{pmatrix} a_{22} & a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$$

Markaa toos la isugu dhufto, waxa la caddayn karaa in $A^{-1}.A = 1$. Markaa, si loo qoro isweydaarka taxanaha A ee laba jibbaarane (2X2), kaasoo (A) $\neq$ 0, waxaynu isku beddeli karnaa ku-tirsanayaasha ku jira (xaglogooyaha door), labada kutirsanee kelna waxa aynu qaadan tabnaantooda, waxa soo baxana waxa aynu ku dhufan $\frac{1}{\delta(A)}$

TUSAALE: Haddii $A = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$, Raadi A^{-1}

FURFURID: Marka hore waa in aynu helaa sugaha A; kaasoo ah $\delta(A) = \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -1 - 6 = 7$

Markaa, $$A^{-1} = \frac{1}{-7}\begin{pmatrix} -1 & -3 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{7} & \frac{3}{7} \\ \frac{2}{7} & \frac{-1}{7} \end{pmatrix}$$

Marka, aad heshid isweydaarka taxane, waa in aad had iyo jeer hubisaa waxa soo baxaa inaanay qalad lahayn.
Hadda, kii bal aynu hubino:

U fiirso $$A^{-1}A = -\frac{1}{7}\begin{pmatrix} -1 & -3 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$$

$$= \frac{1}{7}\begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

Taxane kasta oo laba jibaarane ahiba wuu leeyahay isweydaar, hase yeeshe shardi waxa ah in sugaha taxanuhu aanu le'egaanin eber. Taxanaha isweydaarka leh, waxa la yidhaa weydaarle.

Taxanaha laba jibaarane, ee suguhuna yahay eber, waxa lagu magacaabaa taxanaha weydaarlaawe (a singular matrix).

TUSAALE: Tus in ay $\begin{pmatrix} 3 & 5 \\ 6 & 10 \end{pmatrix}$ yahay weydaarlaawe oo aanu lahayn isweydaar.

FURFURID $\int(A) = \begin{pmatrix} 3 & 5 \\ 6 & 10 \end{pmatrix} = 3.10 - 5.6 = 0$

Markaa, isweydaar ma jiro.
Waayo?

ARAGTIIN: Haddii A iyo B ay yihiin taxanayaal laba jibbaarane oo weydaarle ah (non-singular), markaa, tarantoodu AB waxay leedahay isweydaar, taas oo ah $(AB)^{-1} = B^{-1}.A^{-1}$

CADDAYN: Haddii aynu $B^{1}A^{-1}$ kaga dhufano midigta AB. Waxaynu helaa sidan:

AB . $B^{-1}.A^{1}$ = Xeerka hormagelinta ee isku dhufashada
A $(B.B^{1})A^{-1}$ = Astaanta isweydaarka taxanaha
A.1.A^{1} = Hormagelinta
A(1.A^{1}) = Astaanta asal madoorshe
A.A^{-1} = Astaanta isweydaarka

Sidaas oo kale haddii aynu $B^{-1}A^{-1}$ kaga dhufano bidixda AB, waxaynu helaa tii hore oo kale.
$B^{-1}A^{-1}$, AB = $B^{-1}.1.B = B^{1}\ B = \frac{1}{B} \times B = 1$

Markaa qeexiddii ahayd isweydaarka taxane, waxay ahayd sidan $(AB)^{-1} = B^{-1}.A^{-1}$

ARAGTIDIINA waxa loo adeegsan karaa isweydaarka taranta marka la raadinayo tiro kasta oo taxanayaal weydaarlayaal ah (non-singular).
TUSAALE ahaan, haddii halkan ay yaalaan saddex isir, A, B, iyo C markaa, $(ABC)^{-1} = ((AB).C)^{-1} = C^{-1}(AB)^{1} = C^{-1}.B^{-1}.A^{-1}$.

Waxaynu soo aragnay dariiqada lagu helo ama lagu soo saaro isweydaarka taxane laba - jibbaarane ah, 2x2. Haddaba, runtu sida ay tahay ma fududa sida lagu raadiyo isweydaarka ee taxane leh horsiimo ka weyn 2. Markaa in kasta oo ay jiraan dariiqooyin lagu raadiyaa isweydaarka isaga ah, haddana buuggan r kuma soo hadal qaadayno.

Haddii aad shaqadan fahmi waydo ku nogo baabkii ahaa isweydaarka taxane.

Xaqiiji in ay $\left[\begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}\right]^{-1} = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix}$

FURFURID

a) $\begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix}\ \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1.0+\ 4.1 & 1.2\ +\ 4.0 \\ 0.0+(-2)1 & 0.2\ +(-2)0 \end{pmatrix}$

$\begin{pmatrix} 4 & 2 \\ -2 & 0 \end{pmatrix}$

$\begin{pmatrix} 4 & 2 \\ -2 & 0 \end{pmatrix}^{-1} = 1/4\begin{pmatrix} 0 & -2 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$

Intani waa inta bidixda xigta.

b) $\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}^{-1} = -\frac{1}{2}\begin{pmatrix} 0 & -2 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix}^{-1} = -\frac{1}{2}\ \begin{pmatrix} -2 & -4 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & -\frac{1}{2} \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & 0 \end{pmatrix}\ \begin{pmatrix} 1 & 2 \\ 0 & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0.1+1.0 & 0.2+1.(-\frac{1}{2}) \\ \frac{1}{2}.1+0.0 & \frac{1}{2}.2\ +0.(-\frac{1}{2}) \end{pmatrix}$

$= \begin{pmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$

Intana waa inta midigta xigta.

Markaa, labada dhinac ee isle'egta waxaynu ka helay laba taxane oo isku mid ah, markaa waynu caddaynay in labadu ay isku mid yihiin.

LAYLI Raadi taxanayaalkan soo socda isweydaarkooda haddii aanay qaar isweydaar lahayn, sheeg sababta:

1. $\begin{pmatrix} 2 & 4 \\ 1 & 0 \end{pmatrix}$, 2. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, 3. $\begin{pmatrix} -1 & -1 \\ -4 & 6 \end{pmatrix}$,

4. $\begin{pmatrix} 6 & 0 \\ -3 & 0 \end{pmatrix}$ 5. $\begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix}$ 6. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$,

7. $\begin{pmatrix} 0 & 1 \\ 3 & 1 \end{pmatrix}$ 8. $\begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$ 9. $\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$

Raadi taranta

10. $\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}^{-1}$ 11. $\begin{pmatrix} 0 & 1 \\ 3 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}^{-1}$

Xaqiiji in ay

12. $\left[\begin{pmatrix} 1 & 2 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}\right]^{-1} = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}^{-1}\begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}^{-1}$

HABDHISKA ISLE'EGYADA TOOSAN

Taxanayaashu waxay inoo sheegaan [illegible] raadiyo furfurista habdhiska isle'egyada toosan.
Bal u fiirso isle'egyadan soo socda.

$a_{11}x + a_{12}y = c_1$

$a_{21}x + a_{22}y = c_2$

Markaa haddii aynu raacno qeexiddii isku dhufashada taxanaha, waxaynu odhan karnaa:

$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$, taas oo isirka hore ee ku jira qaybta bidixda lagu magacaabo taxane-weheliyaha (the coefficient matrix) habdhiska.

Si gaaban isle'egta waxa loo qori karaa sidan AX = B, taas oo A tahay taxane laba jibbaarane, nXn, X iyo Bna yihiin taxanayaal joog u tax, nX1.

OGSOONOW: Halkan waxaynu ku fiirinay taxane weheliye ah 2X2 iyo taxanayaal joog u tax oo ah 2X1, maxaa yeelay waxa la ina siiyay laba isle'eg oo toosan oo ay ku jiraan laba doorsoome.

Haddaba marka la haysto n isle'egyo toosan oo ay ku jiraan n doorsoome, waxaynu isticmaalaynaa nXn taxane weheliye.

Bal dheeho isle'egtan guud ahaaneed nX = B; haddii a tahay taxane weydaarle (non-singular matrix),markaa labada dhinac ee isle'egta waxaynu bidixda kaga dhufan karnaa A^{-1}, si aynu u helo: $A^{-1}.AX = A^{-1}.B$ ama $(A^{-1}.A)X = \bar{A}^1.B$.

Kol haddii $A^{-1}.A = 1$, waxaynu helaynaa $1.X = A^{-1}.B$

$$\therefore \quad X = \bar{A}^1.B$$

HADDIIBA X iyo $\bar{A}^1.B$ ay isle'eg yihiin, kutirsane walloo ku jira x wuxuu le'egyahay ku-tirsanaha ku beegan ee ku jira $\bar{A}^1.B$. Markaana ku-tirsanayaasha $\bar{A}^1.B$ waa xubnaha (solution set) ee habdhiska toosan ee la ina siiyay.

OGSOONOW: Haddii A ay tahay taxane weydaarlaawe (singular), markaa ma yeelan karo isweydaarba, sidaa daraadeed habdhiskuna malaha furfurisba, ama furfuristu ma aha madi.

TUSAALE: RAADI urur-furfurada adoo adeegsanaya taxanaysal.

$$5X + 2Y = 12$$
$$4X - Y = 7$$

FURFURIS: Isle'egtan waxaynu u qori karnaa sansaankan

$$\begin{pmatrix} 5 & 2 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}$$

Markaa, waxa aynu raadinaynaa sugaha taxanaha weheliyaha (Co-efficient matrix):

$$S = \begin{vmatrix} 5 & 2 \\ 4 & 1 \end{vmatrix} = -5-8 = 13$$

Haddana waxa aynu raadinaynaa isweydaarka taxanihii

$A = \begin{pmatrix} 5 & 2 \\ 4 & -1 \end{pmatrix}$ oo ah $A^{-1} = \frac{1}{-13}\begin{pmatrix} -1 & -2 \\ -4 & 5 \end{pmatrix}$

Isle'egta taxane, dhinac walba xagga bidixda kaga dhufo A^{-1}, sidan oo kale

$$\frac{1}{-13}\begin{pmatrix} -1 & -2 \\ -4 & 5 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-13}\begin{pmatrix} -1 & -2 \\ -4 & 5 \end{pmatrix}\begin{pmatrix} 12 \\ 7 \end{pmatrix}$$

ama $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-13}\begin{pmatrix} -2 & 6 \\ -1 & 3 \end{pmatrix}$

$= \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ Markaa halkan waxaynu ka helay in

$x = 2,\ y = 1$.

Marka, urur-furfurada habdhiskan la ina siiyayna waa $\{2, 1\}$

TUSAALE Furfur habdhiskan toosan

$$2x + 3y = 5$$
$$6x + 9y = 10$$

FURFURIS : Marka u horraysa waxaynu u qoraynaa habdhiska sida isle'eg taxane.

$$\begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$$

Talaabada ku xigtaa waa in la raadiyo ama la helo sugaha taxanaha weheliye, sidan oo kale:

$$\delta\begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix} = \begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix} = 18 - 18 = 0$$

Sida aad u aragtidba suguhu waa eber, markaa taxanuhu waa weydaarlaawe (singular), oo malaha isweydaar.

Haddii aad u fiirsatid weheliyayaasha x iyo y, waxa aad ku arki kartaa in weheliyaasha ku jira isle'egta dambe ay saddex laab ka weyn yihiin weheliyaasha ku jira isle'egta hore.

Markaa, waxaabay yihiin saamigal ama way saamigalsan yihiin. Hase yeeshe tirooyinka ku jira dhinacyada midigta ee isle'egyadu saamigalkoodu ma aha sida weheliyaasha bidixda ee isle'egyada (toban laba jeer oo kaliya ayay ka weyn tahay shan). Markaa, haddiiba weheliyayaasha x iyo y ay saamigalsan yihiin, oo aanay tibxaha sugani ṣaamigalsanayn, markaa habdhisku haba yaraatee malaha furfurid. Isle'egyadaas oo kale waxa lagu magacaabaa surmaseegto.

Tan waxa aad looga arki karaa garaafka. Garaafyada isle'eg yadaasuna waa barbarro, bar ay wadaagaana haba yaraatee ma jirto.

TUSAALE

Furfur $2x + 3y = 5$
$4x + 6y = 10$

Bal adigu samee garaafka
labada xarriiqood barbarro ayay
noqonayaan, barna ma wadaagi doonaan

FURFURIS: Sugaha taxanaha weheliye waa

$$\begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 12 - 12 = 0$$

Markaa taxanuhu malaha isweydaar. Weheliyayaasha isle'egta labaad oo idil waa laban laabka weheliyayaasha ku beegan ee isle'egta hore. ((weheliyayaasha oo idili way isu saamigal-san yihiin))

Labadaa isle'eg waxa lagu magacaabaa siyaab .

Furfuridda midkood ayaa ah furfuridda ama jawaabta labadoodaba. Markaas, waxaynu nidhaa habdhiskaanu wuxuu leeyahay furfuro tirobeel ah ama tiro beelay.

Garaafyada isle'egyadaasuna way isdul dhacaan, ama way isdul fuulaan.

TUSAALE: Furfur habdhiskan

$$3x + 4y = 2$$
$$2x + y = 3$$

FURFURIS :

Halkan waxaynu ku aragnaa in weheliyayaasha ku jira isle'egyadani in aanay ahayn saamigal, dabeed halkaa waxa ka cad in habdhisku leeyahay furfurid.

Isle'egta taxane waxay tahay sidan:

$$\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Haddana sugaha taxanaha weheliye wuxuu yahay:

$$\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix} = 3 - 8 = 5$$

Markaa, taxanuhu wuxuu yeelanayaa isweydaar; haddii uu isweydaar yeeshaan maxaad filaysaa in uu yeesho?

Isweydaarkuna waa: $-\frac{1}{5}\begin{pmatrix} 1 & -4 \\ -2 & 3 \end{pmatrix}$

Markaa, $\begin{matrix} x \\ y \end{matrix} = \frac{-1}{5}\begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} =$

$$\frac{1}{5}\begin{pmatrix} -10 \\ 5 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix};$$

taas oo $x = 2$, $y = 1$

urur-furfurkuna wuxuu yahay :

$$\{2; -1\}$$

LAYLI: Raadi furfurista habdhisyadan soo socda ((Haddii aanay lahayn furfuris sheeg sababta))

1. $2x - 3y = -1$
3. $6x - 2y = 4$
   $3x - y = 1$
2. $2x - 3y = 0$
   $2x + y = 16$
4. $3x - 4y = 2$
   $6x + 12y = 36$

5. $4x + 4y = 4$
   $x + 3y = -4$
6. $3x + 2y = 4$
   $5x + 3y = 0$
7. $3x + 3y = -1$
   $x - 4y = -2$
8. $10x + y = 5$
   $x + y = -4$
9. $2x - y = 3$
   $6x - 3y = 9$

XEERKA GARAAMAR

Marka la adeegsanayo taxane siyaabaha lagu furfuro habdhisyada ee isle'egyo toosan, waxay ku lug leedahay ama ay ku xidhan tahayba isweydaarka taxanaha weheliye. Haddii taxanaha weheliye uu yahay taxane ah 2x2, wax alaale wax dhibaato ahi ma jirayso.

Hase-yeeshe taxanayaasha leh adimo waaweyn, siyaabaha loo raadinaayo iswaydaarkoodu aad buu u dhib badan yahay. Si haddaba aanay dhibaatadaasu u jirin ayaa waxa aynu adeegsanaynaa sugayaal, kuwaas oo aynu ku shaqaynayno xeerka Garaamar, inaga oo aan adeegsan taxanayaal.

Bal haddaba eeg sida loo dhisay xeerka Garaamar.
U fiirso habdhiska soo socda:

$$a_{11}x + a_{12}y = c_1$$
$$a_{21}x + a_{22}y = c_2$$

Aan u qaadano in D ay u taagan tahay sugaha taxanaha weheliye.

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

X ku dhufo (adeegsane astaanta 4aad ee sugayaasha).

$$XD = X \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} a_{11} \times a_{12} \\ a_{21} \times a_{22} \end{vmatrix}$$

Imminka, ku-tirsaneye waloo ku jira joog u taxa labaad ku dhufo tiro maangal ah y; waxa soo baxana u gee ku tirsanaayaasha ku beegan ee joog u taxa hore (waxaynu adeegsanaynaa astaanta 7aad markaa waxaynu helaynaa sidan:

$$XD = \begin{vmatrix} a_{11}x + a_{12}y & a_{12} \\ a_{21}x + a_{22}y & a_{22} \end{vmatrix}$$

Ugu dambayn, haddiba $a_{11}x + a_{12}y = c_1$

$$a_{21}x + a_{22}y = c_2$$

waxaynu haysanaynaa sidan:

$$XD = \begin{matrix} c_1 & a_{12} \\ c_2 & a_{22} \end{matrix} \quad \text{taas oo (haddii } D \neq 0)$$

aynu helayno sidan:

$$X = \frac{\begin{vmatrix} c_1 & a_{12} \\ c_2 & a_{22} \end{vmatrix}}{D} = \frac{\begin{vmatrix} c_1 & a_{12} \\ c_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

Si gaaban haddii aynu u qorno waxay noqonaysaa sidan $X = \frac{DX}{D}$.

Ogsoonow: Sugaha DX waxa aynu ku helay marka joog u taxa hore ee ku jira D halkeedii la dhigo tirooyinka madoorsoomayaasha ah kuna beegan ee ku jira dhinacyada midigta ee isle'egyada layna siiyay.

Sidan oo kale waxaynu tusi karnaa in $Y = \frac{\begin{vmatrix} a_{11} & c_1 \\ a_{21} & c_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{DY}{D}$

DY waxaynu ka helay D, taas oo joog u taxa labaad ee D ama sugaha halkeedii aynu dhignay madoorsoomayaasha isle'egta. Dariiqadan waxa lagu addeegsan karaa ama lagu shaqayn karaa habdhisyada isle'egyo kasta oo toosan kuwaas oo ay ku jiraan doorsoomayaal badan. Dariiqadaa iyada ah waxa lagu magacaabaa xeerka Garaamar. Waxa uu xeerkaasi karti inoo siinayaa in aynnu ku furfurro habdhisyada toosano ha yeeshee shardiga ku xidhani wuxu yahay in aanu sugaha taxanaha weheliye noqonin eber.

Tusaale:

Adeegso xeerka Garaamar si aad u furfurtid habdhiska:

$$\begin{aligned} -4x + 2y - 9S &= 2 \\ 3x + 4y + S &= 5 \\ x - 3y + 2S &= 8 \end{aligned}$$

FURFURIS: Marka hore qor taxanaha weheliye, raadina suganihiisa

$$A = \begin{pmatrix} -4 & 2 & -9 \\ 3 & 4 & 1 \\ 1 & -3 & 2 \end{pmatrix}, \quad D = \begin{vmatrix} -4 & 2 & -9 \\ 3 & 4 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= -4\begin{vmatrix} 4 & 1 \\ -3 & 2 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - 9\begin{vmatrix} 3 & 4 \\ 1 & -3 \end{vmatrix} = -4.11 - 2.5 - 9(-13) = 63$$

Markaa sidii aynu horeba u soo sheegnay, waxaynu ku tirsanaayaasha ku jira joog u taxa uu horreeya halkooda dhigaynaa madoorsoomayaasha ku beegan ee ku jira isle'egta 2, 5, 8. Bal eeg:

$$DX = \begin{vmatrix} 2 & 2 & -9 \\ 5 & 4 & 1 \\ 8 & -3 & 2 \end{vmatrix} = 2\begin{vmatrix} 4 & 1 \\ -3 & 1 \end{vmatrix} - 2\begin{vmatrix} 5 & 1 \\ 8 & 2 \end{vmatrix} - 9\begin{vmatrix} 5 & 4 \\ 8 & -3 \end{vmatrix} =$$

$$2.11 - 2.2 - 9 \quad (-47) = 441$$

Sidii oo kale ku-tirsanayaasha ku jira joog u taxa labaad ayaynu halkooda dhigi doonaa madoorsoomayaasha ku beegan ee ku jira isle'egta. Madoorsoomayaashaas oo ah 2,5,8; markaa waxa aynu helaynaa:

$$DY = \begin{vmatrix} 4 & 2 & -9 \\ 3 & 5 & 1 \\ 1 & 8 & 2 \end{vmatrix} = 4\begin{vmatrix} 5 & 1 \\ 8 & 2 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - 9\begin{vmatrix} 3 & 5 \\ 1 & 8 \end{vmatrix} =$$

$$-4\ (2) - 2(5) - 9(19) = -189$$

Imminkana halkii joog u taxa saddexaad baynu dhigi madoorsoomayaasha isle'egta.

$$DS = \begin{vmatrix} -4 & 2 & 2 \\ 3 & 4 & 5 \\ 1 & -3 & 8 \end{vmatrix} = -4\begin{vmatrix} 4 & 5 \\ -3 & 8 \end{vmatrix} - 2\begin{vmatrix} 3 & 5 \\ 3 & 8 \end{vmatrix} - 2\begin{vmatrix} 3 & 5 \\ 1 & 8 \end{vmatrix} + 2\begin{vmatrix} 3 & 4 \\ 1 & -3 \end{vmatrix} =$$

$$= -4\ (47) - 2(19) + 2(-13) = -252$$

Markaa haddii DX, DY iyo DS aynu u qaybino D waxa aynu helaynaa X, Y,S, Sida ay u kala horreeyaan:

$$X = \frac{DX}{D} = \frac{441}{63} = 7 \quad Y = \frac{DY}{D} = \frac{189}{63} = -3$$

$$S = \frac{DS}{D} = -\frac{252}{63} = -\frac{4}{}$$

Dabadeed, habdhiskan urur furufurkiisu waxa uu noqonayaa:
(7, -3, -4)

TUSAALE: Furfur habdhiskan

$$\begin{aligned} x - 2y + s &= -1 \\ 3x + y - 2s &= 4 \\ y - s &= 1 \end{aligned}$$

FURFURIS: $D = \begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & -2 \\ 0 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} - 3\begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix}$

$$= 1 - 3\ (1) = -2$$

(Sugaha waxa lagu didinayaa joog u taxa ugu horreeya)

Tusaalihii hore waa in aad raacdaa:

$$DX = \begin{vmatrix} -1 & -2 & 1 \\ 4 & 1 & -2 \\ 1 & 1 & -1 \end{vmatrix} = -1\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} + 2\begin{vmatrix} 4 & -2 \\ 1 & -1 \end{vmatrix} + \begin{vmatrix} 4 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= -1 + 2\ (-2) + 3 = -2$$

$$DY = \begin{matrix} 1 & -1 & 1 \\ 3 & 4 & -2 \\ 0 & 1 & -1 \end{matrix} = \begin{matrix} 4 & -2 \\ 1 & -1 \end{matrix} - 3\begin{matrix} -1 & +1 \\ 1 & -1 \end{matrix} = -2 - 3\ (0) = -2$$

$$DS \begin{vmatrix} 1 & -2 & -1 \\ 3 & 1 & 4 \\ 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} = -3\begin{vmatrix} -2 & -11 \\ 1 & 1 \end{vmatrix} = -3 - 3(-1) = 0$$

Markaa waxaynu helaynaa sidan

$$X = \frac{DX}{D} = \frac{-2}{-2} = 1, \quad S = \frac{DS}{D} = \frac{0}{-2} = 0$$

$$Y = \frac{DY}{D} = \frac{-2}{-2} = 1$$

OGSOONOW: Haddiiba sugaha taxanaha weheliyaha aanu la mid ahayn eber, xeerka Garaamar wuxuu kaa caawinayaa sida loo furfuro habdhisyo isle'egyo toosan. Haddii D ay le'egtahay eber markaa habdhisku haba yaraatee malaha furfuris.

Ama habdhisku wuxuu leeyahay furfurisyo badan (taasu waa marka weheliyayaasha oo dhammi ay saamigal yihiin).

LAYLI

Adoo isticmaalaya xeerka Garaamar furfur habdhisyadan toosan:

1. $2x + y = 4$
   $x - 2y = 0$

2. $2x - 3y = 12$
   $x - y = 3$

3. $2x - y = 0$
   $3x - 4y = 5$

4. $2x + y + s = 1$
   $x - 2y - 3s = 1$
   $3x + 2y + 4s = 5$

5. $3x - 2y + 5s = 6$
   $4x - 4y + 3s = 0$
   $5x - y + s = -5$

6. $2x + 5s = 9$
   $4x + 3y = -1$
   $3y - 4s = -13$

7. $x + y + s = 0$
   $2x - y + 2s = 1$
   $3x + 2y - s = -1$

8. $x - 2y + 3s = 0$
   $-x + y - 2s = 5$
   $2y - s = -3$

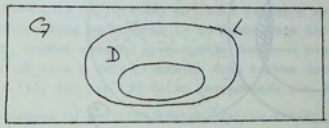
WAKIILINTA WEEDHAHA (REPRESENTING STATEMENTS)

Dhammaan ardaydii dedaashay wey ku liibaaneen imtixaankii lagaga baxayey dugsiyada sare. Weedhan waxaynu ku wakiilin karnaa jaantus fen-Euler.

G = {dhammaan ardaydii u fadhiisatey i.l.d.s.}

L = {dhammaan ardaydii ku liibaantay i.l.d.s.}

D = {dhammaan ardaydii dedaashay}



(Shaxan 1)

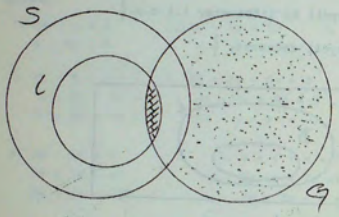
Weedhu waxay malagelineysaa (implies) in D ay hormo u tahay L sida ka muuqata shax.1, oo sida runtu tahay inna tusaya weedha dhammaan ardaydii dedaashay wey ku liibaaneen i.l.d.s.
Bal aan imminka fiirinno mitaal ka sii adayg badan (harder example). Aan weedha dhammaan laydiyadu waa barbarroolayaal iyo laydiyada qaarkood waa kooro (trapezoids) ku wada wakiilino jaantus fen-Euler oo keliya.

L = {Laydiyada dhammaantood}

S = {Barbaroolayaal dhammaantood}

G = {Kooraha dhammaantood}

Labada weedhood waxa loo qori LCS iyo LnG ≠ 0, dheeliga LnG ≠ 0 wuxuu summad ahaan kuu sheegayaa hubaasha ah "waxa jira laydiyaal kooro ah", oo ah weedh been ah, haddii laga eego xagga joomatriga.

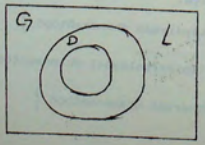


(Shaxan 2)

Gobolka dhibceysan (dotted region) ee ka muuqda shaxan 2 waxa ka suurtooba labada arrimood ee kala ah in uu noqdo gobol madhan, iyo in uu ka gedisnaado ∅.

Asaas-garaado, goammo iyo jaantusyada fen-Euler

Shax. 3 wuxuu wakiil ka yahay weedha "dhammaan ardaydii dedaashay way ku liibaaneen i.l.d.s." haddaba haddii aynu adeegsano jaantuskan fen-Euler, waxa aynu gaadhi karnaa go'aanka runta ah "arday keliya oo dedaaley ma dhicin".



Barashada ururadu waxay faa'iido gaar ahaaneed u leedahay doodaha loojigga, si aan taas u guda galno, aan tixgelino sida xeerarka ururada iyo jaantusyada fen-Euler ay inooga kaalmeeyaan saafidda doodo loojig oo gaar ah.
Bal tixgeli doodan:

Dhammaan labajibaaranayaashu waa laydiyo..... (1)
Dhammaan laydiyadu waa barbarrooleyaal..... (2)
Markaa dhammaan labajibaaranayaashu waa barbarroolayaal (3)

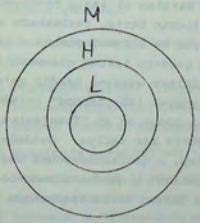
Weedhaha (1) iyo (2) waxa la yiraahaa asaas-garaado ama afeefo, weedha 3-na waxa la yiraahaa go'aanka haddeynu saafno doodeenan gaarka ah waxaynu ogaaneynaa in go'aanka laga soo dheegi karo afeefaha, markaa waxeynu oran doonaa doodeenu waa mid dhisan. Haddeynu si kale u dhigno waxaynu oran karnaa dooda ka kooban weedhaha (1), (2) iyo (3) qiime rumeedkeedu waa (Run).

Doodan waxaynu ku wakiilin karnaa jaantus fen-Euler sida ka muuqata shax. 4 oo

L = {Dhammaan labajibbaaranayaasha}

H = {Dhammaan laydiyada}

M = {Dhammaan barbaroolayaasha}

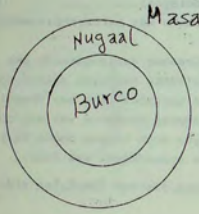


(Shaxan 4)

Sida ku muujisan shaxan 4, L waxay hormo quman u tahay H, H-na waxay hormo quman u tahay M, macnee LCH ∧ HCM, markaa LCM (xeerka dhexidda ee hormonimada ururada).
LCM waa si gaaban oo loo qori karo go'aanka (3).

Dood dhigan iyo Go'aan Been ah

Burco waxay ku taal Nugaal.
Nugaali waxay ku taal Masar
Markaa Burco waxay ku taal Masar.



(Shaxan 5)

Doodda noocani waxay faa'iido gaar ahaaneed ku leedahay caddaynta xisaabta (mathematical proofs). Matalan si aan u caddayno in taanjentka goobo iyo gacanka ka jeexan barta, taabashada ay isku qotomaan, caddeynteena waxaynu ka bilaabi doonaa u qaadashada ah "Taanjentka goobo iyo gacanku iskuma qotomaan dabeedna marka aynu isticmaalo garaadeyn dhisan waxaynu gaadhi go'aanka ah "Taanjentku goobada wuxuu ka gooyaa laba barood" go'aankani wuxuu burinayaa qeexda taanjentka goobo oo ah "Taanjentka, goobo waa xarriiq goobada ka taabata bar keliya. Haddaba Maadaama uu go'aankeenu been yahay, u qaadashadeenuna waa been, markaa waxa run ah diidmada (negation) u qaadashadeena oo ah "Taanjentka goobo iyo gacanka ka jeexan barta taabashada waa ay isku qotomaan.

Caddeyn xisaabeedda sare oo la yiraaho caddaynta dadban waxa si fiican looga fahmi karaa sheekadan.

Cali ayaa lagu soo oogey denbiga ah in uu Xamar nin ku diley 15kii Agoost 1976. Markii la horkeeney, maxkamaddii ayuu qareenkiisi (yiri "sidee buu Cali denbiile u noqon karaa iyadoo ay jiraan markhaatiyaal sheegaya in uu Cali joogey dalka Ruushka maalintii uu denbigu dhacay. Haddii aynu saafno dooda qareenka, Cali ama waa denbiile ama maaha denbiile.

Bal ka soo qaad in uu denbiile yahay. Cali waa denbiile waxay mala gelineysaa Cali wuxuu joogey Xamar maalintii denbigu dhacay (15kii Ogoost 1976). Laakiin waxay markhaatiyaal lagu kalsoon yahay xaqiijiyeen in uu Cali joogey dalka Ruushka maalintii denbigu dhacay; haddaba maadaama aanay suuragal ahayn in uu Cali maalin keliya wada joogo dalka Ruushka iyo Xamar, u qaadashadeeni hore waa been waxa se run ah diidmadeeda oo ah Cali denbiile maaha.

Matalan waxa aad rabtaa in aad caddayso in xarriijinta AB ay le'eg tahay xarriijinta CD. Waxa suuragal ah seddex xiriir: AB $>$ CD, AB $<$ CD, ama AB $=$ CD, haddii aad xaqiijin kartid in AB $\not>$ CD iyo AB $\not<$ CD, markaa waxa aad ku doodi kartaa AB $=$ CD. Haddaba caddeyn xisaabeeddan dadbani waxay inna fareysaa in aan marka hore taxno dhammaan go'aamada suuragalka ah, dabeedna aan xaqiijino in dhammaantood mid mooyaane ay wada been yihiin, markaa ka aynaan caddayn in uu been yahay ayaa run ah.

LAYLI

Ku wakiili doodan soo socota jaantus fen-Euler

1. Haddii ragga qaarkii ay dhaadheer yihiin oo dagaal yahannadu dhammaantood ay dhaadheer yihiin markaa ragga qaarkii waa dagaal yahanno.

2. Adoo adeegsanaya jaantus fen-Euler, hubi (test) dhisnaanta doodan.

b) Dhammaan kooruhu (trapezoids) waa afargeeslayaal.
Dhammaan barbaroolayaashu waa afargeeslayaal.
Markaa dhammaan barbaroolayaashu waa kooro.

t) Naasleyda qaarkood wax ay ku nool yihiin biyaha.
Dhammaan wixii biyaha ku nooli waa kaluun.
Markaa, naasleyda qaarkood waa kaluun.

j) Dhammaan seddexagalladu waa geesoolayaal.
Seddexagallada qaarkood waa labaalayaal.
Dhammaan seddexagalada labaalayaasha ahi waa geesoolayaal.

3. Waa maxay go'aanka laga dheegi karo weedhahan soo socda:

Dhammaan libaaxyada dadcunka ah waxa dhaqda boqor Cali. Libaaxyadu kabaha ma xirtaan ayaga oo dadcun ah mooyaane. Boqor Cali malaha libaaxyo midabkoodu cas yahay.

4. Sharrax caddayn xisaabeedda dadban?

5. Sharrax sidii aad u caddayn lahayd in xarriiqaha l iyo m ay iska gooyaan bar keliya.

DOOD JABAN IYO GO'AAN RUN AH

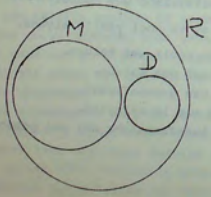
Dadka qaarkii waa dhaadheer ..... (1)
Dadka qaarkii waa caato ..... (2)
Markaa dadka dhaadheer qaarkii waa caato.....(3)
Waxa hubaal ah in weedhaha (1), (2) iyo (3) aan midna la diidi karin hase yeeshee dooda ka kooban (1), (2) iyo (3) waa mid jaban (invalid).

Haddii M = {dadka dhaadheer}

D = {Dadka caatada ah }

R = { Dadka dhammaantii }

Waxa ka muuqata Shax.6 in ay suuragal tahay MnD = Ø ; macnee waxa suurtoobi karta in aanay jirin dad dherer iyo caatanimo isku darsaday. Haddii aan si kale u dhigno, lagama maarmaan maaha in go'aanka (3) uu ka yimaado ama lagaba soo dheego weedhaha (1) iyo (2).



(Shaxan 6)

Dhismaha (structure) doodan waxa aynu ku muujin karnaa inaga oo isticmaala xeerarka ururada sida hoos ku muuqata:

R U M = R ∧ R U D = R
∴ (RUM) n(RUD) = RnR = R
Laakiin waxa aynu tusaaleyney in
(RUM) n(RUD) = RU(MnD)
∴ RU (MnD) = R
Laakiin R U Ø = R

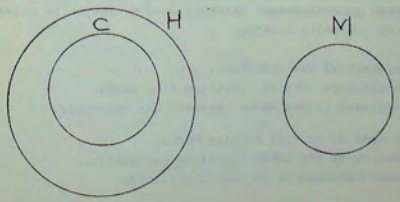
Markaa MnD waxa ay noqon kartaa Ø ;
macnee in M iyo D ay dhextaal yeeshaan maaha wax lagama maarmaan ah.
Waxa suuragal ah in go'aan run ah laga gaadho dood dhisan oo leh laba asaas-garaad (premises) oo been ah. Matalan labada weedhood ee beenta ah 3 > 5 iyo 5 = 2 waxa laga gaadhi karaa go'aanka 3 > 2 oo run ah.

Bal aan tixgelino doodan:
Dhammaan carruurtu waa farxaaniin.
Dadka farxaaniinta ahi ma dheelaan shaxda.
Markaa carruur shaxda dheesha ma jiraan.
Haddii H = {dadka farxaaniinta ah }

C = {Carruurta }

M = {Dadka shaxda dheela }

Waxa shax.7 aynu ka arki karnaa in doodu ay dhisantay oo ay leedahay qiime rumeedka R (Run).



(Shaxan 7)

Haddii aynaan isticmaalin jaantus fen-Euler, waxa aynu qori karnaa:

$C \cap H = C \quad H \cap M = \emptyset$

$C \cap M = (C \ H) \cap M$

$= C \cap (H \cap M)$

$= C \cap \emptyset$

$= \emptyset$

Markaa ma jiraan dad carruurna ah shaxdana dheelaa; macnee ma jiraan carruur shaxda dheeshaa.

LAYLI

Doodahan soo socda haddii ay lagama maarmaan tahay in uu go'aanku ka yimaado asaas garaadyada qoro R(Run); haddii kale, qor B (Been). Dood kasta u samee jaantus fen-Euler oo aad ku xaqiijiso jawaabtaada. Bal isku dey in aad dhisnaanta ama jabnaanta dood kasta ku ogaan karto isticmaalidda xeerarka ururada.

1. Geesoolayaasha qaarkood waa laydiyo. Geesoolayaasha qaarkood waa labajibbaaranayaal. Markaa laydiyada qaarkood waa labajibbaaranayaal.

2. Geesoolayaasha qaarkood waa barbarroolayaal. Barbarroolayaasha qaarkood waa qardhaaso. Markaa geesoolayaasha qaarkood waa qardhaaso.

3. Shan geesoolayaasha qaarkood waxa ay leeyihiin xaglo isle'eg. Shan geesoolayaasha qaarkood waxa ay leeyihiin dhinacyo isle'eg. Markaa shan geesoolayaasha qaarkood waxa ay leeyihiin dhinacyo isle'eg iyo xaglo isle'eg.

4. Wiilasha qaarkood waa dhaadheer.
Wiilasha qaarkood waxa ay leeyihiin timo madow.
Markaa wiilasha timaha madow qaarkood waa dhaadheer.

5. Nayroobi waxa ay ku taal Gobolka Mudug.
Mudug waxa ay ka mid tahay gobollada Soomaaliya.
Markaa Nayroobi waxa ay ku taal Soomaaliya.

6. $x < 1$
$y < 1000$
$\therefore x < y$

7. $x < 2$
$x > 7$
$\therefore \quad 7 < 2$

8. $y < x$
$y^2 > x^2$
$\therefore x < 0$

Tusaalooyinkan hoose mid kasta, sheeg go'aan run ah oo laga gaadhi karo asaas-garaadyada ogaalka ah:

9. Ma jiraan dad waxgarad ah oo caroo badani
dadka wax garadka ah qaarkii way tima madowyihiin.

10. Dahabka qaarkii waa qaali
Alaabta qaaliga ahi dhammaanteed waa quruxsan tahay.