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Capital Theory Debates

Current Relevance of the Cambridge Controversies: A Historical and Analytical Overview

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ABSTRACT

This study deals with the capital theory controversies from both a historical and an analytical standpoint. First, some basic analytical and methodological issues underlying the neoclassical theory are examined; attention is focused on how the problem of a measure of capital arises in economies with heterogeneous capital goods. Secondly, we survey some of the most relevant contributions to the Cambridge capital theory debates in the light of the salient results of ‘reswitching’ and ‘reverse capital deepening’, and show how the implications of these results – which undermine the factor substitution principle – brought about different strategies pursued by neoclassical scholars to overcome theoretic problems in a first phase of the controversies. Thus, it is argued that despite efforts made at the time by renowned economists to minimise the relevance of the debates, the neoclassical approach was compelled to give way to the criticisms which emerged from the controversies. Then we clarify some misunderstandings raised in a second phase of the debates and argue that the shift from a notion of capital as a single factor to a ‘Walrasian’ treatment of capital as a complex of physically heterogeneous goods in the wake of the capital debates was the last ditch dug by the dominant theory to escape the criticisms without giving up the supply and demand framework. The historical-analytical reconstruction of the most salient results of the debates, which touch the foundations of the theory and hence are of a general character, led us to deem as reasonable the argument that capital problems are present in the contemporary versions of the theory. Moreover, even if we put this issue aside, we may still have serious concerns about the usefulness of these versions, whose radically different method – in comparison with the traditional ones – makes it difficult to study real economies. Thus, we conclude that it is hard to accept the seemingly accepted idea in the discipline, that contemporary versions of the theory (‘Walrasian’ treatment of capital) are free of capital problems, while at the same time the capital debates (and their implications) are absent in the current economic literature.

CHAPTER 1

Introduction: Why a study on the capital theory controversies?

mainstream / 'meɪnstri:m / n [the+S] the main or most widely accepted way of thinking or acting in relation to a subject
Longman Dictionary of Contemporary English (12th impression 1991) p. 630.

1.1. Capital as a factor of production and the capital theory debates.

Beginning in the mid-1950s and for the following twenty years, a debate concerning the neoclassical treatment of capital became noticeable in the discipline. This gave rise to a series of interchanges between scholars associated with Cambridge, England, and Cambridge, Massachusetts, (US). This debate is broadly known in the literature as the 'Cambridge capital theory controversies'.¹

The relevance of this controversy lies in that the criticisms of neoclassical theory raised by Cambridge (England) concern the foundational premise underlying the dominant supply and demand approach, namely the factor substitution principle. In the controversies it has been revealed that this principle cannot in general be postulated in order to explain the distribution of the social product in terms of supply and demand. This result, discovered by means of the analysis of the relation between prices and distribution in economies with heterogeneous capital goods, has been revealed as theoretically irrefutable, and, as this study will argue, concerns the hard core of the neoclassical or marginal theory² both in its traditional (capital in value terms) and in its contemporary formulations (capital as a set of heterogeneous goods).

¹ This label was christened in print by the first survey published by Harcourt in (1969). See also n.9 below.

² In this study we shall refer to the neoclassical or marginal approach indistinctly – though the author should like to emphasise that the label 'neoclassical', used in the literature to refer to a theory that is if any *anti*-classical, is a misfortune in the history of economic ideas.

Paradoxically, this debate is hermetically concealed in current economics teaching; as can be seen by leafing through any undergraduate or graduate economic theory textbook, neoclassical theory is the only one theory taught. At best, we are sometimes informed by the mainstream of economics that the capital debates would only be relevant to an “old” or incomplete version of the theory that treats capital in value terms and not for the contemporary versions. However, the results drawn from the controversies are also relevant for a better understanding of the present state of neoclassical capital theory, because, as this study will argue, it has been due to the implications of the debates that dominant theory shifted from the traditional to the contemporary notion of capital.

Controversy in capital theory was not novel in the marginal approach. Neoclassical capital theory has been one of the most controversial areas of economic science since its inception in the discipline at the turn of the twentieth century. While former controversies arose out from different viewpoints on capital *within* the marginal approach, the Cambridge controversies, on the other hand, entailed a criticism of the whole marginalist analysis.³ At the time of the Cambridge controversies, capital as factor of production was still conceived as a single magnitude in value terms among the data of the theory. Although serious misgivings regarding this notion of capital started to arise in the late 1930s and over the 1940s, still the centre of economic theorising conceived capital as a single magnitude in value terms.⁴

Capital as a single magnitude has been the notion on which, under conditions of free competition, traditional theory had to resort to explain the distribution of the

³ See Garegnani (1970a) pp. 424-7; and ch.4 below. For the different controversies on capital theory within the neoclassical approach in the twentieth century see Kurz (1987), Huth (1998), and Kaldor (1937). According to Kurz (1987) the two former controversies *within* the neoclassical theory involved J.B. Clark and E. Böhm-Bawerk at the turn of the twentieth century, while the second one, in the 30s, mainly involved Knight, Hayek, and Kaldor.

⁴ Although this study has not systematically dealt with the general situation of neoclassical capital theory *before* the Cambridge controversies, we shall assume, by relying on Garegnani (1976a, 1990) and Petri (2004, section 5.4), that the notion of capital shared by the centre of economic theorising was, in the main, the traditional quantity of capital, despite the fact that the increasingly reservations arisen in the period referred to as to the legitimacy of this notion. It is interesting to recall that Knight’s entry “*Capital and Interest*” in the *Encyclopaedia Britannica*, which considers capital as a single magnitude expressed in value terms, was reprinted in 1950 by the American Economic Association (see Knight, 1950), thus revealing that until those years capital as a single magnitude appeared to be the central notion. Cf. ch.2 section II below; Dewey (1965) ch.8, in particular p. 114, and Robertson (1957). Problems associated with the conception of capital in value terms started to be recognised by neoclassical economists, such as Hicks (1946) who argues to dispense with the traditional notion of capital altogether, as Garegnani (1976a) has showed.

social product between wages and profits (and rents) in terms of supply and demand – in other words, by applying the factor substitution principle. As we shall see in the following chapter, capital in value terms not only is needed in order to allow for factor substitutability, but also to determine a uniform rate of profits on the supply prices of the several capital goods. The condition of a uniform rate of profits on the capitals' supply prices was a universally accepted condition by the founders of marginal analysis. Satisfaction of this condition ensures that the equilibrium determined by supply and demand forces could conceivably be taken as a *long period* equilibrium in that the persistent nature of the conditions of the economy ensures that the theoretical variables become gravitational centres of actual (trends or average) ones over time. Thus, that notion of capital⁵ allows determining a normal position of the economy's quantities and prices characterised by a uniform rate of profits on the capitals' supply price under free competition both in consumption-goods' and factors' markets. For that determination it is crucial that the theory accounts for downward-sloping demand curves for factors. As we shall see in this study, the Cambridge controversies have demonstrated that the belief on an inverse relation between demand for factors and their rewards cannot in general be postulated.

1.2. The relevance of this conflict to contemporary economic analysis.

The importance of this controversy for the discipline is indubitable. To the vast amount of contributions which were published in that period by many scholarly journals on economic theory⁶ – not to mention the oral debates⁷ – we have to add the ensuing reviews and surveys from both sides assessing the significance of this

⁵ This conception of capital as a single magnitude in value terms is the way in which capital was traditionally conceived as a factor of production and perhaps it might have been seen by some scholars as an aggregate factor. But it will be seen below that this conception has nothing to do with using “aggregate” production functions. Capital in value terms as traditionally conceived is analysed in ch.2. Endless misunderstandings in the controversies were created as to whether the critique only bears on versions of the theory using aggregates production functions. On this issue *cf.* ch.4.

⁶ Mainly in *The Quarterly Journal of Economics*, *The Review of Economic Studies* and *The Economic Journal*. See bibliography below.

⁷ For instance, the oral debates between P. Samuelson and P. Garegnani in 1961-2 in Cambridge, Massachusetts. This oral interchange later gave rise to Garegnani (1970a). See ch.4 below and also Samuelson (1962) p. 193 n.1 and Robinson (1970a) p.311.

remarkable episode.⁸ Hardly could any practitioner of the discipline deny the fact that over more than twenty-five years the controversy in the theory of capital occupied a prominent magnitude in contemporary history of economic analysis.⁹ However, after more than fifty years since the first interchanges between Cambridge¹⁰, we might perhaps be asked by a marginalist scholar something like “Why to rekindle this debate, to-day?” Well-trained in mainstream ambiances he would remind us that the *completely developed* theory of capital is the one which treats “capital” as a physically heterogeneous capital goods’ vector.

According to this view, the Cambridge debates would have petered out once neoclassical theory started to rely on a physical conception of capital (‘Walrasian’ treatment)¹¹ thus rendering, apparently, the supply and demand approach immune from capital theoretic problems. Thus, neo-Walrasian authors argue that the debates would have *no relevance* to the current mainstream; in fact neoclassical scholars deem the theory of supply and demand relying on a Walrasian capital as *the* one and only theory, without even showing the slightest effort to inform students of this

⁸ See Harcourt (1969, 1972, 1976), Bahduri (1969), Robinson (1970a), Garegnani (1976a) for assessments from the critically oriented sides of the debates and Blaug (1974), Hicks (1974), Stiglitz (1974), Ng (1974) for neoclassical surveys. More recent assessments from the critical side also include Cohen and Harcourt (2003, 2005), Garegnani (1990, 2003 “Appendix II”), Petri (2004, ch.6; 2007), while for the neoclassical side we should refer to Hahn (1982) Burmeister (1980 ch.4, 2000), Bliss (2005). See also Bliss (1975, ch.15), Hahn (1972, 1975a), Birner (2002) and the entries provided by Kurz (1987), and Ahmad (1998).

⁹ Though the Cambridge-Cambridge divide has not always matched a clear division of the two different approaches in the history of economic analysis behind these controversies, still it is acknowledged in the literature that the American group may be regarded as the neoclassical side of this debate, while the British one as the non-neoclassical or the critical side. This is so because, broadly speaking, the English school at that time hosted critics of the traditional approach, such as A. Bahduri, K. Bahradawaj, P. Garegnani, N. Kaldor, L. Pasinetti, J. Robinson, P. Sraffa (just to name some of the most important contributors from the critical side), while the MIT (U.S.) was home of important neoclassical economists, notably, P.A. Samuelson and R. Solow. There have of course been many other contributors that worked elsewhere at that time but were associated, in terms of theoretical membership, to either the neoclassical or the critical side. We keep to the geographical divide suggested by the literature in this study. Also, the label “neo-neoclassical” was sometimes used by some to refer to the fourth generation of neoclassical economists that not only includes the ones referred to above but also K. Arrow, C. Bliss, C. Ferguson, F. Hahn, D. Levhari, J. Stiglitz, among others; cf. Robinson (1962b, 1971a). In this work we shall refer to critics as the “critically oriented side” or “critical side”. Note also that some literature refer to the critically oriented side as the “neo-Keynesians”, Harcourt (1972); “neo-Marxist-Keynesians”, Ng (1974); “Post-Keynesians”, King (2002); or “Anglo-Italian critics”, Bliss (2005).

¹⁰ There is almost complete agreement that Joan Robinson’s (1953) article kicked off the debates. See ch.5 below.

¹¹ It has been preferred to use the term ‘Walrasian’ treatment of capital because Walras, differently from his contemporary peers, treated “capital” as a collection of physically heterogeneous capital goods, and not capital as a single magnitude; cf. ch.2 and n.210 ch.4 below.

episode¹² that is the capital theory controversies, which have serious implications on the contemporary versions of the theory, as we shall see in this study. But, on the other hand, we can see today, for instance in neoclassical growth theory such as *Real Business Cycle* and *Endogenous Growth* theories, that “aggregate production functions” are still used. This reveals that a part of the discipline¹³ accept the supply and demand approach treating capital as a single magnitude *as if nothing had ever occurred in recent history of economic analysis*. So a very important question arises: *What happened to the Cambridge debate that today, it seems, nobody talks about it any longer?*

1.3. A first phase in the capital controversies.

The answer that will be attempted here considers a separation of the period of the debates into two *phases*. The first phase (1953-1970/1) regards the period when salient discoveries like ‘reswitching’ and ‘reverse capital deepening’ started to gain notoriety in the discipline¹⁴, after the first interchanges between the two Cambridge much centred on the use of aggregate production functions in economic growth models. The second phase (1970/1-1976)¹⁵ regards a situation which might be characterised, for reasons we shall analyse below, as an obscure, unclear chapter in the history of the debates where many misunderstandings damaged communication between the two sides. In this second phase dominant capital theory had already shifted from the traditional notion of capital in value terms to the Walrasian specification of it and this is why some of the interchanges in this phase were at cross-purposes, as will be seen below. From a historical standpoint, our identification of two stages in the controversies goes along with a dividing line in the development

¹² Cf. Hahn’s (1975a) global assessment of the debates: “Great charade” (p. 364). See ch.6 where we deal with this author. See also Sen (1974).

¹³ See Felipe and Fisher (2003) who have called attention on why economists still use aggregate functions in theoretic and empirical research despite the capital controversies. Cf. Barro and Sala-i-Martin (1995). See ch.5 below.

¹⁴ See chs 3 and 4 below.

¹⁵ As will be seen later (par.4.14 and ch.6), we have considered 1970/1 as the end of the first phase because in 1970 Bliss (1970) replied to Garegnani (1970a) on the grounds of a different notion of equilibrium and thus misunderstanding Garegnani’s criticisms of the theory at stake which was, and is, the traditional long period supply and demand equilibrium theory. As to 1971, it is due to Robinson’s (1971b) suggesting title “The End of the controversy”. On the other hand 1976 has been chosen as the ending date of the second period for reasons described in the main text.

of the debates as has been documented by the literature of the critical side of these debates.

Regarding the first phase it has been natural for the present author to consider one of the most frequent references in the literature, namely Harcourt's 1972 survey.¹⁶ This work has spread the controversies in the discipline and is the most cited by the bulk of practitioners when referring to *the first phase* of the debates. In Harcourt (1972) we find a roughly all-inclusive review of the main contributions to the debates from both sides, which, starting from Joan Robinson's 1953 kick-off, follows a chronological account of the controversies.¹⁷ Harcourt (1972), following Joan Robinson's target of criticisms, focuses on the so-called "aggregate production functions" rather than on the problems for the theory owing to the notion of capital in value terms.¹⁸ Moreover, from the criticisms centred on the aggregate production function, Harcourt, together with Robinson, draw methodological implications in that these authors argue that equilibrium comparisons are not good tools for analysis of changes.¹⁹ But, as we shall see later, phenomena like 'reswitching' and 'reverse capital deepening' cast serious doubt on the theory independently of whether it uses aggregate functions thus bringing about important reactions from the neoclassical side that tried to save the basic postulates of the theory.

Thus, even if Harcourt points out that "double switching" and "capital reversal" damage the foundations of the marginal approach, it is yet not clear, in Harcourt (1972), what consequences of these results may have on the versions of the theory relying on a Walrasian treatment of capital. Then, when neoclassical writers started to

¹⁶ Harcourt (1972) is an extended version of Harcourt (1969).

¹⁷ Beginning with Joan Robinson's kick-off 1953 article criticising the use of the aggregate production functions, Harcourt's account continues by reviewing later replies that Robinson's article stirred up. His survey focuses on issues regarding aggregates production functions, vintage models, technical progress and economic growth models, the debate on the rate of return involving Solow and Pasinetti; then it continues by presenting the 'reswitching' debate, and concludes by summarising the Cambridge alternative distribution theory as represented by the works of Kaldor (1955), Robinson (1962a) and Pasinetti (1962). Though we shall deal with Robinson and her immediate contestants in ch.5 on issues related to the debates on capital theory, the so-called Cambridge theory of distribution is not considered by us here.

¹⁸ Harcourt's (1972) emphasis on the aggregate production function is particularly clear when this author states that the aggregate production "is indeed by far the most common version" of the neoclassical theory (p. 3). But, as we shall see later, the problem of capital does not necessarily entail using aggregate production functions. For instance, Wicksell's system does not use aggregate production function and yet has to resort to a quantity of capital in value terms. Cf. our discussion of Wicksell's general equilibrium system in Appendix B, below.

¹⁹ See ch.5 where we deal with Robinson's critique.

argue against the critics by contending that a “general equilibrium model of capital does not require aggregation”²⁰, counteractions and replies from the critical side were not unified. It is my opinion that when these arguments started to be raised by neoclassical participants in the debates, a second phase in the controversies started. Our contention can be verified by some different views in this regard in some relevant literature of the critical side.

1.4. A second phase in the capital controversies.

If we now turn to Harcourt’s updated account of the debates in 1976 we shall notice a very different opinion regarding whether the results of ‘reswitching’ and ‘reverse capital deepening’ drawn from the first phase of the controversies bear on the versions which rely on a Walrasian treatment of capital. Thus, we find in Harcourt (1976) a very different assessment of the relevance of the debates:

“We now conclude: if the arguments of this article are accepted, while general equilibrium *may* emerge logically intact as ‘the theory of inter-temporarily efficient paths and their price duals’ [Hahn, 1972, p.3] it is *not* the theory which is relevant for the issues raised in the Cambridge controversies. The attempts to use the other versions of neoclassical theory flounder both on the results of the reswitching and capital-reversing debates, with which is allied the problem of measurement of ‘capital’, and the (more fundamental) criticisms that stem from the distinction between comparisons and changes.”²¹

²⁰ Bliss (2005) p. xii. This is the position that Bliss also maintains in his (1975). We shall argue later (ch.6) whether Bliss’s contention is due to a supposedly “more complete” analysis which takes as data as many capital goods as there are in the economy (*i.e.* a Walrasian treatment of capital) or to another reason. Note also that neoclassical authors have almost always taken Robinson’s target of criticisms at aggregate production function in order to contend the critics. On this, see ch.6.

²¹ Harcourt (1976) p. 58, emphasis in the original, underlines added. Harcourt divides the theory into two versions: the aggregate versions (“the other versions of neoclassical theory” in the passage) and the “general equilibrium” one. Harcourt intends the latter as the neo-Walrasian approach of simultaneous equations. This separation of different versions between an “aggregate” one and “general equilibrium” is misleading since it does not take care of Wicksell’s system which is a general equilibrium, is disaggregated and, however, needs of a quantity of capital in value terms (not an “aggregate production function” which is what Harcourt seems to suggest in the passage) for the determination of equilibrium. On the meaning of an intertemporal general equilibrium, see ch.6.

This different appreciation of the results of the debates on the “intertemporal general equilibrium” theory as viewed by Harcourt (1976)²² has induced the present author to delve into this second part of the controversies and place at the centre another view on the relevance of the critique from the critically oriented side.

In fact, in Garegnani (1976a) we find that his appraisal of the controversies goes in a different direction *vis-à-vis* Harcourt’s. Thus, while Harcourt (1976) suggests that the neo-Walrasian theory “may emerge logically intact” from capital theoretic problems, Garegnani argues that after Hicks’ (1946[1939]) *Value and Capital* an increase proportion of the marginal approach started to shift to a “short period general equilibrium.”²³ What is important for our purposes to notice is that the consequent abandonment of the long period equilibrium method

“has not been due to weaknesses of the method as such, but rather to weaknesses of the dominant theory of distribution, and in particular, of the conception of capital it relies on.”²⁴

According to this view, the marginal theory was forced to adopt a short period general equilibrium method – due to the Walrasian specification of capital – as a *last refuge* from the results of the controversies regarding the conception of capital which the *long period* equilibrium theory need rely on. Although in Garegnani (1976a) “the question of whether the two kinds of analysis [intertemporal and temporary equilibria] have in fact overcome the deficiencies of demand-and-supply theory with respect to capital [is left] aside”²⁵, once we take into account that a radical shift in method undergone by the theory has taken place, then it is hard to accept that the new versions of the theory may emerge *logically intact*, as Harcourt (1976) concludes.

Thus, there does not seem to be agreement in the critical literature on whether the results of the Cambridge debates would have forced the theory – in its attempts to

²² Similar positions can be found in Cohen and Harcourt (2003, p. 207; 2005, p. xxxiv). See also the protestations of Pasinetti *et. al.* (2003) on Cohen and Harcourt (2003); and see Bellino (2005), Reati (2006) and Petri (2007) for comments on Cohen and Harcourt (2005). *Cf.* n.176 in ch.4.

²³ Garegnani (1976a) p. 26.

²⁴ *Ibid.*

²⁵ *Id.* p. 38. Garegnani (1990, 2000, 2003) has taken up this issue and has showed that the same kind of difficulties regarding the conception of capital in the traditional versions of the theory relying on a notion of capital in value terms also reappears in the new versions in the investment and savings markets. Garegnani’s arguments are briefly dealt with in ch.6 below.

preserve the supply and demand approach and to overcome the deficiencies of capital – to adopt a method which is radically different from the traditional one and hence whether the debates bear on the new versions of the theory.²⁶ But, it is interesting to note that if the controversies compelled the theory to adopt a method of short period general equilibrium²⁷, which lacks the necessary persistency of the equilibrium²⁸ and was in the main confined to a minority group of mathematical economists, then that shift would be representing the strength of the critique. Thus, we submit, it is not the case that Walrasian theory is *not* the version of the theory which is relevant for the issues raised in the Cambridge debates; the shift to Walrasian theory, with all its methodological implications, is the result of the capital theory debates.

These different views also indicate a gap within the critical side with respect to whether ‘reswitching’ and ‘reverse capital deepening’ are the “more fundamental” criticisms in the debates.²⁹ In fact, Harcourt (with Cohen) still argue that the shift from “aggregate” versions to a theory relying on a Walrasian capital would have represented a shift to ‘more disaggregated analysis.’³⁰ They thus neglect other critics’ views such as Garegnani’s (1976a), who argues that the key to understand the shift to a Walrasian specification of capital is to realise that the theory had to adopt that way – which had already been provided by Hicks’ (1946)³¹ – due to the deficiency of the

²⁶ This apparent halfway house situation of the controversies is also recorded by Backhouse (1985, p.328) who states: “the implications of reswitching for disaggregated neoclassical models [of the Arrow-Debreu type] were still not clear.” Note that “disaggregated neoclassical models” are meant as the neo-Walrasian equilibrium theory.

²⁷ Cf. Garegnani (1976a, 1990); Milgate (1979) for distinctions between short period and long period general equilibrium in marginal theories.

²⁸ Among other problems of the theory which relies on a Walrasian treatment of capital. See ch.6 below.

²⁹ Harcourt (1976) p. 58. It is apparent that in this work Harcourt has given more importance to the “criticisms that stem from the distinction between comparisons and changes.” It is then no surprise that this scholar, following Robinson too, has regarded the ‘reswitching’ and ‘reverse capital deepening’ debates as a “Dead End” (*id.*); cf. in particular Robinson (1975a).

³⁰ This idea is implicitly suggested in Cohen and Harcourt (2005, p. xxii). But this view seems not to have considered that *e.g.* Wicksell’s analysis is as disaggregated as were analyses conducted by Böhm-Bawerk (1891), or by Hicks (1932), thus broadcasting a misleading idea, as Petri (2007, p. 604) has recently remarked, that the sole ‘disaggregated’ analysis would appear to be the new versions of neo-Walrasian theory. In addition, Cohen and Harcourt (*id.* p. xxiv) suggest that the “characterisation of simultaneous equations [in disaggregated analysis of neo-Walrasian theory] is correct”, without however discussing the economic meaning underlying those formal systems. In ch.6 below, we shall discuss the economic meaning of Walras’s system of capitalisation.

³¹ In this study we have considered Hicks (1946), the 2nd ed. of Hicks’ (1939) *Value and Capital*.

traditional notion of capital as revealed by the implications of the results of the debates.³²

One of the aims of this study is to show how the Cambridge capital theory controversies endanger the notion of capital the traditional marginal theory relies on, in order to contribute for a better understanding of the results of the capital debates and their implications for the neoclassical theory nowadays, which relies on a Walrasian treatment of capital.

1.5. Radically different views on the meaning of the critique for the contemporary versions of the theory.

At the turn of the 1970s, as we shall see below, neoclassical authors started to counteract the critique by contending that capital theory does not need aggregation³³ thus suggesting that the contemporary versions of the supply and demand approach would not be subject to theoretic capital problems as in the traditional versions (capital in value); at the same time, however, the critical side did not immediately provide a unified critique of the above neoclassical contention.

It is the author's opinion that neoclassical economists took advantage of the non-unified position with respect to the implications of 'reswitching' and 'reverse capital deepening' on the new versions of the theory and that these neoclassical authors, by disseminating misleading ways to treat the issues,³⁴ have put the debates in a quite *obscure* chapter of contemporary history of economic analysis, which suggests to regard this period as the second phase of the Cambridge controversies. As will be seen later³⁵, this obscurity in the debates, we submit, were brought about by marginalist authors in their attempt to *reverse* the sign of the critique by introducing misleading interpretations of previous results (first phase) and reinforcing further

³² See ch.6 below. Garegnani's (1976a) insights into the issue at stake, that is, how far neoclassical theory relying on a Walrasian capital is immune from capital theoretic problems, gave rise to a series of contributions attempting to clarify some issues of the controversies in capital theory in its second phase. See: Petri (1978, 2004); Milgate (1979); Eatwell, (1990).

³³ This is the position shared by Hahn (1972, 1974, 1975a, 1975b, 1982); Bliss (1970, 1975, 2005); Dixit (1977).

³⁴ For instance, the misleading way in which neoclassical economists received the critique was as if it concerned the so-called aggregate production functions. *Cf.* the works cited in n. 33 above and see ch.6 below.

³⁵ See ch.6.

misunderstandings that “have marred the controversies on capital.”³⁶ Thus, misleading ways used by some participants in the second phase of the debates have fogged communication and might have therefore detained a full appreciation of the strength of the implications of the controversies that touch the general principles of dominant theory.

In order to understand the main issues debated over the capital theory debates, misunderstandings included, the method adopted in this study embraces, on the one hand, a historical standpoint, in that we deal with historical aspects of this conflict in the discipline. However, on the other hand, we have not chosen to proceed in a chronological order: we have rather preferred to place at the centre the analytical arguments necessary to understand the implications of the phenomena of ‘reswitching’ and ‘reverse capital deepening’ which, from a chronological viewpoint, were publicly recognised in *last* stages of the first phase of the debates. And this decision has been made for a very important purpose: to rescue the true nature of the critique produced in the controversies in order to contribute to a better understanding of the current situation of neoclassical capital theory.

1.6. Sketch of this study.

This study is divided into two parts: while the first part is devoted to introduce the analytical and methodological issues in neoclassical economics, the second part analyses and discusses the historical aspects of the Cambridge controversies in the light of the analytical elements provided by the first part.

As the present author will argue that the capital controversies touch the basic premises of the neoclassical theory, essential notions of the marginal approach are introduced in the first part of this study. Thus, in the next chapter (chapter 2), one of the first notions examined is the factor substitution principle, which is explained in very simple terms (corn-consumption, corn-capital and labour economy). Following the exposition method developed in Garegnani (1990) the chapter aims at providing the basic analytical elements to understand the importance of the factor substitution principle for the approach so as to grasp the notion of a downward-sloping demand curves for ‘factors of production’ which is key to derive a plausible explanation of

³⁶ Garegnani (2005) p. 411.

distribution in terms of supply and demand (ch.2, section I). Then, we turn to economies with heterogeneous capital goods and the notion of capital in value terms is examined, by surveying some marginalist authors' views on the matter. In fact in economies with heterogeneous capital goods a problem of measurement of capital arises. In this connection we shall approach that problem from two angles: we distinguish a supply-side or supply-aspect of the problem – which is related to the inescapably need to regard “capital” as a factor in value terms – from a demand-side or demand-aspect of the problem, which is related to the possibility to conceive that factor's demand such that owing to changes in methods of production and changes in outputs the quantity of that factor demanded could be conceived to be inversely related to its rate of remuneration. The chapter also introduces the meaning of the persistent circumstances underlying the method of the long period equilibrium, that is, the equilibrium notion that was at the heart of the controversies. (Ch.2, section II.)

Once the basic notions of the theory are laid down, chapter 3 introduces the meaning of the most relevant results of the Cambridge controversies: ‘reswitching’ and ‘reverse capital deepening’. The purpose of this chapter is to introduce the reader to the concepts and, more importantly, the implications of these phenomena which were at the centre of the most salient interchanges between the two sides.

In the second part of this study – composed of three chapters (4, 5, and 6) – we turn attention to some historical aspects of the capital debates. While chapters 4 and 5 deal with the themes, issues and episodes developed in the first phase of the controversies, chapter 6 considers the questions and problems of the second phase of the debates. The present author should like to state that chapters 4 and 5 do *not* follow a chronological order. Rather, we have preferred to firstly analyse and discuss the group of contributions providing the true nature of the criticisms and the neoclassical replies. We thus begin with (chapter 4) some of the most important contributions in these debates by surveying Samuelson's (1962) intervention and the limits of his particular defence of the marginal theory. The chapter then goes on to analyse the attempts made by Samuelson's PhD student Levhari (1965), who tried to find out the conditions to rule out ‘reswitching’. These interventions were very important from a historical perspective of the controversies, because they subsequently gave rise to the celebration of a famous Symposium on Paradoxes in Capital Theory held by *The Quarterly Journal of Economics* in 1966. Thus, the importance in the development of this conflict is analysed by bringing to the fore the interventions raised in the

Symposium by Pasinetti (1966a) and Garegnani (1966). Yet, as we shall argue, neoclassical scholars, *e.g.* Ferguson (1969), though recognising theoretic difficulties with respect to capital for the whole approach, attempted to minimise or downplay the relevance of the results produced in the capital debates by invoking the “empirical” way-out. Moreover, it is worth recalling an important question in the aftermath of the Symposium: while the heart of the controversies have referred to the traditional, long period equilibrium version of the marginal approach, we shall see that some reactions relying on a short period general equilibrium started to appear; hence the present author has deemed as appropriate to briefly deal with Bliss’s first intervention in 1970 against Garegnani (1970a), in order to point out the way adopted *at that moment* by neoclassicals to overcome the problems concerning capital.

The following chapter (chapter 5) turns back in our historical reconstruction of the controversies: we survey Robinson’s (1953, 1956) first interventions in the Cambridge controversies and her immediate contestants Champernowne (1953), Solow (1955, 1956, 1957), Swan (1956). It will be argued there that over the first interchanges between 1953 and 1960 the debate rather centres on problems regarding measurements of “aggregate” capital in “aggregate” production functions, without however pointing to the general problems of capital for the marginal theory – although, as we shall see, some scholars had tangentially faced the issues of ‘reswitching’ and ‘reverse capital deepening’.

So far the study devotes to what the author has above referred to as the first phase of the debates. Thus, in chapter 6, we present and discuss the particular reaction from the neoclassical side that has tried to avoid the criticisms by returning to a treatment of capital in physical terms (Walrasian treatment). Because the relevant literature regards Bliss and Hahn as the most notably exponents of this reaction we shall deal with how these authors faced the results of the controversies. Since one of the most frequent misunderstandings is the confusion between uniformity of the rate of profit on the capital goods’ supply prices (the *traditional* rate) and uniformity of commodities-own rates of interest, that is why in this chapter problems related to the issue of such different rates are tackled by discussing Walras’s system of capitalisation³⁷ and the critique provided by Garegnani (1958, 1962, 1990). Also, it

³⁷ Walras (1954).

will be argued that the problem of the non-uniformity of the rates of return poses the problem of which way-outs could be pursued by dominant theory if the theoretical framework of supply and demand were still to be maintained: either capital is expressed in value terms, or the traditional method of long period equilibrium has to be abandoned. Thus, following Garegnani (1976a), we shall see how these authors took refuge from the criticisms by confining the analysis to short-run general equilibrium and pinpointing that the Cambridge critique *only* bears on the “aggregate” versions (read it as the “aggregate” production function) but *not* on the modern versions of the theory, thus starting to create misunderstandings and confusions which did not help communication and which in fact characterises this second part of the debates. However, as it will be argued, short period general equilibrium versions of the theory represent a radical transformation of the dominant approach in the wake of the capital controversies. Finally, the last word will be devoted to a provisional examination regarding whether or not ‘reswitching’ and ‘reverse capital deepening’ bear on the *new* versions. Accordingly two positions will be confronted: Bliss (who has attempted to overcome the implications of ‘reswitching’ on the intertemporal theory) and Garegnani (who has extended the critique to the new versions.)³⁸

The study finally ends in chapter 7, where some conclusions are suggested.

³⁸ The confrontation is very provisional and relies mainly on Bliss (1975) and Garegnani (2000, 2003).

PART I:

***ANALYTICAL AND METHODOLOGICAL ISSUES IN
NEOCLASSICAL ECONOMICS: SETTING THE GROUND TO
UNDERSTAND THE CAMBRIDGE CAPITAL
CONTROVERSIES***

CHAPTER 2

Understanding the foundations of neoclassical economics

SECTION I

The principle of factor substitution. From ‘employment’ functions to demand functions for the factors of production.

2.1. The data of the Marginal Theory. The direct mechanism of factor substitution.

In order to understand the basic features of the critique of capital theory, it is worth considering first the foundations on which the principle of factor substitution relies. To begin with, it can be useful to report the three groups of the data taken as independent variables –as ‘given’– by the marginal theory in order to determine prices, output and distribution. Since the data are exogenous, their determination falls outside the purview of analysis of the theory (*e.g.* tastes and preferences are rather studied by psychological and sociological theories). The endogenous variables to be determined, on the other hand, depend crucially on those data; therefore the data must not change while the endogenous variables are being determined.³⁹

The three groups of data taken as independent variables by marginal theory are:

³⁹ *Cf.* par. 2.9 below.

- (i) Tastes and preferences of consumers.
- (ii) Alternative techniques of production (methods of production available) at a certain state of technical knowledge.
- (iii) The endowment of each ‘factor of production’ available in the economy.

According to marginal theory, we can determine, by using these data, the relative prices, output, and distribution simultaneously. It must be now seen how this determination comes about, and which roles the above data play in it. In what follows, by introducing a very simple economy, it will be explained how the two mechanisms of factor substitution, which underlie the whole marginal approach, operate.

Let us assume an economy where the only consumption good is corn and the capital good is corn too.⁴⁰ Corn is produced by means of labour and corn-capital in continuously variable proportions, which are given by group (ii) of the data. There is free competition, land is free, and entrepreneurs are the capitalists. All transactions are assumed to occur in terms of corn. To simplify matters, both productive factors – labour and corn-capital – are assumed to be in fixed supply.

As is well-known, each capitalist is faced with a curve of the net ‘marginal product’ of labour, which results when the variable quantities of labour s/he may hire is applied to the given quantity of corn-capital s/he possesses. We know that, given the quantity of corn-capital, the net ‘marginal product’ curve of labour will be, first, constant (because the entrepreneur applies the exact amount of corn-capital per unit of labour so as to optimise the combination of both inputs in order to obtain the maximum profit, not implying therefore to use “all” the given quantity of corn-capital) and then decreasing (otherwise the increment of output due to hiring one more unit of labour would always be higher; therefore diminishing returns –by keeping fixed one of the factors– must prevail within each firm of the industry)⁴¹. (See Appendix A)

⁴⁰ This simplifying assumption could be replaced by the one establishing that the composition of the output – actually composed of many goods – is given.

⁴¹ The importance for the marginal theory of the diminishing returns principle is expressed *e.g.* by Marshall when he (1959, p. 337) states: “The notion of the marginal employment of any agent of production implies a possible tendency to diminishing return from its increased employment. Excessive applications of any means to the attainment of any end are indeed sure to yield diminishing returns in every branch of business.”

The *net* marginal product of labour curve indicates the entrepreneur how many labourers it would be profitable to hire with his capital at any given real wage. Thus this curve becomes an ‘labour employment’ curve. Then, by adding up the individual ‘labour employment’ curves it is easily obtained a ‘labour employment’ schedule for the whole economy at any given real wage. If distribution changes (*e.g.* the wage rate falls), cost-minimising firms tend to adopt *production methods* that utilise a higher proportion of labour relative to corn-capital. Therefore, by allowing group (ii) of the data to work, we have illustrated the first, *direct*, mechanism of factor substitution.

2.2. The indirect mechanism of factor substitution.

Now let us introduce a second consumption good, ‘cloth’, produced with labour and corn-capital. We now however assume that each of the two goods are produced with only one method of production (fixed coefficients of production), with ‘cloth’ requiring the higher proportion of corn-capital to labour, that is: $K_{cloth} / L_{cloth} > K_{corn} / L_{corn}$. Now it is consumer choice through group (i) of the data that plays a similar role to producer choice and marginal products in providing the basis for a decreasing ‘labour-employment’ schedule.

In fact, any fall in the wage rate will make the price of ‘corn’ fall relative to ‘cloth’. If one introduces the analysis of consumer choice it will in general follow that more corn will be consumed.⁴² Therefore, given the quantity of corn-capital employed in the economy – a crucial assumption to derive demand functions, as we

⁴² A simple example in which the utility of a single consumer –subject to his budget restriction– is maximised according to a well-known type of utility function (*i.e.* a function which allows the principle of marginal utility to rule, as was traditionally assumed) may help us understand how consumer choice determines – through variations in demand – the new level of factors employed as distribution changes (in our example in the main text, a fall in the wage rate). Suppose now, X_1 and X_2 are corn and cloth respectively, whose respective relative prices are p_1 and p_2 . R is the amount of available income, which is at the consumer’s disposal to be expended on both goods. For every specification of prices and income level, we shall show that, as the wage rate falls, the price of corn (P_1) falls and the demand of corn (X_1^d) rises. So we have: $MaxU = X_1 X_2$ *s.t.*

$$R = p_1 x_1 + p_2 x_2, \quad \text{hence} \quad L = U - \lambda(p_1 x_1 + p_2 x_2 - R). \quad \text{The FOC establish that}$$

$$\frac{\partial L}{\partial x_1} = X_2 - \lambda p_1 = 0, \quad \frac{\partial L}{\partial x_2} = X_1 - \lambda p_2 = 0, \quad \text{and} \quad \frac{\partial L}{\partial \lambda} = p_1 x_1 + p_2 x_2 - R = 0. \quad \text{Therefore: } X_1^d = \frac{R}{2p_1},$$

so when the price of corn falls (due to a fall of the wage rate and because corn is more labour “intensive” than cloth) the quantity of corn demanded increases.

shall see below⁴³ – a higher proportion of it –of that constant corn-capital – will be employed in the production of corn, with a consequent increase in the quantity of labour employed in the economy as a whole, since, by assuming fixed production coefficients, corn needs more labour for any quantity of corn used as capital than cloth does.

The important point to notice here is that, even when there is no alternative methods of production available (that is group (ii) of the data only provides one fixed-coefficients production method), the existence of substitutability among consumption goods can give rise to decreasing employment curves for factors, analogous to the employment factor curves derivable from technical substitution, because the substitution among consumption goods implies an *indirect* substitution among the factors employed in their production.

If we reintroduce the assumption of variable coefficients of production, it can be seen that the concepts of marginal utility and marginal productivity will reinforce each other in ensuring the downward sloping labour-employment schedule.

2.3. From ‘employment’ functions to ‘demand’ functions (I).

To use the ‘factor-employment’ curve as a basis for demand functions, we need find a justification for assuming that the amount of *corn-capital employed tends to remain constant*. Or, more precisely, as Garegnani has pointed out:

“we need establish that the amount of ‘corn-capital’ employed tends to vary in accordance with its ‘supply function’ as the wage rate, and therefore the interest rate, vary.”⁴⁴

In the meantime, let us continue to assume that the supply of corn-capital is a fixed one. So, a further requirement to ensure that the quantity of labour ‘demanded’ and ‘supplied’ are equal at *any* (whatever the level may be) level of the wage rate and the interest rate is to assume some kind of *flexibility* of both variables. What we need, in other words, is that the *shape* of those ‘employment functions’ will not be in

⁴³ Cf. par. 2.4.

⁴⁴ Garegnani (1990) p. 6.

contradiction with the conception of free competition. Accordingly, it is assumed that workers will tend to bid down their wage so long as some are unemployed, whereas capitalists unable to hire the quantity of labourers they want will bid it up⁴⁵. This implies that those employment-curves' are downward-sloping.⁴⁶

By establishing the plausibility of the concept of free competition, supported by, and in accordance with, the *shape* of the functions, as showed in the graphs, we have arrived at the resulting explanation of distribution of the social product in terms of the *forces of supply and demand*. The economy, consequently, will *gravitate* around the wage rate w^* and the interest rate i^* of equilibrium. It is actually this *gravitation* that allow us to assert that demand and supply schedules determine both w^* and i^* . (See figures 1 and 2 below.)

⁴⁵ As Wicksell wrote (1934, pp. 113-4, emphasis in the original): “So long as wages are materially *lower* than the marginal product of the sixteenth labourer [the last], it will be to the advantage of every landowner [being the entrepreneur] to employ more than sixteen labourers [the maximum]. But all the landowners cannot simultaneously succeed in this object, and consequently their endeavour must result in a rise of wages. Again, if wages are *higher* than the marginal product, each of the landowners will content himself with less than sixteen workers, which will result in unemployment and a fall in wages through the competition of the unemployed (...) The final wage must therefore lie somewhere between the marginal product of the sixteenth and that of an imaginary seventeenth labourer.”

⁴⁶ Consider figure 1. Suppose that the market wage rate is w' , the number of labourers employed is L' , which is lower than L^* (the quantity of labourers seeking employment). Now, the assumed fall in the wage rate will lead to such a situation that the quantity L^* will be employed at w^* , because the existence of unemployment u (that is $u=L^*-L'$) will push workers to bid w down, as assumed by free competition. By the same token, at an initial wage rate at w'' , the tendency of capitalists to bid the wages up, when they are unable to hire the workers they want, will lead to the point (w^*, L^*) .

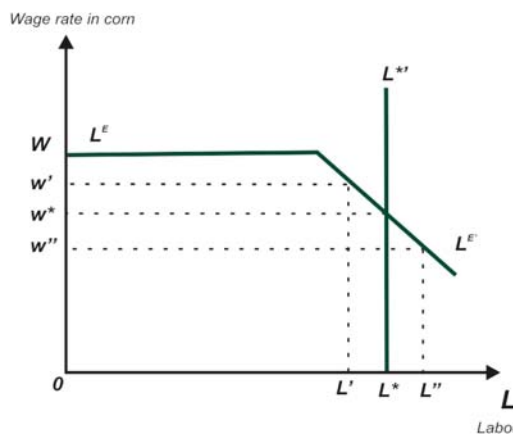


Figure 2.1 Labour employment curve

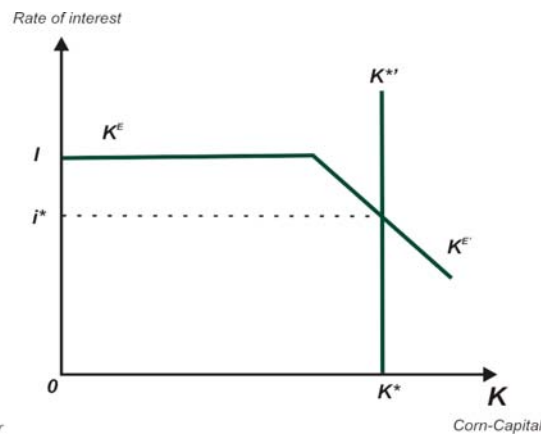


Figure 2.2 Capital employment curve

2.4. From ‘employment’ functions to ‘demand’ functions (II).

The marginal scheme is not yet complete. It was said above that, as w varies, we need the quantity of corn-capital employed not to change with the wage, or to change in accordance with the labour-employment schedule, in order to derive the resulting increase in labour employed and therefore, through the direct and indirect substitution mechanisms, to derive an explanation of equilibrium by the forces of supply and demand.

In fact, if we assume that quantity of corn capital employed in the economy to tend to remain constant as the *wage rate falls*, corn will be demanded in a higher proportion relatively to cloth: hence a higher proportion of corn capital will be employed in corn (we assume fixed coefficients of production) because we need produce more corn. Then, since corn is more ‘labour intensive’ than cloth, a consequent *increase in the quantity of labour* employed in the economy is reached. This is a substitution mechanism derived from tastes and preferences.

As the wage rate falls, the production of cloth becomes more expensive relatively to corn production because the production of a unit of cloth requires more corn-capital than labour relatively to a unit of corn output. Through consumer choice corn is demanded in a higher proportion to cloth. Production of corn must be increased to satisfy the new consumer demand. But this assertion makes sense so long as the corn-capital *employed* in the economy remains constant. The following diagram illustrates our argument.⁴⁷

$$\downarrow w \Rightarrow \downarrow P_{corn} \Rightarrow \uparrow P_{cloth} \Rightarrow \uparrow Corn^D \Rightarrow \uparrow Corn^Y \Rightarrow \downarrow K^D_{cloth} \Rightarrow \uparrow K^D_{corn} \Rightarrow \uparrow L^D_{corn} \Rightarrow \uparrow L$$

Diagram 2.1: Labour market when w falls

Not only the assumption of constant employment of capital, or, more generally, that the quantity of capital changes in the way expressed by its supply function (compatible with the labour employment schedule), is plausible because the

⁴⁷ In *diagram 2.1*, P_{corn} and P_{cloth} are the relative prices of corn and of cloth respectively, $Corn^D$ is the demand for corn as a consumption good, $Corn^Y$ is the output of corn, K^D_{cloth} and K^D_{corn} are the demand for corn-capital from the cloth industry and from the corn industry respectively, L^D_{corn} is the demand for labour from the corn industry, w is the wage rate in terms of corn, and L is the quantity of labour employed in the economy. Upwards arrows indicate “increase in”, while downward arrows “decrease in”.

entrepreneurs are the capitalists. Even if workers (or a third party) are the entrepreneurs that assumption is plausible, since a mechanism similar to the one corresponding to labour market will also be at work in the corn-capital market⁴⁸.

In fact, it is “the law of constant returns”⁴⁹ that ensures that the wage and the interest rates will be not affected by which person acts as entrepreneur. Both labourers and capitalists are free to employ their labour or corn-capital on their own account or to hire it out to others. If the share, say of labour, in the product is different in the two cases (that is $w(r) \neq w$), the difference will be soon cancelled out by competition. And it is similar for the corn-capital.⁵⁰

⁴⁸ In other words, suppose $i(r)$ to be the interest rate that capitalists as entrepreneurs earn for each unit of corn-capital employed, and i to be the interest rate of the market –that is the interest rate which the capitalists would obtain by supplying corn-capital in case the *workers* (or a third party) were the entrepreneurs. Due to competition, therefore, $i(r) = i$ in equilibrium. Otherwise, *e.g.*, if $i(r) > i$, then every capitalist would find it more profitable to act as an entrepreneur than to supply corn-capital to firms managed by workers or a third party. But this would contradict our hypothesis of the existence of a corn-capital market, since corn-capital, in this case, would not be supplied. Competition prevents this contradiction from taking place. Consider the following diagram.

$$\uparrow i \Rightarrow \uparrow P_{cloth} \Rightarrow \downarrow P_{corn} \Rightarrow \downarrow Cloth^D \Rightarrow \downarrow Cloth^Y \Rightarrow (given L) \Rightarrow \downarrow K^D_{cloth} \Rightarrow \downarrow K$$

Diagram 2.2 Capital market

In *diagram 2.2*, $Cloth^D$ is the demand for cloth, $Cloth^Y$ is the output of cloth, K^D_{cloth} is the demand for corn-capital from the cloth industry, i is the interest rate in terms of corn, and K is the quantity of corn-capital employed in the economy.

⁴⁹ Wicksell (1934) p. 129.

⁵⁰ The profits of the entrepreneur must therefore be zero. In fact, if the entrepreneur devotes his human capacities to the management of production, then he must receive a wage like any other worker. If this person also employs corn-capital in production, then he will receive interest. If, on the contrary, he might obtain a *share* of product not based on either corn-capital or labour, then everybody would like to enter this “easy” business. Therefore, the profits of entrepreneur must be nil. For this, we need the condition of constant returns, or, as Wicksell (*op. cit.*) p. 126, puts it that “large-scale and small-scale operations are equally productive, so that, when all factors are increased in the same proportion, the total product also increases exactly proportionally”. Constant returns can therefore be seen as the necessary condition for an explanation of distribution according to marginal productivity principles; were it otherwise (*i.e.*, constant returns do not prevail at the level of the industry) the levelling influence of competition could no longer be invoked by the theory. Consider the following example. Suppose P to be corn-output, K corn-capital, and L labour. (Symbol \oplus stands for “together with”, and symbol \rightarrow stands for “produce”). Suppose, further, that one unit of P can be obtained with hundred units of each factor: $100L \oplus 100K \rightarrow P$. Then suppose a 1% increment on each factor: $101L \oplus 100K \rightarrow P + P_L$ and $101L \oplus 101K \rightarrow P + P_L + P_k$. As is well known, the marginal productivity of the last unit of each factor, in equilibrium, equals the respective rates of remuneration (both expressed in terms of P).

With constant returns, hence, we have $P_L + P_k = \frac{P}{100}$ which immediately implies that

$100P_L + 100P_k = P$ so that the wages of $100L$ and interests of $100K$ together exactly correspond to the whole product; the latter being distributed between both factors according to their marginal
(*cont.*)

2.5. The capital supply function.

To use the labour employment curve as a basis to derive the demand function for labour, we need the amount of corn-capital employed tend to vary in *accordance* with its ‘supply function’ as the wage rate, and with it the interest rate, vary. The point here is to ensure that the supply function of corn-capital be not in contradiction with the demand schedule for labour. However, in this regard, a very important question arises: Is the assumption on constancy of capital employed coherent? Or, more generally, can we *a priori* establish that the supply function of capital will always be compatible with the downward-sloped shape labour employment schedule?

In other words, when w falls (and i rises)⁵¹, what we need is a supply function which allows us transferring a part of that *constant* corn-capital from the cloth industry to the corn industry, thus ensuring that the labour employed in the economy will be increased. For this to be so, however, the supply schedule of corn-capital must *not* be in contradiction with that equilibrium process. The compatibility of the capital supply function with the downward-sloped shape of the labour employment schedule is ensured, if the supply function of corn-capital is a fixed one.⁵² Moreover, the

contribution to production. Now, on the other hand, consider that an increment of 1% on each factor will yield an increment of P higher than 1%, (that is $(1 + \alpha)\%$, with $\alpha > 0$):

$101L \oplus 101K \rightarrow P + \frac{P}{100} + \frac{\alpha P}{100}$. It follows, accordingly, that the rates of remuneration to pay for

cannot be paid from the total output, for the marginal contribution of the last unit of each factor – which governs the remuneration’s equilibrium levels– is higher than the product: $100P_L + 100P_K = (1 + \alpha)P$. Consequently, to pay for P_L and P_K , the entrepreneur would lose $(-\alpha)$. (Similarly, with decreasing returns, the entrepreneur would earn extra-profits).

⁵¹ As we have argued in the main text, constant returns to scale ensure that the wage rate and the interest rate will be not affected by *which person* acts as entrepreneur. Furthermore, due to competition, factor payments will exhaust the product in equilibrium, so if the wage rate in terms of corn decreases, the interest rate increases. In the case with fixed coefficients that inverse relationship can be easily seen. With alternative methods of production, let $Y=F(L,K)$ be the production function, with Y corn-product, L labour, K corn-capital; let F_L and F_K indicate the marginal products of labour and corn-capital respectively equal to the wage rate and the interest rate. Then $dF = F_L dL + F_K dK$. Moreover, constant returns to scale imply $F(L, K) = F_L L + F_K K$ and by differentiation of both sides: $dF = dF_L dL + F_L dL + dF_K dK + F_K dK$. Hence, in equilibrium, $dF_L dL + dF_K dK = 0$ so dF_L and dF_K are of opposite sign and therefore, also with variable coefficients does the interest rate increase as the wage rate falls.

⁵² More generally, the supply function of capital should not be less negatively inclined than is the demand curve at any level of the interest rate i .

downward-sloped shape of the demand curves would guarantee the equilibrium stability. The capital supply function must be *pre-determinable*.⁵³

Summing up. In deriving the ‘employment curve’ for a factor (firstly labour in the example) the reasoning does not require that the other factor (corn-capital) be fully employed; it only requires that the second factor’s employment be given. However, since, at *whichever level* the second factor’s employment (corn-capital) is given, *a tendency toward the full employment of the first factor* (labour) will be at work, and since this holds true for *either factor*, it can be argued that the economy will be able to reach the full employment of both factors.

Then, in general, the assumption that the second factor (corn-capital) is fully employed comes out to be the correct one, when one wants to determine the price (wage) of the first factor (labour) which competition will tend to establish. Thus the marginal product curve of a factor (or, what is the same, the ‘employment’ factor curve), derived under the assumption of full employment of the other factor, and under free competition, can then be seen as a demand curve, since the assumption of constancy of one of the factors when distribution changes is already entailed by the full employment of such a factor.

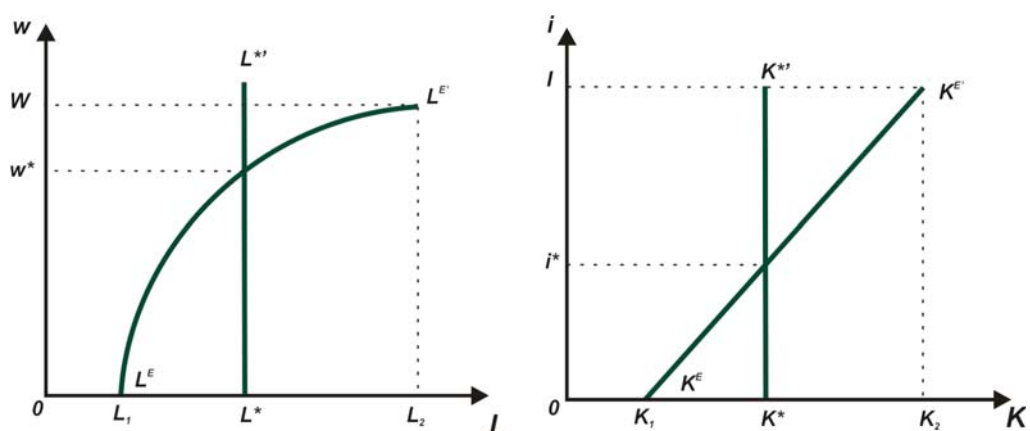
(The analysis we have so far undertaken can be extended to *any number* of factors of production).

2.6. Key role of the downward-sloping shape of demand curves for factors.

So far it was shown the role played by the factor substitution principle to derive a plausible idea of downwards-sloping demand functions for “factors of production”. The downward-sloping shape of those curves do in fact play a key role in the definition of free competition assumed in the marginal theory (if there is some unemployed labourers, they will bid down their wages) and in the *determination of a*

⁵³ Apart from both an upward-sloped supply and a fixed supply function, it would be possible for the theory to encounter some difficulties in that, as distribution changes, let us assume the wage rate falls, the quantity supplied of the capital moves in such a way that the quantity of labour employed may not be increased – even though a huge number of labourers seeking a job can be assumed to exist. If that were the case, one may guess a negative answer to the question posed above. Nevertheless, we do not concern ourselves with these further probable difficulties that the theory might come across. The same reasoning can be applied to the compatibility condition between the supply function of labour (L^*L^* in figure 2.1 above) and the downward-sloped shape
(*cont.*)

supply-and-demand explanation of distribution of the social product between wages and interests. Moreover, the tendency towards full employment of productive factors in an economy crucially depends on the negative-sloping shape of the demand functions.⁵⁴ So if the shape of those demand functions were not inversely related to its factor price, or what is the same, if the shapes of the demand functions for factors were upwards-sloping, the explanation of the distribution of income between wages and interests in terms of supply and demand so as to conceive a *unique and stable* equilibrium would be at serious risk.



Figures 2.3-2.4: Employment function of labour and capital with upwards-sloping shapes

Now a very important implication can be drawn from this succinct analysis of the basic mechanisms at work underlying the marginal theory, which are decisive to be borne in mind to assess the controversies in the theory of capital between Cambridge. The implication is that two fundamental pillars of the marginal approach to explain the determination of distribution in real economies through the forces of supply and demand are:

- (i) the downward-sloped shape of the demand for labour *vis-à-vis* the real wage rate.
- (ii) the downward-sloped shape of the demand for capital *vis-à-vis* the interest rate.

of the demand for corn-capital. In brief, the supply functions of factors must reflect their *relative* scarcities in *accordance with* the demand functions for them.

⁵⁴ We shall see below (ch.3) how, for particular ranges of values of the rate of interest, the shape of the demand function for capital may not be downward sloped and therefore, it would no longer be possible to make the assumption *e.g.* that unemployed labourers would bid wages down indefinitely so long as they remain unemployed -a basic definition of free competition.

SECTION II

An overview of the problem of capital in value terms

2.7. Problems connected with the heterogeneity of capital goods.

In the simple economy described in section I above, capital consisted of a single good and therefore, a problem of measurement did not arise, since the units in which capital was expressed were the same in both roles namely – a role describing the alternative methods of production (group ii) of the data) and the role defining the “factor endowment” (group iii) of the data). Turning the analysis to more realistic economies it must be assumed that capital is no longer homogeneous both in the alternative processes of production and in the production of two consumption goods. Accordingly it is worth providing here the basic characteristics of those commodities normally treated in the literature as capital goods:

- (a) They are reproducible goods – quite different in comparison to land (or to labour) in that either of the latter cannot be reproduced.
- (b) They are utilised for further production both of consumption and of capital goods.
- (c) They wear out within a sufficiently short period of time.⁵⁵

⁵⁵ One can better regard the long-life capital goods as rent goods, as Wicksell (1934, p. 118, emphasis added), puts it: “capital in many cases also advances rent. A farmer who breeds cattle for meat, for milk, (...) must pay rent for his pasturage *for many years before* he can employ the animals in question”. Likewise, some pages later we find in Wicksell’s *Lectures* the following distinction: “nearly all so-called fixed capital (houses, durable machinery, etc.) are, economically speaking, on the border line between capital in the strict sense and land...[T]he operation of the laws of capital depends upon the assumption of a constant adjustment of concrete capital goods in an endless repetition of the same process of investment and production. But this is only of practical importance in *capital investments of relatively short duration.*” (*ibid.* p. 186, emphasis added). Nowadays one might compare the depreciation that, say, a lathe would undergo through its
(*cont.*)

When capital consists of heterogeneous goods bearing the properties just reported, a contradiction between the roles pointed out above arises. This contradiction lies in the fact that, on the one hand, the capital goods which enter in the argument of the individual production functions defining the methods of production are (and must) be expressed in technical units to define the alternative methods of production⁵⁶, while on the other, the endowment of “capital” in the economy, as it can be traced back in the history of marginalist economic analysis since its inception at the turn of the twentieth century, has been expressed in value terms. If this is so, there is a serious problem for the theory in that the “factors of production” should be *measured* – and hence known in advance – *before* knowing distribution, which is the aim of the theory.⁵⁷

Traditionally, as we shall see remainder of the chapter, capital as a “factor of production” has been however conceived in value terms. Moreover, in the derivation of supply and demand equilibrium functions for productive factors capital has been incorporated in traditional general equilibrium analysis as a single magnitude in value terms. The definition of the endowment of capital is not independent of distribution; thus a contradiction between those roles arises.⁵⁸

A very important question arises: What are the consequences of treating capital in value terms when it is used as part of the argument to derive *supply and demand*

utilisation life with the depreciation of a dam or a windmill in order for the former to be replaced through a “constant adjustment in an endless repetition”.

⁵⁶ Both these roles of capital can be seen in Wicksell’s general equilibrium system developed in his *Lectures* (See Appendix B to this chapter.) In that system, capital is, on the one hand, expressed into the production function in technical units (saved-up labour and saved-up land used in previous periods for the production of capital goods necessary for the production of the consumption good, *cf.* equation (1.B) in the appendix already referred to). On the other, together with the equilibrium of supply and demand functions for original factors (labour and land expressed in technical units) are the supply and demand equilibrium functions of capital.

⁵⁷ Note that the identification of factor’s endowments can be easily satisfied by the factors traditionally grouped as labour and land. What we call “labour” is actually composed of *different* kinds. However, it can be assumed that, for long periods of time, the hierarchy of wages relating to the different kinds of labour does not steeply change, so one can regard the different kinds of labour as different powers of a standard labour. But even if wages of different kinds of labour cannot be taken as constant, those kinds of labour can always be dealt with *without resorting to value magnitudes*, by treating *each* as a factor to which there correspond separate supply and demand functions. Similarly, different kinds of “land” can also be dealt with in the same way as we have seen for the case of different kinds of labour when the wages of those kinds cannot be taken as constant.

⁵⁸ As Garegnani (1990), p. 10, emphasis added, has pointed out: “if we cannot define the quantity of a factor before knowing distribution and relative prices, *we shall clearly be unable to use the theory*, which allows us to determine distribution and relative prices only when we know the methods of production and the factor endowment of the economy in question.”

equilibrium functions for the productive factors, and why capital was treated in value terms in the definition of the endowment?⁵⁹

Since the Cambridge critique is connected with the role of capital as a “factor of production” by marginal theory, it might be useful to approach the difficulties associated with the question raised above from two angles. On the one hand, a ‘supply-side’ role of capital, which is associated to the identification problem of the capital endowment, and on the other, a ‘demand-side’ role of capital, which is associated to the demand conditions of capital.

2.8. Supply-side role of capital.⁶⁰

Some difficulties associated with the measurement of capital can be seen from a “supply-aspect” or a supply-role of capital. The question is, how to define, and hence to measure, the given ‘capital’ as an endowment of an economy independently of distribution if it does not consist of a single good? Should we regard in a world of heterogeneous capital goods each single capital good as one of the endowments of an economy, as we did before when capital consisted of a single good? If one goes back in time to see how traditional economists had considered capital among the data to determine the equilibrium of an economy, we shall however see that economists such as S. Jevons, E. von Böhm-Bawerk, K. Wicksell, J.B. Clark, T.N. Carver, or A. Marshall (among others) conceived of capital as a *single factor* in value terms susceptible to changing form without however changing its value.⁶¹

For instance we have the case of Marshall who in his time-honoured *Principles of Economics* states:

⁵⁹ The problems, which arise out by treating capital in value terms as arguments of the production function, are analysed in ch.5 below, where Joan Robinson’s participation in the Cambridge controversies is analysed.

⁶⁰ See Appendix B where the supply-role of capital is treated in some more detail by considering the general equilibrium system which Wicksell put forth to analyse production in economies with capital goods- that is “capitalistic production” as he called it (Wicksell, *op. cit.*, p. 144).

⁶¹ These examples are not unrelated to the broad theoretical work undertaken by other traditional marginalist authors like Jevons (1879), von Böhm-Bawerk (1891) Clark (1965[1899]), Carver (1904). Cf. equation (4.B) of Wicksell’s system, Appendix B, below. For the treatment of capital by Pigou see Hennings (1990) p. 166. Along this work we shall return to this topic, that is why these marginalist authors conceived of capital in value terms to define the equilibrium of supply and demand- and not as a vector of the heterogeneous capital goods which we may come across in an economy as is the case of Walras’s treatment of capital.

“the chief trouble [regarding ‘the general rate of interest’] comes from the fact that the income derived from capital (...) invested in particular things (...) is properly a quasi-rent and can be regarded as interest only on the *assumption that the capital value* of the investment has remained unaltered.”⁶²

On the same page, the following conclusion is raised by the renowned English economist:

“thus then interest (...) tends towards an equilibrium level such that the aggregate demand for capital in that market, at that rate of interest, is equal to the aggregate stock forthcoming there at that rate.”

Marshall’s ideas were not isolated. Wicksell, too, regarded ‘capital’ (the endowment) as a single factor in value terms susceptible to adopting any physical form. In fact, when he deals with “the concept of Capital”, he states:

“Capital also includes the raw materials (...) and other commodities which must be saved-up. (...) This, of course, is *the commonly accepted sense of the term*. [A]ll these [different capital goods] (...) have only one quality in common, namely that they represent certain quantities of exchangeable value so that (...) they may be regarded as *a single sum of value*, a certain amount of the medium of exchange, money.”⁶³

Both authors – together with the most representative traditional economists – conceived capital, the endowment, as a substance in value terms. Thus, this “single sum” is taken as *given* in the group (iii) of the data; capital, a quantity measured in a single unit (a consumption good for instance, or also in terms of a capital good)⁶⁴ is taken as *datum* in the determination of output, prices and distribution in terms of

⁶² Marshall (1959[1890]) p. 443, emphasis added. The adjective which I chose to use in the text refers to the several reprints of Marshall’s book.

⁶³ Wicksell (*op. cit.*) pp. 144-5, emphasis added. Cf. also par.B.3 (Appendix B), below.

⁶⁴ If the value of a particular capital good were made constant by using the capital good in question as its own unit value, still the problem would evidently be there for all other capital goods.

supply and demand functions.⁶⁵ What was the justification of such measure in marginal theory?

Although this measurement does not satisfy the requirement of independence from changes in prices and distribution, it was traditionally believed that owing to group i) and ii) of the data the theory would somehow determine distribution according to the factors substitution principle establishing an inverse relation between the quantities of a factor of production – relative to another one– and its rate of remuneration. In a world of heterogeneous capital goods, capital was conceived as single factor in value terms which could adopt the physical form according to the equilibrium conditions of the economy, which were thought to be sufficiently persistent so as to regard the theoretical variables determined by the theory as gravitational centres of actual variables. The determination of the physical form of the stock of capital was envisaged as a process for which a *uniform* rate of profit (interest) on the supply prices of the several capital goods could be ascertained.

The idea underlying this process is to conceive that the several capital goods can embody different amounts of the substance “capital”; in other words, “capital” is a kind of stuff in value terms which can change form without change its total quantity in value terms. For example, imagine for a moment a non-equilibrium situation; we can envisage an “old” and a “new” situation, before and after some change in the conditions (*e.g.* a change in a pattern of consumption or a change in the quantity of people seeking for jobs due to a process of immigration): in that *equilibrating* process the physical composition of that substance would be endogenously determined, if, given a sufficient length of time to adjust supplies and demands, the specific physical capital goods to be produced corresponding to the *new* conditions of the economy could be produced by the most profitable use of the available resources, which must go in accordance with the factors’ substitution principle. In fact, a change in the “form” of the single factor “capital” was thought to be brought about by the employment of factors, namely – labour and “capital”, which may had been previously used to produce the *no longer* capital goods, in the production of other or new kinds of capital goods. Thus the *new* capital goods produced, are produced by the

⁶⁵ The outstanding exception among traditional authors is Walras, who conceived “capital” as a physically heterogeneous capital goods vector existent in the economy. *Cf.* Walras (1954); see also Garegnani (1960 part II; 1962). We shall deal with this author later (ch.6) and see the problem arising out from such a conception of capital (par.6.2).

same economic resources of the economy which produced the hitherto capital goods and therefore, would have the *same total cost of production* and hence the same *total value* as the no longer produced capital goods.⁶⁶ This complex process was thought somehow to work through the sufficiently persistent of the data of equilibrium.

As was noted above, capital in value terms not only ensures the uniformity of the rates of profits on the supply prices of the capital goods, but also it allows for the substitutability of the factors of production.⁶⁷ In the wake of some change in the conditions of the economy, the economy would reach a new position of equilibrium by the supply and demand forces which work through the data of the theory. The new position to be reached implies a process where production, demands, prices, quantities are in dis-equilibrium; however, after certain length of time the supply and demand forces would put the economy again in balance. For such a result, it is key to assume that the capital “substance” be expressed in a single value magnitude susceptible to change form but not changing in quantity in order to let the supply and demand functions work throughout. Yet that “quantity” in value, like any other value magnitude, depends on the variable which theory attempts to determine – the rate of interest; thus a problem, which we may identify as the “supply-aspect” problem, which regards the measure of capital as the data consistent with the logic of the theory, arises.

⁶⁶ The clue of such an explanation provided by marginalist authors (that is the justification of the principle of factor substitution) can be traced in the attempt at a generalisation of the principle of intensive rent applied by Ricardo however to “land” and “labour *and* capital” to explain profits: the generalisation was then applied to *any* number of factors, in particular to labour and “capital” (assuming free land) such that both labour and “capital” can be seen as receiving their marginal product. Therefore the analysis is thought to apply symmetrically to factors of production, land, labour, and “capital”, and it is re-interpreted as evidence of the two substitution mechanisms we have considered in the main text. Accordingly, were it not for conceiving that “capital” in *constant value* terms susceptible to adopting any physical form as the economy conditions change, an explanation of distribution relying on ‘factor’ substitution –from which in turn we can derive the equilibrium supply-and-demand functions– would not be possible. Were it otherwise, this would entail the impossibility to derive demand functions for factors from employment factor’s curves, as we have already argued in our simple economy’s example (*cf.* par. 2.2 - 2.4 above.) This, as we argue in the main text, also explains why the physical composition of the stock of capital is left to be determined endogenously, while the value of that stock does not change. For an analysis of Ricardo and the classical political economy, see in particular Garegnani (1960, part I; 1984a).

⁶⁷ See Appendix B where the issue of the uniformity of the rate of interest of capitals’ supply prices is discussed within the framework of Wicksell’s general equilibrium system.

2.9. *Persistence character of the determining circumstances: The traditional method of ‘normal positions’.*⁶⁸

We have seen above the data taken as given by the theory to determine the dependent variables output, distribution and prices simultaneously; how the forces of supply and demand work in the factor markets have been explained in the first part: those forces are supposed to work in real economies, since they are thought to be grounded on the basic principle of factor substitution. We have lastly seen that the factor substitution can be brought into analysis both through alternative techniques of production (direct mechanism of substitution), and through tastes and preferences of consumers (indirect mechanism of substitution). The resulting equilibrium (let us suppose to be unique for the sake of exposition) of supply-and-demand (*i.e.*, the level of the dependent variables at which it is arrived through the working of those forces) is ensured to be stable – provided the “right” shape of the demand curves for factors. But to apply this approach to real-world economies the data from which one arrives at equilibrium must account for some *basic* characteristics: the data must be thought of as having a *persistence nature* through time in that they are supposed to be those variables on which the working and the results of the mechanisms of factor substitution rest.

In actual markets, however, prices, quantities, the level of the wage rate are magnitudes that depend on circumstances which are at work due to many but many “day-by-day” causes. A specialised journalist, say in agriculture products’ prices, does every morning try to provide his audience (in general perhaps investors) with the level of those prices due to the variables he considers to be the most relevant at that

⁶⁸ The term “normal positions” has been adopted by Garegnani (1990) in order to refer both to the marginal long period equilibrium of supply and demand, which defines a “long period position” of supply and demand, and to the “normal” or “natural” positions of the classical economists (Smith, Ricardo), which are *not* derived from supply and demand forces, like in the marginal theory. The latter school of economic analysis, which has been revived by Sraffa (1960) and the successive works undertaken by the surplus approach, has based the explanation of normal or natural position of the economy by relying on the key notions of a subsistence wage (determined by social and political circumstances) and the existence of a surplus, which arises after the necessities of the workers and the replacement of means of production are deduced from the social product. (See Garegnani, 1984a) Though in this study we shall deal with the marginal approach, it is worth reminding that, nevertheless both theories have relied on entirely different forces to explain distribution of income, they have shared the method of normal positions – although the marginal theory has since recent decades shifted to a different notion of equilibrium. We shall argue that this shift has been one of the consequences of the debates on capital theory, which is analysed in ch.6.

particular moment in that particular place. They run from the drought of agriculture fields in American prairies to the level of the oil price, on which the production costs of corn depend and which in turn is conditioned by the political situation in some oil exporter's countries.⁶⁹ But these particular situations, at a theoretical level, are not thought to be *persistent* circumstances on which those prices ultimately depend. So, in general, supply-and-demand equilibrium prices (and quantities, and distribution), as envisaged by the theory, are not "day-by-day" prices, but rather *average*, "*normal*", "*natural*" ones. The actual variables, or better still, the average of actual variables, if the theory were correct, would follow the trend described by those theoretical equilibrium values. But to arrive at such averages does take time: it is thanks to the persistence of the conditions of economic phenomena that ensure that gravitation process, where divergences between the values of the theoretical and actual variables are cancelled out over time, so long as repetition of transactions in markets is guaranteed⁷⁰; (following the analogy, temporary droughts in American prairies are not a persistent causes in the determination of corn prices). Thus an explanation of distribution of the social product in real economies in terms of equilibrium of supply and demand is grounded in the persistent nature of economic forces.

From the foregoing description of the persistent nature of economic conditions, a question arises: Does persistency imply that the data of the theory must not change?

Traditional marginalist authors did obviously not deny that the data might change. However, thanks to the persistence of the causes of economic phenomena, they estimated that their changes were generally either pretty *slow* enough to be negligible relative to the presumable *speed of adaptation* of demand and supply for factors and products, or if they were rapid, changes were assumed to be once-and-for-all changes, whose effects could be dealt with through comparative statics, that is by assuming a tendency to the corresponding *new* equilibrium. In other words, the same persistent nature of the causes allows the theory to deal with changes by relying on the method

⁶⁹ Cf. the following argument raised by Marshall (*op. cit.*), V, p. 291: "The actual value at any time, the market value as it is often called, is often more influenced by passing events, and by causes whose action is fitful and short lived, than by those which work persistently. But in long periods these fitful and irregular causes in large measure efface one another's influence so that in the long run persistent causes dominate value completely."

of comparison of *long period* equilibrium positions.⁷¹ Let the present author explain it a little further, if only briefly.

Suppose an economy where a process of immigration of people at the age of working is taking place. Now this process will certainly bring about a change in the quantity of labour supplied. There was an equilibrium in the labour market before that process, say (w^*, L^*) , to which there also corresponded a determined set of equilibrium prices and outputs. At the very moment of immigration, imagine that workers' wages remain at w^* . However, population at the age of getting a job is changing. As time goes by, population is (re)adapting itself to the conditions of the labour market: some new arrived people go to some firms to seek jobs, other "old" ones go to other firms, both new and old labourers are modifying the labour supply: however, it is necessary *time* for that supply to arrive at a determined level compatible with demand and free competition. In that process, there of course is a "quantity of capital" in value terms which is slowly changing, as those forces are pushing the system towards equilibrium, that is – as the physical composition of that "quantity" is being adapted to the new conditions. In that very process the theory assumed that quantity to remain constant. That period of time must be a long one, in order to let the physical composition of capital be adjusted, even though, individually perhaps, the period required by a "new" arrived person (or family or group) to get a job was short due to specific circumstances (*e.g.* s/he or they was/were given a job in advance before arriving *in situ*) and so on. But, at a general level, the period of time must be long. How then is the equilibrium reached?

As the population at the age of getting a job changes, the supply of labour will change (some of the data of the theory *change*).⁷² Accordingly, the wage rate falls, and the quantity employed rises. But for this we need assume "capital" employed to

⁷⁰ The variables that the theory determines cannot be the actual magnitudes observable moment by moment in the market, which are influenced by many accidental factors, since those factors could not be known in advance. See, Garegnani (1990) p. 46.

⁷¹ The usefulness of the method of long period equilibrium lies in the fact that theory provides a logic framework to analyse changes through an analysis of comparison of different "normal positions" by means of the mechanisms pointed out in the main text. It is however very important to bear in mind that for this method to be used the capital must be conceived in value terms, thus an illegitimacy is entailed. Yet we will return along this work to these uses of the theory, in particular, when we come to approach the "new" versions of the general equilibrium theory in its intertemporal or temporary versions.

⁷² The theory can give an appropriate picture of the process if the data are stable in comparison to the speed of adjustment. For a detailed methodological analysis, see Schlicht (1985), §1.2.7, p. 6.

be constant (in fact in full employment). However, the *composition* of that capital will also be changing, as the more “labour intensive” methods of production come to be adopted in that very process of adjustment (a time consuming process from disequilibrium towards equilibrium). In between, that is after the immigration in our imaginary economy and before the *new* equilibrium levels are reached, transactions, contracts, etc., take place at non-equilibrium values (with prices changing very gradually, that is almost imperceptible) but, however, they take place at the same time by thinking of a tendency to a future level of equilibrium.⁷³ As the immigration process takes place, more “labour-intensive” processes of production are adopted – given the technical knowledge of our imaginary economy. When the composition of “capital” is then adjusted to the new conditions of the economy, we would have reached a *new* long period equilibrium. Once the uniform rate of return on the supply prices of the capital goods is reached, that composition is endogenously adjusted; thus ensuring persistence of the equilibrium.

In conditions of free competition in the capital market, and due to the characteristics of the capital goods, (in general, products of short duration life) investors will, in the wake of some change in demand conditions, seek the most profitable capital goods wherein to invest their capital. The resulting equilibrium prices of capital goods will *tend* towards equality with their costs of production or supply price and thus the rates of return over the costs of the capital goods will tend to be uniform: the differences in profitability among the several capital goods will cause some investors to sell their capital, whilst others to buy other capital goods. This process can then be seen as an equilibrating process in that the several goods will pay over costs a uniform return, independently of the particular kind of capital good concerned. Thus provided, we may see this basic reason why in traditional writers the endowments of the several capital goods cannot be taken as part of the

⁷³ A logical necessity of the traditional method – which attempts to explain actual magnitudes as oscillating around centres of gravitation – is that these gravitational centres must *not* be affected by the “day-by-day” disequilibrium production and consumption activities undertaken by the economic agents. Traditionally, it was thought that in the definition of long period equilibrium of supply and demand, prices’ changes could be ignored. The argument for such a thesis is the assumption that, when adjustments are taking place, the allocation of savings among capital goods could not appreciably influence the equilibrium position. See Garegnani (1990) p. 51; *cf.* Appendix B below, in particular equation (3.B) regarding Wicksell’s treatment of savings.

data of the equilibrium⁷⁴, if a process involving disequilibrium adjustments are taking into account: in general those adjustments need long periods of time.

2.10. Demand-side role of capital. Conclusion.

In the example about immigration above, we have said that, once the composition of the stock of capital is endogenously determined a new long period equilibrium will be reached by satisfying the condition of uniformity of the rates of profits on the capitals' supply prices. The substitution of factors when capital consists of a family of heterogeneous capital goods entails, in general, that it is *not* a change in the *proportions* of "factors" that allows theory to arrive at a new equilibrium but a change in the *kinds* of the capital goods due to the new methods adopted for production.

In fact, whether an economy would produce particular *kinds* of capital goods will depend a) on the methods of production available in the economy and b) on outputs. In other words, production of the specific capital goods (investment) depends upon the *demand conditions* of the economy. In the wake of some change in the demand conditions, marginal theory still needs a treatment of "capital" as analogous to that of any "factor of production" in the direct and indirect factor substitution mechanisms, to permit that the *demand* for "capital" be a *decreasing* function in relation to its "price" – that is the interest rate. This is the demand-role or demand-aspect of capital. This side concerns two related aspects, which are very important to bear in mind when we come to approach the results of the controversies in the theory of capital (chs 3 and part II).

"Capital" could be treated as any physical factor to ensure that the substitution mechanism work in the "right" direction, *i.e.* the demand function for capital is inversely related *vis-à-vis* the rate of interest.

The conception of "capital" of variable 'form' in turn makes it possible to assume the existence of a *sufficiently* substitutability between "capital" and labour (or land). This sufficient substitutability is compatible with a conception of a quantity of "capital" in value terms but incompatible with a conception of the physically heterogeneous capital goods as data. The importance of this sufficient substitutability between "factors of production" can be traced *e.g.* in Hicks' 1932 *The Theory of*

⁷⁴ With the outstanding exception of Walras as noted earlier.

Wages, where he argues that such a substitutability would be impossible to conceive, in the long period, for single capital goods:

“In the short period it is reasonable to expect that the demand for labour will be very inelastic, since the possibility of adjusting the organisation of industry to a changed level of wages is relatively small; (...) Since the whole conception of marginal productivity depends upon the variability of industrial methods, little advantage seems to be gained from the attempt which is sometimes made to define a ‘short period marginal product’ – the additional production due to a small increase in the quantity of labour, when *not only the quantity, but also the form*, of the co-operating capital is supposed unchanged. It is very doubtful if this conception can be given any precise meaning which is capable of useful application.”⁷⁵

In fact, that “variability” of the form of “capital” cannot be conceived as an instantaneous process; actually, at each moment of time, “capital” is “crystallised” in specific capital goods, and can take the best form adapted to demand conditions: changes in demand due to changes in consumer patterns, technical progress, etc. But that change in “form” is *gradual*: once the existing capital goods are depreciated the reinvestment of these allowances into *new* capital goods which are different from the depreciating ones. Thus “capital” – the value substance– embodied in the old capital goods becomes *free* to be *re*-invested in the new, different ones. This is the idea of a “capital” as a single factor susceptible to changing form. Thus, the “marginal product of capital” (see the basic scheme in section I above) can manifest itself in investment in the new plants: the flow (investment) of *free* capital would meet the labour “freed” by the closure of the old capital goods’ plants. This linking is what allows us to consider the investment demand as a manifestation of a demand for capital.⁷⁶ Then if

⁷⁵ Hicks (1932) pp. 20–1, emphasis added.

⁷⁶ That is to say: the supply and demand functions for “capital” as a *fund* were envisaged by traditional authors to analyse the forces operating in the investment and savings market – where capital appears as a *flow*. To understand this it is sufficient to think about the traditional supply and demand functions for capital the single factor. As it was showed in the text the physical composition of the capital demand function will be determined by the production methods and outputs and hence are unknowns of the problem. On the other hand the physical composition of the capital supply function cannot be but what is commonly known as the stock of the economy. In general, both composition will differ; this incompatibility between both compositions is resolved itself precisely in the investment savings market: what is underlying the capital supply curve is that the physical composition of the stock appropriate to the equilibrium will be assumed by the existing stock *over a period of time* when part of the capital goods has to be replaced by
(cont.)

this linking is admitted, and, provided the “right” shape of the demand curve for capital, the interest rate could be then seen as the price *bringing into equality* the demand for and supply of “capital” the *flow*.

To conclude. The introduction of “quantity of capital” the single factor into the general equilibrium supply and demand functions makes it possible to arrive at long period equilibrium, while leaving the endowments of the *several capital goods* to be *endogenously determined* by the *tendency* toward a uniform rate of return on the supply prices of the capital goods through factor substitution. This allows the theory to apply the method of comparison of normal positions for an analysis of changes. Notwithstanding the supply-side problem regarding the illegitimacy of conceiving capital in value terms to determine distribution⁷⁷, it was once believed by the founders of the neoclassical approach that, in order to get an equilibrium position as a result of applying supply and demand general equilibrium functions, the quantity of capital per worker in the economy had to be inversely related to the rate of interest, by means of changes in demand associated with groups i) and ii) of the data of the theory.

This “belief” regards the demand-aspect of the question: that aspect resolves itself in the necessity for the theory to have *downward-sloped* demand curves for the productive factors. If downward-sloped demand curves for factors of production can be rigorously demonstrated, then the way to explaining distribution in terms of supply and demand through the mechanisms of substitution of “scarce” factors would be open.

reinvesting the depreciating allowances into new capital goods. As Garegnani (2000, pp. 395-6) has argued, it would seem that the traditional authors’ focus was on analysing the forces of the *fund* concept of capital rather than the *flow*, in order to analyse the “substitution of factors in a purer form”. We shall see the link between the savings-investment market with the fund capital market in more detail later, ch.6.

⁷⁷ This is the most evident contradiction which arises from the measurement of capital in value terms for the marginal theory, since it is that the data from which we should depart in order to arrive at the equilibrium values of output, prices, and hence the very distribution. Therefore, the circularity in the reasoning is not avoided; however, it is capital conceived in value terms that allows the theory to leave the physical composition of the stock be endogenously determined, thus being coherent with the traditional method of a uniform rate of profits on the capitals’ supply prices (see the case of Wicksell in appendix B).

Later in this work we shall also present the *other* treatment of capital namely that of Walras who, even sharing with his contemporary peers the traditional method, considered the capital endowment in the data of equilibrium as a vector of heterogeneous capital goods of the economy: this led Walras to contradict his own general equilibrium system, since in general the arbitrary
(cont.)

In the next chapter, we shall introduce some main results drawn from the capital controversies in order to see whether the marginal theory may determine distribution on the grounds of an inverse demand curve for “factors” in economies with heterogeneous capital goods. What is key to keep in mind is the role of the downward-sloping demand curve for “capital” as traditionally conceived – a single magnitude in value terms – and with it the supply and demand aspect of the problem of capital. The role of the slope of the demand curve is decisive for uniqueness and stability of the equilibrium.

composition of capital in his treatment will prevent the rates of return of the several capital goods from being uniform on the supply prices of that goods. See ch.6.

CHAPTER 3

‘Reswitching’ and ‘Reverse Capital Deepening’: Introduction to the heart of the controversies and their implications

3.1. Some premises.

In the previous chapter it has been introduced the key role of the downward-sloping demand curves for factors, which can be derived according to the factors’ substitution principle. This principle can univocally establish, *e.g.* in a labour and corn (both as consumption- and as capital-good) economy, an inverse relation between factors’ intensity and factors’ rates of remuneration. However, can we derive the same kind of relations when one of the factors is “capital”? As sketched above, marginalist authors have conceived capital as a single magnitude in value terms susceptible to adopt the physical form of the several capital goods – and hence their quantities as well – determined by the equilibrium condition of a uniform rate of profit on the supply prices of the several capital goods. Nevertheless the illegitimacy of the notion of capital in value (the supply-side aspect), the founders of the neoclassical approach somehow believed that, through changes in demand due to changes in the production methods and outputs – associated with groups i) and ii) of the data – a negative relation between the rate of interest and the demand for “capital” could in general be postulated. In other words, it was believed that owing to changes in demand the factor substitution principle could be also taken for granted even when factors were measured in value terms so as to derive a monotonic relation between changes in distribution and factors’ intensities, *i.e.* to derive a negative relation between “capital” and the rate of interest. This is the demand side of the problem. As we shall see in this chapter, the discovery of ‘reswitching’ and ‘reverse capital deepening’ in the Cambridge controversies has destroyed that neoclassical belief.

‘Reswitching’ and ‘reverse capital deepening’ started to gain prominence in the Cambridge controversies in the 1960s, particularly after Sraffa’s (1960) famous book

which was followed by a series of contributions that started to spread these discoveries over the whole decade.⁷⁸ Rather than following a chronological description of the episodes around the Cambridge debates, it has been preferred in this part of the study to bring to the foreground the meaning and implications of ‘reswitching’ and ‘reverse capital deepening’, because these results touch the hard core premises of the marginal theory – despite that these phenomena started to be acknowledged in a later stage of the first phase of the controversies. The aim of this chapter is to introduce the reader to the meaning of such phenomena and, more important, their implications for the marginal theory. Thus this chapter provides the ground on which further exchanges between the two sides evolved.⁷⁹

In a heterogeneous capital goods economy, is it possible to derive a univocal relation between “capital” intensity and changes in distribution? Neoclassical traditional theory believes that, at a low rate of interest (r) and from a spectrum of alternative techniques (say A and B both using heterogeneous capital goods), producers –e.g. of a single final consumption good⁸⁰ – would be led to use a “more capital intensive” technique (say A) so that this technique would, in competitive long-run equilibrium, minimise the production costs of the final good. It was also believed that a rise in r might lead producers to change (switch) to a “less capital” intensive technique (B) – so long as this technique minimised costs – leading the economy to account for a lower quantity of “capital” per worker. But it was also believed that before a further rise in r the *old* technique could *not* be adopted again because that technique would have been categorised as a “more capital” intensive one, which could have never been chosen *again* at a higher rate of interest. In the Cambridge

⁷⁸ See Appendix C. Among the several contributions to the ‘reswitching’ debate we should cite: Sraffa (1960) and the articles published in the 1966 Symposium: Pasinetti (1966a), Bruno *et. al.* (1966), Morishima (1966), Garegnani (1966), Levhari and Samuelson (1966), Samuelson (1966a). That Symposium was the result of the intervention from the neoclassical side of Levhari (1965). We deal with the historical aspects of the Symposium in the next chapter. The content of the present chapter is mainly drawn from Samuelson (1962) and Garegnani (1970a).

Note, however, that phenomena of ‘reswitching’ and ‘reverse capital deepening’ had been noticed in the literature in the controversies before 1960, in particular in Robinson (1953) and Champernowne (1953); however these authors gave these phenomena little prominence at the time (*Cf.* Robinson (1953) p. 106 where ‘reswitching’ is treated as a “curiosum”; and also Robinson, (1956) p. 109 where the Cambridge author regards the “curiosum” as an “intricate piece of analysis which is not of great importance.”) Nevertheless we shall deal with these authors in ch.5. Finally note that, after Sraffa (1960), the first work where the terminology of “switching” commenced to be used was – to the author’s knowledge – McManus (1963).

⁷⁹ See ch.4.

⁸⁰ Or a given bundle of final commodities.

controversies it was showed that *that* possibility cannot be excluded *a priori*. Since capital goods are reproducible and also require further capital goods in their production, it could be the case that at a higher r the technique that had been used at lower values of r could be chosen again – why? because this technique might also minimise costs at the new situation of prices and distribution. This theoretical possibility has been known in the debates as ‘reswitching’ of techniques. Thus, it would seem, a technique *a priori* considered as “more capital” intensive is chosen for a range of values of r and also is chosen for other range of values of r , while another technique is chosen in between.⁸¹ Evidently, there is no monotonic relation between changes in distribution and “capital” intensities. The idea of an ordering of techniques according to “more” or “less” capital intensity disappears with changes in distribution, for a change in distribution entails a change in the way in which – if existed – those techniques were ordered according to relative intensities. But, if that were the case, then there would be no theoretic support to think of an inverse relation between factors’ demands and their rates of remuneration, and hence of the factor substitution principle, which is key to the theory. Thus, with ‘reswitching’, hard core premises of the approach are at stake.

‘Reverse capital deepening’ indicates, in economies with production involving heterogeneous capital goods, the possibility that the demand for capital be in a *direct* rather than in an inverse relation to the rate of interest. Why? Thanks to ‘reswitching’, as we shall see below⁸², the *value* of capital per worker in the economy may rise with a *rise* in r as the economy ‘reswitches’ from one technique to the one that had been previously used at lower values of r . But also, even without ‘reswitching’, ‘reverse capital deepening’ may arise. In fact, due to changes in distribution it might be the case that the value of capital per worker changes in the opposite direction as conceived traditionally.⁸³

The novelty of these results, revealed by the critical side of the Cambridge debates, lies in the fact that even if the illegitimacy of the supply side is put aside, and by taking into account the very premises of the theory, serious difficulties would arise in order to derive downward-sloping demand curves by relying in demand changes,

⁸¹ Or that technique may be “more capital” intensive for some ranges of values of r and “more labour (or other factor)” intensive for other ranges of r .

⁸² See par.3.4.

⁸³ See Appendixes C and D.

because also this demand aspect is undermined by ‘reswitching’. Hence it turns out to be impossible to derive any measure of capital in value terms in a *univocal* direction with changes in distribution, but, for reasons already explained this notion is central for the whole approach. (*cf.* ch.2 section II.)

In what follows the demonstration of results of ‘reswitching’ and ‘reverse capital deepening’ is introduced by using very simple graphical analysis as used by the participants in the debates.⁸⁴

3.2. *Economies with heterogeneous capital goods. The wage curve.*

In order to show the results of ‘reswitching’ and ‘reverse capital deepening’ it is assumed an economy with the following characteristics.

- Yearly cycles of production of the consumption good and of the capital good.
- Capital is circulating.
- Constant returns to scale.
- No scarcity of natural resources.
- The wages are paid at the end of each productive cycle.
- There exists a surplus obtainable over the pure replacement of machines: that is to say, the economy is viable, hence the proportion of the capital good entering the production of the capital good itself must be lower than unity (*i.e.* $c_c < 1$).⁸⁵
- The capital good enters the production of both commodities.
- Some labour is required directly or indirectly to produce both commodities.

Let us first consider production with only one system of production. Each system consists of two “methods of production”, that is, one method to produce the

⁸⁴ For the present exposition we shall follow Samuelson (1962) and Garegnani, (1970a). System of production involves two methods of production: the one for the consumption good and the other for the capital good. It is assumed that for each system the capital good is produced by itself and labour which is a very simple case chosen by Samuelson (*op. cit*) in the controversies, then also used by Hicks (1965) and fully studied and criticised by Garegnani (*op. cit*). For more general models, see Sraffa (1960), Pasinetti (2003, ch.6), Garegnani (*op. cit*, pp. 417-21).

⁸⁵ Lower than one because we assume capital goods to be circulating. (See Garegnani (1970a, p. 408, n1.) We shall not deal with fixed capital since our aim is basically to show these results of the debates as clear as possible. The reader may be referred to Sraffa (1960, ch.10). For an analysis of the historical aspects of fixed capital and its distinction from circulating capital see Varri (1998).

consumption good, A , by means of fixed quantities of labour (l_a) and of the capital good, C , (c_a), and the other method to produce the capital good by means of fixed quantities of labour (l_c) and of the capital good itself (c_c).⁸⁶

The imposition of a uniform rate of interest and a uniform wage will enable us to write the following two price equations:

$$\begin{aligned} p_a &= 1 = l_a w + c_a p_c (1+r) \\ p_c &= l_c w + c_c p_c (1+r) \end{aligned} \quad (3.1)$$

where p_a and p_c are the consumption good's and the capital good's prices respectively. As usual, w is the wage rate in terms of the integrated industry of the consumption good and r is the interest rate in the same terms. The consumption good has been considered as our *numéraire*, hence $p_a=1$.

There are two equations and three unknowns (p_c , w , r). The degree of freedom will allow us to take one of the distributive variable as independent (henceforth r), and then both w and p_c will be univocally determined for any given value of r .

From this system the following relation between the wage and the interest rate corresponding to the current system of production can be obtained:

$$w = \frac{1 - c_c(1+r)}{[l_a + (l_c c_a - l_a c_c)(1+r)]} \quad (3.2)^{87}$$

From equation (3.2), and for any value of r , there corresponds a unique value of w , hence (3.2) is a function. Moreover, both of the distributive variables are negatively

⁸⁶ Other systems of production involve, apart from homogenous labour that is used by the different systems, *different* capital goods for the production of the consumption good A .

⁸⁷ From the second equation in (3.1) we obtain $p_c = \frac{l_c w}{1 - c_c(1+r)}$ (3.3). Then by substituting p_c

on the Right Hand Side (RHS) of the first equation of (3.1) for (3.3), we obtain (3.2), which is equivalent to $w = \frac{1 - c_c(1+r)}{l_a + D(1+r)}$ (3.2'), where D is the determinant of the productive coefficient

matrix, that is $D = \begin{vmatrix} l_c & l_a \\ c_c & c_a \end{vmatrix}$. This determinant is central to the entire analysis, in particular, for the

graphical representation of the relation between w and r which allow us to easily grasp many of the results of 'reverse capital deepening' and 'reswitching'.

related.⁸⁸ A rise in the rate of interest (while the other elements remain constant) will make the numerator of (3.2) fall, whereas the denominator will rise: consequently w will fall.⁸⁹

To calculate the value of capital per worker it may be useful to introduce the integrated industry of the consumption good. This imaginary industry is such that the proportion, K , of the capital industry relative to the consumption good industry is the one which ensures the *replacement of the capital goods* consumed in the integrated industry. Accordingly, this industry must re-integrate the quantity of the capital good necessary (c_a) for producing the consumption-good directly, and the quantity which is necessary for producing itself (c_c) – that is the proportion of the capital good consumed in the production of the proportion of the capital good necessary for producing the consumption good. In formal terms:

$$K = c_a + c_c K$$

Then,

$$K = \frac{c_a}{1 - c_c} \quad (3.5)$$

Equation (3.5) provides the quantity of capital of the integrated industry of the consumption good. Recourse to the integrated industry is necessary if the *net* product (that is the product to be divided between wages and interest) is to consist of a

⁸⁸ Cf. equation (3.9) n.93 below.

⁸⁹ Likewise, it can be verified that the price of the capital good (p_c) is *positive* for every $0 \leq r \leq R$ (where R is the maximum rate of profits obtainable when $w=0$ then $R = \frac{1}{c_c} - 1$). In fact from

(3.2') and (3.3) we can obtain $p_c = \frac{l_c}{l_a + D(1+r)}$ (3.4). Then if $D \geq 0$ hence $p_c > 0$ for $r \geq 0$.

And if $D < 0$, then $p_c > 0$ too. In fact, proving by contradiction we may consider p_c be negative; in this case denominator of (4) should be negative (since l_c is positive throughout). Then

$r > -\frac{l_a}{D} - 1$ but we know that $r \leq R$ so $R > -\frac{l_a}{D} - 1$, then

$\frac{1}{c_c} - 1 > -\frac{l_a}{D} - 1$ hence $D < -l_a c_c$ and hence finally $l_c c_a < 0$, but this is impossible since l_c and

c_a are both positive. So $p_c > 0$ for every $0 \leq r \leq R$.

physical quantity of the consumption good. Therefore, the *net* physical product (of the integrated industry) per worker⁹⁰, q , can be measured as follows:

$$q = \frac{1}{l_a + l_c K} \quad (3.6)$$

Then by replacing (3.5) in (3.6):

$$q = \frac{1}{l_a + \left(\frac{c_a}{1-c_c}\right)l_c} = \frac{1-c_c}{l_a(1-c_c) + c_a l_c} \quad (3.7)$$

But, (3.7) will be equal to (3.2) or (3.2'), whenever $r=0$.

Hence:

$$w(r=0) = W = \frac{1-c_c}{l_a(1-c_c) + c_a l_c} = q \quad (3.8)^{91}$$

If r is zero then the segment OW measuring the maximum wage rate will also measure the net physical product of the integrated industry per labourer. If the wage rate is w_1 and the rate of interest r_1 , the value of capital per worker can be measured in the following way:

Since $w_1 W$ indicates the quantity of consumption good received as profits for each worker employed in the integrated industry, and Or_1 on the other hand indicates the level of the interest rate, then the value of capital k can be obtained as the ratio between⁹²:

⁹⁰ Consumption-good production is normalised to one so q is one unit of *net A* divided by the quantity of labour employed both directly and indirectly in the production of such a unit of net physical output.

⁹¹ Since $w = \frac{1-c_c(1+r)}{[l_a + (l_c c_a - l_a c_c)(1+r)]}$ with a zero r becomes equation (3.8), where W is the maximum wage, and equation (3.8) is equal to equation (3.7).

⁹² Mathematically, the way to obtain it is by measuring the trigonometric tangent of the angle θ —of the triangle formed by points P , W and w_l . (See figure 3.1). We have in the abscissa the interest rate value Or_1 , while in the vertical axis, correspondingly, the value of the interest per worker
(cont.)

$$k = \text{Interests per worker} / \text{Rate of interest}$$

The system of production gives the relationship between r and w .⁹³ We shall label this relationship as the *wage-curve*⁹⁴, whose graphic can be obtained from equation (3.2), and is drawn in figure 3.1 below.

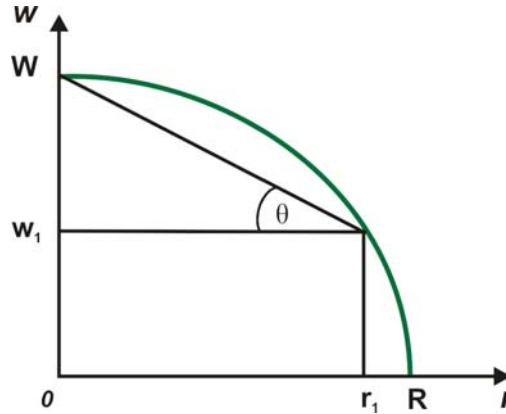


Figure 3.1: The *wage-curve*: OW is the net physical product per worker; $\tan(\theta)$ is the value of capital per worker (k) when the wage rate (w) is $0w_1$.

In figure 3.1 above, it has been assumed $D < 0$. If $D = 0$, then the function (3.2), or (3.2'), would define a straight line. If $D > 0$, the price of the capital good will fall as the rate of interest rises, and the form of the wage curve will be convex.⁹⁵

(w_1W) . Then, the trigonometric tangent of angle θ will give us:

$$\tan(\theta) = \tan(w_1PW) = \frac{w_1W}{w_1P} = \frac{w_1W}{0r_1} \quad (P \text{ is point } w_1r_1), \text{ and then, } \tan(\theta) = \tan(w_1PW) = \text{interest per}$$

 worker / rate of interest = value of capital per worker, and this is the result at which we wanted to arrive.

⁹³ The inverse relationship between w and r in (3.2) is shown by its graphic representation. Some formal (and economical) considerations are in order: if, as we have already seen, $r=0$ then $w=W$. On the same assumptions, as r rises from zero, w falls reaching zero when r reaches R – its maximum value. And this is so because of the assumptions here assumed for the economy, which ensure that w be a continuous differentiable function of r . In fact, the inverse relationship between w and r can be shown by differentiating (3.2'):

$$\frac{dw(r)}{dr} = \frac{-c_c[l_a + D(1+r)] - [1 - c_c(1+r)]D}{[l_a + D(1+r)]^2} = \frac{-c_c l_a - D}{[l_a + D(1+r)]^2} = \frac{-l_a c_a}{[l_a + D(1+r)]^2} \quad (3.9)$$

which is *negative* provided that the production coefficients (l_a , c_a) are both positive.

⁹⁴ Garegnani (1970a). Other authors have called this relationship in different ways, e.g. Samuelson (1962) who named it “factor-price frontier”.

3.3. The wage-frontier. Economies with more than one system of production.

Thus equipped with the fundamental relationships between the wage rate, the interest rate, the value of capital and the net output per worker, it may be considered two systems of production, for instance α and β . Each system involves two methods of production – one for the consumption good, one for the capital good.⁹⁶

Under long period conditions, the producer's tendency regarding which technique⁹⁷ s/he might chooses depends on the costs associated with the production of the consumption good. The cost of the consumption good will depend on the price system which happens to be in use.

⁹⁵ In other words, the form of the function will crucially depend on the sign of determinant D (i.e. on the relative factor coefficients in the production of each of the goods). In fact, by differentiating equation (3) with respect to the rate of interest we obtain: $\frac{dp_c}{dr} = \frac{-l_c D}{[l_a + D(1+r)]^2}$ (3.10).

The case $D > 0$ (equation 10 is negative and will indicate that the price of the capital good per worker increases as the rate of profits falls) was called in the debates a *negative* “price Wicksell effect”. Instead, if $D < 0$, the price of the capital good will *rise* as the rate of interest *rises*, and the form of the w curve will be concave. This was called a *positive* “price Wicksell effect”. Lastly, if $D = 0$, the price of the capital good does not change as the rate of interest changes, and the form of the wage curve will be a straight-line (a *neutral* “price Wicksell effect”). The terminology here adopted is that used by Burmeister (1987). On the other hand Harcourt (1972, pp.39-45) has instead preferred to call a *negative* price “Wicksell effect” the case when $D > 0$ and *vice versa*. The Wicksell effect was a term coined during the controversies by Robinson (1953), who actually picked it up from Uhr (1951). See also Swan (1956), Robinson (1958), Bahduri (1966), Pasinetti (1978), Garegnani (1984b). The connection between changes in the value of capital when distribution changes with the name of Wicksell can be seen in terms of the RHS of equation 4.B in Appendix B; for instance if w rises – assume that the rate of rent does not vary for the sake of exposition – then the interest rate must fall. The point is that the absolute increment in wages may be higher, or lower (or equal) than (to) the absolute decrement in interest and hence the value of capital (on the RHS of the equation) may increase or decrease (or remain unchanged) as the rate of interest fall, even when the physical quantities there involved vary in a direction in accordance with the theory (A_l rises, etc.) These problems regarding changes in the value of capital due to changes in distribution are discussed by Wicksell in his *Lectures* (pp. 149-*passim* and pp. 180-1) where the Swedish economist argues against Von Thünen's theorem which holds that the “marginal productivity of capital” equals the interest rate. Wicksell shows in those pages that due to “unproductive absorption” of the increased output von Thünen's theorem is false.

⁹⁶ In Alpha system the coefficients regarding the methods of production of the consumption good are $(l_{a\alpha}, c_{a\alpha})$ and the ones regarding production of the capital good are $(l_{c\alpha}, c_{c\alpha})$. For the case of Beta system we have $(l_{a\beta}, c_{a\beta})$ and $(l_{c\beta}, c_{c\beta})$. The curvature of each wage-curve corresponding to each system crucially depends on the value of the determinant D , which actually depends on the value of the production coefficients of each system. For example if $c_{c\alpha}/l_{c\alpha} > c_{a\alpha}/l_{a\alpha}$, that is a rise in interest cost affects the production of the capital good more than that of the consumption good – then $D < 0$. Note that we can confront both of the relations referred to, since the capital good used in both sectors is the same, that is $c_{c\alpha}$ and $c_{c\beta}$ are proportions of the same capital good C^α (the capital good associated with system α).

If system α is in use, for a given r we have w^α and $p_{c\alpha}$, then $p_{c\beta}^{(\alpha)}$ and $p_{a\beta}^{(\alpha)}$ can be calculated at that r and at w^α (prices of capital good and consumption good using β -technique at prices and w of system α). If on the other hand β is in use instead of α at that level of r , we have w^β and $p_{c\beta}$, then $p_{c\alpha}^{(\beta)}$ and $p_{a\alpha}^{(\beta)}$ can be calculated at that r but at w^β . In other words the costs of the consumption good will be different according to which system happens to be in use. However it has been demonstrated that producers' tendency to switch from one system to another more convenient (cheaper) one in the existing price situation will bring them to the one giving the *highest wage rate* at the given rate of interest.⁹⁸

⁹⁷ The term *technique* is used as synonymous of system involving two methods of production.

⁹⁸ Assume system α to be in use:

System α

$$\begin{aligned}
 p_{a\alpha} &= 1 \\
 p_{a\alpha} &= l_{a\alpha}w^\alpha + c_{a\alpha}p_{c\alpha}(1+r) \\
 p_{c\alpha} &= l_{c\alpha}w^\alpha + c_{c\alpha}p_{c\alpha}(1+r) \\
 p_{a\beta}^{(\alpha)} &= l_{a\beta}w^\alpha + c_{a\beta}p_{c\beta}^{(\alpha)}(1+r) \\
 p_{c\beta}^{(\alpha)} &= l_{c\beta}w^\alpha + c_{c\beta}p_{c\beta}^{(\alpha)}(1+r)
 \end{aligned} \tag{3.11}$$

The last two equations of system α give us the estimated costs both of the consumption and of the capital goods with system β but under the prices and wage of system α .

In case β is in use we have:

System β

$$\begin{aligned}
 p_{a\beta} &= 1 \\
 p_{a\beta} &= l_{a\beta}w^\beta + c_{a\beta}p_{c\beta}(1+r) \\
 p_{c\beta} &= l_{c\beta}w^\beta + c_{c\beta}p_{c\beta}(1+r) \\
 p_{a\alpha}^{(\beta)} &= l_{a\alpha}w^\beta + c_{a\alpha}p_{c\alpha}^{(\beta)}(1+r) \\
 p_{c\alpha}^{(\beta)} &= l_{c\alpha}w^\beta + c_{c\alpha}p_{c\alpha}^{(\beta)}(1+r)
 \end{aligned} \tag{3.12}$$

The last two equations of system β give us the estimated cost of both the consumption and the capital goods with system α but under the prices and wage of system β . The producer knows that if system α is in use, and $p_{a\beta}^{(\alpha)} < p_{a\alpha}$ is verified, then there would exist a convenience to change to the method of producing A given by system β , since the cost of producing the consumption good is lower than the current price $p_{a\alpha}$. On the other hand, if technique β is in use, and $p_{a\alpha}^{(\beta)} < p_{a\beta}$ is verified, then he would change technique β for technique α . So it would seem to appear that, *a priori*, given two methods of production, the choice will depend upon where the producer happens to be initially.

However, if we consider each system separately, for instance system α , we shall have four independent equations to solve for five unknowns. The problem here is that we cannot determine the value of those unknowns, since $p_{a\beta}^{(\alpha)}$ and $p_{c\beta}^{(\alpha)}$, as such, are qualitatively different from the remainder unknowns ($p_{c\alpha}$, w^α) because the system used to produce both A and the capital good, that is the system underlying unknowns $p_{a\beta}^{(\alpha)}$ and $p_{c\beta}^{(\alpha)}$, is β , not α . The same applies to both $p_{a\alpha}^{(\beta)}$ and $p_{c\alpha}^{(\beta)}$ in case we consider system β .

Even though we cannot determine the value of the unknowns, we can determine the *ratios* between them, since both systems are *structurally* identical. By considering the relations between the unknowns:

$$p_{a\alpha} : p_{c\alpha} : p_{a\beta}^{(\alpha)} : p_{c\beta}^{(\alpha)} : w^\alpha = p_{a\beta} : p_{c\beta} : p_{a\alpha}^{(\beta)} : p_{c\alpha}^{(\beta)} : w^\beta \tag{3.13}$$

and by discarding each of the first equation of both systems, we have:

(cont.)

Since producers will change systems according to this rule, it can be generated an *outer envelope* (Northeast envelope) by intersecting wage-curves of the two systems of production considered in the present exposition, and will represent the relation between r and w . This curve may be labelled as the *wage-frontier*, which has a “corner” where a switch occurs (“switching point”), and in the corner itself the two systems can co-exist (“adjacent”⁹⁹ systems). (See figure 3.2).¹⁰⁰

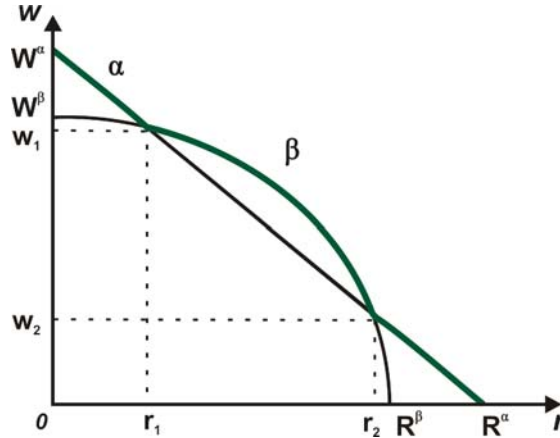


Figure 3.2. The *wage-frontier* and ‘reswitching’ of techniques. At $r=r_2$ producers switch back to technique α .

3.4. Implications for the basic premises of the theory. Conclusion.

With the aid of the wage-curves and the wage-frontier the intuitive explanation of ‘reswitching’ introduced in this chapter may be reinforced by analysing the relations between the net physical product, the wage rate, and the value of capital per worker as the rate of interest changes.¹⁰¹

$$w^\alpha/w^\beta = p_{a\alpha}/p_{a\alpha}^{(\beta)} \quad (3.14)$$

$$w^\alpha/w^\beta = p_{a\beta}/p_{a\beta}^{(\alpha)} \quad (3.15)$$

Whenever the ratios (3.14) and (3.15) are equal, the cheaper system will be the same at both wage rates and price systems. Thus if $w^\alpha > w^\beta$, then $p_{a\beta} > p_{a\beta}^{(\alpha)}$ i.e. system α is cheaper for producing the consumption good than β , and allows at the same time to have a higher wage. This is the objective method. Note that in the process of switching, entrepreneurs who first happen to switch to the most profitable technique might get some extra profits. But when the economy has already adopted the cheapest method no more extra profits can be obtained. See: Garegnani (1970a), Pasinetti (2003), Kurz and Salvadori (1997).

⁹⁹ Samuelson, (1962) p. 198; Garegnani, (1970a) p. 412.

¹⁰⁰ This envelope could also be generated by intersecting a *family* of systems of production. See ch.4 below.

¹⁰¹ It is also possible to consider w as the independent variable, and the results will be as similar as those reached when we regard r as the independent variable.

For our purposes here it will be sufficient to consider what happens to the value of capital per worker as the rate of interest changes. Assume that, as required by the theory, an *a priori* criterion to establish which technique is more (less) capital intensive might be chosen. Assume that a method is “more” capital intensive because it is the method yielding the highest net product per worker of the integrated industry. Assume that α gives the highest net product and is a straight-line curve (*i.e.* $D=0$)¹⁰², while the second technique, β , which is concave in its entire trajectory (*i.e.* $D<0$), yields a lower net product and intersects system α at two points.¹⁰³ Then according to the method explained above (see n.98) when $0 < r < r_1$ technique α (“more capital” intensive, as assumed) is chosen; then as r rises further such that $r_1 < r < r_2$ technique β (“less capital” intensive) is chosen; so far neoclassical long period technical choices go in direction as hypothesised by the theory namely – the higher the rate of interest the lower the “capital” intensity. However, if r increases further such that $r > r_2$ it will be found that the “more” – and not the “less” – capital intensive technique is chosen according to the rule governing the choice of techniques.¹⁰⁴ (See figure 3.2.) Thus, even if a criterion could be adopted so as to order “intensities” according to different levels of distribution, the ordering itself will change with changes in distribution.¹⁰⁵

In the foregoing analysis capital-inputs have been measured in technical units independent of distribution; yet there is no guarantee that changes in the rate of interest will cause changes in the opposite direction in the quantity of capital per

¹⁰² The purpose pursued by Samuelson (1962), who introduced this case, are analysed in ch.4 below.

¹⁰³ On the issue of the number of switch-points *cf.* Bharadawaj (1970).

¹⁰⁴ In the main text it has been illustrated ‘reswitching’ undermining the “direct” factor substitution mechanism, which is underlying group (ii) of the data. As to the “indirect” mechanism, consider two goods (corn and cloth) and decide, whichever the criterion adopted, which consumption-good is more ‘capital’ intensive. Then, whichever the more ‘capital’ intensive consumption-good, there will be one or more ranges of values of the interest rate, where an *increment* of r makes the more ‘capital’ intensive good relatively cheaper, implying a shift of the composition of consumption in the direction of an *increase*, rather than a *decrease*, of the average ‘capital’-intensity of the demand for consumption goods. *Cf.* Appendix D where it is also illustrated “perverse” behaviour when there is *no* ‘reswitching’.

¹⁰⁵ In the following chapter we shall survey the views of critics and defenders of the theory at stake on these results. In the controversies many examples have been built (*cf.* Symposium 1966: Pasinetti (1966a), Garegnani (1966), and see also Levhari (1965)) in order to generate concave or convex, or concave and convex curves if each system involves one or more capital goods in such a way as to give rise to more than one intersect between different wage curves. We shall also see in that chapter, that Samuelson’s (1962) attempt at rescuing neoclassical theory implies the fact that each wage curve intersects only once each other.

worker¹⁰⁶ employed in the economy. However, some might argue that the substitution mechanisms work “in the right way” when capital is measured in value terms, thus raising the claim that the rate of interest might bring investment (capital demand) into equality with full-employment savings (capital supply).¹⁰⁷ However, in the light of ‘reverse capital deepening’ and ‘reswitching’ there seem to be no guarantee to derive a downward-sloping demand curve (or employment curve for capital). To show that the demand for capital might be *directly* related to r , suffice it to use the graphical device below (figure 3.3) which corresponds to values of figure 3.2.

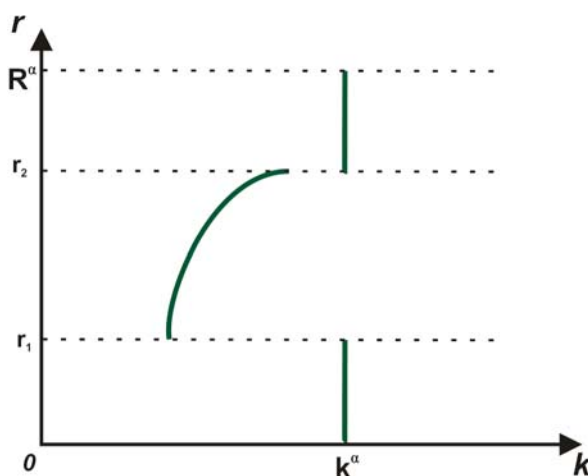


Figure 3.3. ‘Reverse capital deepening’: The value of capital, k , is not inversely related to r .

As can be seen from figure 3.3 above, the “employment curve” for capital (a basis to derive the demand function for capital) is no longer inversely related to the rate of interest.^{108,109}

¹⁰⁶ Measured in terms of the consumption good of the integrated industry.

¹⁰⁷ Cf. above par.2.10.

¹⁰⁸ For $0 \leq r \leq r_1$ there is a vertical curve indicating a constant value of capital due to the straight-line wage curve (α is in use, cf. figure 3.2) which corresponds to these levels of the rate of interest. Then, when $r=r_1$, along the wage frontier there is a switch point; hence both systems can co-exist (adjacent systems) and the value of capital per worker assumes different levels at that rate. Then, as r increases from r_1 up to r_2 , system β is in use and the value of capital per worker gradually increases because the value of each successive tangent along the β wage curve increases as r increases until r_2 . At $r=r_2$, there is ‘reswitching’ of techniques. At that point both techniques can co-exist, and that is why the value of capital assumes different levels at that rate. Then, as r rises from r_2 up to R^α the value of the capital is constant, since the α wage curve associated to that range of values of r is a straight-line – hence α is being used *again*, hence ‘reswitching’.

¹⁰⁹ Some (e.g. Solow, 1967, p. 30) have argued that the form of the demand curve with ‘reverse capital deepening’ and the discontinuities in the values of k is due to *discrete* technologies; so if the number of techniques becomes larger and larger, switch from a technique to another would
(cont.)

So far it has been seen that even if the illegitimacy of the supply aspect of the problem of capital (the necessity to conceive of capital in value terms to explain *long period supply-and-demand equilibrium*, expressed, in the capital market, through the equilibrium condition of the uniformity of the rates of return on the capital goods' supply prices) is put aside, 'reswitching' and 'reverse capital deepening' undermine the demand aspect so that it turns out at least doubtful to believe in an inverse relationship between the rate of remuneration and the quantity of the factor employed. But, if we are reminded the key role played by the downward-sloping demand curves so as to provide a plausible basis for the theory to postulate a stable and unique long period equilibrium then it would not be so difficult to envisage the scope of the implications of the critique based on 'reswitching' and 'reverse capital deepening'.¹¹⁰

The results described in this chapter cast doubt on the issue concerning how to explain the distribution of the social product between wage-earners and profit-earners. But these results do not arise out from outside the neoclassical assumptions. On the contrary, the critical side in the arguments between Cambridge has showed, by using analysis of production theory (minimisation of costs), that distribution might be explained by forces other than the supply and demand forces – the explanation of which we have seen in ch.3 above. *On the grounds* of supply and demand approach the demand curves could be of any form, not necessarily negatively sloping, in economies with heterogeneous capital goods. As we have also seen in ch.3 above, supply and demand forces were conceived by the founders of the marginal theory to operate in markets under free competition by relying on the belief of the factors' substitution principle. We thus conclude this first part of this study with the following question: Could then we expect from this theory predictions and forecasts on real

imply an infinitesimal change in the production coefficients ($l_{a\alpha}$, $c_{a\alpha}$, $l_{a\beta}$, etc.) so as to ensure such a continuous relation between k and r (and q) along the technical frontier (the wage frontier) that the neoclassical relations could be sustained. However, it has been demonstrated (Bellino, 1993) that the same wage frontier – which entail “enveloping” wage curves and would have ensured changes in the coefficients so as to keep neoclassical relations among variables – could be also generated by *other* wage curves, without however implying changes in those coefficients, and which not necessarily entail a neoclassical relation between k , r and q . Cf. also Pasinetti (1969) p. 523 and Garegnani (1970a) pp. 412-4.

¹¹⁰ The author would like to highlight that the negative implications of these results for the theory have nothing to do with the utilisation of any “aggregate” production function, as Joan Robinson has continuously insisted. See ch.5 below where Joan Robinson's role in the Cambridge debates is surveyed. Cf. also par.4.6 below.

economies, if *e.g.* demand curves are not downwards-sloping? Can we rely on the factor substitution principle in order to determine the distribution of the social product between wages and interests?

As we shall see in the second part of this study, the theoretic possibilities of ‘reswitching’ and ‘reverse capital deepening’ have serious implications for the dominant theory, which one may appreciate as soon as it is realised why notable neoclassical scholars, such as Samuelson, entered the capital debates. As the following chapter will discuss, this author’s participation together with the replies and counteractions it gave rise constitute, from a historical standpoint, the heart of the Cambridge controversies in its first phase of its development.

PART II:

***THE CAPITAL THEORY DEBATES IN A HISTORICAL
PERSPECTIVE AND THE TRANSFORMATION OF DOMINANT
THEORY***

CHAPTER 4

A first phase in the controversies:

Samuelson's 1962 intervention, the 1966 Symposium, neoclassical reactions and the highest point of the debates

"No alchemist can turn one capital good into the other"

P. Samuelson (1962) p. 196.

4.1. Introduction.

In the previous chapter, it has been introduced one of the most relevant results of the controversies, namely: 'reswitching' of techniques and 'reverse capital deepening'. We have seen that these phenomena seriously concern the core foundations of neoclassical theory: it cannot in general be postulated a negative relation between demand for capital and the rate of interest. The aim of the previous chapter, it is fair to say it, has been to introduce the analytical ground on which the controversies evolved, so as to understand the arguments raised in the successive debates in the light of those results.

Although by the beginning of the 1960s the most salient arguments raised in the debate were not clear with respect to the traditional role ascribed to "capital" as a "factor of production" in deriving the equilibrium supply-and-demand functions for that "factor", serious doubts regarding its theoretical tenability had started to arise.¹¹¹ As we shall see, the way in which Robinson sparked the ensuing controversies in her

¹¹¹ Eventually the period previous to 1960 (*i.e.* Sraffa's publication date) may be characterised as a "pre-reswitching" period of the debate – although, of course, the phenomenon had already been noticed by Robinson (1953; 1956) and Champernowne (1953), but little importance was given to 'reswitching' by these and other authors at that time. See ch.5 where we treat Robinson's role in the controversies as well as Champernowne's contribution.

1953 article much characterised the debates that immediately followed it.¹¹² In particular the first critiques in the controversies were mainly concerned about the use of aggregate production function as an instrument to determine distribution by the partial derivatives of that construct.¹¹³ Yet it had not been discussed in a clearer ground the consequence for the marginal theory of distribution the fact that, as Sraffa demonstrates in the 1960 book, it is impossible to talk about ‘more’ or ‘less’ capital-intensive production processes independently of distribution. Indeed, the marginal theory relies on the factor substitution principle, a theoretical principle that should indicate the cheaper or the dearer production processes of net output according to ‘relative factor’s intensities’ as distribution changes. But, due to the phenomena examined above (ch.3), it turns out to be impossible to conceive, independently of distribution, which production process is more or less ‘capital’ intensive among the alternative productive processes, because the very classification of techniques according to “factor intensities” changes as the distribution of income between wages and profits changes. This problem is totally independent of the aggregate production function and shows that it cannot in general be postulated a negative relation between ‘capital’ and the rate of interest, thus casting serious concerns on the determination of a ‘quantity of capital’, which is essential to the traditional theory. That is why the present author has deemed as more appropriate to deal with the first phase in the capital debates by discussing the contributions which appear in the 1960s decade, rather than having adopted a chronological order in the discussions. (We shall turn back to Robinson’s criticisms and her contestants of the 1950s in ch.5).

The aim of this chapter is to put at the centre the salient interchanges that surround the phenomena of ‘reswitching’ and ‘reverse capital deepening’ since Samuelson’s intervention in the controversies in 1962.¹¹⁴ In the wake of Samuelson’s article the debates followed an interesting path both from a historical as well as from an analytical viewpoint. As to the first aspect, the article appeared soon after Sraffa’s book; as to the second, Samuelson laid the ground for the discussion by presenting systems of production with heterogeneous capital goods – though from a particular

¹¹² Robinson (1953).

¹¹³ Champernowne (1953) tried to save the production functions from being a multi-valued relation due to value measurement of capital by the ‘chain index’ measurement of capital, a measurement which, however, is also dependent on distribution. See ch.5, in particular par.5.6 below.

¹¹⁴ Samuelson (1962).

viewpoint as we shall see below. Also interestingly is to note that Samuelson's article triggered off a critique by Garegnani, who had criticised the former in an oral intervention around 1961-2 in MIT where a manuscript of Samuelson's article circulated. It was there that Garegnani developed his criticisms, which finally reached readers somewhat eight or nine years later in (1970a).¹¹⁵ (par. 4.2-4.6).

Also, this chapter deals with the reactions of neoclassical economists to the criticisms raised in the 1966 *Quarterly Journal of Economics* Symposium on paradoxes in capital theory, which came into being thanks to the intervention of one of Samuelson's *go-between* at that time namely – Levhari, who in 1965 pretended to have showed that 'reswitching', though possible at the level of single industries, cannot arise for the economy as a whole. (par. 4.7-4.10) This was contested in such a detail and accuracy by the critical side that 'reswitching' turned out to be issue for the discipline. Some successive reactions (Samuelson, Ferguson, Bliss) after the Symposium and Garegnani's 1970 critique are analysed (par.4.11-4.15).

4.2. Samuelson's (1962) model with heterogeneous capital goods.

Although Samuelson's contribution is a direct response to Robinson's concerns about the theoretical tenability of the production function –in fact Samuelson dedicates his paper to Joan Robinson's "memorable 1961 visit to MIT"¹¹⁶ – it is important to emphasise the fact that in Samuelson's contribution one can find the discussion of the relation between distribution and demand for factors in heterogeneous capital goods economies on a ground which is clearer than the ground on which Robinson (and her immediate contestants) had put it.¹¹⁷ Indeed that ground was the one on which Sraffa had already laid the argument of choice of techniques in his 1960 book, where the important results of 'reswitching' and 'reverse capital deepening' emerged as a result of the analysis of the relation between prices (choice of techniques) and distribution. (See appendix C below.)

That we have decided to deal with Samuelson (1962) in this chapter (which is the first of the second part of the study) is vindicated by the fact that in that article it is

¹¹⁵ See n.134 below.

¹¹⁶ P. Samuelson (1962) p. 192, n.1.

possible to find one of the most serious attempts from the neoclassical side to deal with heterogeneous capital goods without any recourse to a production function and that in Samuelson's surrogate note there is the subtle message that heterogeneity of capital goods could *not* bring headaches to the traditional theory. A similar view can be found in King (2002) who writes:

“Ironically the first to expose the depth of the Sraffian critique were defenders of the neoclassical theory. (...) [It was] Samuelson's attempts to solve [Sraffa's results on *reswitching* and reverse capital deepening which were pointed out by former PhD student in Cambridge, P. Garegnani] that initiated the Cambridge Controversies.”¹¹⁸

Samuelson starts his note by pointing out that capital theory can be rigorously developed *without* using any “Clark-like” concept of aggregate capital¹¹⁹. The attempt pursued by Samuelson is to show that, in order to determine distribution, economists can *use* a “new concept”, the “surrogate production function”, which is *supposed to consider* the heterogeneous capital goods involved in the different techniques of production. On the other hand he claims that by using the surrogate production function and the associated “surrogate capital” one can predict *exactly* how “complicated heterogeneous capital models will behave”.¹²⁰

Thus, Samuelson begins with the introduction of a heterogeneous-capital-good model:

“I begin with a concrete model in which there are a great variety of capital goods: call them: alpha, beta, ..., 999 or anything else.”¹²¹

¹¹⁷ For the way in which discussions evolved over the first stages of the debates see ch.5 below. See ch.3 above, where we have introduced the ground for the forthcoming discussions in these debates. See also n.134 below.

¹¹⁸ King (2002) p. 95.

¹¹⁹ “Clark-like” concept of “capital” refers to the idea that the production of net output of an economy can be represented by a “production function” whose argument contains homogeneous labour and “capital” (in terms of output) such that relative rates of remuneration can be determined by the partial derivatives of such a function with respect to each factor. Concepts like “Clark-Ramsey parable”, “Clark parable” and the like refer to the same idea.

¹²⁰ Samuelson (*op. cit.*) p. 194.

¹²¹ *Ibid.*

and in the same page some rows below he explains (emphasis in the original):

“One need never speak of *the* production function, but rather should speak of a great number of separate production functions.”

In Samuelson’s model there are concrete different capital goods that can produce an output composed of a consumption-good and of a capital-good in fixed proportions with homogeneous labour. Samuelson’s model is, apparently, quite similar to that present in Sraffa (1960) though there are quite important differences which are worth mentioning: In Samuelson’s model there is a fixed coefficient production process for each type of output. Both production processes comprise the system of production of the economy where constant returns to scale and perfect competition prevail. Samuelson generalises the fixed-coefficient model for any system of production, so that the consumption good can be produced with homogeneous labour and a particular capital good associated with each system of production, while each particular capital good is produced by itself and labour. Then he assumes that each capital good does not enter the production of any other one capital good except for the production of itself.¹²²

As is well known, different production systems will in general imply different wage-curves or “Factor-price frontiers” as Samuelson have chosen. Different wage-curves may or may not intersect, depending crucially upon the *relative* production coefficients. Now we must discuss a crucial assumption in Samuelson’s model.

4.3. Samuelson’s ‘special sub-class of realistic cases’.

To be coherent with the hypothesis of production with strictly heterogeneous physical capital goods, Samuelson puts forth a “special sub-class of realistic case”¹²³ in which every capital good is different according to the different systems of production, and enters the production process in fixed proportions with labour. In this

¹²² Samuelson’s model is similar to that discussed in ch.3 with the difference that we have considered only circulating capital whereas Samuelson considers fixed capital, though his treatment of fixed capital evades the problem of joint production. See Garegnani (1970a) p.408, n.1.

¹²³ *Id.* p. 196.

regard Samuelson makes a highly restrictive *assumption*, namely, that the proportion of labour to means of production for the system of production concerned must be the *same* for all goods produced, *i.e.* for the final consumption good and for the capital good ($l_d/c_d=l_c/c_c$).¹²⁴

No doubt, then, that the “factor-price frontier” built for a single capital-good technique where the capital-labour ratio is constant as distribution changes must result in a straight line. The straight line connects two points, namely, the maximum wage rate for the process at zero profit rate and the ratio of technical surplus over replacement to means of production when the wage rate is zero. As the labour-means of production ratio is assumed to be equal in both sectors of the economy, there will only be *one* possible capital-labour ratio in the economy as a whole. Then the value of capital is constant independently of changes in distribution. Samuelson finally extended his restrictive assumption to all systems of production in the economy (“book of blueprints”).¹²⁵ Then, as already explained, the points on the “north-east frontier” envelope, *i.e.* the *wage-frontier*, are of equilibrium. Because of the particular assumption made by Samuelson this envelope – for a finite number of techniques – has straight parts (when one technique is the one most profitable) as well as corner

¹²⁴ *Id.* pp. 196-7. *Cf.* ch.3, above. The case of the straight-line wage curve is $D=0$. It must be noticed that not only is Samuelson’s model quite arbitrary because of the *equality* of the capital-labour ratio in each industry, but also because he assumes that every activity needs just only one particular capital good – a very unrealistic case. The fact that in Samuelson we have switch points in his very special cases is due to the existence of only one good common among techniques, also a very unrealistic case; on this issue see Baldone (1984).

¹²⁵ Thus one can plot in the wage-profit rate layout the different straight-line “factor price frontiers” according to each system of production so that, if a range of production systems (each involving only one capital-good) is considered, each with either a higher proportion of “capital” to labour, the “envelope” (Samuelson, *op. cit.*, p. 198.) of the outer portions of the straight-lines will depict the most profitable techniques at any profit rate. (See figure 4.1.)

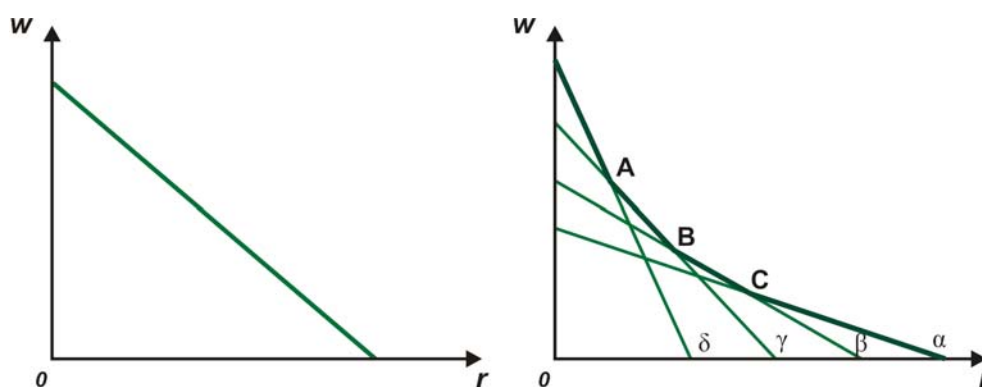


Figure 4.1: A straight wage-curve and a family of straight wage-curves

points (“Switch points”, “adjacent techniques”) where two techniques can coexist. (Cf. points A, B, or C of the right-side graph in *figure 1*) The switch points on Samuelson’s envelope have the basic neoclassical property of being all “forward switches”, *i.e.* as the profit rate rises the capital-labour ratio falls.

The successive step that Samuelson took was to associate the “Clark-Ramsey parable” with his “more realistic” heterogeneous capital-good model. Samuelson, after having drawn the envelope corresponding to the book of blue prints (see *figure 1.b* above), states that the “envelope” generated by straight-line wage curves is “generally similar” to the function relating the wage rate and the profit rate *derived* from the “Clark-Ramsey parable”.¹²⁶ Samuelson then claims that

“if we *invent* the right fairy tale, we can come as close as we like to duplicating the true blue-print reality in all its complexity.”¹²⁷

4.4. The neoclassical ‘parable’: ‘Surrogate’ capital and the ‘surrogate’ production function.

Did Paul Samuelson succeed in inventing the “right fairy tale”? Samuelson believed in having found a rationalisation of the “Clark parable” because the envelope generated by the latter (see *figure 4.2.c*) is generally similar to the envelope derived from his supposedly “real” heterogeneous capital goods model. (Cf. *figure 4.1* above.) Thus, he pretended to have supplied a demonstration that the formal properties of the “parable system” are the same as those derived from his “quasi-realistic *complete* system of heterogeneous capital goods”. That is why Samuelson believes that his “new concept”¹²⁸ of “Surrogate” production function, with its associated “Surrogate” capital¹²⁹, which replicates the “parable”, could be as a useful concept to analyse heterogeneous capital models.

¹²⁶ Samuelson (1962) p. 201.

¹²⁷ *Ibid.*, emphasis added.

¹²⁸ *Ibid.*

¹²⁹ Samuelson (*Id.*, p. 200) assumed that labour and homogeneous physical capital (jelly) produce a flow of homogeneous national product, which consists of consumption goods or of net capital formation “the two being infinitely substitutable on a one-for-one basis”

This similarity between the formal properties of both constructs – that is an “imaginary” surrogate function (or parable)¹³⁰ and the “real” economy of straight wage-curves – quickened Samuelson to state:

¹³⁰ *Id.* p. 200. The “Clark parable” entails using a production function, which represents the whole techniques of production of the net output that is produced with labour and “capital” homogeneous with the output. Samuelson (*id.* p. 201) thinks that “K” is surrogate capital “that is to go into the Surrogate Production Function that will predict all behaviour.”

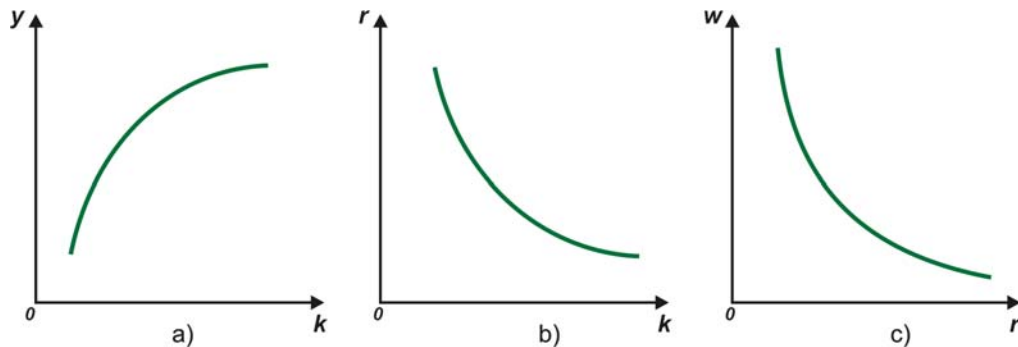


Figure 4.2: The “parable system”

Samuelson pretended to analyse the relations between the wage rate, the profit rate and the net output per worker that prevail in the “real” economy by analysing the properties of the surrogate production function. In formal terms we have:

$$Y = f(K, L) \quad (4.1)$$

because of constant returns we may re-write the function as follows:

$$y = f(k) \quad (4.1')$$

where y is net output per worker and k is capital per worker.

Then, as Samuelson assumed, from this information the family of the wage-curves can be derived, once wages and profits are determined by the marginal productivities of each factor respectively, so that:

$$w = f(k) - kf'(k) \quad (4.2)$$

$$r = f'(k) \quad (4.3)$$

If we derive both of these equations with respect to k we obtain:

$$\frac{dw}{dk} = -kf''(k) \quad (4.4)$$

and

$$\frac{dr}{dk} = f''(k) \quad (4.5)$$

If we divide both sides of (4.4) and (4.5) we finally get:

$$\frac{\frac{dw}{dk}}{\frac{dr}{dk}} = \frac{dw}{dr} = -k \quad (4.6)$$

(cont.)

“[T]he Surrogate Homogeneous Capital gives the same result as does the shifting collection of diverse capital goods in our more realistic model.”¹³¹

(Again, it is worth looking at *figure 4.2* above.)

So, apparently, Samuelson would have dealt with heterogeneous capital goods economies and then would have showed that, by studying the formal properties of the surrogate production function, one could analyse the relations between w , r , and q (*i.e.* the net physical consumption output of the “real” economy).¹³² Even more, these apparently similar formal properties of the “parable system” and of the “real” model seem to have made Samuelson state that

“[the surrogate function] is not an approximation but a rigorous equivalence. This justifies the Surrogate Production Function as a useful summarising device.”¹³³

4.5. *What did Samuelson actually demonstrate?*

Before answering that question, it is interesting to note to what extent Samuelson parable had been accepted by the discipline at that time. In this regard it is worth citing the following views from a well-known protagonist of the controversies, Joan Robinson:

“At first the neo-neoclassicals were happy to accept this parable [the surrogate production function] (...) For some years they remained cooped up in this position.”¹³⁴

which is the slope of the envelope (family of wage-curves) and represents the capital-labour ratio of the economy. (*Cf. Id.* p. 202 where Samuelson develops the surrogate function.)

¹³¹ *Id.* p. 201.

¹³² In fact Samuelson derived from equations (4.4) and (4.5) a functional relation between w and r , which would be the wage-frontier (envelope) representing the whole competitive techniques. Then, as output and surrogate capital are homogeneous, Samuelson concluded that the envelope is a *convex curve* and therefore could be compared with the wage-curves derived from Samuelson’s “quasi-realistic” cases where, in actual fact, every single wage-curve is a straight line. As we well know, if we consider an infinite number of straight wage-curves, then the “envelope” will be a smooth convex envelope (see *figure 4.1* and *cf. figure 4.2.c*).

¹³³ *Id.* p. 203.

Five years later Robinson provides further information on the historical aspects surrounding Samuelson's participation in the controversies:

“For several years, everyone (except Piero Garegnani) was somewhat baffled by the surrogate production function.”¹³⁵

Joan Robinson's appraisals indicate that among the critical contributions to the Cambridge controversies one may find in Garegnani's (1970a)¹³⁶ article a critique of Samuelson's construct. Let us see why.

As we saw (par.4.3-4.4), Samuelson's analysis assumes the existence of a “real” economy, one feature of which being an “envelope” function relating the wage rate and the profit rate, derived from the alternative systems of production. Then he introduced the “surrogate” production function¹³⁷, and showed that the results derived from this “new” device are the same as those obtained from the “realistic” model. Is that true?

Following Garegnani, to come to grips with the particular defence of traditional theory attempted by Samuelson with his Surrogate function we need set out a more familiar scheme of traditional theory: it will be necessary to assume that each system

¹³⁴ Robinson (1970a) p. 311. Robinson is thinking of Ferguson (1969) when she asserts that the neo-neoclassicals “were happy” with the surrogate function. See par.4.12 below, where we deal with Ferguson (1969).

¹³⁵ Robinson (1975a) p. 37.

¹³⁶ Garegnani (1970a). The article finally published in 1970 is an extension of a formerly accepted paper submitted to the editors of *The Review of Economic Studies* in April 1963, which had been accepted subject to revision. Garegnani was a visiting Rockefeller Fellow at MIT in 1961-2, where he started writing the manuscript finally published in 1970, that is to say, in the period immediately previous to the publication of Samuelson's “Surrogate” article (1962). In the discussions with Samuelson, Garegnani already pointed out the extreme assumptions on which the surrogate function is based. Though Samuelson acknowledges Garegnani for having saved him “from asserting the false conjecture that my extreme assumption of equi-proportional inputs could be lightened and still leave one with many of the Surrogate propositions” (Samuelson, *op. cit.*, p. 202, n.1), yet that did not prevent Samuelson from asserting that the Surrogate function is a “rigorous equivalence” (*Id.* p. 203) of the “real” heterogeneous capital goods model. The issue under dispute has been precisely how far that “real” model actually represented production with heterogeneous capital goods. This issue was finally settled in Garegnani's 1970 article. Thus Garegnani's (1970a) became a very important reference, from the critically oriented side of the controversies, in the history of the contemporary capital debates, as is recognised *e.g.* by Joan Robinson. On the influence of Garegnani's criticisms of Samuelson *cf.* Bahduri (1969) p. 536 and n.172 below.

¹³⁷ The “as if” aggregate production function, Samuelson (*op. cit.*) p. 194, n.1.

of production can change continuously with r . By this it is meant *i*) that any change in r , however small, causes a change of system, and *ii*) that one system only is in use at each possible level of r . Accordingly, the family of systems¹³⁸ does not show any corner-point, and the wage-curve of each system contributes only points to the wage-frontier and not stretches. Thus, it is guaranteed that this wage-frontier becomes a smooth envelope¹³⁹ enclosing the whole family of the wage-curves from above, determining a continuous function between the wage rate and the rate of interest, that is

$$w = e(r) \tag{4.7}$$

As a matter of fact, we perfectly know that the higher the rate of interest the lower the wage rate ($\frac{de(r)}{dr} < 0$), but its *curvature* can be of any shape.

In general this function *envelops* several wage-curves of different curvatures, and may therefore alternate convex with concave segments. For example, if all the wage-curves are straight lines or convex ones the envelope will be convex, whereas the latter will be concave or straight when the wage-curves are concave.¹⁴⁰

As it was assumed that at any possible value of r a single system is in use, a given physical net output and physical capital per worker corresponding to each system can therefore be found for any given r . This net product per worker, q , is therefore function of r :

¹³⁸ The assumptions underlying the present discussion are those present in par.3.2 above.
¹³⁹ This smooth envelope entails avoiding switch points.
¹⁴⁰

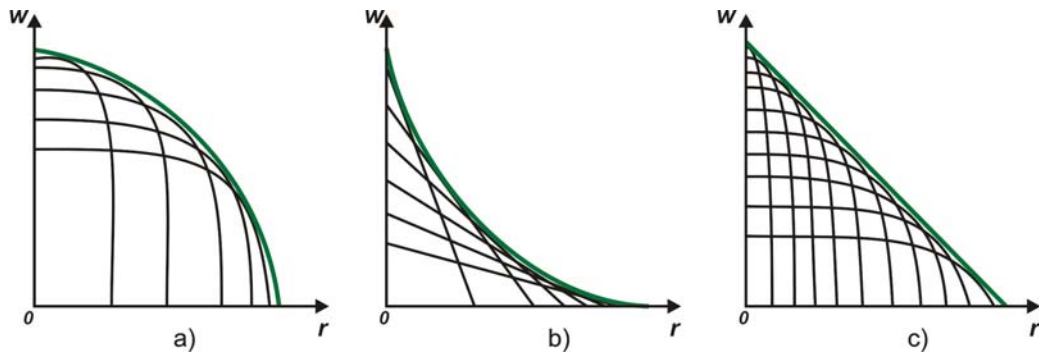


Figure 4.3: Concave, convex and straight “envelopes”

$$q=q(r) \tag{4.8}$$

It is important to notice that equations (4.7) and (4.8) reported above belong to the heterogeneous capital goods economy (the actual “real” economy), and must have, as Samuelson hypothesises, a parallel with equations (4.2) and (4.3) above, which belong to the imaginary economy (the “Clark parable”). It is a confrontation of the meanings associated with both groups of equations that will actually allow us to see how “rigorously equivalent”, as Samuelson so decidedly claimed, the Clark system and the heterogeneous capital-goods model are. Since according to Samuelson both the “Clark system” and his heterogeneous capital-goods model are “rigorously equivalent”, then heterogeneous capital-goods might be expressed as quantities of homogeneous capital. The core of Samuelson’s attempt was to apparently determine the relations (in the “real” economy) between r , w , and q through a production function, the imaginary parable.

In analytical terms, we are asked to find a production function homogeneous of the first degree:

$$S=S(J,L) \tag{4.9}$$

with S as the quantity of net product, J that of capital, and L that of labour. This is the “surrogate function”, and must satisfy the following *two conditions*:

$$i) \frac{\partial S}{\partial L} = e\left(\frac{\partial S}{\partial J}\right) \tag{4.10}$$

The marginal product of labour $\left(\frac{\partial S}{\partial L}\right)$ must be equal to w , which must be equal to the ‘envelope’ function $e\left(r = \frac{\partial S}{\partial J}\right)$ of the “real” economy at the given r corresponding to that equilibrium.

$$ii) S/L = q\left(r = \frac{\partial S}{\partial J}\right) \tag{4.11}$$

The net product per worker, defined for the “parable”, must be equal to the function q for the real economy defined above.

It is useful to re-write equation (4.9), by using Euler's theorem:

$$\frac{S}{L} = \frac{\delta S}{\delta L} + \left(\frac{\delta S}{\delta J}\right) \frac{J}{L} \quad (4.12)$$

Since in the imaginary economy the equilibrium profit (interest) rate must equal the 'marginal productivity of capital', then, by using condition *i*) noted above, equation (4.12) can be written as follows:

$$\frac{S}{L} = e(r) + r \frac{J}{L} \quad (4.13)$$

So far we have obtained functions expressing the net output per worker in the imaginary economy. Now let us turn to the capital-labour ratio of that imaginary economy. If we differentiate function (4.12) with respect to $\frac{\delta S}{\delta J}$ we obtain¹⁴¹:

$$\frac{J}{L} = - \frac{d\left(\frac{\delta S}{\delta L}\right)}{d\left(\frac{\delta S}{\delta J}\right)} \quad (4.14)$$

But expression (14), by using again condition *i*), is:

$$J/L = - e'(r) \quad (4.15)$$

¹⁴¹ If we derive (4.12) with respect to $\frac{\delta S}{\delta J}$ we immediately obtain

$$\frac{d\left(\frac{S}{L}\right)}{d\left(\frac{\delta S}{\delta J}\right)} = \frac{d\left(\frac{\delta S}{\delta L}\right)}{d\left(\frac{\delta S}{\delta J}\right)} + \frac{J}{L} + \frac{d\left(\frac{J}{L}\right)}{d\left(\frac{\delta S}{\delta J}\right)} \frac{\delta S}{\delta J};$$

by multiplying and dividing the LHS of this equation by $d\left(\frac{J}{L}\right)$

$$\text{then } \frac{\delta S}{\delta J} \frac{d\left(\frac{J}{L}\right)}{d\left(\frac{\delta S}{\delta J}\right)} = \frac{d\left(\frac{\delta S}{\delta L}\right)}{d\left(\frac{\delta S}{\delta J}\right)} + \frac{J}{L} + \frac{d\left(\frac{J}{L}\right)}{d\left(\frac{\delta S}{\delta J}\right)} \frac{\delta S}{\delta J};$$

from which $0 = \frac{d\left(\frac{\delta S}{\delta L}\right)}{d\left(\frac{\delta S}{\delta J}\right)} + \frac{J}{L}$ that is equation (4.14).

Function (4.15) establishes that the slope of the “envelope” curve measures the capital-labour ratio of the economy.¹⁴² But now let us compare this result with the information we can trace from our real economy thanks to knowledge of the curvature of the envelope.

In effect we know that in any real economy the envelope may be concave to the origin in parts or throughout if the wage-curves are concave. Therefore, J/L would rise with $r = \frac{\partial S}{\partial J}$ as long as the envelope is concave, but this implies that the marginal product of “capital” rises as J/L (the ratio of ‘capital’ to labour) rises¹⁴³ and, therefore, “the function S could not be a production function”¹⁴⁴ because equilibrium in the “imaginary” economy requires an inverse relation between “marginal product of capital” and the capital-labour ratio.

Furthermore, should we consider some straight-line part of the ‘envelope’ –which is theoretically admissible when we have concave wage-curves– the contrast with the “Clark-Ramsey parable” would be even more striking, for the “marginal products” would change when the ratio of ‘capital’ to labour *does not*. This would also imply that condition *i*) above cannot be satisfied with any differentiable homogeneous function as it is impossible to talk about marginal changes when factor’s proportions do not vary (they cannot vary by definition of a straight line envelope). The foregoing considerations are sufficient to dismiss the existence of such a surrogate production function, but it is still of some interest –as Garegnani points out– to consider the case of convex envelopes.

When the “envelope” is a convex curve we have two alternatives, either the wage-curves are convex or they are straight lines. But S/L will equal $q=q(r)$ of the real economy only when the wage curve is a straight line but *not* when the wage curve is a convex one.¹⁴⁵ So the only one case in which function (4.16) correctly estimates q

¹⁴² Cf. equation (4.6) above

¹⁴³ In concave parts of the envelope we have $e''(r) < 0$, and accordingly we would have that J/L increases when r rises. In fact we must rule out concave wage-curves because they imply that, even when there is only one system of production from which a concave wage-curve is derived, the value of capital increases as the profit rate increases, a behaviour that goes in contradiction with the marginalist premises establishing an inverse relationship between demand for “capital” (investment) and the interest rate.

¹⁴⁴ Garegnani (1970a) p. 415.

¹⁴⁵ To see that a convexity relation between the wage rate and the profit rate does *not* ensure the existence of the surrogate function, it is necessary to re-write equation (4.12), which, combined with result (4.15), becomes:

(cont.)

while function (4.9) satisfies conditions *i*) and *ii*) is when the wage-curves are all straight lines, and only in this particular case does a surrogate function exist.¹⁴⁶

4.6. There is no heterogeneous capital goods model in Samuelson’s (1962).

From our knowledge of the systems of production involving heterogeneous capital goods a straight wage-curve actually means that the proportion of capital goods to labour is the *same* both in the consumption industry and in the capital good industry. It is in this particular case that the “marginal product of capital” (the partial derivative of the production function with respect to “capital”) would measure the exact level of the rate of profit. No doubt, now, that Samuelson has pretended to generalise the “parable” to apparently “real” heterogeneous capital goods models by precisely turning some particular heterogeneous models into the parable system.

In this regard Garegnani clears the ground:

“[B]ut for the arbitrary choice of the capital-good unit, the input coefficients of the two industries are identical.”¹⁴⁷

$$\frac{S}{L} = e(r) + r[-e'(r)] \quad (4.16)$$

In fact, function (4.16) underestimates q as can be seen from the following graph. Function (4.16) will *not* equal q when the wage curve is convex either. In this case that function will over-estimate the real value of q .

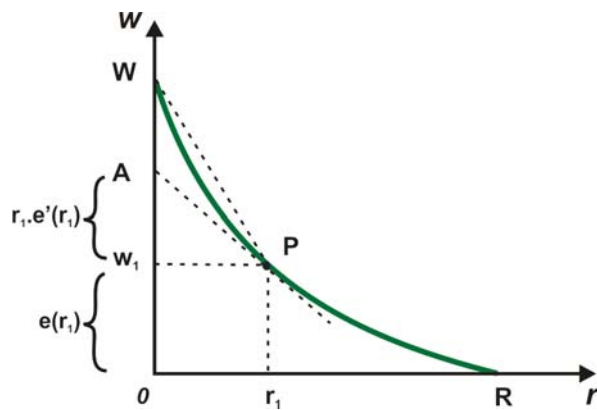


Figure 4.4: q is not equal to S/L when the wage-curve is convex

¹⁴⁶ It is important to note that Samuelson’s way of measuring the relative shares of labour and capital through the “elasticity” of the envelope is incorrect for general cases (i.e. for convex and concave curves). For example, consider the convex wage-curve of figure 4.4: the elasticity on point P is $0w_1/w_1A$, whereas the relative share actually is $0w_1/w_1W$.

¹⁴⁷ Garegnani (*op. cit.*) p. 415.

Since heterogeneity of commodities can be defined only as differences in the production conditions (that is different systems of productions involving different capital goods), a straight wage-curve does actually mean that the *consumption good is produced by itself and labour*. So Samuelson's system (*i.e.* a straight-line wage-curve representing the production methods of the consumption good and of the capital good) becomes *indistinguishable* from one where the consumption good is produced by itself and labour. Samuelson's quasi-real model in which input coefficients are equal in both sectors is not more real than the jelly world he believes in. This jelly world is the only one world in which an aggregate production function or surrogate production function can exist.

Garegnani also noted¹⁴⁸ that the aggregate production function is arrant nonsense, because if there is only one consumption commodity in the economy, there is no need to "aggregate", since there is no more than a single commodity in that economy and hence there is no aggregate production function. If, on the other hand, there are many consumption goods as well as capital goods, the aggregate production function does not exist either, because it can only be built if and only if the national net output is composed of only one commodity produced by itself and homogeneous labour. In a word, Samuelson's claim – establishing that to deal with heterogeneous capital goods economies there is no need to recourse to a "quantity of capital"¹⁴⁹ – is absolutely dependent upon the highly restrictive assumptions of equi-proportionality of inputs, as Garegnani has showed it.¹⁵⁰

In Samuelson's jelly world the demand aspect problem of capital (see ch.2 above) in the context of the traditional method of long period equilibrium or normal positions is totally saved, because at any given r , the ratio J/L and the 'slope' of the envelope is at the same time the 'slope' of the wage-curve tangent to the envelope at that r , so

¹⁴⁸ *Id.* pp. 421.

¹⁴⁹ Samuelson (*op. cit.*) p. 200, wrote: "An interesting point (...) is that even in our discrete-activity fixed-coefficient model of heterogeneous physical capital goods, the factor prices can still be given various long-run marginalism (*i.e.*, partial derivative) interpretations. *And all this without our ever having to pretend there is any quantitative aggregate homogenous 'capital' that itself truly produces anything*" (emphasis added).

¹⁵⁰ The following conclusion raised by Garegnani seems to go in accordance with what we have said above, with the help of Robinson's views, regarding whether and how far the parable system had been accepted by most of the practitioners of the discipline at that time: "Samuelson's
(*cont.*)

that the capital per labourer demanded is independent of changes in the profit rate. Moreover, in Samuelson's model the switch points would all be "forward" switches, *i.e.* neoclassical switches establishing a lower value of capital per labourer as the profit rate increases since each technique is single-valued in the capital-labour that it contains (the value of capital is always independent of changes in distribution).

Therefore, where is the "rigorous equivalence" between the "Clark system" and the quasi-real heterogeneous capital-good model that Samuelson would have demonstrated? The point is that, in our opinion, a "rigorous equivalence" must take into account the general cases (convex, linear, concave wage-curves) not only a very particular one: indeed both "worlds" turn out to be equivalent because Samuelson disguised the "Clark parable" into his "quasi-real" model – though, actually, linear and hence homogeneous. Samuelson would have demonstrated, therefore, that the "parable" provides the same properties of a heterogeneous capital-good model if and only if there had existed only one good produced by itself and labour but then there is no heterogeneous capital good economy at all. The "Samuelson parable" is not less unrealistic than Clark's. Thus in Samuelson's model the crucial problems of 'reswitching' and 'reverse capital deepening' *cannot occur by definition*: the consumption good is produced by itself and labour thus the systems of production under use (each using heterogeneous capital goods) are indistinguishable.

To conclude, it is true that in Samuelson's world "No alchemist can turn one capital good into the other" (1962, p. 196), not because there are physically heterogeneous capital goods, but, on the other hand, because in that world the alchemist, if existed, would not have any work to do at all, since there exists only one capital good homogeneous with the consumption good. The alchemist would therefore find that his *métier* in *that world* is no longer profitable, and will resign himself to pass his mysterious task on neoclassical economists. Samuelson's 1962 model actually avoided dealing with heterogeneous capital-goods economies.

It is logical to follow our visit to the controversies by looking at some neoclassical reactions and critical counteractions over the period 1965-1970.

'surrogate production function' is thus nothing more than the production function, whose existence in such an economy *no critic has ever doubted*." (Garegnani, *op. cit.*, p. 416, emphasis added)

4.7. *Levhari (1965) and the 1966 Quarterly Journal of Economics Symposium.*

In 1966 the MIT-based *Quarterly Journal of Economics* gathered together a series of contributions which conclusively demonstrated that ‘reswitching’ and ‘reverse capital deepening’ are phenomena which can in general exist either in economies with indecomposable or decomposable technology matrices.¹⁵¹ In terms of Sraffa, ‘reswitching’ is possible in economies where all commodities are basic (an indecomposable technology matrix) as well as in economies where not all the commodities are basic but for at least one. This story regarding the possibility of perverse behaviour with indecomposable or decomposable technologies is due to the claiming carried forward by Levhari, who, in 1965¹⁵², published an article in which he pretended to have proved that ‘reswitching’ is not possible at the economy level for indecomposable techniques. Levhari’s analysis is briefly analysed here below, while the important criticisms raised in the Symposium by Pasinetti (1966a) and Garegnani (1966) will be analysed later (par. 4.8-4.10).

Levhari’s purpose has been to demonstrate that

“though it is quite possible that we could use some technique of producing good *i* at a high rate of interest, then switch to another technique and, as we reduce it further, return to the old technique, this cannot happen with the whole matrix.”¹⁵³

His “whole matrix” refers to a technology wherein every commodity enters directly or indirectly in the production of all the commodities of the economy. In terms of Sraffa that means that all commodities are basic. The input-output matrix representing this type of economies is indecomposable. So while Levhari recognises the possibility of ‘reswitching’ for single industries, it is not possible in an economy as a whole where all commodities are basic. Levhari himself acknowledges this conjecture to Samuelson.¹⁵⁴ Because Levhari’s article gave rise to the Symposium, and because the issue of ‘reswitching’ became publicly acknowledged in this

¹⁵¹ Pasinetti (1966a), Bruno *et.al* (1966), Garegnani (1966), Morishima (1966), Samuelson (1966a), Levhari and Samuelson (1966).

¹⁵² Levhari (1965)

¹⁵³ *Id.* p. 103.

¹⁵⁴ *Ibid.* n. 5.

Symposium, in what follows the main argument carried out by Levhari will be very briefly reproduced.

Levhari assumes an economy where all capital is circulating and is in stationary equilibrium; all commodities are basic in the sense of Sraffa (so technological matrices are indecomposable), and prices are expressed in terms of labour commanded, that is $w=1$. Wage-curves represent different systems of production in terms of real-wages ($w/p=1/p$) and the rate of discount λ ($\lambda=1/1+r$). So, for a given rate of profit, the technique will be chosen according to which one provides the lowest price (analogous to the highest wage). To simplify matters assume that there are only two techniques, a and b which produce n basic commodities. Each technique differs from the other in at least one activity (method of production). Row-vectors a_0 and b_0 refer to the labour input associated with each technique respectively. As is well known, we can write the price equations as follow:

$$\begin{aligned} p_a &= a_0(\lambda I - a)^{-1} \\ p_b &= b_0(\lambda I - b)^{-1} \end{aligned}$$

The condition for a to be chosen is that $p_a \leq p_b$, so

$$a_0(\lambda I - a)^{-1} \leq b_0(\lambda I - b)^{-1}$$

then

$$a_0(\lambda I - a)^{-1} (\lambda I - b) \leq b_0$$

and, with some manipulation we can finally get:

$$a_0(\lambda I - a)^{-1} (a - b) \leq b_0 - a_0 \tag{4.L}$$

If techniques switch from a to b , then $p_b < p_a$ or what is the same $a_0(\lambda I - a)^{-1} (a - b) > b_0 - a_0$. If a is *not* to return, as speculated by Levhari, then this inequality must continue to hold. Levhari attempted to prove this by finding sufficient conditions such that $a_0(\lambda I - a)^{-1} (a - b)$ be a monotone non-decreasing function of

λ . This crucially depends on the sign of the difference between matrices a and b , *i.e.*, the sign of $(a - b)$.

As “all the elements of $a_0(\lambda I - a)^{-1}$ are monotonic decreasing functions of λ ”¹⁵⁵ the elements composing the difference $(a - b)$ must be negative or zero to ensure $a_0(\lambda I - a)^{-1}(a - b)$ to be a monotonic non-decreasing function.

Levhari then introduces a key condition for his whole argument:

“Now, for two positive indecomposable matrices, there exists a semi-positive vector x such that either $(a-b)x \geq 0$, $(a-b)x \leq 0$, or $(a-b)x = 0$.”¹⁵⁶

The economic meaning of the above condition is expressed by Levhari in the following way:

“there exists some activity level x such that we need more circular capital of all goods either with a or with b , or we are indifferent.”¹⁵⁷

The above mathematical condition would ensure, Levhari believes, that either technique a is in use and b is never to be chosen ($(a-b)x \geq 0$) or that the economy switched from technique a to technique b but a is never to return ($(a-b)x \leq 0$). Before discussing the validity of this conjecture, which has been precisely the issue of the Symposium, let us introduce the last part of Levhari’s argument.

Levhari’s next step was to multiply both sides of inequality (L) – the inequality showing that system a is at least as profitable as system b – by the semi-positive vector x^* :

$$a_0(\lambda I - a)^{-1} (a - b) x^* \leq (b_0 - a_0) x^* \quad (4.L')$$

¹⁵⁵ *Id.* p. 104. This Levhari has already demonstrated in his proof of the non-substitution theorem, an issue we are not going to deal with in this study. See Levhari (*op. cit.*) pp. 100-2.

¹⁵⁶ *Id.* pp. 104-5. In Levhari’s article these inequalities are written as strong inequalities; but we have here introduced them as reported by Pasinetti (1966a) p. 511, who had been told by Levhari that that had been a misprint.

¹⁵⁷ *Id.* p. 105.

The left side of the above inequality is called by Levhari function $\psi(\lambda)$. If we differentiate this function with respect to λ we will get:

$$\psi'(\lambda) = -a_0(\lambda I - a)^{-2}(a - b)x^*$$

Hence if $(a - b)x^* > 0$, $\psi(\lambda)$ is monotonic decreasing; whereas if $(a - b)x^* < 0$, $\psi(\lambda)$ is monotonic increasing.¹⁵⁸ Finally if $(a - b)x^* = 0$, $\psi(\lambda)$ is a constant, actually zero.

Levhari then argues that in a case where system a has been used at a given λ and *then* at a higher λ (lower r) the economy switches to system b , we need that $\psi(\lambda)$ be monotonic decreasing in λ so as a not to return, and hence $\psi'(\lambda)$ must be negative at the switch point.

So if Levhari's argument were true, 'reswitching' would, therefore, take place only for single goods production but not for an economy as a whole. Thus Levhari attempted to minimise the critical strength of 'reswitching' and, once more, to rescue received theory. Now we turn to see whether Levhari's argument is valid by visiting some key works gathered in the Symposium.

4.8. Pasinetti (1966a).

Levhari's –and Samuelson's– conjecture was firmly contested by some participants of the controversies in the already introduced Symposium.

The first participant in the capital theory debates that replied Levhari was Pasinetti, who, in 1965, had already presented a refutation of Levhari's lemma at the Rome Econometric Society meeting by providing a counter-example which rebutted the "non-reswitching" theorem. Pasinetti's contribution to the Symposium is an improved version of that paper, as Pasinetti himself recalls in his article.¹⁵⁹

¹⁵⁸ Every element of $-a_0(\lambda I - a)^{-2}$ is negative. In the article (p. 105) Levhari makes a mistake when he establishes that $\psi'(\lambda)$ is either monotonic increasing or decreasing according to whether $(a - b)x^* > 0$ or $(a - b)x^* < 0$. What is relevant, instead, is function $\psi(\lambda)$.

¹⁵⁹ Pasinetti (1966a), p. 503, n.(*). Note, however, that in Pasinetti's example the underlying technological matrices are not indecomposable. See *id.* p. 505.

The first part of the article is again devoted to his counter-example which invalidates Levhari's lemma.¹⁶⁰ Pasinetti concludes in the first part that, since the example which "satisfies all the requirements of Samuelson-Levhari analysis can yield results that contradict their conclusions", his illustration "is sufficient to disprove their theorem".¹⁶¹ Albeit this would have sufficed to show that Levhari's argument is totally unfounded, Pasinetti also provides a "direct logical criticism" of that construction.

In the discussion about Levhari's mathematical construct, we have seen that, for two given techniques that differ in only one activity, the elements of the matrix corresponding to the difference between both of them are all zeroes except for those located in one column, which will contain both positive and negative elements. "Thus", Pasinetti explains,

"the matrix elements which are responsible for making all prices change, when the switch takes place, are *all* contained in that nonzero column."¹⁶²

However in order to get Levhari's theorem right, it is necessary that the elements of the semi-positive vector x^* , which are multiplied by the non-zero column of $(a - b)$, be all zero. This is the one trivial case for which Levhari's argument would stand. But the implications of such a trivial case are quite strong.

As Pasinetti argues, if the column responsible for the switch –to which we have referred above– is multiplied by the zero elements of x^* , then

"this will simply eliminate the column. And if precisely that column is eliminated which contains all the differences between the two matrixes, it will become impossible to tell which of the two is more profitable. (...) Levhari's device would thus lead us to the straight equality $(a - b)x^* = 0$, and to the false conclusion that matrix a and b are indifferent for all levels of the rate of profit."¹⁶³

¹⁶⁰ *Id.* pp. 504-10.

¹⁶¹ *Id.* p. 510.

¹⁶² *Id.* p. 511.

¹⁶³ *Id.* p. 512.

Therefore the exercise of comparison between two series of prices after eliminating the components which actually make them differ is totally absurd. Pasinetti shows that Levhari's theorem is false.

4.9. Garegnani (1966).

Garegnani (1966) demonstrates that if technique a is more profitable than b then the strong inequality in (4.L) must hold at least for one sector. But, because of having multiplied inequality (4.L) by a semi-positive vector x^* , we may, therefore, have multiplied by zero one or more relations in (4.L') to which the strong inequality previously applied, hence getting $0=0$ in their place. Then "the sign = would apply to all the relations"¹⁶⁴ in (4.L'). The same reasoning can be applied when it is technique b at a higher λ that is the most profitable, and again the sign = would apply to all the relations in (4.L'). Then, assuming that $r' < r''$ and that at $r=r'$ a is the most profitable technique, but at $r=r''$ b is so, Garegnani concludes,

"[since] $\psi(r'') = \psi(r')$ and the fact that $\psi(r)$ is a monotonic function (in this case, a zero constant) does not exclude that producers may switch back to A as r rises beyond r'' ."¹⁶⁵

So the validity of Levhari's theorem requires "stricter conditions"¹⁶⁶, namely, that nonzero elements of vector x^* be "applied to *at least one* of those industries for which, at r' or r'' , the methods of systems A and B are not indifferent".¹⁶⁷

Despite the fact that these results would have sufficed to invalidate Levhari's conclusion, Garegnani in addition points out which Levhari's "main error" is. In fact Garegnani shows that Levhari's proposition, which establishes that there exists a semi-positive vector x such that either $(a-b)x \geq 0$, $(a-b)x \leq 0$, or $(a-b)x = 0$, cannot "be proved since it is *false*".¹⁶⁸ What Garegnani shows is that, for two given techniques (a and b) which produce, say, two commodities, and both techniques differing in two

¹⁶⁴ Garegnani (1966) p. 558.

¹⁶⁵ *Ibid.* Garegnani uses function $\psi(r)$ instead of $\psi(\lambda)$, as did Levhari. Remember that Levhari uses $\lambda = 1/1+r$.

¹⁶⁶ *Id.* p. 559.

¹⁶⁷ *Id.* p. 558-9, emphasis in the original.

methods (activities), then, after multiplying the matrix containing the elements of the differences of both techniques (matrices a and b) by a semi-positive vector x^* , the resulting vector $(a-b)x^*=y^*$ can exhibit elements such that one of them is negative and the other positive, thus contradicting Levhari's lemma *i.e.* $(a-b)x \geq 0$, $(a-b)x \leq 0$, or $(a-b)x=0$.¹⁶⁹ Levhari's argument is, therefore, false: 'reswitching' (with all its implications) can exist in economies where techniques are either indecomposable or decomposable. Samuelson's conjecture –as Levhari recalls in his article– is, once more, proved wrong.

4.10. The meaning of the 1966 Symposium: A turning point in the controversies.

Both Pasinetti and Garegnani end their respective contributions to the controversies by turning attention to the implications of 'reswitching' and 'reverse capital deepening' for the marginalist theory. Thus, they conclude, once it is showed that those phenomena can exist either in decomposable or indecomposable technologies, as well as in any single industry, it has to be recognised that there is no logical ground on which supporting the traditional premise of marginal theory, which establishes that there is a negative relationship between the rate of profit and the quantity of capital per labour.

Pasinetti (1966a, p. 514):

“The general conclusion is that, at any given state of technical knowledge, switches of techniques due to changes in the rate of profit do not allow us to make any general statement on changes in the ‘quantity of capital’ per unit of labour. The new technology may require a lower ‘quantity of capital’ per unit of labour, or it may require a higher ‘quantity of capital’ per unit of labour, whether ‘capital’ is measured in terms of value or in terms of any chosen physical unit, whether we consider any single industry or the economic system as a whole.”

¹⁶⁸ *Id.* p. 561, emphasis added.

¹⁶⁹ See *Ibid.* n 9; and Bruno *et. al.* (1966, p. 543), where these authors give Solow credit for having correctly “pointed out that one cannot in general find a semipositive vector x such that either $(a-b)x \geq 0$, $(a-b)x \leq 0$, or $(a-b)x=0$.”

Garegnani (1966, p. 564, emphasis added):

“In reality, once we correctly recognize how the value of capital goods depends on distribution, the very possibility of ordering the techniques according to the proportions of capital to labour slips from our hands; that order may change as prices and distribution change. And no way-out of this difficulty can be found by relating capital per man to some magnitude independent of distribution, which would then be taken as the measure of capital intensity: the ‘return’ of a technique shows that any such measure, *even if it could be found*, would lead to contradicting the principle of an inverse relationship between rate of interest and capital intensity.”

Both authors raised very strong concerns about the *admission* of these results in the hitherto literature.¹⁷⁰ On the one hand we have Pasinetti, who argues that “the whole theory of capital seems to have been caught in the trap of an old mode of thinking”.¹⁷¹ Garegnani, on the other, concludes that those results were admitted only “sporadically” and that, as he also pointed out in his long gestated reply to Samuelson’s Surrogate note, the mainstream approach to economics has ever had the tendency to regard those possibilities as “exceptions”.¹⁷²

¹⁷⁰ Both authors refer to Hicks (1965) p. 154.

¹⁷¹ Pasinetti (*op. cit.*), p. 516.

¹⁷² Garegnani (1966), p. 565. We have written “long gestated” reply to Samuelson partly because the reasons we have already seen (see n.136 above). By the way, the present author would like to call attention on the issue about when ‘reswitching’ became publicly important in the debate. For the reasons we explain in this chapter, that importance arose in the wake of the 1966 Symposium. Nevertheless, this does not mean that previous to that date ‘reswitching’ has not been used by critics as a powerful instrument of critique of the marginal approach nor does it mean that neoclassical economists have not realised their implications. An example is precisely the case of the oral controversy between Garegnani and Samuelson then crystallised in the 1970 paper. However, in the literature on the controversies, there does not seem to be total agreement on this. Recently, Birner (2002, p. 107, n. 2) writes that when he asked Paul Samuelson (in an interview in April 1989) whether or not the latter had read Sraffa (1960) in the time of Garegnani’s visit to MIT in 1961-2 and whether or not Garegnani had orally used ‘reswitching’ in his critique of Samuelson, “Samuelson replied that he did not want to say that (at that time) he understood all the things that were in this cryptically written book. But he was rather emphatic in his recollection that ‘reswitching’ had not come up in Garegnani’s oral criticism”. Actually what Birner wants to argue in that passage, as well as in the chapter referred to the controversy between Garegnani and Samuelson (*id.* pp. 66-83) is that, by relying on the support of Samuelson’s answers to Birner’s questionnaire, ‘reswitching’ “did not play a major part in Garegnani’s criticisms” (p. 77) of Samuelson. It is hard to go along with Birner’s position once it is realised that, despite the fact that “*reswitching* is only mentioned in a footnote of [Garegnani’s] manuscript” (*Ibid.*) – which is, by the way, the only one reason other than Samuelson’s answers pointed out by this author for
(*cont.*)

Pasinetti affirms that the traditional principle (inverse relationship between demand for capital per worker and the profit rate) has been taken for granted

“without any justification, except that this is the way economists have always been accustomed to think.”¹⁷³

On the other hand Garegnani points to the problematic consequence that traditional theory would undergo had those negative results been admitted. This may allow us to envisage what could happen to the discipline if the problematic issues like ‘reswitching’ and ‘reverse capital deepening’ had been *frankly* admitted¹⁷⁴:

“[T]he consequences of admitting this, however, are far-reaching, because on that principle has been erected the dominant theory of distribution.”¹⁷⁵

What Pasinetti and Garegnani unveil in their interventions in the Symposium is that the columns upon which the edifice of the dominant theory has for so long been built are not so strong, as it was once believed. The problem of capital undermines even the idea of thinking of that building (and re-building) of those columns because there is no appropriate – even less definable – material for such a construct. It is therefore hard to conceive an explanation of *profits* determined by the *scarcity of capital considered as a factor of production*.¹⁷⁶

justification of his thesis – the ‘reswitching’ case is, just to argue in the same terms as does Birner, actually illustrated by Garegnani at the outset (p. 411) of section II of (1970), which had already been sketched in the 1961-2 MIT manuscript. That ‘reswitching’ became *publicly* important some years *after* 1961-2 (*i.e.*, precisely in the Symposium of 1966 owing to Levhari’s 1965) is not under dispute (see this chapter). But this is radically different from saying that ‘reswitching’ “did not play a major” role, as Birner (*ibid.*) conjectures, in the criticisms raised by the Italian economist against one of the most important representative of the marginal school in the world, as is Samuelson. (See par.4.5 above for the consequences that Samuelson’s surrogate would have created among the discipline and how that situation was viewed by Joan Robinson).

¹⁷³ Pasinetti (*op. cit.*), pp. 516-7. As Sraffa (1960, preface) clearly points out the classical political economy had already been “submerged” by the marginal revolution at the end of the nineteenth century. The “way economists have always been accustomed to think” to which Pasinetti refers is the marginal theory. *Cf.* also Keynes (1936) p. viii.

¹⁷⁴ The word *frankly* was italicised for reasons that will appear immediately below (See Samuelson’s reactions).

¹⁷⁵ Garegnani (1966), p. 565.

¹⁷⁶ For the reasons we have here examined, the importance of the Symposium in the Cambridge controversies is indubitable. However, recently, Harcourt and Cohen (2003) seem to have forgotten the turning point of this episode by completely ignoring it in their accounts of the
(*cont.*)

4.11. Neoclassical reactions: Samuelson (1966a).

The 1966 Symposium conclusively showed that the traditional principle establishing a negative relation between the rate of interest and the “quantity of capital” could not be postulated in economies with heterogeneous capital goods. Defeat was then admitted by defenders of marginalism. Let us see now how this admission has appeared soon after the Symposium.

An authoritative neoclassical view, as represented by Samuelson, admitted this by pointing out that, in the light of the results of the Symposium, there exist serious problems for the theory. Thus Samuelson (1966a), in the well-known “Summing up”, recognises that ‘reswitching’ is a theoretically possible phenomenon:

“Pathology illuminates healthy psychology. Pasinetti, Morishima, Bruno-Burmeister-Sheshinski, Garegnani merit our gratitude for demonstrating that reswitching is a logical possibility in any technology, indecomposable or decomposable. Reswitching, whatever its empirical likelihood, does alert us to several *vital* possibilities.”¹⁷⁷

One of these possibilities for Samuelson is that there would no longer be any guarantee to ensure an inverse relationship between consumption per capita and the interest rate. However, Samuelson totally minimises the relevance of ‘reswitching’ and ‘reverse capital deepening’ by pointing to a problem of a “dynamic stability” in steady-state equilibria.¹⁷⁸ Quite incomprehensibly, when referring to the consequences of ‘reverse capital deepening’, he writes:

“going to a lower interest rate may have to involve a *disaccumulation* of capital, and a surplus (rather than sacrifice) of current consumption, which is balanced by a

controversies. In fact the lack of this issue in their review immediately provoked the reactions of Pasinetti, who argues that the Symposium “was the central point of the controversy and which is inexplicably ignored by the authors” (Pasinetti in Pasinetti *et. al.* 2003, p. 227). This recent exchange on issues which took place around forty years ago reveals the current importance of setting in perspective both the Cambridge debates in general and, specifically, the 1966 Symposium.

¹⁷⁷ Samuelson (1966a), p. 582, emphasis added.

¹⁷⁸ *Id.* p. 578.

subsequent perpetual reduction (rather than increase) of consumption as a result of the drop in interest rate.”¹⁷⁹

Samuelson of course is taking for granted that accumulation is created by “sacrifice of current consumption”, that is by full employment savings. He does not seem to be worried, none the less, about how the accumulation of capital, in the light of ‘reverse capital deepening’, would be brought about: in the presence of ‘reverse capital deepening’, a situation showing savings exceeding investment would need a *rise* in the interest rate so as to increase investment to the level of the former, whereas the tendency in the savings-investment market, according to supply-and-demand analysis, should be to lower that rate. This argument is what we have illustrated above by introducing the meaning – and their importance in economic terms – of ‘reverse capital deepening’ and ‘reswitching’. (See ch.3.)

Despite the fact that Samuelson admits the problems for the theory as a result of the definitive critiques raised in the Symposium, his admission does not mean abandoning the theory, as it is the theory relying on factor’s substitution what is showed to be unfounded. Then it is no wonder the way in which the famous MIT economist receives the criticisms:

“If all this causes headaches for those nostalgic for the old time parables of neoclassical writing, we must remind ourselves that scholars are not born to live in an easy existence. We must respect, and appraise, the facts of life.”¹⁸⁰

Did successive theorists appraise *and* respect “the facts of life”, as Samuelson stated to the practitioners of the discipline?

As we shall see below, some Samuelson’s disciples, and even Samuelson himself, actually tried to judge the validity of the criticisms, increasingly relying on the

¹⁷⁹ *Id.* p. 581, emphasis in the original. A similar position can also be found on p. 579, when Samuelson writes: “[Due to the possibility of reverse capital deepening] after sacrificing present consumption and accumulating capital goods, the new steady-state equilibrium can represent a rise in interest rate!” Interestingly, Samuelson’s position is not isolated: a similar attitude can be found in *e.g.* Burmeister (2000) p. 310.

¹⁸⁰ *Id.* p. 583

empirical “importance”, so as to minimise their relevance, rather than to respect them – let alone to *accept* them. Ferguson (1969) is a good example (see below).

Needless to repeat that Samuelson, and many others (*e.g.* Ferguson, 1969; Blaug, 1974), when admitting the possibilities of irrefutable discoveries like ‘reswitching’, have had the common tendency to associate those results, which undermine the “aggregation” of capital, with a problem of “aggregate” production function: in fact they characterise these results as elements raising “suspicion” of the neoclassical “parables”¹⁸¹, a term that in the controversies was associated with the aggregate production function “version”. Despite the fact that this form of the theory has clearly been shown to be theoretically secondary in Garegnani’s critique of Samuelson as far as the building of the roots of received theory is concerned, it is a matter of persistent misunderstanding that the damaging implications of ‘reswitching’ on the *conception* of capital as a single magnitude (which in a sense might be seen as a problem of ‘aggregation of capital’)– which is what actually is relevant for the long period supply and demand theory – is misread as damaging the “aggregate” production function, as if the results of the controversies would have only affected a “less rigorous” version of the theory. These misunderstandings were mostly created by neoclassical economists – though sometimes owing to responsibility to some critical economists like Joan Robinson.¹⁸²

4.12. Further neoclassical reactions after the Symposium: Ferguson (1969).

We deal with Ferguson (1969), because this author has been one of the most notable representatives of the “neo-neoclassicals cooped up by the surrogate function” to whom Robinson refers in some above citations.

At the outset of his book, Ferguson recognises that the Cambridge criticisms are valid. This does not mean, nevertheless, that he has not attempted to minimise the significance of the criticisms of the theory he tried to shield:

“[The validity of the] Cambridge Criticism is unquestionable, but its importance is an empirical matter that depends upon the amount of substitutability there is in the system.

¹⁸¹ Samuelson (1966a) p. 574.

¹⁸² *Cf.* ch.5, below.

Until the econometricians have the answer for us, placing reliance upon neo-classical economic theory is a matter of faith.”¹⁸³

The terms in which Ferguson disputes the relevance of the Cambridge criticism are hard to accept. Firstly, because he reduces the problem of the basic principle on which the theory relies to an “empirical” matter. But the point is, on the contrary, that marginal economic theory is *not* grounded on the probabilities of having downward-sloping demand curves for factors. When the theory meets insurmountable *theoretical* flaws some of its defenders appeal to the empirical “way-out”; still the downward-sloping demand functions were basically derived from logical deductions, in turn derived from self-evident facts.¹⁸⁴ Secondly, Ferguson *dodges* the problem at all, because, since what matters is the empirical probability to derive “well behaved” demand curves, we must look forward to the “econometricians’ answer” in order for us to assess whether or not one can rely on the traditional supply and demand functions. In the meantime the author proposes to “have faith” in the neoclassical theory, hence in the downward-sloping demand curves, etc.¹⁸⁵ Thirdly, it is worth noting that the problem is the “substitutability” itself, not just a problem of “amounts of” as Ferguson believes. The substitutability principle is a basic premise of the

¹⁸³ Ferguson (1969) p. xvii. Later on the book Ferguson also claimed: “The crucial point to emphasize is that the validity of neoclassical theory is an *empirical*, not a *theoretical*, question” (*Id.* p. 258, emphasis in the original).

¹⁸⁴ These facts, alternative methods of production and tastes and preferences, allow deriving logical deductions resulting in inverse relations between quantities of “factors of production” used and their rates of remuneration. What is behind the consideration of the above mentioned two facts as part of data of the theory is that these data might directly and/or indirectly guarantee the substitution of factors as distribution changes. This *general* principle is what is so necessary for the theory rather than the “empirical probability”. To try to ascertain the probabilities of “perverse” behaviour would mean to try to ascertain the probabilities of “non-perverse behaviour” that is of downward-sloping demand functions for factors. However, this attempt would imply a “drastic change in the nature of the theory”, as Garegnani (1990, p. 72), pointed out, because “that shift of basis would occur in a field like economics, where experiment is impossible and the material is so highly complex as generally to limit considerably the weight one can attribute to any observed, purely empirical regularity”. Though we do not deal in detail with this drastic consequence that the theory would undergo if the “empirical” foundation were adopted, it is worth noticing that all these attempts appeared soon after the controversies in capital theory.

¹⁸⁵ The “empirical” route in the wake of the controversies was not only taken by the neoclassical side. We also have to note that some critically oriented authors in the controversies somehow attempted to ascertain the “probabilities” of ‘reverse capital deepening’ in some heterogeneous capital-goods models. See Mainwaring and Steedman (2000), D’Ippolito (1987). The “empirical issue” as viewed by these authors is not examined in this study, since we have preferred to focus on the neoclassical reactions against the critical Cambridge. For a discussion of these works, see Ciccone (1996) and Petri (2000; 2004, ch.6).

theory, a basic principle that, being necessary for the theory, had long been taken for granted since the inception of marginal economics at the turn of the twentieth century (see ch.2 above). Yet this extreme opinion is quite different from some other Ferguson's views on the debates:

“Shortly after this [Samuelson's ‘surrogate’ article] was completed, the ‘Symposium on Capital Theory’ appeared in the *Quarterly Journal of Economics*; and it then became quite apparent that the Cambridge Criticism, as I call it, must be accorded more careful consideration.”¹⁸⁶

One who happens to come across Ferguson's reaction grounded in terms of “faith” would probably have expected for another kind of “more careful consideration” of the “Cambridge Criticism”. We are not arguing that basing economic principles on “faith” is unfair, but Ferguson does not provide any convincing argument in order for economists to have faith, and his only attempt to that purpose is “to invoke the *weight of Samuelson's authority* as represented by [the following words]:”¹⁸⁷

“Until factors cease to have their rewards determined by bidding in quasi-competitive markets, I shall adhere to generalized neoclassical approximations (...) somewhat limited factor substitution can fail to have some of the simple properties of the idealized J.B. Clark models.”¹⁸⁸

These words reveal an embedded belief of neoclassical economists, namely – that the sole way to envisage a process of “bidding in quasi-competitive markets” is by invoking marginal, or neoclassical, supply and demand functions and not any other alternative. To logically derive such functions it is indispensable the factor substitution principle, which would act through the data of the theory. But the critiques of marginal theory precisely showed how groundless is to postulate a general principle by relying on “factor substitution” when one of those “factors” is measured in value terms – capital. That principle is the ultimate foundation on which

¹⁸⁶ Ferguson (*op. cit.*) p. xvii.

¹⁸⁷ *Id.* p. xviii, emphasis added.

the determination of “rewards in quasi competitive markets” relies – namely “well behaved” supply and demand functions, which are what Samuelson means when he speaks of “bidding in markets”.

Ferguson later introduces “Samuelson’s authority” again in his book¹⁸⁹ when he attempted to give the criticisms “more careful consideration”. Ferguson’s purpose is to show that “a small deviation from Samuelson’s [surrogate] model” will result in a “slope of the frontier [which] does not give the capital-labour ratio [of the economy].”¹⁹⁰ The small deviation alleged is that the relative factor production coefficients are the same in both sectors (Samuelson parable), but the absolute magnitudes of the coefficients differ. This is called by Ferguson the “Hicks case” because, according to Ferguson, Hicks had emphasised this case “so strongly”.¹⁹¹

In Samuelson parable the wage-curve is $w = \frac{1}{l_a} - \frac{c_c(1+r)}{l_a}$ hence $-\frac{dw}{dr} = \frac{c_c}{l_a}$,

which is equal, if the absolute magnitudes are the same in both sectors, to $\frac{c_a}{l_c} = \frac{K}{L}$,

which is of course equal both to $\frac{c_c}{l_c}$ and to $\frac{c_a}{l_a}$. (In the example it is being considered

only circulating capital). Now Ferguson by introducing his “Hicks case” argues that

$\frac{c_c}{l_a} \neq \frac{K}{L}$. However this is a more general condition. In fact since in Samuelson’s

¹⁸⁸ Samuelson (1966b) pp. 444-5. This passage is reproduced in Ferguson (*op. cit.*) p. vi (as a “flyleaf”) and p. 250.

¹⁸⁹ Ferguson (1969) pp. 251-70.

¹⁹⁰ *Id.* pp. 259-60.

¹⁹¹ *Cf. Id.* p. 259. Ferguson refers to Hicks (1965) pp. 148-59. However, it is hard to go along with Ferguson here as well, since Hicks’ chapter Ferguson refers to actually treats cases in which the relative factor proportions in the “corn” and in the “tractor” industries may differ. On the point Ferguson introduces, Hicks (*op. cit.* p. 153) pointed out that, in cases where both factor proportions are equal and by comparing two straight-line wage curves, “a fall in the rate of profit would lead to a shift in technique such as (...) the technique [chosen] must become more capital intensive.” Hicks justified that by arguing that as the profit rate falls the technique chosen must account for a higher “capital-capital coefficient” and a lower “labour-consumption good coefficient” – in other words, as r falls, cc/la must increase. Of course, Hicks here does not realise that he is actually dealing with homogenous systems since, as we have seen above with regards to Samuelson’s parable and the criticisms raised by Garegnani, a straight-line wage curve actually means production of “corn” by means of “corn” and labour. And this is valid for any number of techniques whose wage equations are linear. In fact, then, Hicks (p. 154) considers cases in which relative factor proportions are not equal in both productive sectors but minimised the relevance of the possibility of ‘reswitching’ by pointing out that “such cases [could be regarded] as exceptional.”

model the relative factor production coefficients are equal in both sectors, then the capital-labour ratio of the economy will be equal to the relative coefficient and, moreover, will be constant as distribution changes because the price of the capital per labourer is constant throughout. The capital-labour ratio of the economy, when absolute magnitudes differ, will be equal to $\frac{c_c}{l_a} \left(\frac{l_a}{l_c} \right)$ – that is a physical magnitude multiplied by a value magnitude, which is, in Samuelson’s model, equal to the relative ratios of quantities of labour in both industries.¹⁹²

Ferguson’s conclusion is that “the factor price frontier may not yield the neoclassical relation between factor proportions and relative factor prices even in the *absence of reswitching*.”¹⁹³ Of course ‘reswitching’ cannot arise in Ferguson’s model. *For linear wage curves* there is no heterogeneous model at all. A linear wage curve actually implies that the consumption good is produced by itself and labour; now if a second linear wage curve is introduced, though apparently involving a heterogeneous capital good, it is implied that the same consumption good is being produced and therefore, since this technique is depicted by a linear curve, again, the consumption good is produced by itself and labour. Independently of Ferguson’s explicit minimisation of the ‘reswitching’ results for the theory, his analysis fails from its very beginning, as he thinks of having dealt with heterogeneous capital goods economies with systems of production described by *linear* wage-curves. Yet, as we have seen in previous part of this study, the problems for the theory not only involve ‘reswitching’ but also ‘reverse capital deepening’, a problem, that is, which does not seem to disturb so much Ferguson.

Ferguson like many, if not most, neoclassical economists, downplayed the relevance of ‘reswitching’ by incessantly pointing out that the Cambridge criticisms only “invalidate the *simple* Clark fairy tale” (p. 269), show that “the Clark parables may not hold” (*ibid.*), “point up a definite potential weakness of *simple* neoclassical theory” (p. 270), and that “*simple* neoclassical results concerning the relation between

¹⁹² For the sake of the argument assume an economy where $\frac{c_c}{l_c} = \frac{c_a}{l_a} = 4 = \frac{K}{L}$. Then

$\frac{c_c}{l_a} l_a = \frac{c_a}{l_a} l_c$ and from this equation we can obtain: $\frac{c_c}{l_a} = \frac{c_a}{l_a} \frac{l_c}{l_a}$ so that $\frac{K}{L} = 4 = \frac{c_c}{l_a} \frac{l_a}{l_c}$.

¹⁹³ *Id.* p. 260, emphasis in the original.

production and input and output markets may not hold” (p. 265). Is there, perhaps, a *more robust* or more sophisticated version than the “fairy tale”, on which Ferguson is relying his “faith”? The point is that the criticisms destroy or invalidate the “simple relation” from which “all neoclassical results follow”, as Ferguson himself put it: “the lower the rate of interest, the greater the capital intensity of production.”¹⁹⁴ Ferguson recognises this, but manifests his concerns by pinpointing “that economists may be unable to make any statement concerning the relation of production to competitive input and output markets.”¹⁹⁵ However, Ferguson believes that economists *can* still use the theory under attack, and, once again, he justifies his claiming on his own faith. He argues that all that is needed to establish the neoclassical parables is the existence of a sufficient substitutability establishing capital intensity uniqueness, but the latter is “an econometric question susceptible of resolution in a probabilistic sense.”¹⁹⁶ Hence, “until the econometricians have the answer for us, placing reliance upon neoclassical economic theory is a matter of faith”, as we noted above. This was the way chosen by Ferguson to rebut the criticisms of capital.¹⁹⁷ As noted, Ferguson’s reactions are very extreme. Still it would seem that many practitioners of the discipline have taken up this route, as it is pretty evident the great deal of work done in econometrics relying on the traditional theory – and all this as if nothing had ever happened in the theory of capital.¹⁹⁸

4.13. Garegnani’s 1970 critique of the marginal theory.

Garegnani’s (1970a) article not only provided the analytical elements necessary to definitely dismiss Samuelson’s surrogate function that “somewhat baffled everyone” – as Robinson pointed out¹⁹⁹ – but also shows that expressing the production conditions in terms of a production function (either aggregate or individual)²⁰⁰ with

¹⁹⁴ *Id.* p. 252.

¹⁹⁵ *Id.* p. 269.

¹⁹⁶ *Id.* p. 270.

¹⁹⁷ It is worth noticing that the ‘empirical’ way out was emphasised by some commentators from the neoclassical side such as Blaug (1974), pp. 39-40.

¹⁹⁸ See n.185 above.

¹⁹⁹ See par.4.5 above.

²⁰⁰ It must be said that nor can individual production functions exist such that they should represent the production of a single commodity by means of labour and “capital”, because of the same reasons of why it cannot exist an aggregate function for the economy as a whole, that is to say, they can only exist if and only if the net consumption output is produced by itself and labour,
(*cont.*)

“capital” as one of its arguments, while the other one being labour, has been a “feature of only *some* versions of the traditional theory.”²⁰¹ In fact what is being argued by Garegnani is that marginal theory reduces its core explanation of distribution to the functions of supply and demand for productive factors. The basic premise on which the traditional theory ultimately relies is the “belief” that a fall of the profit rate cheapens the more “capital-intensive” method either by means of the alternative methods of production available (direct factor substitution mechanism) or by changes in consumer substitution in favour of the “more capital intensive” goods. And this problem is quite independently of considering the systems of production as represented by an aggregate production function.²⁰²

In addition it is interesting to note how Garegnani uses the implications of ‘reswitching’ and ‘reverse capital deepening’, when he argues that

“[Theoretical] examples do not seem to indicate that the conditions in which a fall of r results in a relative cheapening of the less capital-intensive productive processes are any less plausible than those in which the opposite would be true. This appears to undermine the ground on which rests the explanation of distribution in terms of demand and supply for capital and labour.”²⁰³

The fact that the factor substitution principle is undermined by the phenomena of ‘reswitching’ and ‘reverse capital deepening’ implies that, in systems of production with heterogeneous capital goods, a *fall* in the rate of profit may well bring about a *fall* rather than a rise in the capital-labour ratio of the economy. Thus, with ‘reverse capital deepening’ there would not be any support to the claim that it is only by postponing consumption (that is by saving) that investment can be brought about, because as the profit rate falls it is a *less* “capital intensive” method which is chosen

so this single commodity would be produced by itself and labour, hence the question which arises is: What kind of representation of different systems of production – involving heterogeneous capital goods – can these individual production functions be? Yet these striking conclusions seem not to disturb the authors who are devoted to use aggregate production functions in *e.g.* modern growth theory, see Barro and Sala-i-Martin (1995).

²⁰¹ Garegnani (1970a) p. 422, emphasis added.

²⁰² The particular target of attack at production functions pursued by Robinson in the controversies is analysed in ch.5 below.

²⁰³ Garegnani (*op. cit.*) p. 425.

(because it is the most profitable at the new distributive situation) with a *lower* value and *not* a higher ‘quantity of capital’ per worker.²⁰⁴

We have seen above (ch.1), the consequences for the traditional marginal theory of having upward-sloping demand curves for productive factors. In this regard, Garegnani’s article also provides a clear-cut example²⁰⁵ which conclusively demonstrates that “a change, however small, in the ‘supply’ or ‘demand’ conditions of labour or capital (saving) may result in *drastic changes* of r and w .”²⁰⁶ These drastic changes in distribution actually mean that the profit rate might fall to *zero* while wages to *infinite* or *vice versa*, as supply and demand for each respective “factor” will not be brought into equality. The point is that, had one ever detected in any real economy, for example, a zero wage rate with unemployment, the variables determined by the theory would have served as theoretical guides of actual ones, and explanations in terms of supply and demand would have resulted in a plausible theory of distribution. But, as Garegnani concludes

“no such instability of an economy’s wage- and interest-rates has ever been observed. The natural conclusion is that, in order to explain distribution, we must rely on forces other than ‘supply’ and ‘demand’ (...) [T]he theory becomes implausible once it is admitted that this principle [that a fall in r would always raise the proportion of ‘capital’ to labour in the economy] is not always valid.”²⁰⁷

What, Garegnani asks, then to an economic theory which cannot describe *any* actual economy? In a word, with ‘reswitching’ and ‘reverse capital deepening’ we would have to admit, as theoretical possibilities, extreme values of distributive variables; an infinite value of, say, the interest rate – owing to an upward-sloping demand function for “capital” – would imply zero wages: it is hard to conceive any

²⁰⁴ Cf. Samuelson’s reactions in the Symposium, par.4.11 above.

²⁰⁵ Garegnani (*op. cit.*) p. 424.

²⁰⁶ *Id.* p. 426. emphasis added.

²⁰⁷ *Ibid.* Garegnani in this passage talks about the existence of a neo-classical “belief” in the traditional principle. “Belief” is here taken to mean that, independently of the supply aspect of the problem of capital (that is the illegitimacy of conceiving of capital in value terms), the idea of an *inverse relation between demand for “capital” and the profit rate* would somehow have been taken for granted by the theory; the “belief” being “supported” by two groups of the data: the alternative methods of production, and tastes and preferences of consumers. This “belief” is what is behind the demand aspect of the problem of capital, and it is this belief that is totally destroyed by ‘reswitching’.

explanation establishing an equilibrium in terms of supply of, and demand for, labour resulting in a zero wage rate. In the light of these results it is clear that the traditional belief does not work, and therefore the forces “in order to explain distribution”, as Garegnani strongly stressed²⁰⁸, must be sought in a theoretical approach alternative to marginal theory.

4.14. Bliss’s 1970 intervention. Prelude to the shift to neo-walrasian equilibria and the beginning of a second phase in the debates.

In the same issue of *The Review of Economic Studies* where Garegnani (1970a) published his critique of Samuelson, there appeared the sole reply that Garegnani’s article ever received for the record. It was the case of neoclassical Christopher Bliss, who, in a brief comment²⁰⁹, completely misunderstood the theoretical background in which Garegnani’s argument relied – the conception of capital in value terms. In fact Bliss takes Garegnani’s argument as if the latter would have referred to a short-period, *neo-Walrasian* equilibrium.²¹⁰

Bliss argues that one of the most striking Garegnani’s conclusions – the possible non-existence of equilibrium in the savings-investment market due to ‘reverse capital deepening’ – is contradictory with Debreu’s results, who, according to Bliss, would have considered the same required conditions as did Garegnani for *the existence of*

²⁰⁸ *Id.* pp. 424-7.

²⁰⁹ Bliss (1970)

²¹⁰ Bliss (*op. cit.*) refers to a “momentary equilibrium”. We call “neo-Walrasian” equilibrium the notion of equilibrium professed by the modern versions of the theory (temporary/momentary equilibria or intertemporal equilibria) which, as we shall explain in ch.6 below, take as datum of the equilibrium the physically heterogeneous capital goods of the economy instead of the single magnitude “capital” as it was traditionally conceived (with the exception, as is well known, of Walras.) The label “neo-Walrasian” is *not* misleading, because it indicates that latest versions of the theory (*i.e.*, the temporary and intertemporal equilibrium versions associated with the names of Arrow, Debreu, Hahn, Bliss) conceived, as Walras did, the data regarding capital as the heterogeneous vector of the several capital goods (the “Walrasian”, so to speak, “arm” of that label); while “neo” refers, instead, to a completely different notion of the equilibrium owing to the not satisfaction of the condition of a *uniform* rate of returns on the supply prices of the capital goods, thus renouncing to the traditional long period (or normal position) method, which had characterised received theory until recent decades (*Cf.* Garegnani, 1976a): this Walras was very concerned on, because despite he had conceived of capital as a heterogeneous vector, he shared, on the other hand, the traditional method of the normal positions, but the latter was contradictory with the conception of capital as a collection of heterogeneous goods. It is important to notice that Debreu and other economists of mathematical orientation pursued the research programme commenced by Hicks in *Value and Capital* (1946[1939]), where the author after having criticised
(*cont.*)

equilibrium.²¹¹ But, in a Debreu's economy, capital is conceived as the heterogeneous capital goods vector of the economy. In Debreu's words:

“The total resources of an economy are the given quantities of commodities that are available. They include the capital of the economy at the present instant, *i.e.*, all the land, buildings, mineral deposits, equipment, now existing. All these are a legacy of the past, they are a priori given.”²¹²

Then, Bliss goes on to argue that “[w]ith more than one capital good current demand for investment goods will depend not only upon present prices but also upon future expected prices.”²¹³ Evidently Bliss is taking Garegnani's argument regarding investment as if it were a short-run problem of expectations.²¹⁴ Bliss's misunderstanding is finally completed by his conclusion:

“the major fault in [Garegnani's] argument is the illicit importation of long-period equilibrium theory into the analysis of a short run situation.”²¹⁵

But, as it has been described in the present chapter, the background of the controversies was completely another one. In fact Garegnani immediately replied²¹⁶ Bliss, pointing out that he had never treated in his argument the discussion of such short-period equilibrium and argues that:

“the demand and supply explanation of distribution seems faced, *at the present moment*, by two alternatives. On the one hand, there is a long-run analysis undermined by the inconsistencies of the notion of capital it must use. On the other hand, there are short-period analyses where the attempt to avoid these inconsistencies (by referring to

the traditional method of long period equilibrium proposes a “temporary” equilibrium by relying in a Walrasian conception of capital. *Cf.* particularly par.6.1 below.

²¹¹ Bliss is referring to the well-known work of Debreu (1959). These conditions are continuity of demand functions and convexity of technology.

²¹² Debreu (1959) pp. 74-5.

²¹³ Bliss (*op. cit.*) p. 438.

²¹⁴ Bliss points out that “Garegnani's error” would have been due to an overlook of “expected changes [of prices] over time” (*Ibid*). But prices change “over time” was a matter that the traditional approach of long period equilibrium took into account. *Cf.* ch.2, par.2.9 above.

²¹⁵ *Ibid*.

²¹⁶ Garegnani (1970b).

heterogeneous capital goods) leaves room for the doubts about the fruitfulness of the whole approach. In this situation, the temptation might be strong to use the short-period approach to provide some rationalisation of results which derived their plausibility only from a long-run analysis.”²¹⁷

Unfortunately Garegnani’s arguments were not replied by Bliss or by any other marginalist scholar *at that moment*. Although it cannot be assumed that communication between the different sides of the debates has been an easy matter for the participants, because communication is *per se* a question with many ramifications, yet this kind of neoclassical reaction, which no doubt *obscures* the argument for an otherwise better communication, is raised for a purpose. While it is worth noticing the misunderstanding surrounding Bliss’s *appreciation* (thought, it may seem, not *respect*, in the light of Samuelson’s view after the Symposium²¹⁸) of the results of the controversies, on the other hand Bliss is pointing to a way-out to those negative results, the reason of which Garegnani has already envisaged clearly in his brief rejoinder.

These kinds of exchanges reflect what then turned into an increasing attitude by defenders of the neoclassical theory soon after “more careful consideration” – to use Ferguson’s wording – the phenomena of ‘reverse capital deepening’ and ‘reswitching’ raised in the debates was “accorded”. As we shall see in ch.6 in much more detail, “the temptation to use the short-period approach”, as Garegnani points out, was in fact the way out adopted by the neoclassical approach to the inconsistencies of the traditional theory unveiled in the controversies by the critics.

4.15. The zenith of the debates.

It has been seen at the end of the last chapter the theoretic problems surrounding the notion of capital for the theory. In this chapter, in addition, we have seen how defenders of neoclassical theory reacted against the threats of ‘reswitching’ and ‘reverse capital deepening’. In this regard, it is worth mentioning Samuelson’s influence in stimulating the debates: on the one hand we have seen how Samuelson’s

²¹⁷ *Id.* p. 439, emphasis added.

²¹⁸ See par.4.11 above.

Surrogate article resulted in what we can here call one of the cornerstones of this controversy – namely Garegnani’s criticisms of the neoclassical theory, and his struggling for an alternative. On the other hand, we have again Samuelson – though indirectly – trying to defend dominant theory but at the same time bringing about one of the most notorious results of these controversies for the discipline as a whole –the 1966 Symposium. It is interesting to look at a retrospective view on Samuelson’s role in the controversies from another authoritative voice from MIT, R. Solow:

“Samuelson had the notion that universal diminishing returns, together with the assumption that every line of production involves every commodity directly or indirectly, would rule out the paradoxes of reswitching even in models with many capital goods. (...) [T]he notion is false. But sometime in 1964 Samuelson planted it in the mind of David Levhari. With such a gardener it would be hard not to grow something; in 1965 Levhari published a proof of the notion. Since the notion is false, the proof contains a slip. Counterexamples were soon produced and reswitching became an issue instead of a curiosum, like the *Giffen* good.”²¹⁹

The Symposium in paradoxes in capital theory definitely revealed that the seeds planted by Samuelson – to follow Solow’s analogy – were totally *sterile* to fence in the traditional theory from the critical Cambridge. The lesson to be learned from the Symposium is that ‘reswitching’ and more generally paradoxes in neoclassical capital theory cannot be ruled out in any one of the imaginable worlds proposed by neoclassical authors. The debates were gaining momentum and defeat from the neoclassical side had to be admitted – though, as noted, hitherto sporadically. The critically oriented side of the controversies was winning the debate in the theory of capital by showing how groundless the idea of conceiving capital as a factor of production is.

In the next chapter we make a step back in the history of the debates. We shall discuss whether and how the question of capital appeared in the literature prior to 1960. Then, chapter 6 returns to the consequences of the tensions created by the implications of ‘reswitching’ and ‘reverse capital deepening’.

²¹⁹ Solow (1983) p. 184. In ch.5 below, Solow’s participation in the controversies is analysed.

CHAPTER 5

A step back in the debates

Joan Robinson's role in the controversies and first neoclassical reactions

5.1. Introduction. ²²⁰

Joan Robinson was a renowned participant in the Cambridge controversies for various reasons. One of them is her famous and provocative article published in 1953 by the *Review of Economics Studies*, which has been considered as the *kick-off* for the so-called Cambridge debates in capital theory.

It has been said above that we have preferred to place at the centre the results of 'reswitching' and 'reverse capital deepening' and their implications because these phenomena allow us to clearly grasp the relevance of the debates. Now, in the light of the implications of the critique, we turn to the analysis of the first arguments raised in the Cambridge controversies by Robinson and her immediate contenders. The object of this chapter is not to exhaustively account for the large amount of contributions of the Cambridge economist but rather to critically focus on two points that we have deemed as the most relevant for the problems of capital. While the first point is focused on why Robinson might have chosen the aggregate production function as her main target of attack, the second one is to explain as simple as possible Robinson's analysis in order to understand the kind of criticisms she raised and how far those criticisms touch the heart of the problem of capital as we have seen above (chs 3 and 4). Lastly some neoclassical reactions raised against Robinson critique in the mid-1950s (Champernowne, 1953; Solow, 1955, 1956; Swan, 1956) will be reviewed.

²²⁰ We have considered for the part regarding Robinson the following works: Robinson (1953), (1956) chs 8-10 and "Diagrams", and (1962). See also Robinson and Naqvi (1967); Harcourt (1972) ch.1.

5.2. Robinson's critique: Aggregate production functions as her main target.

Joan Robinson starts her critique of the neoclassical capital theory in her vividly written article of 1953. In that article Robinson accuses the neoclassical economist of not making clear, when he teaches students the production function $O=f(L,C)$, “in what units C is measured.” In fact, Joan Robinson's critique of capital has almost always been associated with the problem of considering this “factor” as an argument, together with labour, of an aggregate production function, in which, according to this economist, “the relative prices of the factors of production [*i.e.* the wage and interest rates] are exhibited as a function of the ratio in which they are employed.”²²¹

The aggregate production function Robinson criticises is a very particular, and theoretically secondary, representation of the neoclassical long period equilibrium theory, where output and capital per worker are assumed to be homogeneous.²²² In this representation it is assumed that the physical composition of capital is instantaneously adjusted by the supply and demand mechanisms. So the use of aggregate functions already entails accepting the supply and demand approach to value and distribution. But Robinson does not focus her critique on the supply and demand *mechanisms* as such, but rather on the long run equilibrium conditions which must prevail, always according to Robinson, in order to use that production function.

Robinson is criticising the fact that in real life the composition of capital cannot be instantaneously adjusted due to problems of *uncertainty* and *expectations* underlying problems of effective demand. Relying her arguments on Keynesian premises, Robinson believes that the production function is a very poor instrument to analyse process of accumulation occurring in real life because the “pseudo-production function”²²³ (and hence the neoclassical theory, according to Robinson)²²⁴ could be only used in conditions of long run equilibrium, which imply that expectations about the uncertain future must be realised.

²²¹ Robinson (1953) p. 81.

²²² *cf.* Samuelson's construct in ch.4 above.

²²³ Originally Robinson labelled the production function as a “pseudo-productivity curve” (*Id.* p. 106). See par.5.4 below.

²²⁴ Robinson (1970a) considers that both Wicksell and Walras do not account for a theory of distribution at all and that the sole neoclassical theory (“neo-neoclassical” in her terms) is that represented by the aggregate production functions.

In fact Robinson sees the problem of capital only limited to this representation of the long period equilibrium, when she states that, in equilibrium, the value of capital could be measured either in terms of costs or current purchasing power or future earnings

“but to do so, we have to begin by taking the rate of interest as given, whereas the main purpose of the production function is to show how wages and the rate of interest are determined by technical conditions and the factor ratio.”²²⁵

The citation makes it clear that for Robinson it is only in the aggregate production function representation that the problem of valuation might arise; but this view is devoid of a critical examination of the problematic issue entailed by a value measure of the endowment of capital for the theory independently of the aggregate production function. In fact, as we have seen above, it is neither necessary nor sufficient the existence of any such construct to realise the *necessity* of a value measure for capital in marginal, long-period equilibrium, theory.²²⁶ On the other hand, it would seem that the strength of her arguments crucially lies in her notion of long-run equilibrium.

5.3. *Equilibrium and expectations.*

Robinson conceives equilibrium as a situation where the composition of the physical capital stock is fully instantaneously adapted. The core of her arguments on equilibrium lies precisely in Keynesian premises regarding investment, expectations and uncertainty. In this regard it is worth quoting her definition of equilibrium, which will characterise all her further works on the controversies:

“Equilibrium requires that the rate of profit ruling today was expected to be ruling today when investment in any plant now extant was made, and the expectation of future profits obtaining today was expected to obtain today. Thus the value of capital in existence today is equal to its supply price calculated in this manner. The heavy weight which this method of valuing capital puts upon the assumption of

²²⁵ *Id.* p. 81.

²²⁶ Our visit of Wicksell’s general dis-aggregated long-period equilibrium system has illustrated this issue. See Appendix B (below) and ch.2 above.

equilibrium emphasizes the impossibility of valuing capital in an uncertain world. In a world where unexpected events occur which alter values, the point of view of the man of deeds, making investment decisions about the future, and of the man of words making observations about the past, are irreconcilable, and all we can do is botch up some conventional method of measuring capital that will satisfy neither of them.”²²⁷

According to Robinson, equilibrium of realised expectations in an uncertain world implies that the capital stock composition was adjusted to the conditions expected in the past by investors when investment was made. The “equilibrium rate of profit” today (which will equal the interest rate) can be used to value capital in terms of current purchasing power, but, also, in terms of the costs of production because that rate “was expected to be ruling today” when investment (in the past) was made; then it will be also equal to the value of capital in terms of future earnings, because that rate of profits prevailing today, is expected to prevail in the future.

But the corollary of Robinson’s message would be that in a state of *realised* expectations the theory would *not* be faced with *any* problem regarding the issue of conceiving capital in value terms for the determination of a neoclassical equilibrium. Moreover, according to Robinson,

“In a position of equilibrium ... there is a quantity [of capital] which can be translated from one number to another by changing the unit. This is the definition of equilibrium. It entails ... that the human beings in the situation are expecting the future to be just like the past.”²²⁸

These citations suggest that the theory would meet the *question* of capital (the illegitimacy of conceiving of capital in value terms to explain prices and distribution in terms of supply and demand, as it has been analysed chs 2, 3 and 4 above) only in a state of *not* realised expectations. The problem of having one of the *datum* in value terms for the determination of equilibrium – outside the one-commodity aggregate

²²⁷ Robinson (1953) p. 96.

²²⁸ *Id.* p. 82

production function– is totally discarded in Robinson’s analysis. Rather, as Robinson also informs us in her 1956 book, in equilibrium capital can be given a meaning:

“Under the protection of our assumptions [of equilibrium], there is no difficulty in measuring the stock of capital within any one economy.”²²⁹

Under realised expectations (and free competition) Robinson believes that the traditional adjustment mechanisms of supply and demand work properly:

“an accidental increase in the stock of capital above the equilibrium quantity would depress the rate of profit, and cause the additional capital to be consumed.”²³⁰

In other words, nevertheless Robinson criticises the neoclassical theory on the grounds that it does not provide the appropriate tools for an analysis of accumulation owing to the problems regarding expectations and uncertainty, on the other hand the neoclassical relation between accumulation of capital and the rate of interest seems not to be put under dispute by the English economist.

5.4. *Understanding Robinson’s way: The ‘Pseudo-production function’ and the ‘curiosum’.*

The production function is considered by the Cambridge economist as her main target of attack because she believes that the analysis proposed by a production function is nothing more than an exercise of static *comparison* of long period equilibrium positions of the economy, where the profit rate is what it is because it has for a long time been so, and it is expected to be at that level in the (infinite) future as well. As early as her 1953 article, we are informed by Robinson that this kind of exercises are not suitable for an analysis of *changes* (of factor’s supplies, of inventions, of technical knowledge) running in time.²³¹ As she persistently insisted, to

²²⁹ Robinson (1956) p. 117.

²³⁰ Robinson (1953) p. 87.

²³¹ Cf. Robinson (1953) p. 100: “In short, the comparison between equilibrium positions with different factor ratios cannot be used to analyse changes in the factor ratio taking place through
(cont.)

compare two equilibrium positions on the curve due to a shift in some parameter does not tell us so much about how those positions were reached, because as we pass from one point to another one all the *history* (the form of the function, the position of the function) is being changed.²³²

To show that the production function is ridiculous for an analysis of changes, Robinson analyses the relation between output, labour, capital, distribution and technical knowledge in an economy under equilibrium conditions.²³³ In fact it is only in equilibrium of full realised expectations and knowing distribution in advance that that construct can be built. That is why Robinson called it the “pseudo-production function”. Outside equilibrium of full employment that construct is impossible to build, Robinson thinks. It is in connection with the building of that construct – which is only useful to make comparisons but not for analysing changes – which Robinson’s analysis tangentially touches the question of capital. To such constructs we now turn.

To begin with it is necessary to introduce the “productivity curve”. This curve, which has been used by Robinson since 1953, is similar to a neoclassical one-good production function, with the difference that it is drawn for a *given* value of r .

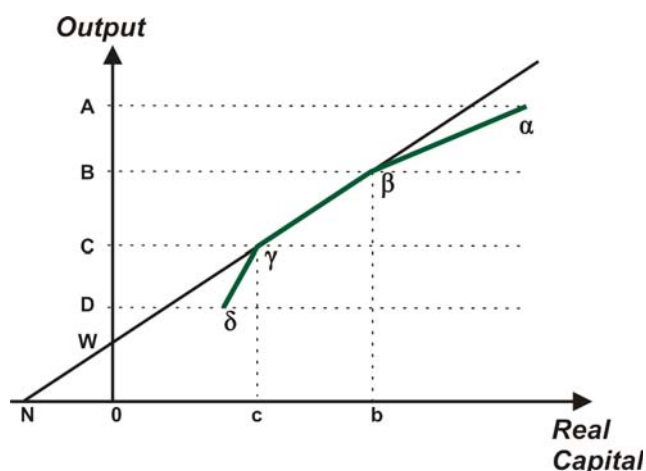


Figure 5.1: The productivity curve

time, and it is impossible to discuss changes (as opposed to differences) in neo-classical terms. The production function, it seems, has a very limited relevance to actual problems.”

²³² Cf. e.g. the following critique extracted from Robinson (1974, p. 58): “The problem of the ‘measurement of capital’ is a minor element in the criticism of the neo-classical doctrines (...) What they pretend to offer is nothing but an error in methodology – a confusion between comparisons of imagined equilibrium positions and a process of accumulation going on through history.”

²³³ See Robinson (1953) pp. 85-7.

This curve represents, for a given r , the relation between the (net) output per man (vertical axis) and the value of capital goods per man (horizontal axis) in terms of wage-units (labour commanded), or as Robinson preferred to call it, “real capital”.²³⁴ For the case illustrated, each corner on the curve represents the use of a determined technique ($\alpha, \beta, \gamma, \delta$, etc.); while stretches on it represent combination of techniques. For the given r (I/ON , represented by the slope of the tangent drawn) only the stretch where γ and β are used are of equilibrium, the other techniques being not eligible at that r .

*The ‘pseudo-production function’*²³⁵

Robinson draws, for each feasible value of r , a determined productivity curve. Then by generalising the productivity curves for all values of r we may draw what Robinson calls the ‘pseudo-production function’²³⁶. This generalisation is done by choosing the maximum rate of (net) output at the minimum costs (real capital) for all feasible values of r . The pseudo-production function is the result of joining all equilibrium segments for every r .

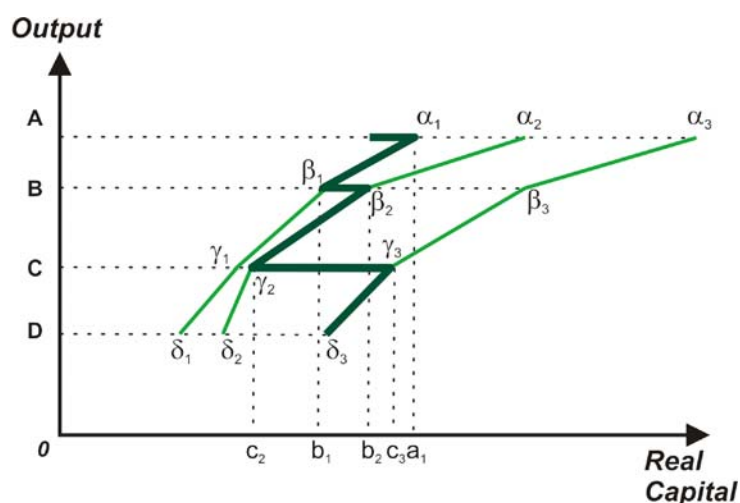


Figure 5.2: The ‘pseudo-production function’

²³⁴ Notice that the two aggregates are expressed in different *numéraire*. Yet it is not a problem, since Robinson is just measuring those magnitudes in a long-run equilibrium in order to compare the different positions on that curve at a *given* rate of interest (profits) – and hence a given wage rate. On this see Salvadori (1996).

²³⁵ Robinson (1953, 1956) also calls it factor-ratio curve.

²³⁶ Once we have got these productivity curves corresponding to the whole possible values of r we must draw tangents from every level of the wage rate onto the former curves. Any point lying both on each tangent and on the corresponding productivity curve represents equilibrium positions.

It is the author's opinion that the above analysis lacks of clarity. In fact the stretches of the productivity curves which do *not* belong to the pseudo-production function²³⁷ have no economic interpretation and so it is hard to understand them with the analytics of Robinson. The construct of the pseudo-production function crucially depends on the *relation* between one productivity curve and the next one as r changes – relation which depends on the reaction of the cost of equipment to differences r . Again, this depends, we are told by Robinson, on the “gestation period” and “length of life of equipments”. But, since in Robinson analysis the production conditions of capital goods associated with each technique are not specified, her productivity curves are quite arbitrary. In fact, Robinson in her analysis prefers to take the traditional premises for granted in that the value of capital (in terms of output) in the economy will be higher as the rate of profit falls due to the use of a “more mechanised” technique, which becomes profitable for lower rates of profits.²³⁸

To overcome the deficiency around Robinson's construction of the pseudo-production function, we can precisely use the analysis we have described in ch.3 regarding wage curves and the wage frontier (or envelop). My reinterpretation of Robinson's construct in the light of the analysis of wage curves will allow us to draw the pseudo-production function, reflecting the relation between output and capital per man, without showing points *out* of equilibrium in some stretches, which might result misleading for a reader. In fact by drawing on the analysis provided by the wage curves, which can supply us with the relationships between value of (net) output per man, capital per man, and distribution, a pseudo-production function can be rebuilt. We shall analyse two cases: *i*) a typical neoclassical case, and *ii*) ‘reswitching’ (Robinson's “curiosum”).)

i) Neoclassical case. Four techniques are considered. Every production techniques have values of productive coefficients such that $D > 0$. (*cf.* par.3.2 above)

²³⁷ It is the measurement of capital in wage terms associated with non profitable techniques at a given r .

²³⁸ Regarding the relation between one productivity curve and the next, Robinson (1953, p. 94) comments that “there is little to be said *a priori*” and she adds, on the other hand, that “it is reasonable to suppose that the most mechanised techniques are the most sensitive to the rate of interest, so that the family of curves fans out laterally [to the right] as it rises.” *Cf.* also Pasinetti (1978) p. 186 n2.

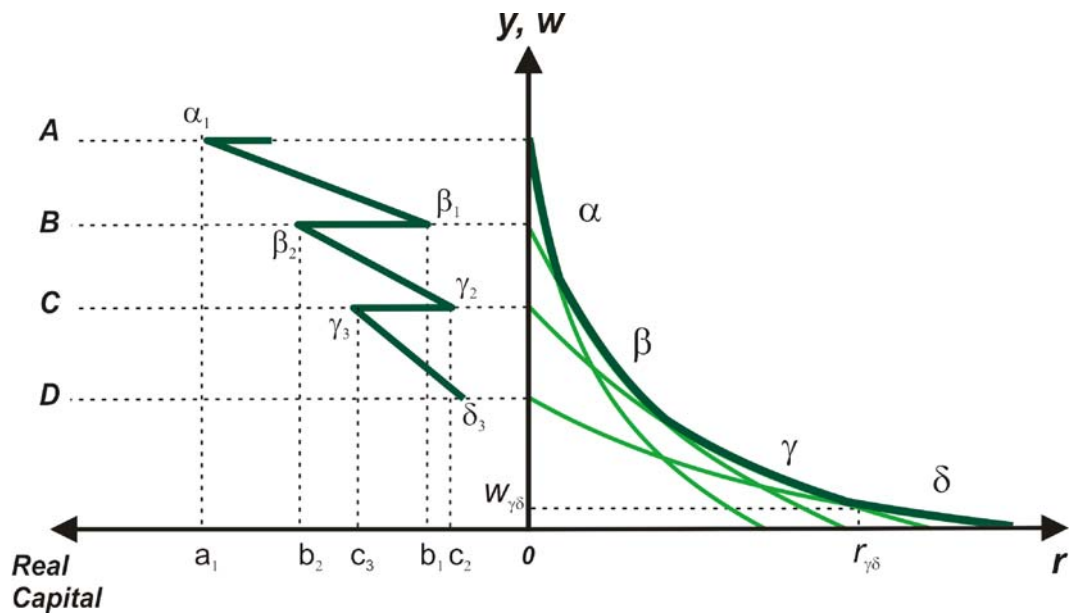


Figure 5.3: Wage curves and the “pseudo-production function” (neoclassical case)

On the right hand we draw four techniques, α , β , γ , δ . All these techniques show the neoclassical property that the value of capital in terms of output corresponding to each technique increases as r falls. This is the example with which Robinson exemplifies her analysis.

Assume that equilibrium conditions entail $r=r_{\gamma\delta}$ then we can *univocally* pass from our measure of capital in terms of output to the value of capital in wage units (real capital). We know that at that r both γ and δ are equi-profitable. At that point the value of capital in terms of output varies from the value associated with δ to the value associated with γ , which is higher than the former. As we pass to measure capital in wage units, real capital increases from the value associated with δ to the one associated with γ at $w=w_{\gamma\delta}$: that is the value in wage units is changing due to a *change in the composition* of the stock of capital from δ to γ at that given r and w . The pseudo-production function is built by joining each value of real capital to the corresponding level of output: for example the minimum value of real capital is associated with level D , and the maximum real capital at that rate of profit corresponds to C . So when we have a switch point we have *inclined stretches* in Robinson’s pseudo-production function.

As we move from right to left onto the “envelope” (from higher to lower values of r) we find that it is only γ the only profitable technique. Then, at some r just a little higher than $r_{\gamma\delta}$ the value of capital in wage units decreases as w at which γ is now

profitable is higher. For higher w , γ technique continues to be the only profitable one and therefore the value of capital in wage units decreases, thus showing a *horizontal* bar joining different real capital levels to the corresponding level of output associated with γ , that is C . So when we have dominance of a technique over the rest we have a *horizontal bar* in Robinson's pseudo-production function

At a lower r such that γ and β are both profitable, the value of capital in wage units varies from the value associated with γ to the value associated with β . Again we can see that real capital at that r increases due to the change in the composition in the stock from γ capital-goods to β capital-goods.

This analysis may be extended to lower values of r where β is the only profitable technique; then we can see that at an even lower r , α is also profitable and so on.

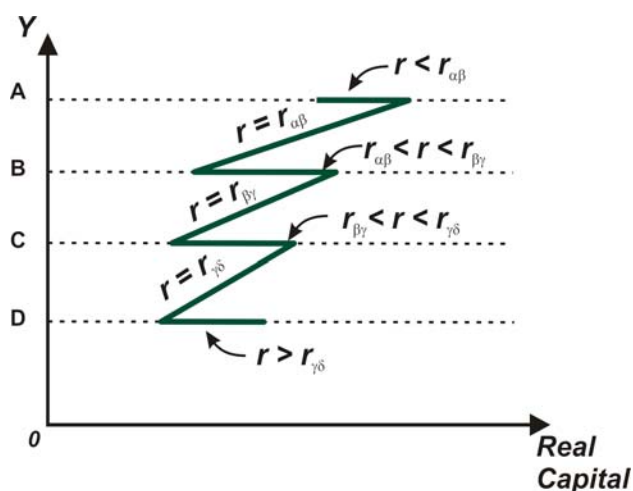


Figure 5.4: The “pseudo-production function” (from fig. 5.3)

Thus, thanks to this re-interpretation a pseudo-production function can be built by the analysis of choice of techniques in terms of the wage curves. Because Joan Robinson tried to show the equilibrium positions, the relevant curve is the following “pseudo-production function” (and not figure 2)²³⁹:

²³⁹ The important thing to note is that we need not draw the productivity curves which are *not* competitive. For example the stretches $\gamma_3-\beta_3-\alpha_3$ or $\beta_2-\alpha_2$ in figure 5.2 are not drawn on an objective basis. Robinson might have drawn them with a determined slope to show that those positions are *not* of equilibrium (and that the techniques associated with those points are not profitable) but no objective method can support that drawing, because they are arbitrarily drawn on the basis of having already assumed that a determined technique is cost minimising at a given r while others are not. The value capital goods (like value of any other produced commodity) might not have a univocal relation with changes distribution. What is interesting to notice is that
(cont.)

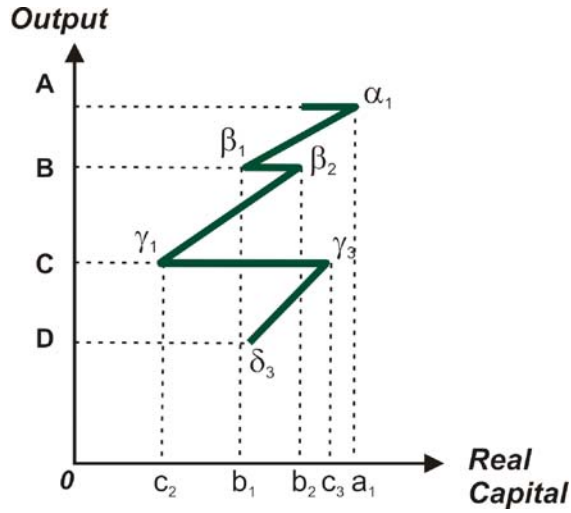


Figure 5.5: Robinson's 'pseudo-production function' amended

ii) 'Reswitching' of techniques or Joan Robinson's "Curiosum". There are two techniques such that $D_\alpha=0$, $D_\beta<0$ ($W^\alpha>W^\beta$ and $R^\alpha>R^\beta$ for every r and w)

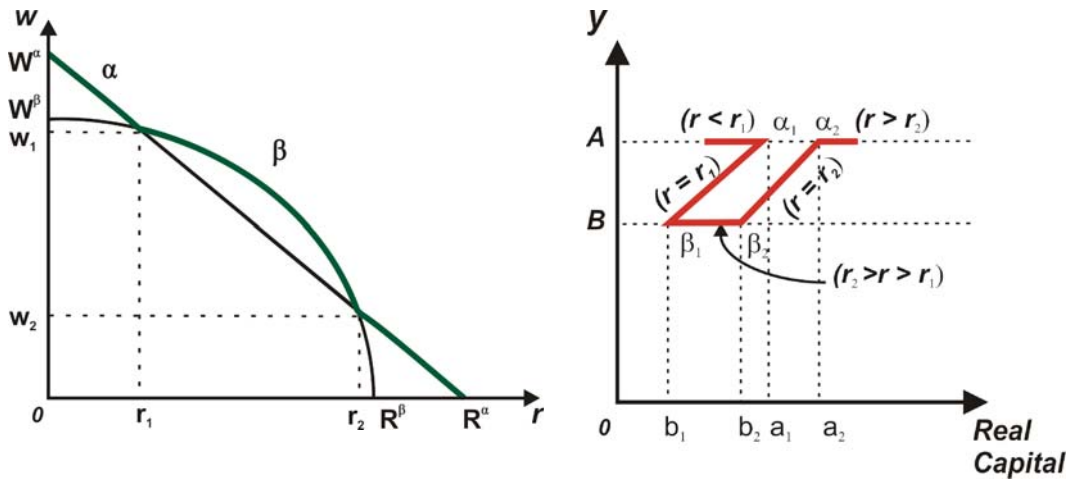


Figure 5.6 and 5.7: Wage-curves ('reswitching') and the "pseudo-production function" (from fig. 5.6)

Robinson, even until 1969 (*Accumulation of Capital*, 3rd edn) when the 'reswitching' debate had got already started, continued to use the productivity curves as her main instruments. Cf. Robinson (1969) "Diagrams" p. 427.

When $r < r_1$ α is more profitable than β and this is reflected by a horizontal bar measuring different values of capital in wage units corresponding to net output level A. Since the value of capital in terms of output when α is used is a constant value for any level of r so that as r falls, real capital falls. When $r = r_1$, we have co-existence of techniques. At that rate of profit real capital falls, provided that entrepreneurs are changing the composition from α capital-goods to β -ones. When r increases further, but does not reach r_2 yet, β is the only profitable technique. We have a horizontal stretch here representing a higher real capital as a result of dividing an increasing value of capital by a lower and lower w . When $r = r_2$ we have ‘reswitching’, since α is also profitable at this r and will continue to be so as that r increases further and further. When $r > r_2$ only α is the most profitable technique. Real capital increases as a result of dividing a constant value of capital in terms of output by a lower and lower w .

The form of the factor-ratio curve resembles a U-like line shape.²⁴⁰ This is what Robinson labelled a “Ruth Cohen’s curiosum”.²⁴¹

²⁴⁰ We could draw the pseudo-production function in terms of capital measured in terms of output by plotting in the output-capital (in output terms) diagram the values of capital and net output obtained from the wage curves. Basically, the function will have a triangle-like form. As the rate of profit rises from its minimum ($r=0$) to its maximum (R^c), points (y, k) move anticlockwise.

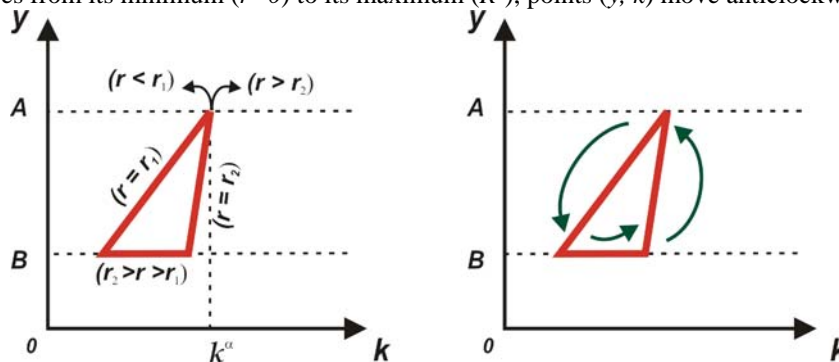


Figure 5.8: The “pseudo-production function” in terms of capital per worker in terms of output

²⁴¹ Robinson (1953) p. 106, n.1. This phenomenon, later known as ‘reswitching’, was ascribed to the agricultural economist Ruth Cohen by Robinson; however, Robinson (1970a, p. 309) later revealed that it was a “personal joke” and recognised that the critical arguments raised in her article were a “preview” of Sraffa’s (1960). Also, Robinson (*ibid.*) recognises the influence of the famous Italian economist, since the clues for her arguments had been “picked up from Sraffa’s Preface to Ricardo’s *Principles*.”

5.5. Conclusion.

The main question which emerges from our brief visit to some main criticisms raised by Robinson is: Does her particular way of criticisms point to the heart of the Cambridge controversy, as we have seen in above chapters?

What emerges from our review of some Robinson's works is that she might have chosen the aggregate production function as her main target of attack because she might probably have thought that it was a didactical way to spread the critique against this particular representation of the long period neoclassical equilibrium. In addition, her remarks reveal the intuitiveness of her arguments used to broaden the view of traditional theory by Cambridge, England.²⁴² Lastly, Robinson's arguments also reveal her revival of Keynesian economics at a time when some neoclassical scholars, such as Samuelson, Modigliani, and Hicks²⁴³, had started to incorporate Keynes within the neoclassical theoretic framework.²⁴⁴

Notwithstanding Robinson's harsh criticisms of the neoclassical production function for analysis of changes, it is hard to find in her writings an actual critical assessment of the supply and demand mechanisms which underlie the core of the theory she attempted to deny. In particular, on the relation between "capital" and the rate of interest Robinson states:

"The production function has a very limited relevance to actual problems and after all these labours we can add little to the platitudes with which we began: in country Gamma, where the road builders use wooden shovels, if more capital had been accumulated in the past, relatively to labour available for employment, the level of real wages would probably have been higher and the technique of production more mechanised, and, given the amount of capital accumulated, the more mechanised the technique of production, the smaller the amount of employment would have been."²⁴⁵

²⁴² Cf. "The student of economic theory ... is instructed to assume all workers alike, and to measure [labour] in man-hours; he is told something about the index-number problem involved in choosing a unit of output; and then he is hurried on the next question, in the hope that he will forget to ask in what units *C* [capital] is measured. Before ever he does ask, he has become a professor, and so sloppy habits of thought are handed on from one generation to the next." Robinson (1953) p. 81

²⁴³ e.g. Hicks (1937)

²⁴⁴ And this could also be appreciated by recalling the post-Keynesian literature which arose in the wake of the controversies. See Pasinetti (1962), Kaldor (1955), Robinson (1956; 1962a).

²⁴⁵ Robinson (1953) p. 100.

where the relation between “accumulation” and the interest rate is taken by the critical economist on traditional grounds. Moreover, in Robinson’s analysis it is hard to find the analytical roles of capital conceived as a single magnitude for a determination of equilibrium of supply and demand. The necessity of capital in value terms to determine a uniform rate of profits and the key role of a downward-sloping demand curve for that factor are aspects that are not discussed by Robinson, even when Robinson herself faced with what later became known as ‘reswitching’, which destroy the belief on an inverse relation between capital and the rate of interest. In fact she seemed to downplay by considering as “curiosum” a phenomenon which is general. The curiosum case is regarded by Robinson as unimportant.

“A good deal of exploration is needed before we can say whether the above [curiosum] is a mere theoretical rigmarole, or whether there is likely to be anything in reality corresponding to it.”²⁴⁶

Nevertheless this economist did not bestow much importance to this line of criticism and that her preferred way of attacking neoclassical theory reveals weaker in comparison with the arguments of ‘reswitching’ and ‘reverse capital deepening’, it was due to Robinson’s vivid intervention that later on stirred up a series of replies from the neoclassical camp. To some of those first reactions we turn in what follows.

5.6. Champernowne’s (1953) contribution.

David Champernowne’s 1953 article was the first echo of Joan Robinson’s kick-off for the debates. Champernowne’s endeavour was to provide a kind of measure of capital alternative to the one adopted by Robinson (“real capital”), because he considered her measure as “not convenient”, for two “anomalies” might arise. What Champernowne rejects, if capital is measured in “Joan Robinson units” (in wage terms), are two possible phenomena: *i*) that the same physical capital equipment may

²⁴⁶ *Id.* p. 106. In Robinson (1956) she totally dismisses the pursue of this line of critique: “[The curiosum is] a somewhat intricate piece of analysis which is not of great importance” (1956, p. 109, n.1).

have different values at two different “stationary” positions due to the *two* different values both of w and of r associated with each position, and *ii*) that the production function may not preserve a *1-1* relationship between output per capita and capital per capita. The last possibility may lead to the “paradoxical result” that the relation between the output per capita and the capital per worker might be negatively related.²⁴⁷ Champernowne wants to overcome Robinson’s puzzle in order for the production function to determine distribution.

For such a purpose Champernowne built a “chain-index” measure of capital. The “chain-index” is a device to measure the capital as an argument in the production function in such a way as to make it possible to determine the distribution of product between wages and profits through the partial derivatives of that function.²⁴⁸ To form such a construct in a historical sequence²⁴⁹, the index of capital is increased at each step of the sequence by the proportion in which the cost of capital at current w and r at the end of the step *exceeded* the cost of capital at the beginning of that step, calculated at those w and r . This construct has the apparent advantage, Champernowne believes, of removing the effects of the interest rates on the costs, and thus “paradoxical results” or “anomalies” would be prevented.²⁵⁰

But how sure was Champernowne that his device would have removed the “glaring difficulties”? As we may speculate now, in the light of ‘reswitching’ and ‘reverse capital deepening’, there is no univocal relation between the costs of capital

²⁴⁷ Champernowne (1953) p. 112. In terms of figure 2.2 above, the first “anomaly” pointed out by Champernowne can be seen in the horizontal bars of Joan Robinson’s “pseudo-production function”, where only one technique is being used but bear different values owing to changes in r and w . The second anomaly, on the other hand, occurs when to identical values of “real capital” there correspond *different* levels of net output per worker – hence the “pseudo-production function” is actually *not* a function for some intervals of real capital.

²⁴⁸ Cf. “[T]he use of [the chain index] removes the more glaring difficulties in the way of regarding aggregate output as a function of the amounts of labour and capital employed. In particular, we shall show that the rewards per unit of the factors are once again given by the partial derivatives of the aggregate production function if stationary state conditions with perfect competition are assumed.” Champernowne (*op. cit.*) p. 115.

²⁴⁹ Champernowne regard the long period equilibrium positions as “sequence of equilibria” or as historical sequences of equilibria.

²⁵⁰ Intuitively, Champernowne’s chain-index attempts to eliminate, so to speak, the “zags” (the first anomaly in Robinson’s pseudo-production function) and to diminish the slope of the successive “zigs” (the second anomaly) in Robinson’s production function. The author of this study owes the “zig-zag” analogy to Harcourt (1972, ch.1).

and changes in distribution. However, what is interesting to notice is that, in order to construct the chain-index, this author had to assume that²⁵¹:

a) if any one technique, having been the most (or equi- most) profitable technique at a given r (or range of values of r), could never reappear again at another value (or range of values); and that

b) of two techniques, which are equiprofitable at a given r , it is the one with higher output per capita and higher capital per capita that is more profitable (and hence chosen) at lower values of r .

In other words the only way to keep the value of capital per worker to move in accordance with the traditional principles, and thus achieving a one-to-one relation between output and capital per worker, is by *a priori* assuming that phenomena like ‘reswitching’ (point *a*) and ‘reverse capital deepening’ (point *b*) not to arise.

So, in general, Champernowne’s construct cannot avoid “anomalies”. In fact, he supplied examples of “exceptional cases” which may lead to “paradox results” (a triple valued production function) and points out that

“formidable difficulties arise, unless we rule out cases in which a lowering of interest rates can cause the introduction of techniques with a *lower* productivity than those used up until then.”²⁵²

Nevertheless Champernowne recognises that these cases cannot be ruled out “on logical grounds”, he states that “intuition suggests that the excluded case is unrealistic” (p. 119). It is interesting to note that it *is* “intuition” what might have led the author to believe in the basic, foundational, premise of the theory *i.e.* that “a gradual fall in the rate of interest would entail increases in the quantity of capital per head”²⁵³ namely – a belief that it has for long been taken for granted in the discipline at the time of the first blows of this controversy.

In conclusion, the ‘chain-index’ measure of capital appears to be designed only to analyse a process of capital accumulation by constructing the path of sequence

²⁵¹ Champernowne (*op. cit.*) pp. 115-6.

²⁵² *Id.* p. 130 (emphasis in the original)

²⁵³ *Id.* p. 118.

equilibria²⁵⁴ by already assuming that the neoclassical theory works in the “correct” way; it does not, however, seem to address the *basic question* on which the issues of the measure of capital turns: whether or not the marginalist supply-and-demand functions provide a logically valid explanation of prices and distribution.²⁵⁵ On the contrary, this construction as well as the so-called aggregate production functions, assumes the explanation in terms of supply and demand to be *valid*.²⁵⁶ Finally, it is worth noticing that Champernowne, like Joan Robinson, did not give his “unrealistic” case (that is Robinson’s Ruth Cohen’s *curiosum*) much importance.

5.7. Further replies to Robinson: Solow (1955, 1956) and Swan (1956).

Besides Champernowne’s response to Robinson’s first complaints are two further authors from the neoclassical side who contributed to the debates in the first interchanges. Anyway the attempts of these responses have not been to solve for the problems that Robinson had raised, but merely to dismiss them altogether.²⁵⁷ We consider here Solow (1955 and 1956) and Swan (1956).

Solow (1955) considers that the process of production can be visualised as a process divided into two stages: first, an “index” quantity of capital, K , can be a function of the physically heterogeneous capital goods, say, C_1 and C_2 . Secondly, the consumption good, Q , is produced by means of labour and the services of the several

²⁵⁴ As Champernowne calls the different long period equilibrium positions of the economy.

²⁵⁵ It might be also pointed out, as Petri (2000, p. 31) has suggested, that Champernowne’s chain index assumes that changes of technique involve the rate of profit stopping at a value in which two techniques are equiprofitable. But changes of techniques, in disequilibrium adjustment process, do not necessarily involve that the change in the rate of profit has to stop at the level in which two techniques are competitive. And, what is more, when economies change techniques (production systems), the “form” or kind of capital goods will in general change, and so will the value of capital. Hence the chain index is not a “reliable measure of the quantity of capital”, since the theory needs this “factor” susceptible to changing form but keeping the quantity unaltered.

²⁵⁶ On the issue of capital as an argument of the production function, Garegnani (1960) has pointed out: “[If the] aggregate of capital goods is measured so as to appear as a variable ‘quantity of capital’ when distribution changes, it will no longer be possible to establish a *univocal* relation between the quantity of capital and the physical quantity of the product. And it turns out to be so, after measuring capital in value terms: given the quantity of the other factors of production, both the same technique and the same physical quantity of output would be compatible with more than one quantity of capital.” (p. 85, emphasis in the original, my translation.) Once a production function is so built, its partial derivatives, which *should* have resulted in the ‘marginal products’, are however an *entirely different construction* from the traditional meaning given to the ‘marginal products’ of the theory, which assumes a physical measurement of all factors.

capital goods through function F . However, if the first function can be logically built, then Q will be a function both of labour and of “index” capital. Solow justifies this procedure by resorting to Leontief’s weak separability condition (the substitution rates between the factors that one wishes to aggregate – here C_1 and C_2 – must be independent of the quantities employed of the other factors, L), which is necessary and sufficient for the representation of the production function of the form $Q=F(L, C_1, C_2)$ into the form $Q=H(L, \phi(C_1, C_2))=H(L, K)$, if $K=\phi(C_1, C_2)$. However, there are two points worth noticing: first, *a priori*, there is no justification why C_1, C_2 should be services of heterogeneous capital goods. Rather, they may stand for amounts of original factors such as (different types of) labour or land both measured in physical units. So this is sufficient to say that Solow’s attempt does not answer to Robinson’s problems. Secondly, in terms of real economies, it is very unlikely to find situations such that the marginal substitution rate of C_1 and C_2 are independent of the quantity of labour. In fact Solow himself commented on this: “[the condition of independence of the marginal substitution rate from the quantities employed of the other factors] is not satisfied [e.g.] for one-ton trucks and two-ton trucks; technical substitution possibilities will depend on the number of drivers available.”²⁵⁸ However, Solow believes that “there is a whole class of situations in which the condition may be expected to hold ... For example, we can imagine that C_1 and C_2 to be two kinds of equipment for generating electricity which is then used in further production. [E]lectric power would be an index of the capital inputs” (*ibid.*). To conclude. Solow’s attempt is nothing but a clear-cut defence of the validity of the marginalist propositions. Nevertheless, for such a defence, he cannot overcome the difficulty entailed by the fact that any index of capital depends on changes in distribution.²⁵⁹ Moreover, and more important, this relation between the index and changes in distribution is not univocal.

²⁵⁷ To dismiss the problem that distribution cannot be determined by the marginal productivities of the factors. Solow dismisses it by considering the problem raised by Robinson as “antiquated”, see Solow (1955) p. 101 and Robinson (1955) p. 247.

²⁵⁸ Solow (1955) p. 103.

²⁵⁹ Actually, in the article, Solow (1955, p. 106) recognises that his “quantity-of-capital-index” is also the result of the transformation of a “price index”. Certainly, in the literature, we might come across many instances in which “prices and quantities indices come in pairs” (*ibid.*); cases like this are known as the so-called ‘duality problems’. Solow (*ibid.*) comments: “Apart from the comforting thought that at this late date no one should be surprised to find price-quantity dualities, I have no explanation to offer”. But the problem is well another one: if capital is expressed in
(*cont.*)

One of the most known works from the neoclassical side that indicates that the first interchanges much centred around aggregate production functions has been Solow (1956). In that work Solow represents an economy that produces a single output by means of itself and homogenous labour. The commodity produced can be used either as consumption good or as an input of itself. It is assumed continuous variability in the proportions of the factors necessary for producing the output. Full employment of labour is guaranteed, provided a sufficient flexibility of the wage in perfect competition. In fact, since it is produced only one commodity there is no indirect mechanism of factor substitution. The only mechanism available therefore is the direct one: through technical conditions of production. Since technical conditions are assumed to be variable, “capital” can always adapt itself as to employ the whole quantity of labour of the economy. Savings determine investment: the consumption forgone is saved and hence to be added to the stock of capital. Savings, investment and capital are all measured in the same technical units. (There is, under these restrictive assumptions, no problem regarding the irreversibility of investment – another of Robinson’s concerns in the 1953 article).

Of course, there is no problem regarding the measure of the marginal productivity and of the measure of capital. Robinson’s concerns about capital are plainly dismissed. We are in an economy just as the same as the one we have described at the beginning of this work in ch.2: the only way to save the neoclassical propositions, in economies with heterogeneous capital goods as between techniques of production for the consumption good, is to assume that the output, actually composed of heterogeneous goods, is produced *as if* it were a single commodity which is produced by itself and labour. His belief in aggregate production functions ultimately relies on the acceptance of the basic premises of the theory.²⁶⁰

value terms (and it has to be so, if we keep in the long period tradition), there is of course no guarantee to “justify [Solow’s] statement: there is a perfectly definite and consistent sense in which it can be said that the relative factor price ratio P_K/P_L is a decreasing function of the ratio of capital to labor K/L ” (1955, p. 105).”

²⁶⁰ In Solow (1957), where he attempts to apply the (1956) model to US data for the period 1909-1949, his belief in aggregate production function (and hence in neoclassical theory) is expressed in the following way: “Were the data available, it would be better to apply the analysis to some precisely defined production function with many precisely defined inputs. One can at least hope that an aggregate analysis gives some notion of the way a detailed analysis would lead.” Solow (1957) p. 312, n.1. We have, however, seen that even when the heterogeneous capital goods are measured in technical units according to the different techniques for producing the consumption good, there is no guarantee that capital employed in the economy (gross investment of full-
(cont.)

Swan (1956)

Swan is concerned with Robinson's notion of capital as a factor of production which is accumulated over a period of time by savings. In his long appendix to that article, he regards capital as "the number of meccano sets". This "meccano" can adapt itself instantaneously and without costs to any form, according to economic profitability and technical progress. This attempt too avoids the irreversibility of investment.²⁶¹

Swan assumes the existence of more than one commodity, but he assumes that *prices are constant* when distribution changes. This is not very different from Solow's (1956) model: the only way to assume that prices do not change when distribution does, is by assuming an economy where the commodities, which compose the output, are produced *as if* it were a single commodity produced by itself and labour – that is Solow's model.

Swan affirms that his device does not solve for Robinson's problems; instead, they can directly eliminate them. That is to say, Swan takes the assumption of a "malleable"²⁶² capital ("meccano" sets) as a "scarecrow" assumption in order "to keep off the index-number birds and Joan Robinson herself". Nevertheless, this author considered that, due to the assumptions on constant relative price (including that of the capital goods "meccano"), accumulation – the level of which is, in equilibrium, equal to savings and investment²⁶³ – will exactly be the current output of "meccano" sets, and accordingly can *always* be measured in terms of any kind of consumption good.

On the other hand, Swan agrees with Robinson insofar as she was complaining that the neoclassical tradition contains no indication of how a technical unit may be devised in order to measure capital, but, however, he concludes: the neoclassical theory, when aiming at explaining the marginal variations about the stationary position, does not depend on the existence of such a unit in which measure 'capital',

employment savings) be an inverse function of the rate of interest. (*Cf.* the discussion of Samuelson's "surrogate" function in ch.4 above.)

²⁶¹ Swan (1956) p. 344.

²⁶² *Id.* p. 343. Similar other names were used during this period in the literature concerned with economic growth models to refer to "capital" in aggregate production functions such as *steel, leets, butter, ectoplasm.*

²⁶³ *Cf. Id.* p. 349.

because the “natural unit of Capital is simply an equilibrium dollar worth regardless the physical variety of capital goods.”²⁶⁴ This last statement can hardly be satisfactory, because of the problems that we have already seen when capital in value terms is considered as argument of production functions. (*Cf.* ch.2 above.) And also, because the original meaning of marginal productivity is related to the description of the methods of production of the consumption good –measured in physical terms– according to the different combinations of the factors of production which must be measured in physical terms too. It is by drawing on this, that we can derive the direct factor substitution mechanism as a base for an explanation of the equilibrating forces of supply and demand.

To conclude on these authors we may say that the only way that neoclassical economic growth models can be rigorously formulated is in economies which produce either a single commodity by means of itself and labour (Solow) or a composite good redistributed without costs as between the different uses (Swan). Outside these instances, the shortcomings entailed by the dependence of the value of capital upon distribution cannot be cancelled –nevertheless Robinson trifled the role of a quantity of capital as a determining one for the definition of the supply-and-demand functions for productive factors.

5.8. *Robinson rejoinders.*

On Solow: Robinson does of course not go along with Solow’s view. For her, the economist from Massachusetts does not deal with the problem of capital. She correctly asserts that his endeavour to collapse “ C_1 and C_2 ” into K is rather a way to represent heterogeneous natural resources into a single index and not a way to represent capital goods because “nothing is said about the time which it takes to produce them.”²⁶⁵ In her reply to Solow (1955), as well as in her further interventions

²⁶⁴ *Id.* p. 350. Were a technical unit measure of capital existed, the marginal product of capital, measured in that unit, would be equal to the rate of profit multiplied by the price (in output terms) of a unit of capital (in technical units). But, since Swan assumes that its price does not change, capital could be also measured in value terms, in which case its marginal product equals the rate of profit. However, this does not overcome one of the criticisms raised by Robinson, which concerns the nonsense of talking about marginal productivity in equilibrium, whereas it is only in equilibrium that, Robinson assumes, capital can be aggregated.

²⁶⁵ Robinson indicates how the issue may be considered: “[C]ompare two machines exactly alike in all respects except that one is drawn from an economy with a higher product-wage rate. The
(*cont.*)

in the debates²⁶⁶, there is no reference to the problematic attempt to derive inverse functions between factor's remuneration and factors' (demanded) quantities when capital is measured in value terms –inverse relationships that Solow takes for granted.²⁶⁷

On Swan: Robinson replied to Swan in 1957.²⁶⁸ As to the question of *not* having considered a single-commodity output as Solow did, but an output composed of many goods, Robinson does not accept the assumption that prices can be kept constant in Swan's model, since, if distribution varies, prices will vary too: "Swan must come off the fence, and say which *kind* of good is taking as numeraire."²⁶⁹

On the other hand, Robinson correctly accuses Swan of thinking of "the wage rate as being determined by the physical marginal product of labour, which in turn is determined by the amount of meccano in existence" without, however, stating the units which it is a quantity of.²⁷⁰ Therefore she considers Swan's way as unsatisfactory to answer the questions which she had raised in (1953). According to Robinson, equilibrium is obtained, for a given marginal productivity of labour, if the conditions of the *amount* and *form* of capital, the rate of investment, and the propensity to save are such as to make the wage rate equal to that marginal productivity. This is the kernel of her reply; a reply relying on Keynesian principles about investment and accumulation.

In this chapter we have seen the particular characteristic that the debates took during the first interchanges between the two Cambridge. We have seen that discussions seemed to be related to find a measure of aggregate capital in order to be considered into aggregate production functions, which would in turn determine distribution. Swan's attempt was to consider the malleability assumption (his

value of the two machines is different, and the investment required to create them is different. A difference in value remains if we deflate them by the wage rate (obtaining capital in labour time), for in two economies with different wage rates the rate of profit and the rate of interest are different." (1955) p. 247. Solow (1955) does not address his answer to that issue.

²⁶⁶ In particular *cf.* Robinson (1962b, 1970a, 1971a, 1971b, 1974, 1975a)

²⁶⁷ Solow (1955) p. 105

²⁶⁸ See Robinson (1957)

²⁶⁹ *Id.* p. 104 (emphasis added). *Cf.* Samuelson's very special case in ch.4.

²⁷⁰ *Id.* p. 105. Robinson scathingly concludes her reply to Swan in the following way: "My scheme of analysis [i.e. in *Accumulation of Capital*, 1956] is intended to make it possible to discuss accumulation in spite of the fact that capital cannot be measured in physical units. ... It is not really my fault that capital does not consist of pieces of meccano." (1957) p. 108

“meccano” sets). Solow, instead, plainly regards social output *as if* it were a single commodity produced by itself and labour. Champernowne’s attempt does not overcome the anomalies for which the ‘chain-index’ was built. In sum, in the first reactions to the issues raised by Robinson the analytical problems connected with the notion of capital for the marginal theories are still unclear, and that is why we have introduced them in advance (chs 3 and 4) without following the chronological path of the many issues debated in the Cambridge controversies. From a historical standpoint, our brief review over these first exchanges indicates us that participants in the controversies had to await Sraffa (1960) from the critical side and Samuelson (1962) from the neoclassical one, in order to find more clarity in the issues at stake, as showed in ch.4.

The following chapter restarts by discussing a new phase in the capital debates, which was initiated, as I shall argue, by some neoclassical participants soon after the most salient results analysed in ch.4 were acknowledged. In particular we shall focus on the *way* in which these advocates of dominant theory received the critique; a preview of that way was indicated in ch.4 too, when Bliss (1970), in his reply to Garegnani (1970a), drastically changed the background on which the critique had been conducted (*cf.* par.4.12 above.)

CHAPTER 6

A second phase in the capital controversies and radical changes in the neoclassical theory

6.1. Introduction

In the previous two chapters we have seen the main discussions of the first phase of the Cambridge controversies. In particular attention has been focused on the results of ‘reswitching’ and ‘reverse capital deepening’, the implications of which touch the basic premises of the neoclassical theory of production, value and distribution (chs. 3 and 4). The neoclassical reactions reviewed have evinced reluctance to admit the consequences of the debates: advocates of the marginal theory have tried either to find out the ‘conditions’ to rule out ‘reswitching’ or to minimise their relevance by pointing to the empirical issue. However, the first phase of the controversies clearly shows that phenomena of ‘reswitching’ and ‘reverse capital deepening’ are theoretically irrefutable as some neoclassical contenders were compelled to admit.

In the aftermath of the Symposium and related contributions, dominant theory increasingly started to rely on a different notion of capital and thus of equilibrium.²⁷¹ This indicates that a different phase in the controversies began. In fact, the main characteristic of this second phase is the fact that almost the majority of the core of neoclassical theorists had already *embraced* the path provided by Hicks’ (1946[1939]) *Value and Capital*, who incorporated the Walrasian specification of capital into the supply and demand theory. The history underlying the incorporation of Walrasian capital cannot be developed in full detail in this study. However, by relying on related works²⁷², we have to bear in mind that both before and during the Cambridge controversies theoretical work started to be further developed by

²⁷¹ Cf. par.4.12 above.

²⁷² Garegnani (1976a), Petri (2004, ch.5).

neoclassical economists – the most of them of mathematical inclination²⁷³ – and thus a different version of the marginal theory was, so to speak, available to those willing to use it by renouncing to the traditional notion of capital, but still keeping the supply and demand approach intact. After the repercussion of results in the first phase, which no doubt put the dominant theory at stake, neoclassical participants started to argue against the critics in terms of this version of the theory in order to avoid the problematic issue of capital, by precisely relying on a physical conception of capital. But this shift, as we shall see, represent a radical change in the nature of the theory.

The aim of this chapter is to discuss the way in which the neo-Walrasian theorists received the critique of capital, attempting at the same time to clear up some misunderstandings created in this phase which, as we shall argue, might have damaged both the relevance of the capital debates and the communication between the two sides. For this purpose we deal in the following three paragraphs with Walras's treatment of capital as the collection of physically heterogeneous goods and the inconsistency with the traditional conception of equilibrium (6.2), the suggested way-out to that inconsistency (6.3) and the implications for the approach of adopting the Walrasian treatment of capital (6.4). Then we return to the historical aspects of the controversies in this second phase (6.6 and 6.7).

6.2. Rehabilitation of Walras's treatment of capital as physically heterogeneous goods and Walras's inconsistency.

Because the modern versions of the supply and demand theory rely on a Walrasian treatment of capital we shall here briefly deal with Walras's theory of capitalisation.²⁷⁴ As is well known Walras treats capital as the physically heterogeneous capital goods' vector. Then every single capital good (from ploughs to screws, from lathes to tweezers, etc.) is taken as a "factor of production" in the same footing as "labour" and "land".

²⁷³ *e.g.* Debreu (1959), Morishima (1969).

²⁷⁴ We shall henceforth refer Walras's *Eléments* to the 4th and definitive edition of 1900 translated by Jaffé, and published in 1954. (Actually there was a 5th edition in 1926 that was strictly alike the 4th). In this par. we follow Garegnani (1958, 1960, 1962, 1990).

As has been analysed²⁷⁵ Walras general equilibrium system would admit solutions when capital services are considered as original factor's services. However these solutions do not take into account the fact that capital goods are reproducible and therefore would only refer to exchanges and production without capital.

To be coherent with the fact that capital goods are reproducible Walras introduces their production in the analysis and hence the equilibrium condition of uniformity of rates of returns on capitals' supply prices – implied in the equality signs of the price equations of those goods. However, as soon as production of capital goods is introduced into the general equilibrium system, problems as to economically meaningful solutions of the system arise.

In order to deal with the production of the capital goods Walras introduced the price equations of each capital good (k, k', \dots) reflecting the costs of production (supply prices), which, in equilibrium must equal the demand prices of the capital goods (which is the capitalisation of the service prices p_k). Note that satisfaction of this condition imply determining a uniform rate of return on the capitals' supply prices, i – which is crucial to the whole analysis. On the other hand, there will annually emerge out of the annual output a flow of savings ($F_e(.)$) which must be equal to the investment flow namely – valued quantities of the new capital goods produced ($D_k P_k + D_{k'} P_{k'} + \dots$). All these conditions can be summed up in the following two blocks of equations.

$$F_e(p_p, \dots) = D_k(k_i p_i + \dots + k_p p_p + \dots + k_k p_k + k_{k'} p_{k'} + \dots) + D_{k'}(k'_i p_i + \dots + k'_p p_p + \dots + k'_k p_k + k'_{k'} p_{k'} + \dots) \quad (Y)$$

(Note that the total amount of “capital” *i.e.* the savings (investment) is *endogenously* determined; see equation Y)

$$\begin{aligned} \frac{P_k}{i + c} &= k_i p_i + \dots + k_p p_p + \dots + k_k p_k + k_{k'} p_{k'} + \dots \\ \frac{P_{k'}}{i + c'} &= k'_i p_i + \dots + k'_p p_p + \dots + k'_k p_k + k'_{k'} p_{k'} + \dots \end{aligned} \quad (Z)^{276}$$

²⁷⁵ Garegnani (1958; 1960; 1962; 1990). Cf. also Eatwell (1990).

²⁷⁶ Equations in (Z) are the result of equalising supply prices $P_k = k_i p_i + \dots + k_p p_p + \dots + k_k p_k + k_{k'} p_{k'} + \dots$; $P_{k'} = k'_i p_i + \dots + k'_p p_p + \dots + k'_k p_k + k'_{k'} p_{k'} + \dots$; (cont.)

where $k_t, k_p, k_k, k_{k'} \dots k'_t, k'_p, k'_k, k'_{k'} \dots$ are the quantities of productive services necessary to produce k, k', \dots

(Note that the RHS of equations in Z reflect the costs of production of the several capital goods.)

Though formally this system has solution it is now interesting to see whether the system admits economically meaningful results.

The issue which concerns us here is the economic meaning attached to equations (Z) because these equations establish the equilibrium condition of equality between demand and supply prices of the capital goods produced – hence uniformity of rates of return on the supply prices of the capital goods. Then, to understand the meaning we may consider two cases. First, assume that consumers' tastes led up the yearly gross savings to zero. This being so, according to (Y), we will see that gross investment in k, k', \dots , represented in (Y) by $D_k, D_{k'}, \dots$, must be all zero, since they cannot, of course, be negative amounts. In this case the system is over-determined: in general Walras's capital formation system is inconsistent and hence has no solution.²⁷⁷

This case, which considers gross savings zero, would however be sufficient to argue that there is no general solution to that system because, if a theory endorses the prestige of being general, it must account for general cases, including also the case in

etc. and demand prices $P_k = \frac{p_k}{i + c}$; $P_{k'} = \frac{p_{k'}}{i + c'}$; etc.; where p_k indicates the *gross* price of the capital good k (the price from which a *net* price is obtained after subtracting the amount corresponding to depreciation and insurance, both expressed by Walras as fraction $c > 0$ of P_k – the price of the *new* capital good). We are not going to discuss the incorrect way in which Walras treats depreciation of fixed capital since his formula does not take into account the role of interest upon the very depreciation through time. For a clarification in this regard see Garegnani (1960, part II; 1990, p. 14); Sraffa (1960, ch.10). For the purposes of our present discussion we will use Walras's original formula. Up until the 3rd edition of *Eléments* Walras regarded the raw materials as being consumed as soon as they were produced and not stocked in advance. In the 4th edition circulating capital is treated in a strictly similar way as fixed capital thus assuming for these capital goods a value of $c=1$.

²⁷⁷ Cf. Garegnani (1958, part II; 1962). If a *uniform* rate of interest were determined under savings being zero it would be a very particular case, where the configuration of the relative scarcities of the several capital goods taken as data should have to be appropriate so as to be in accordance with the consumers' tastes and preferences which had determined the prices of the productive services of the capital goods. But in general the arbitrariness of the initial configuration of the capital goods deprives their quantities from being the appropriate ones determined by consumers' conditions.

which savings happen to be zero. However it is interesting to consider the case when savings are positive.

The only possible way that the rates of return be uniform throughout the system lies in fact in the influences of the quantities currently produced of capital goods on the prices of the capitals' productive services. To show as clear as possible that in this more general case (positive savings) there does not exist economically meaningful solutions either, let us begin by imagining that, owing to the arbitrariness of the endowment supply of, say, two distinct capital goods, k and k' , $i' > i$ in (Z) so that we have one more unknown.²⁷⁸ To re-establish equality between number of unknowns and independent equations let us further assume the following relation:

$$R = \frac{D_{k'}}{D_k}$$

Since we assume $i' > i$ free competition of capitals' bearers would make R increase so as to bring about effects on both the costs of production of the capital goods *and* the productive services' prices. Of course those effects will crucially depend on the *relative proportions* of the capitals' productive services in the production of the very capital goods.²⁷⁹ Since those proportions differ, a change in R will make relative scarcity of capital goods change, and with it the productive services' prices and hence the rate of return on the costs of production.²⁸⁰

A rise in R will cause a rise in the costs of production of k' . Hence to re-establish equilibrium, and by keeping constant the gross prices of both capital goods, i' will tend to fall while i will tend to rise, so that the initial disparity between both interest rates will tend to *narrow*.

²⁷⁸ These transformed equations we call (Z'), see below.

²⁷⁹ The particular case in which those proportions might happen to be equal is devoid of significance here, for the effects referred to above would not exist at all (the relative scarcity of capital goods would *not* be modified as R changes). Cf. Garegnani (1962)

²⁸⁰ Let us see this point analytically:

$$\frac{P_k}{i + c} = k_i p_i + \dots + k_p p_p + \dots + k_k p_k + k'_k p_k. \tag{Z'}$$

$$\frac{P_{k'}}{i' + c'} = k'_i p_i + \dots + k'_p p_p + \dots + k'_k p_k + k'_{k'} p_{k'}.$$

On the other hand the effect of a rise in R on the prices of the productive services of the capital goods is *not* univocal. In particular the *sign* of those effects on the referred prices will crucially depend on the technical conditions of production characterising the capital goods. If, in our hypothesised economy, the productive requirements of k' (relatively to k) are higher in the production of k' than in the production of k , then an increase in R – an increase in production of k' relatively to k – will make the relative scarcity of k' rise (because k' needs “more” of itself than of the other capital good – this is the *auto-intensive* case $k'_k/k'_k > k_k/k_k$) hence the price of the service associated with k' will tend to rise as well. If this is so, then the original disparity between both rates of interest will tend to *widen*.

For the sake of completeness we can see that in the opposite case to the one referred above namely, the *hetero-intensive* case ($k'_k/k'_k < k_k/k_k$), the initial disparity in the rates of interest will tend to *narrow* as we make R rise. Still this compensation is not guaranteed, because the initial disparity between the different rates of interest might be of *any width*, since it depends on the arbitrary *given endowments of capital goods* which are at disposal of producers in the economy. The ultimate result is therefore totally uncertain.²⁸¹ Thus, we have a further negative element in Walras system which undermines the equilibrium conditions he set out there – equilibrium conditions in accordance to the long period equilibrium method or normal position. In our example, we have assumed an increase in R – one may even imagine that rise further and further to the point where all the gross savings are invested in k' alone while k approaches zero. At that moment, as a matter of fact, the difference between both interest rates can not be reduced²⁸² since the economy is run out of savings to be invested in k . That the capital good k then would *not* be (re-)produced has important implications that deserve our attention. (See par. 6.3 below.)

To conclude, the Walrasian system is prevented from having an economically meaningful solution and hence fails in its purposes.²⁸³

²⁸¹ Cf. Walras (*op. cit.*) p. 292 where he analyses the effects of the services prices of the capital goods on the costs of capital goods to be produced and their services' supply. Cf. also Garegnani (1962) p. 19.

²⁸² Formally one can get a solution with, say, D_k being negative; but since D_k is gross investment it would be totally meaningless, in economic terms, to account for some “negative” investment.

²⁸³ In the words of Collard (1973, p. 472): “If there is production of new capital goods [in Walras system] the approach is open to the charge of sterility [because] almost anything might happen.”

6.3. Walras's way-out to the inconsistency.

As is well known, Walras did not limit himself only to count the number of unknowns and equations in order to seek economically meaningful solutions of his system. In fact in the part regarding capital formation in the 4th edition of the *Eléments* Walras somehow went by intuition that equations (Z) might undergo some modifications, namely – to substitute the “=” symbol for inequality “≤”.²⁸⁴ Before discussing the meaning of such formal changes it is important to notice that in Walras's system the *data* of the equilibrium were assumed *not* to change as the time-consuming process of adjustment *towards* equilibrium was taking place. This can be noticed when he describes the *tâtonnements* process (to grope towards equilibrium by trial and error):

“Our problem is to reach equilibrium in capital formation *ab ovo* (...) we propose to start by assuming the arbitrary data of our problem to be constant over a certain period of time, and subsequently we shall suppose them to change in order to study the effects of such changes.”²⁸⁵

For Walras it is *crucial* that the data of equilibrium do *not* change over the course of the trial and error actions carried out by agents; that the *tâtonnements* process must be seen as an analytical device for an analysis of an equilibrium is clear when Walras introduced the so-called “tickets” (*bons*) in the 4th edition so that they could allow for transactions to be provisional until equilibrium is reached. But if this is so, namely, if the purpose of Walras was precisely to determine a long period equilibrium (normal

²⁸⁴ See Walras (1954) pp. 286-95.

²⁸⁵ Walras (*op. cit.*) p. 282. Interestingly enough, in lieu of “over certain period of time [pendant un certain temps]” (as written in the 4th edition) we read, in the 2nd edition, “pendant tout le temps que dureront nos tâtonnements” [for the entire period of time that the *tâtonnements* will last; (my translation)]. If we regard the *tâtonnements* as a purely analytical device for an analysis of solutions which were to be stable, it is clear that the data of equilibrium could not change drastically in the very process of time-consuming adjustment towards equilibrium. If doubts, reasonably, still remain, it is worth reading some further words from Walras, which are full of significance in this regard: “Thus equilibrium in capital formation will first be established *in principle* [through the tickets]. Then it will be established *effectively* by the reciprocal exchange between savings to be accumulated and new capital goods to be supplied *within a given period of time*, during which *no change in the data is allowed*. Although the economy is becoming *progressive*, it remains for the time being *static* because of the fact that the new capital goods play no part in the economy until later in a period subsequent to the one under consideration” (pp. 282-3; emphasis in the original).

position) then it is – at least – not clear the suggested way-out put forth by Walras himself in the 4th edition too, as to (meaningful economic) solutions of equations (Z).

In fact alteration in (Z) would imply that when the inequality sign applies then the production of the capital good(s) concerned will be *zero* because the rental gained from investment in such capital good(s) will fall *short* of rentals obtainable from investing elsewhere. *Then* as Walras indicates:

“all the remaining equations of system [Z] will be satisfied after the exclusion of those new capital goods which it was not worth while produce.”²⁸⁶

So Walras seems to suggest that a solution to his system – namely to determine a long period equilibrium of supply and demand in free competition – can be obtained by only regarding those capital goods which can be (re-)produced thus determining the rate of interest at its highest level.²⁸⁷

The fact that a capital good’s demand price falls below its supply price (costs of production) actually means that that capital good is *not* to be then produced. The consequences of this change cannot be overlooked: it will of course entail a change in the composition of the (initial) capital stock of the economy, thus implying changes in prices and quantities in the economy throughout; hence in that process – of groping towards equilibrium – *changes* in the quantities of capital goods will be in contradiction with the *assumed* endowments of capital goods which the theory takes as *given*. Thus the equilibrium reached would not therefore bear the necessary *persistence* that it is however required so as to regard it as a gravitational centre of actual variables.

²⁸⁶ *Id.* p. 294.

²⁸⁷ Walras is coherent when he excludes from production the capital goods whose demand prices would fall below their supply prices. In fact that is the way of the traditional method which, by conceiving of capital as a single magnitude, could endogenously determine the composition of such a stock. Walras keeps the system within the method of the long period equilibrium characterised by a uniform rate of return on costs – and this is what is done when system (Z) is modified by letting only those capitals whose supply and demand prices equal at the highest rate. However, the problem is that the notion of capital as the physically heterogeneous vector as a *given* of the problem is in *contradiction* with that method, which Walras of course shared with all his contemporaries peers such as Böhm-Bawerk, Wicksell, Marshall, Clark. *Cf.* Garegnani (1990) pp. 20-2.

6.4. Implications of the Walrasian specification of capital.

After the results of the first phase of the Cambridge debates, the theory started to increasingly rely on a Walrasian conception of capital.²⁸⁸ The latest versions of the neoclassical theory, which have become dominant since comparatively recent decades²⁸⁹, have tried to overcome Walras's inconsistency by *dropping* the equilibrium condition of uniformity of rates of returns on capitals' supply prices.

But, now, relying on a Walrasian treatment of capital and dropping the condition of the uniform rate of profits have drastic implications for the method of the theory itself. In fact this new notion of equilibrium could no longer refer to the normal positions of the economy, because the equilibrium variables are prone to rapid changes owing to any initial disequilibrium situation such that, when groping towards equilibrium, some of the data will also change, and therefore it would be hard to conceive that equilibrium as a gravitational centre taken by agents in their groping towards equilibrium.

This leads us to the problems of lack of *persistence* of the data of equilibrium. In fact, that lack might prevent the repetition of the markets – transactions – which are however enormously necessary in order to ascertain gravitational centres such that deviations between theoretical and observable – trends or averages in time – variables could be compensated each other. Moreover, the datum of capital in the definition of equilibrium by the theory relying on a Walrasian capital is the physically heterogeneous vector, which is arbitrarily given and hence will not allow the necessary *substitutability* among the capital goods when time-consuming adjustments take place so as to determine their amount required by equilibrium.

Thus, in the wake of the results drawn from the Cambridge controversies, marginal theory abandoned the notion of capital in value terms and adopted the Walrasian specification of capital; however to keep analysis within the supply and demand framework the theory was compelled to *abandon* the long period equilibrium method,

²⁸⁸ It is a misfortune, as Bellino (2005, p.24) and Petri (2007, p.597) have remarked in their respective reviews, that the recent 3-volume collection of works on Capital theory edited by Bliss, Cohen and Harcourt (2005) does not include anything about Walras's theory of capital, important as it is for understating the contemporary versions of the neoclassical theory.

²⁸⁹ As Garegnani (1976a) has suggested the restoration of Walras's conception of capital was not new, since the path adopted after the controversies had already been laid by Hicks (1946[1939]), who proposes to dispense with the notion of capital in value terms because that notion would have only considered steady states. Cf. also the discussion of par.1.4, ch.1 above.

rendering the very theory of doubtful results, as Garegnani (1976a) has showed in his assessment of the results of the controversies. (*Cf.* Harcourt's views, ch.1 above, on the shift to Walrasian theory: a "more disaggregated" analysis, which may emerge "logically intact" from the capital debates).

The point which must be clear is that this new notion – short-period – of equilibrium has to do with capital, and *not* with the long-period method itself. (As Garegnani pointed out in (1976a); *cf.* par.1.4 in ch.1 above.)

Hence the Cambridge debates are relevant to the current theory because it forced the latter to a theoretic apparatus of very doubtful results for explaining actual economies. The controversies put in doubt the whole approach and compelled the theory to adopt a short period general equilibrium method, while at the same time modern marginalist authors tried to reverse the sign of the critique by pinpointing that "general equilibrium does not require aggregation".

Whether or not neo-Walrasian theory is free of capital theoretic problems is an issue we shall deal with in par.6.7, although provisionally; but now we attempt to clear up some misunderstandings raised by neoclassical authors in the second phase of the controversies, who argue that neoclassical capital theory "does not require aggregation" (par. 6.5) and that the *uniformity* condition of the rates of profit on capitals' supply prices is not important (par. 6.6).

6.5. Hahn's (1975a; 1982) and Bliss's (1975) views on the question of capital: A first critical excursus.

The neo-Walrasian theorists have tried to side-step the critique concerning capital and the problematic consequence for the whole approach by entertaining that rigorous theory does not need "aggregates". Hahn and Bliss, as it has been introduced above²⁹⁰, are the most notably representatives of this reaction.²⁹¹

In the wake of results raised in the Cambridge debates Hahn and Bliss reacted in such a way so as to minimise their relevance for the theory under attack. One of their most noticeable reactions of these authors²⁹² is the claim that the results of 'reswitching' and 'reverse capital deepening' would only touch the so-called

²⁹⁰ See ch.1.

²⁹¹ Hahn (1975a, 1982); Bliss (1970, 1975, 1976, 2005)

“aggregate” versions of the theory, meaning by that the particular and theoretically secondary version of a representation of a long period theory of distribution which regards production of commodities by an “aggregate” production function. This reaction however evinces a misapprehension of the traditional meaning attached to the notion of capital. As we shall see below, these authors have, up to-day²⁹³, had the tendency to regard the issue of capital as a problem of aggregation which would not be different from the issue of aggregation of *original* factors like land or labour. By thus arguing, these authors have created a dense fog which did not allow scholars to come to grips with the *actual* critique, which we have visited in previous parts of this study. To attempt to clear up some confusions surrounding the “aggregation” issue we may begin with F. Hahn (1982), who counter-attacked the critics of neoclassical theory in the following way:

“In general, there does not exist a function from the vector of endowments to the scalars such that knowledge of the scalar (and of preferences and of technology) is sufficient to allow one to determine a neoclassical equilibrium. If you put it the other way round, it is even more obvious. In general, the neoclassical equilibrium can be found given the vector of endowment which may have, say, 10^8 components. It would be surprising if there were a single number which gives the same information as the 10^8 dimensional vector. In fact, sometimes and in very special cases, this surprising property holds. But neoclassical economists have shown these special cases to be without interest.”²⁹⁴

In this passage Hahn refers to a “neoclassical equilibrium” where the endowments of the several capital goods are taken as *given* by the theory. He affirms that there is no “single number” that can occupy the place of the endowments of the several capital goods in the determination of that equilibrium. That “single number” is the quantity of capital the single magnitude in value terms which, as we have seen, was at the centre of attack in the debates. He then seems to suggest that “aggregation” would be the *substitution* of the vector of endowments for that single number. This would then suggest that the version that relies in the quantity of capital the single magnitude would not be interesting for the determination of the “neoclassical equilibrium”

²⁹² And others, *e.g.* Dixit (1977), von Weizsäcker (1971)

²⁹³ See *e.g.* Bliss (2005)

because it would be hard to believe that a “single number” may supply “the same information” as does, according to Hahn, the endowments of the capital stock. However, on the contrary, the traditional versions of the theory – long-period equilibrium or normal positions method – could, by relying on a notion of capital in value terms, *endogenously* determine the composition of the capital stock, *i.e.* the several capital goods endowments subject to the condition of the uniformity of rates of returns on capitals’ supply prices. Capital as a single magnitude in value terms among the data of equilibrium is *essential* and *indispensable* so long as the variables to be determined by the theory refer to normal values, thus defining normal positions. Therefore, in the traditional version of the theory the “single number” does not imply to do away with the endowments; the latter are determined according to the former and the satisfaction of the uniformity of rates of returns on their supply prices. The “ 10^8 components” are not erased in the traditional equilibrium framework, but endogenously determined. Yet, to achieve that result, the theory has to rely on an unacceptable conception of capital in value terms, which is what has been uncovered over the controversies. This point seems to be missed in Hahn’s argument.

In fact Hahn, who had actually published the core of his 1982 message against the “Neo-Ricardians” seven years earlier in a Symposium organised by the *Economic Record* ²⁹⁵, and to which we shall come back later (par.6.6), wrote:

“Why do people balk only at aggregation of machines and not of people?”²⁹⁶

where it is clear that the author refers to the question of capital by pinpointing that it would not be a distinct problem from the technical aggregation of different kinds of labour. But the issue of “aggregation” of items of heterogeneous capital goods must not be seen as a problem analogous to that of aggregation of original “factors”. While “aggregation of people” may conceivably be assumed since the *endowment* of each type of labour can be taken as given since they change pretty slowly with respect to the speed of adjustment in their respective markets, “aggregation of machines” on the other hand poses very different theoretic problems since those endowments of

²⁹⁴ Hahn (1982) p. 369.

²⁹⁵ Hahn (1975a). See also Harcourt (1975a).

²⁹⁶ *Id.* p. 364.

“machines” will change with changes in outputs and demand – hence the endogenous determination of the endowments of the stock, hence the essential need for a quantity of capital in value terms.

Hahn’s position has not been isolated at the time of this phase of the debates which started somewhat at the turn of 1970s. In effect his misleading view on the centrality of the notion of capital in value terms for the traditional approach is shared by the second spokesman of the supply and demand approach we are considering here namely – C. Bliss.

In his case, Bliss explicitly states that the problem of “aggregation” he tries to work out refers to

“what is involved in replacing a heterogeneous collection of inputs by an aggregate.”²⁹⁷

Again it is not a question of a simple replacement of the endowment by a single number – the quantity of capital. The point is, on the other hand, a matter that in the traditional versions was by far clearer: the quantity of capital is *crucial* for the definition of the equilibrium in that this notion allows the stock to adapt the form required by the equilibrium conditions entailed, under free competition, by the condition of uniformity of the rates of returns on the capitals’ supply prices. It may be of interest to remind what we have seen in previous chapters regarding the supply aspect of the problem of capital and the demand aspect of it. In the traditional version the “supply” aspect is expressed by the quantity of capital in value terms which has to be taken as datum to determine the very value of the rate of interest. The value of the total stock does not change though its composition of course does according to the equilibrium conditions. This is the only way in which the physical form of the several capital goods can be adapted such that their total demand prices can be made equal to the quantity of capital – a *given* amount of value; yet there will still be an endowment composed of heterogeneous capital goods in that equilibrium position. Bliss’s “heterogeneous collection of inputs” is not erased from the traditional neoclassical blackboard by an “aggregate” but rather *both*, so to speak, live together so long as

²⁹⁷ Bliss (1975) p. 147.

that “aggregate” is taken to mean the quantity of capital the single magnitude in value terms.

Of course, it would have been clearer for us to grasp the centrality of conceiving capital in value terms as datum had Bliss given the crucial reasons why “capital cries out to be aggregated”, as he himself wrote in the outset of his work dealing with the theory of capital.²⁹⁸ However, not only did this author provide no hint at such statement but also, to repeat, takes the problem of capital as a problem of “aggregation” analogous to aggregation of non-produced factors of production.

In fact, Bliss has devoted a good deal of his celebrated 1975 book to broadcasting the misleading idea that “capital aggregation” is of *little importance* for the theory of capital (read it – *the marginal theory needn’t rely on the quantity of capital*). His contention seems to be based in the acceptance of the framework of “aggregation” introduced by Solow in the debates by relying on the so-called Leontief separability conditions²⁹⁹ and then he argues that:

“[T]he investigation has provided no support whatsoever for the idea that the aggregation of capital is relatively difficult. The conditions for general capital aggregation are identical to the conditions for the aggregation of labour, or of output.”³⁰⁰

So what Bliss is saying is that, since “capital aggregation” involves the same nature of technical difficulties – which underlie the strong conditions – as does the aggregation of labour or land or output, then the “aggregation of capital” is actually *not* a problem.³⁰¹ And Bliss concludes:

²⁹⁸ Bliss (1975) p. 8.

²⁹⁹ These conditions establish that the rates of substitution of the factors one wishes to aggregate must be independent of the quantity employed of the other factors, and hence allow technically to aggregate a number of factors into a “single” factor in a production function. As discussed above, there is no reason to assume that the factors to be aggregated be services of capital goods. Cf. ch.5 above where we have seen the reactions of Solow (1955) to some Robinson’s (1953) complaints.

³⁰⁰ *Ibid*, emphasis in the original. See ch.2 above where I have discussed the entirely different meanings of aggregation of different kinds of labour or land with respect to that of capital.

³⁰¹ It might perhaps be interesting to recall that the process of selection of the data of a theory is a matter of the highest importance, which might perhaps have not been entirely grasped by some authors arguing that a notion of “capital” in value terms is *not* “relatively difficult” when is taking as datum of the equilibrium. Cf. the following passage from Schlicht (1985, p. 19, emphasis added) regarding methodological issues in economics: “In freezing some factors by means of the *ceteris paribus* clause, we transform them into the data of our analysis. The choice of these data is
(cont.)

“We may thus conclude that the widespread belief that there is a notable, particular and distinct problem posed by capital aggregation is at best an ill-formulated idea, and at worst is based simply on ignorance.”³⁰²

But, was actually *that* the real problem for traditional theory? The “notable problem” Bliss refers to is precisely the problem to define and to determine and hence to measure the quantity of capital in value terms so long as we keep in the long period equilibrium version of neoclassical theory. However it is worth mentioning that the way in which Bliss refers to the problem of a “capital aggregation” is the way in which Robinson has in her huge collection of critical works raised her concerns about capital – attacking the aggregate production function. But we have also seen that the question of capital as uncovered in these debates, and which touches the core of the traditional approach to distribution, has nothing to do with an aggregate production function (see ch.4). The actual arena where we can appreciate the relevance of the struggling between the two sides of the controversies is *not* that associated with aggregate production functions, but the supply and, particularly, the *demand* functions for productive factors, since what has been conclusively shown by critics in the controversies is the impossibility to generalise, in heterogeneous capital goods’ economies, the factor substitution principle – the basic premise of the theory.

not, however, simply a matter of caprice if the isolation is intended to be of *substantive relevance*. A substantive isolation requires those factors transformed into the data by means of the *ceteris paribus* clause have to be *sufficiently stable* with regard to the process we want to explain such that the movements of the data does not destroy or supersede the relations we are studying.” If one confronts this view vis-à-vis Bliss’s, one may understand that, in the traditional versions of the marginal theory, to regard “capital” in value terms as datum is an exercise of “substantive relevance” when studying the forces explaining equilibrium. On the other hand, the author of this study asks himself what is the “substantive isolation” undertaken by neo-Walrasians in the process of considering as datum the physically heterogeneous capital goods’ vector present in the economy.

³⁰² *Ibid.* It is interesting to notice that these authors have left a lamentable imprint in successive work carried forward by neo-Walrasian theorists when dealing with the controversy in mid-1970s. In particular it is noticeable that authors who followed that path concentrated much of their efforts on presenting the issue of the notion of capital as a single magnitude by confusing it with the issue of “aggregation” and hence contributing to increase misunderstandings on all these matters. For example we have the opinion of Dixit (1977, p. 18), who, in his review of Bliss (1975), stated: “I for one would not wish to make too much out of the fact of the relative *simplicity of capital aggregation*.” However, it is not a *convenient simplifying device*, as Dixit pretends to show, but essential to that theory.

6.6. Uniform profit rates and intertemporal prices. Hahn's 'special case'.

6.6.1. Introduction

In what follows it will be discussed the meaning of own-commodity rates of profits and the (traditional) rate of profit on the capitals' supply prices, because the uniformity of the latter has been at times confused with uniformity of the former.³⁰³ Once we have cleared up those two meanings we shall discuss why this confusion brought "obscurity to the controversies", as Garegnani has pointed out³⁰⁴, which, as we shall argue, has eventually marred the relevance of the debates. In order to clarify the issue a little further we have to define, to begin with, what an own-rate of interest means in an intertemporal general equilibrium.³⁰⁵

6.6.2. Commodity own rates of interest

We shall consider a two-period intertemporal economy (the first from $t=0$ to $t=1$, and the second from $t=1$ to $t=2$; for brevity we shall refer to these periods as $t=0$ and $t=1$ respectively) with two goods (a and b), each being both a consumption and a capital good (we shall assume them to be circulating capital).³⁰⁶ In addition, we assume that production is viable (*i.e.*, there exists a surplus from the pure replacement of that capital), and wages (w) are paid ex-post (after the production cycle) in terms of one commodity.³⁰⁷

In an intertemporal framework, all contracts happen in markets at the initial moment ($t=0$) of the first period. For instance, let us consider $P_{b0}=1$ (good b in $t=0$ is taken as *numéraire*). Imagine that someone wishes to buy one unit of that good in $t=0$ (when the contract takes place) but to be *delivered* in $t=1$. Accordingly, this agent

³⁰³ See Bliss (1987)

³⁰⁴ Garegnani (2000) pp. 392-3.

³⁰⁵ We are not considering the case of a temporary equilibrium, which takes as datum the expectations functions of the agents, since the meaning of own-rates of interest does not change if we consider this version.

³⁰⁶ We are following the model present in Garegnani (2000; 2003) which is the same presented by Hahn (1982).

³⁰⁷ Actually we have in this economy four commodities. In an intertemporal economy there are nT commodities; while n refers to the number of heterogeneous goods, T does to the number of periods the whole economy contains. This has been developed by Debreu (1959), who distinguishes the commodity as to location and time; examples bread in London in 1998 is different from bread in Newcastle in 1998, or bread in London in year t is different from bread in London in say year $t+s$. However we limit the analysis only as to time and do not consider location.

will disburse P_{b1} , *i.e.*, the amount of commodity b disbursed in $t=0$ against the promise that this agent will receive one unit of that good. In other words, this agent has to give up 1 unit of b in $t=0$ to consume $1/P_{b1}$ in $t=1$. However, it is pretty clear that this operation might be regarded as a *loan*. In fact, that operation is equivalent to that where someone lends (in $t=0$) the amount P_{b1} of b to some other agent, who will pay that debt back at the following period by providing one unit of b . Actually, a loan has been done, which consists of an amount (P_{b1}) of commodity b (in $t=0$) against a promise of delivering that commodity at the following period. This operation entails a *rate of interest* which we can call the *own-rate* of interest in terms of good b .

Following the foregoing example, let us now assume that $P_{b1}=10/11$, namely, the amount of commodity b to be disbursed in $t=0$ in order to receive in $t=1$ one unit of that good. In other words, a loan in terms of b has been made in $t=0$, and the rate of interest there involved is 10%:

$$P_{b1}(1+r_b) = P_{b0} \text{ hence } r_b = 10\%$$

So, in general, own-commodity rates will be defined by:

$$r_a = \frac{P_{a0}}{P_{a1}} - 1$$

$$r_b = \frac{P_{b0}}{P_{b1}} - 1$$

6.6.3. Uniform rates of return on supply prices in intertemporal equilibria.

Having defined the two own-rates of return in our economy, a very important question arises: which is the form that the condition of a uniform rate of return on the supply prices of the capital goods takes, when the equilibrium relative prices change over time?

Now let us assume that one unit of commodity a in $t=0$ has a value of 2 (*i.e.*, one unit of a is exchangeable by 2 units of b), but paying for it in the moment of its delivery ($t=1$), it would cost 1.8 (notice that we are assuming this price of good a to be decreasing, and to be undiscounted). The question that arises is how to determine P_{a1} (the *discounted* price over time).

But before going into that problem, we have to face the compelling issue regarding the (traditional) uniformity of rates of interest. In effect, it will first be necessary for us to consider that *basic condition*: the rate of interest, in equilibrium, cannot be different for the different loans made in terms of the different commodities. We have seen that we can obtain with $10/11$ units of b in $t=0$ one unit of itself in $t=1$, the rate of interest there involved being 10% .³⁰⁸ Therefore, such a rate of interest will have to be the *same* as in the case we could obtain $10/11$ units of b by renouncing to the acquisition of commodity a in $t=0$, and with that *unit* of b “saved” (since the $10/11$ were not disbursed) buying commodity a_1 in that period. In that period ($t=1$) we know that one unit of a costs 1.8 of b , therefore, in order to know how much we must pay in $t=0$ for obtaining one unit of a (but to be delivered in $t=1$ namely $-P_{a1}$), we must discount 1.8 at the rate of 10% .

Therefore:

$$P_{a1}=1.636$$

This will be the value of commodity a in terms of b , namely – the amount of b obtained in $t=0$ by selling one unit of a which will be delivered in $t=1$. That amount of b could be lent to someone, then obtaining an interest rate of 10% when the borrower pays back in $t=1$ one unit of commodity b . On the other hand, since we know that $r_a=P_{a0}/P_{a1} - 1$ and that $P_{a0}=2$, then $r_a=22.2\%$.

Clearly both own-rates of interest are different; moreover, the own-rate corresponding to the good a is higher than b 's, for the relative value of the former commodity to the latter is *decreasing* over time, as was assumed above. On the other hand, if the loan were made in terms of b , while its relative price increases over time, it will follow that to compensate those relative advantages (of the lender) and disadvantages (of the borrower) r_b would have to be *lower* (and r_a *higher*) as showed in the example. This is no more than a simple triangular arbitrage operation.³⁰⁹

³⁰⁸ Of course, since we have assumed commodity b in $t=0$ as *numéraire* ($P_{b0}=1$), then both the own rate of return of that good and the (traditional) uniform rate of interest will coincide. It is clear, therefore, that that equality will entirely vary according as the choice of the good taken as *numéraire*.

³⁰⁹ In any intertemporal economy there are as many own-rates of interest as are goods for each pair of “periods”. In our case discussed in the main text we have two own-rates since we have two goods (a and b) for only one pair of periods, namely – the only pair involving $t=0$ and $t=1$. In general, we may account for own-rates of interest by relating to the rates for the succession of adjacent periods linked by the so-called “Hicksian condition”, that is, the connection of two
(cont.)

The above example also shows us that both own-rates of interest would coincide if and only if $P_{a0}/P_{a1}=P_{b0}/P_{b1}$, that is, when both intertemporal prices are equal, from which it can be deduced $P_{a0}/P_{b0}=P_{a1}/P_{b1}$ namely – when both relative contemporary prices are equal. In general, however, relative prices will tend to be different and so will the own-rates of interest. So, when equilibrium prices are assumed to change over time, the condition of a uniform rate of interest on the capitals' supply price will in general entail *different* own-rates of interest. Accordingly it is convenient to call the former uniformity of rate of interest as the *effective* rate as Garegnani has christened it in his attempt at clarification on some misconceptions created in the second phase of the controversies:

“The word *effective* by which we qualified the long-period notion of a uniform rate of return is meant to take care of the fact that the definition of such a uniform rate will entail a non-uniformity of the *own rates of returns* of the several capital goods, when changes in their relative prices are considered.”³¹⁰

Now, having clarified the different meanings between the uniformity of the *effective* rate of interest on the capitals' supply prices and the uniformity of own-rates, it must be seen in this connection what the critical side of the debates unveiled regarding the *non-satisfaction* of the uniformity of the *effective* rate owing to the Walrasian specification of capital namely – to take as datum of the equilibrium the physically heterogeneous capital goods' endowment, instead of the quantity of capital the single magnitude in value terms.

In fact the Walrasian specification of capital is not compatible with the traditional method. Yet in Walras's equations we can explicitly see that the equilibrium condition underlying the traditional method, namely the uniformity of the *effective* rates of profit on the capitals' supply prices, is present as soon as we recognise in those equilibrium price equations the *equality* sign between demand and supply prices

generic own-rates by the following condition: $(1+r_{i(t, t+1)}) / (1+r_{j(t, t+1)}) = (P_{it} / P_{jt}) / (P_{i,t+1} / P_{j,t+1})$ where i and j are two generic commodities. Cf. Bliss (1975) pp. 53-6.

³¹⁰ Garegnani (2000) pp. 392-3, n. 2.

of those capitals.³¹¹ But, because of assuming capital in physical terms, Walras was then compelled to admit solutions with price equations bearing *inequality* signs, instead of equality signs, thus leading some capital goods not to be produced. “Walras’s inconsistency”, as revealed in the controversies, is disregarded by the new versions of the theory, because the latter disregard the traditional meaning attached to the normal positions or long period method entailed by the uniform rate of profit on the supply prices.³¹² The dropping of the uniformity of the *effective* rate of profit is evident when we recognise the inequality sign in the price equations of those versions of the theory.³¹³

6.6.4. Further discussion of Hahn (1975a)³¹⁴

What do all these digressions have to do with the Cambridge controversies? In the course of the debates the criticism of Walrasian theory as contained in Garegnani (1958; 1962) started to be spread by Robinson (1970a).³¹⁵ Robinson’s views were then echoed by Harcourt (1975a) whose arguments became the target of Hahn’s (1975a) attack, as we shall see below. The issue, not to be forgotten, is the uniformity of the *effective* rate of return. The following Harcourt’s message – though literally echoing Robinson’s (1970a) wording – reflects Garegnani’s critique of Walrasian theory³¹⁶:

³¹¹ In fact, as soon as capital goods’ demand prices are equalised with the corresponding supply prices it is assumed both *i*) production of all capital goods and *ii*) the introduction of the traditional condition of uniformity of the rate of profit on those supply prices. Cf. Garegnani (2003) p. 151.

³¹² Of course, it is worth noticing that this path was increasingly adopted by latest versions of marginal theory in the wake of the discoveries of ‘reverse capital deepening’ and ‘reswitching’ in the course of the controversies.

³¹³ See *e.g.* Hahn (1982).

³¹⁴ Part of this paragraph is based on Garegnani (2003) pp. 149-57.

³¹⁵ Also, before 1970, we can read in Robinson (1964, p. 60): “there is a notorious difficulty about introducing a rate of profit into the Walrasian general equilibrium system.”

³¹⁶ To assess the evolution and results of the controversies, it must be clear that Walras’s treatment of capital was exposed and criticised by Garegnani in his Cambridge PhD dissertation (1958), and some years later taken up again to become in a manuscript (1962). The argument was known by Joan Robinson, who was one of Garegnani’s thesis examiners together with L. Robbins, so that Robinson was well acquainted with Garegnani’s work. Also it must be noted that some of Robinson’s closest collaborators, in particular, John Eatwell, made the reading of Garegnani’s dissertation in the Cambridge Library almost compulsory for all his students. (I owe this information to professors P. Garegnani and F. Petri.) Therefore, Robinson’s claim that Walras does not have any theory of the profit rate at all can be taken as an echo of the inconsistency between the condition of the effective uniformity of rates of return on the supply prices of capital goods and regarding capital as the physical stocks. It might be noticed that perhaps the form chosen by Robinson to refer to the traditional authors who conceived of capital in value terms – “Walrasian
(*cont.*)

“the Walrasian model of general equilibrium has the implications that it is not possible to have a theory of the rate of profits.”³¹⁷

where, no doubt, the impossibility in the “Walrasian model” referred to in that passage clearly means the *non*-uniformity of the *effective* rate of return on capitals’ supply prices. This is what Hahn (1975a) took up in his reply to Harcourt (1975a) – anyhow intended to reverse the sign of the critique – to argue that this issue would only affect a “Special case”³¹⁸ of the theory under attack.

Hahn’s contention can indeed be derived from what he states in the following passage:

“General equilibrium theory *is* general and so can discuss the equilibrium of an economy whatever its ‘initial conditions’, *e.g.* outfit of goods inherited from the past. For most such specifications it will not be the case that the equilibrium price of a good for future delivery in terms of the same good for current delivery will be the same for all goods.”³¹⁹

It is thus apparent that Hahn’s “special case” is not but the result of a confusion involving the two kinds of uniformities of the rate of profit that we have previously discussed.

According to Hahn, then, the issue of the impossibility to determine the *effective* rate of profit owing to a Walrasian specification of capital is misleadingly confined to a “case” in which relative intertemporal prices are equal over time. In other words, by recalling the example we have previously used, Walras’s inconsistency would only affect the “case” such that $P_{b1}/P_{b0}=P_{a1}/P_{a0}$ (“the equilibrium price of a good for future delivery in terms of the same good for current delivery will be the same for all goods”) namely – when the *own*-rates of profit happen to be equal owing to the constancy of equilibrium prices over time. However, we have seen that the *effective*

leets” – have misled some participants in the controversies, when it is well known that Walras does not use capital as a single magnitude.

³¹⁷ Harcourt (1975a) p. 350.

³¹⁸ Hahn (1975a) p. 360.

³¹⁹ *Ibid.* emphasis in the original.

uniformity takes the form of *non*-uniform own-rates of profit when we admit changes in the equilibrium prices.

On the other hand there is a further issue. While it is clear that, according to Hahn, “general equilibrium theory” is the neo-Walrasian theory which takes as datum of equilibrium the physically heterogeneous capital vector, it is totally hidden in Hahn’s intervention that Walras’s inconsistency *affects* that “general theory” independently of whether we consider changes in equilibrium prices over time. But Hahn’s hideaway can be easily discovered as soon as we recognise that the *effective* uniformity of the rate of profit of “Wicksell (and others)”³²⁰ is completely misunderstood by the uniformity of *own*-rates of profit due to constancy of prices. Yet the condition of uniformity will be present in the new formulations of the theory as soon as we reckon in the price equations of the capital goods the equality signs.

To rebut criticisms in this second phase of the controversies, Hahn attempted to reverse the sign of the critique by arguing that the *effective* rate of profit in a “Walrasian model” is *not* an issue:

“The crudest empirical observations will convince one that there is no unique rate of profit to be observed in the economy. Do we conclude from *that* that competition is functioning badly? Answer: No. Consult any general equilibrium text. Why is ‘the rate of profit’ an interesting unknown? Ask the neo-Ricardians. If we knew all relative prices from now to doomsday, could we ask more?”³²¹

Note that Hahn has apparently tried to show the existence of divergences in the own-commodity rates in reality; but those observations would be actually indicating different rates of profits in different businesses compared in terms of money or other *numéraire*; but, then, this divergence will have nothing to do with divergences of own-commodity rates because, when prices of the numéraire are changing, the own

³²⁰ *Ibid.* This reference to “Wicksell (and others)” in Hahn’s words is related to the way in which Hahn somehow acknowledges the results of critics in the controversies as raised by Robinson – echoed by Harcourt (1975a) – concerning the Walras’s inconsistency. In fact we can read the entire Hahn’s passage: “to get a uniform rate of profit Wicksell (and others) had to work with Robinsonian leets” (*Id.* pp. 360-1) where it is clear that “Robinsonian leets” refers to the homogeneous capital in value terms, which was actually what Wicksell, Jevons, Böhm-Bawerk, Marshall, Clark had to work with in order to get the *effective* uniformity of rate of profit. Nevertheless Hahn shows in those very passages awareness of the issue at stake, yet he confuses both kinds of uniformities as the main passage quoted in the main text evinces it.

commodity rates will be showing the same rate of profits expressed in different numéraires. So Hahn's contention showed in the passage misinterprets the divergence between own-commodity rates: own-rates may diverge each other, when prices change over time, while the effective rate of profit is being ascertained itself – when the physical composition of capital, in the *traditional* versions, is adjusted. On the other hand, we may have a situation where the *effective* rate of profit is not ascertained, that is the Walras inconsistency due to the arbitrariness of the capital endowments, as the critics revealed in the controversy, and yet account for uniformity in the own-commodity rates (this case being when prices are assumed constant over time).

Notwithstanding Hahn's confusions, what is clear is that the essential role of the uniformity of the *effective* rate of profit is totally minimised by Hahn because he believes that one could not “ask more” if “all relative prices” are known up to the end of the world – what implies to know all the intertemporal prices and accordingly the own-commodity rates those prices would imply.

The “rate of profit” is however “an interesting unknown” in order to determine long period equilibrium of the economy, as the founders of the marginal approach believed. And the meaning of that interesting unknown was at the heart of the controversies. For Hahn this would be a very special case of a general theory. In fact Hahn seems to insist that *his* “general theory” is quite safe and

“that a very extreme specialisation of a general model somehow shows the latter to be inapplicable requires the very summit of incomprehension.”³²²

where that “extreme specialisation” would be the one associated with a uniformity of *own*-rates of profit, which is taken by Hahn, however, to mean the *effective* rate of profit.³²³ With the lenses of modern neo-Walrasian equilibrium theory, which treats

³²¹ Hahn (1975a) p. 361.

³²² Hahn (1975a) p. 360.

³²³ The fact that Hahn has in this intervention confused the uniform *effective* rate of profit with the uniformity of *own*-rates of profit due to constancy of prices over time might perhaps be traced in the fact that in the traditional versions of the theory, when determining the long period equilibrium, the prices were assumed to be constant in that very process; so in that very process of adjustment when prices are assumed constant both uniformities coincide. However this method centred on the persistence of the equilibrium itself, which in turn is ensured by the *effective* (cont.)

capital in physical terms, it would only appear as a “very extreme specialisation” the case with which “Wicksell (and others)” had to deal with – namely capital the homogeneous magnitude.³²⁴

However Hahn’s “general model” can no longer determine the variables as intended not only by “Wicksell (and others)” *but also* by Walras – to determine long period equilibrium or normal positions of the economy.

6.6.5. Conclusion: some obscurity in the debates.

In the hope that the Walrasian conception of capital might have overcome the deficiencies associated with the notion of capital in value terms for the traditional approach, as uncovered in the capital controversies, the neo-Walrasian theorists, as exemplified by Hahn, have *minimised* the results raised by the critics in the debates by confining the issue of the *effective* uniformity of the rate of profit to steady-state equilibrium where relative prices through time are constant. However, the arbitrarily given physically heterogeneous capital goods vector cannot ameliorate the situation under which the marginal theory has fallen, unless the traditional notion of equilibrium be abandoned – what is what actually happened in marginal theory since recent decades.

In this connection – together with our excursus through Hahn’s misleading views on the issue of uniformity of the *effective* rate of profit – one can appreciate the following remark recently raised by Garegnani:

“A question which contributed to the opacity of the capital controversies of recent decades [has been] that inequality of *own rates* has often been confused with inequality of *effective rates on the supply prices of the capital goods*, which is instead due to the arbitrary initial composition of the capital endowment of Walrasian theory. This has

uniformity, allows such an assumption. Not being acquainted with such a key notion of persistence might perhaps misled Hahn into confusing both kinds of uniformities. The point to remark here is that Hahn and other authors, by introducing such a misleading argument, in turn confused the traditional long period equilibrium or normal positions with equilibrium of steady state (constancy of prices). Thus a further misunderstanding was created as well. Cf. Garegnani (1976a, p.32) and Robbins (1930).

³²⁴ In the neo-Walrasian authors’ “comprehension” this extreme case would reveal itself as the constancy of equilibrium prices over time.

had the result of obscuring both the necessity of the concept of a ‘quantity of capital’ for the traditional equilibrium, and the causes of the abandonment of the latter.”³²⁵

As some reflection on communication might suggest, “obscurity” is *per se* the worst appropriate ground for debating. But, sometimes, obscurity might be the arm used by some both to hide the issue at stake and to escape the critiques from their contenders. Yet obscurity brought by Hahn into the debates has had the further purpose to reverse the sign of the critique. In fact, Hahn (1975a) ended his intervention by proposing to substitute one of Harcourt’s (1975a) appreciations of the debates – “The Great debate” – by “The Great Charade”.³²⁶ However, as we have seen, it was the neo-Walrasians who actually produced an endless series of misunderstandings on key questions for the theory. Then, after having visiting the foregoing interchanges in this second phase of the debates, we would not be guilty of slandering if we pointed out Hahn, Bliss, Dixit, von Weizsäcker and many other neo-Walrasian adepts as the ones who *actually* contributed to foster such an image of the final results of the controversies that it might have appeared as a sham.

6.7. Do ‘reswitching’ and ‘reverse capital deepening’ bear theoretic problems on intertemporal general equilibrium? Two views: Bliss and Garegnani.

6.7.1. Bliss.

According to Bliss (1975)³²⁷ the “paradox” of ‘reswitching’ in an intertemporal general equilibrium concerns a key assumption underlying a maximisation problem of the linear programming type³²⁸: the possibility that *two price systems* could be solution to the maximisation problem. Bliss explains that if two prices systems maximise the present value of consumption, a “convex combination” of them should also be solution of the problem, and therefore “it is not possible to have two *disjoint* sets of prices (*i.e.* complete price systems) at each of which a development of the

³²⁵ Garegnani (2000) pp. 392-3, n. 2.

³²⁶ Hahn (1975a) p. 364. A similar position can be found in Hahn (1972) p. 5.

³²⁷ Cf. Bliss (1975) ch.3.

³²⁸ This linear programme maximisation entails a *dual* minimisation of costs problem. Cf. Dorfman, Samuelson and Solow (1958).

economy is a production-price equilibrium.”³²⁹ However, *with* ‘reswitching’, the author also notices, two such price systems “may not be connected.”³³⁰ This phenomenon would undermine the convexity properties underlying price systems solutions of linear programming problems and it might probably have been this feature Bliss is concerned about; nevertheless this author contends:

“Surely part of the excitement caused by the demonstration that double switching was a ‘common’ phenomenon, in the sense that the examples do not depend upon choosing very special values for the parameters, arose because it seemed that the set of values [of the rate of interest] at which a technique might be chosen was not connected. *But this is an optical illusion.*”³³¹

The “optical illusion” Bliss refers to would have been caused because economists have had “the habit of working with a space of price sequences corresponding to constant rates of interest.”³³² Since, according to Bliss, phenomena like ‘reswitching’ may *only* arise when economic analysis is “confined to constant rate of interest price systems”, the “illusion” would then disappear as soon as it is realised that “[firms choose their investment plans] in the light of complete intertemporal price systems.”³³³ Thus, it would seem, by *widening* the range of values of the interest rates (implicitly in a “complete intertemporal price system”), Bliss would have re-established “the property of convexity in the set of solutions” and their connectedness; hence the “paradox” would disappear in the intertemporal versions.

However apart from the fact that ‘reverse capital deepening’ is not accounted for in Bliss’s disputations, on the other hand it is yet *not* clear why ‘reswitching’ would

³²⁹ *Ibid.* (emphasis added). The idea of a convex combination is the following: if p and p' are both price systems (or “weights”, or “discount factors”, in Bliss’s words) which are solution to a maximisation linear programming problem then any *convex combination* of those “weights” $\alpha p + (1-\alpha)p'$, with $0 < \alpha < 1$, will be solution as well. Cf. Bliss (*op. cit.*) p. 238, n.8.

³³⁰ Bliss (*op. cit.*) p. 238. This could be understood as if the same technique, in our well-known wage curves graphs, is in use for two *disjoint* intervals of the rate of profits.

³³¹ *Id.* p. 239 (emphasis in the original). “Double-switching” is used by Bliss to refer to ‘reswitching’.

³³² *Id.* p. 238.

³³³ *Id.* p. 239. Note the way in which this author sees the *long period* rate of interest that is central in the controversies: a simple “constant” rate.

have disappeared in intertemporal general equilibrium theory when the set of values of the rate of interest can be enlarged and connectedness re-established.³³⁴

6.7.2. Garegnani.

The critically oriented side of the controversies has started to reply to advocates of intertemporal general equilibrium theory by arguing that problems of ‘reswitching’ and ‘reverse capital deepening’ are also present in those versions. Garegnani (2000, 2003) is one of the most notably representatives of the critical side for having brought the critique in the intertemporal marginal approach.

The crux of Garegnani’s argument lies in that problems associated with capital in the traditional versions might reappear in the savings-investment markets, which are of course present in the new versions of the theory.³³⁵

³³⁴ Yet it may be interesting to see that even *without* violation of the so-called connectedness property there is cause for concern when, in the case of ‘reverse capital deepening’ without ‘reswitching’ on the “envelope” (where one technique is in use only for one convex interval of the rate of profits), we realise that when producers switch from technique β to γ at $r=r_2$ a higher rate of profits is associated with a *higher* net product per worker. (See graph below.) Bliss could warn us of the fact that we are considering “the constant rate of interest price systems”. But even if in this “subset” of values of rates of interest we may account for “perverse” relations between distribution and net output per worker, how could one be sure that by *widening* the ranges of values of that rate – *i.e.* to regard “complete intertemporal price systems” – these “paradoxes” may not appear?

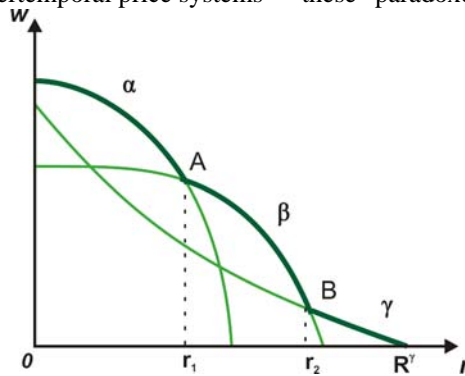


Figure 6.1: ‘Reverse capital deepening’. Each technique is used for convex intervals of r

³³⁵ Garegnani (2000, 2003) shows that despite savings and investment formally disappear in the so-called individual (or collective) wealth equations in the standard formulations of intertemporal general equilibrium, savings and investment still are present in the intertemporal relations. In a two-period intertemporal economy ($t=0, t=1$) where agents consume all output in $t=1$, production will have sense only if it takes place in $t=0$. Savings and investment are realised in $t=0$ too. In $t=0$ savings (S_0) are the result after deducting from the initial endowments of commodities a and b (A_0, B_0 ; both used as consumption and circulating capital) the respective consumption demand (D_{a0}, D_{b0}):

$$S_0 = Y_0 - G_0 = (A_0 P_{a0} + B_0 P_{b0}) - (D_{a0} P_{a0} + D_{b0} P_{b0})$$

$$S_0 = (A_0 - D_{a0}) P_{a0} + (B_0 - D_{b0}) P_{b0}$$

where Y_0 : aggregate output in $t=0$

G_0 : aggregate consumption in $t=0$

(cont.)

How to see the problem? Let us assume, in the intertemporal economy specified above (see n. 326), that all markets other than savings and investment, such that savings exceed investment ($\Delta S = S - I > 0$). In the intertemporal framework the excess of savings over investment entails an excess supply of commodities in $t=0$ and an excess demand of them in $t=1$.³³⁶

$$\dots \dots t=0 \dots \dots \dots t=1$$

$$\Delta s = -(P_{a0}\Delta D_{a0} + P_{b0}\Delta D_{b0}) = P_{a1}\Delta D_{a1} + P_{b1}\Delta D_{b1} \quad (\sigma)$$

where ΔD 's indicates the differences in the respective quantities of goods demanded as consumption between $t=0$ and $t=1$.

If we consider the RHS of the first equality sign of equation (σ) and equalise it to zero, we obtain the equilibrium condition:

$$-P_{a0}\Delta D_{a0} = P_{b0}\Delta D_{b0} \quad (\pi)$$

Equation (π) represents the case in which there is an excess demand of b_0 relative to a_0 , *i.e.* the case of relative excess demand of contemporary commodities. Garegnani argues that (π) represents the equilibrium of *contemporary* consumption but that it is equation (σ) that represents the intertemporal adjustment of savings and investment.³³⁷

In $t=1$ savings will be determined by the following relations:

$$S_1 = Y_1 - G_1 = (L_0w + S_0) - (D_{a1}P_{a1} + D_{b1}P_{b1}) = 0$$

where L_0 is the quantity of labour employed in $t=0$ for producing output to be consumed in $t=1$, and w is the wage rate (in terms of P_{b1}). Note that L_0w becomes available in $t=1$ since w is paid in arrears.

To introduce the so-called individual *wealth equations*, it is appropriate to consider both the income-side and expenditure-side in each period:

$$y_0 = a_0P_{a0} + b_0P_{b0} = g_0 + s_0$$

$$y_1 = l_0w + s_0 = g_1$$

By adding both equations, we obtain:

$$y_0 + y_1 = a_0P_{a0} + b_0P_{b0} + l_0w + s_0 = g_0 + g_1 + s_0$$

hence:

$$y_0 + y_1 = a_0P_{a0} + b_0P_{b0} + l_0w = g_0 + g_1$$

Note that the highlighted equation above is the so-called wealth equation, where savings for each individual have been *cancelled out*, since each individual's savings enters both the expenditure-side in t as well as the income-side in $t+1$. Yet individual (and collective) decisions to save and to invest are present in the intertemporal versions.

³³⁶ Garegnani (2000) pp. 436-9.

³³⁷ *Id.* p. 439.

In order to see the nature of the problem, now consider, as Garegnani suggests³³⁸, three periods in the intertemporal economy, *i.e.*, $t = -1, 0, 1$. Let us assume that a_0 and b_0 are produced with means of productions and labour of $t = -1$.

Now, in the case of the contemporary excess of supply of a_0 relative to b_0 (equation π) there would *not* appear to be strong difficulties to *shift* the labour and means of productions of $t = -1$ freed by excess supply of a_0 in order to produce ΔD_{b_0} , since the inputs needed to produce both consumption goods are similar, that is, labour and the given proportions of a and b composing the means of production. This adjustment takes place does not seem to meet insurmountable obstacles since it is in $t = -1$, and equilibrium is therefore reached *within* the same period due to changes of contemporary relative prices caused by the excess demand of b_0 relative to a_0 .³³⁹

However, regarding the intertemporal adjustment (equation σ), *i.e.* the savings-investment adjustment, the *nature of the problem* is entirely different from that regarding the contemporary adjustment outlined above. In this case the intertemporal equilibrium adjustment ought to be such that the “freed” labour and means of production in $t = -1$ (due to, say, an excess supply in $t = 0$ of a and b) may be *directly* employed in $t = 0$ in order to produce in $t = 0$ quantities ΔD_{a_1} and ΔD_{b_1} ; (*cf.* equation σ). However, this will in general *not* be possible, because labour and the means of productions in $t = 0$, which *directly* produce D_{a_1} and D_{b_1} , are *different* from those “freed” in $t = -1$ due to the excess supply of a_0 and b_0 . The difference lies in that the inputs required to produce the consumption goods (in $t = 0$) are different from the

³³⁸ *Id.* p. 437.

³³⁹ It may be due to this kind of analogy, however formal as it seems, that Arrow (1989) might have written that “[in any study of] intertemporal allocation, the variable are today’s prices for future goods, [so there is] a perfectly consistent story that does not look any different from the story about choosing commodities today.” (p. 155). The root of seeing the problem of intertemporal allocation *as if* it were a problem of contemporary consumption goods’ adjustment might be realised by confronting two radically different principles guiding consumption and investment (savings). Some basic economic reflection may help us understand that from the consumers’ viewpoint, who wants to satisfy his needs, specific commodities satisfy specific needs (for instance it would be as futile to overcome hungry by coal as it would be to consume bread in order to get rid of the winter cold.) However, in the case of the savers, who *demand* for *future net income*, capital goods in which investment is done are perfectly *substitutable*: it does not matter whether the capital good is a lathe or a loom or a plough: what matters is the expected rate of return that the investment might yield, independently of the particular *type* of the capital good. Then, the savers expect to obtain the highest rate of return on the value of the investment. If this is so, as Garegnani argues, then, the quantity of capital – and its problems – will be also present in the latest versions of the theory.

inputs required to produce the means of productions which, in turn, are required to produce the consumption goods.

Consequently, to re-establish equilibrium, the only way will be to increase production in $t=0$ of ΔD_{a1} and ΔD_{b1} will be that of *increasing the productivity* of the already fully employed labour in $t=0$ by *increasing* the quantity of the means of production cooperating with that labour which *directly* produce a_1 and b_1 . Hence there should be a production of ΔI_a and ΔI_b (investment) in $t = -1$ by decreasing production of quantities ΔD_{a0} and ΔD_{b0} . Accordingly, the way to motivate these *increments in investment* will be by means of a *rise* in the intertemporal price P_{b0}/P_{b1} so as to make agents give up consumption in $t=0$ (ΔD_{a0} and ΔD_{b0}) in order to obtain some extra in the future $t=1$. But a rise in such a price is a *fall* of r_b . Assuming that the latter rate is also the *effective* rate of interest (r), we can no longer be sure that a *fall* in r will make investment rise, as the capital controversies have revealed (see above chs 3 and 4). Thus, Garegnani concludes, ‘reverse capital deepening’ and ‘reswitching’ undermine the intertemporal versions of the theory as they do in the case of the traditional marginal theory.

6.7.3. Confrontations.

The negative implications of ‘reswitching’ and ‘reverse capital deepening’ on the contemporary intertemporal version of the theory have not been completely considered by many if not most modern neoclassical advocates. So far, however, we have seen how Bliss might have believed in overcoming capital theoretic problems in his framework without solidly argue his point. Moreover, our brief assessment of Bliss’s treatment of the issue seems to suggest strong reservations so as to accept, *without further inquiring*, the claim that “most observers consider [Bliss’s 1975 book as] the definitive neoclassical treatment of capital theory that *ended* the Cambridge controversies.”³⁴⁰

On the other hand, the critically oriented side of the debates has recently started to bring the critique to the intertemporal land by strongly arguing that capital theoretic problems may reappear in the savings-investment markets, as Garegnani maintains. Thus, in the light of this recent investigation, it is hard to go along with many voiced

³⁴⁰ Cohen and Harcourt (2003) p. 206 emphasis added.

interpretations, like the one quoted above, suggesting that Bliss's 1975 is a "definitive treatment of capital theory" that would have therefore led the controversies to an end – read it as a neoclassical end.³⁴¹

³⁴¹ This interpretation is also suggested by Solow (1983) p. 184.

CHAPTER 7

CONCLUSIONS

This study reveals the importance of the Cambridge capital theory debates for a better understanding of the present state of neoclassical capital theory, and thus contributes to rescuing the relevance of the controversy from the apparently inconclusive state at which it might have been led in its second phase.

Among the many issues debated over the capital controversies, this study focused on the most salient and relevant phenomena of ‘reswitching’ and ‘reverse capital deepening’. These results cast serious doubts on the hard core premises of the neoclassical theory. In fact, once the interdependency between relative prices and distribution is taken into full consideration, it will be realised that the implications of those results drawn from the debates undermine the factor substitution principle, namely the general principle on which dominant theory (traditional and contemporary) founded an explanation of distribution in terms of supply and demand. That is why we purposed in this study to deeply analyse and put at the centre these phenomena: to reveal how they undermine the marginal approach (*cf.* ch.3).

Then what I thought to do in this study was to reconsider some different positions in these debates by taking as the main line of argument the results and implications of ‘reswitching’ and ‘reverse capital deepening’. As I showed in this study, neoclassical advocates did not remain silent when the serious threats started to be perceived by neoclassical theorists. They attempted different strategies in order to preserve neoclassical theory from the tension created by the capital theory debates. As we saw (*cf.* ch.4), the range of strategies rehearsed by neoclassical participants have been i) to build up the so-called parables (Samuelson) – but this resulted in a totally unsuccessful theoretical device to save their theory; ii) then, once the negative implications started to be acknowledged, they tried to find the conditions to rule out ‘reswitching’ (Levhari-Samuelson), but this too was proved wrong altogether; iii) to minimise the implications of the results by pointing to the empirical way-out (Ferguson) – an extreme position that completely dodge the question raised by capital theoretic problems, and finally (*cf.* ch.6) we find a *shift* to Walrasian theory and hence a shift in the notion of equilibrium. What this reveals, as I argued, is that the

counteractions from the neoclassical side arose out of the fact that the implications of ‘reswitching’ and ‘reverse capital deepening’ were perceived as very strong concerns, and not simply as ‘curiosa’ as Joan Robinson remarked (*cf.* ch.5). As illustrated, the fact that advocates of the traditional approach – Nobel awardees included – have tried to salvage the supply and demand theory by recurring to these strategies indicates the tension caused by the capital debates. Still this tension forced the theory to make all the possible efforts at their disposal to keep alive a theory whose basic premises cannot theoretically be supported on its own inner logic. Thus, the historical-analytical reconstruction developed in this study shows the relevance of these controversies.

On the other hand, we have a word to say about the critical side as well. In fact, despite that the bulk of the critics placed themselves in opposition to the marginalist arguments raised in the first phase of the debates, it must be pointed out that some critical participants chose to emphasise a kind of critique that is secondary to the kinds of problems entailed by the question of capital. As I argued in ch.5, an outcome of my analysis was that the overemphasised critique of the aggregate production function pursued by some participants could have contributed to foster misunderstandings that still persist (*cf.* Bliss’s 2005 views in ch.1). Moreover, the phenomena of ‘reswitching’ and ‘reverse capital deepening’ have been confined to “dead end” by some critical authors as well (*cf.* Harcourt’s views in ch.1). Perhaps some critics might have believed that the critique centred on the aggregate production function would have supplied a ‘more evident’ line of argument against dominant theory (*cf.* Robinson’s passages in ch.5). However the disadvantages of this line of criticism turn evident when, in the second phase, the neoclassical side started to escape the critique by claiming that the theory does not need “aggregates”, thus supplying a defensive line to neoclassical advocates to totally shun the implications of the controversies.

However, as I discussed (ch.6), the purpose of these neoclassical theorists in the second phase was far from *debating*; in fact they diverted the central focus from the question of capital to a question of “aggregates” as if the former question were a problem of considering an “aggregate” capital in the same footing as labour or land (*cf.* par. 6.5). This view, as we saw, regards the issue of the quantity of capital as a matter of “simplicity” in presenting the theory; but the role of the quantity of capital for the traditional theory is essential to the determination of a long period equilibrium

(*cf.* ch.2). Thus rather than debating some authors might have preferred to bewilder the ground by ignoring the role of the ‘quantity of capital’ in the traditional versions and by entrenching the theory to short period general equilibrium so as to evade the problematic issue of capital, (*cf.* ch.4, par. 4.10 and 4.15.) But this confirms what Garegnani pointed out in (1976a): a new notion of equilibrium had become dominant in marginal theory due to inconsistencies in capital, rather than a problem of the traditional, long period method itself (*cf.* par.1.4). However there was not a unified reply from the critical side in this second phase.

In fact, as I showed in the discussion of the Walrasian rehabilitation of capital (par.6.4), the critically oriented side of the controversies received neoclassical counteractions in different ways. Thus, the obscurity brought by marginal advocates arguing in misleading ways – together with a not rapid and unified reply from the critical side in the context of the Walrasian theory – might have contributed to lead the relevance of the Cambridge controversies, in the second phase, to an apparent inconclusive point. In the effort to understand the reasons for the gap within the critical side, this study argued that there seems not to be reasons for taking the neo-Walrasian theory as the theory that is *not* relevant for the issues raised in the controversies (*cf.* Harcourt’s (1976) passage and recent positions of this author in par.1.3 and 1.4.) On the contrary, what turns out to be relevant is that the results of the controversies touch the core premises of dominant theory, thus it is the critique that has to be relevant to the theory and not one or another “version” of the theory for the issues raised in the debates.

The clarifications made (ch.6) suggest that the fact that today nobody seems to talk about the capital debates is *not* because of the capital theory debates actually ended (*cf.* 6.7.) Some would argue that the issues raised in the debates do not concern the modern versions of the theory like the intertemporal general equilibrium theory, however, advocates of these versions have not been able to show the opposite, as analysed if only provisionally in par. 6.7. Moreover, on the other hand, the provisional assessment of the recent critique of these versions of the theory could also be taken as a confirmation that there would not be grounds to *dispense* with the critique by neoclassical theory relying on Walrasian capital. Rather the threat for the theory that problems regarding capital may reappear in the savings-investment markets would indicate that neoclassical theory would have to face this issue – if that were the case. For this, however, communication between the two sides must be re-

established; given the present situation (*cf.* Bliss's 2005 views) we are still far from that. Although this study has partially dealt with the situation of capital theory in the new versions of the theory, our attempt at clarification, it is hoped, is intended to nurture future communication.

This research also revealed the necessity to further study how the problems of 'reswitching' and 'reverse capital deepening' are dealt with by neoclassical authors within the framework of the intertemporal and temporary versions. These issues were only partially dealt with by this study, since they exceeded the original scopes I had originally proposed here.

APPENDIX A

*The curve of the net marginal product of labour*³⁴²

A.1. In a labour and corn-capital economy, the competitive capitalist will minimise costs when hiring the number of labourers for which the *net* marginal product (an amount of corn) is equal to the corn-wage he finds ruling in the market.

To make the concept of a *net* marginal product clear, let us imagine that the *gross* marginal product of labour (M_gP_L) is represented by the following function:

$$\begin{cases} M_gP_L = 4 & \text{if } 0 \leq L \leq 2 \\ M_gP_L = 4 - (L - 2)^2 & \text{if } 2 < L \leq 4 \end{cases}$$

For well known reasons, we know that the relevant part of the M_gP_L curve is the negatively inclined stretch (the second component of the above function), since, given the quantity of corn-capital, the product may be increased with additional quantities of labour –that is the resulting area under that curve is larger the higher the quantity of labour employed. Of course, the limit of such a quantity will be determined both by the supply of labour and by the wage rate ruling in the market. Given the quantity of the other factor, the rate at which the output increases is a decreasing one, as the negatively inclined part of the curve indicates. But let us return to our construct of the *net* marginal productivity curve.

Let us assume that for producing a unit of *net* corn output it is necessary to replace a quantity $\alpha > 0$ of corn-capital. This means that there will be a quota (αK) of the given quantity of corn-capital which must be replaced for producing a given level of *net* output given by the technical conditions of production. We assume $\alpha = 0.1$ (whichever the level of corn-output produced), the given capital is $K = 2$ units of corn, and that the labour available in the economy is not higher than 4. The maximum *gross* average product of labour, which is four, is the maximum value that can be reached by the *gross* marginal product of labour. Lastly we assume that half a unit of

³⁴² Appendix to ch.2, par. 2.1.

labour is combined with 0.5 of corn-capital, one unit of labour is combined with one unit of capital, 1.5 of labour with 1.5 of corn-capital and so on up until labour reaches 2 units which is combined with 2 units of corn-capital. All these combinations yield a *gross* marginal product of 4. (See figure A.1 below).

Then what is relevant to our purposes here is to show that the “share” of the amount of corn-capital to be replaced (αK) out of the gross output will be *diminishing* as the output increases for the negatively inclined part of the $M_g P_L$ curve – that is when the area under the $M_g P_L$ curve is being increased by adding quantities of labour higher than 2. By subtracting that “share”, for all relevant values of labour employed and given the quantity of the other factor, from the $M_g P_L$ function, we shall obtain the *net* marginal product of labour.

Let us make it sure we understand this construct by using the above function and the assumptions of the relevant variables. Imagine, now, that the $M_g P_L=3$ hence $L=3$.

In this hypothetical situation the gross output is given by $8 + \int_2^3 4 - (L - 2)^2 dL$ which is

equal to $\frac{35}{3}$. But that marginal product is *gross*, not *net*. To find the *net* $M_g P_L$ it is

necessary to calculate the “share” of the replacement of corn-capital when *gross*

output is $\frac{35}{3}$. This “share”, $\frac{\alpha K}{\frac{35}{3}}$, amounts to $\frac{\frac{1}{5}}{\frac{35}{3}} = \frac{3}{175}$. Then the *net* marginal

increment in output when the labour employed is approaching 3 units will be given by

$3 - \frac{3}{175} = 2.982$. Now, in order to see that that quota out of the gross output is a

decreasing function of the quantity of labour employed (and hence of the output),

imagine that $M_g P_L = \frac{7}{4}$ hence $L=3.5$. In this situation gross corn output is given by

$8 + \int_2^{3.5} 4 - (L - 2)^2 dL$ which amounts to 12.875. Accordingly $\frac{\alpha K}{12.875} = \frac{\frac{1}{5}}{12.875} = 0.0155$

which is lower than $\frac{3}{175} = 0.017$, and the net marginal increment in output when the

labour employed is approaching 3.5 units will be given by $\frac{7}{4} - 0.0155 = 1.7345$. It can

be noted that, so long as we move from north-west to south-east on the negatively inclined part of the *gross* marginal productivity curve, the *net* marginal productivity curve will be approaching the former, as the labour employed is increased up until its maximum value – assumed to be 4 only for the sake of exposition.

On the other hand, for values of labour which correspond to the part of the *gross* marginal product curve near to its constant part, the respective levels of the *net* marginal product will be near the *net* marginal product of labour for the constant part of the gross marginal product curve, which is a constant value too. This value, due to our example, is equal to $\frac{1}{40}$, resulting from, when $L=1$, $\frac{1}{40} = \frac{\alpha K}{4} = \frac{0.1}{4}1$, and when $L=2$, $\frac{1}{40} = \frac{\alpha K}{8} = \frac{0.1}{8}2$.

Thus equipped we can derive the following *net* marginal productivity function of labour:

$$\left\{ \begin{array}{l} NetM_g P_L = \frac{1}{40} \\ \\ NetM_g P_L = \frac{\frac{1}{5}}{8 + \int_{l_0}^{l_1} 4 - (L - 2)^2 dL} \end{array} \right.$$

The first component of the function is applied when $0 \leq L \leq 2$, while the second one is applied when $2 < L \leq 4$, such that $l_0 < l_1$ with $2 \leq l_0 < 4$ and $2 < l_1 \leq 4$.

A.2. Therefore we have built the *net* marginal product curve, which provides the *net* marginal increment in output as an infinitesimal unit of labour is increased. Then, the capitalist, who is the entrepreneur, will minimise costs when hiring the number of labourers for which the *net* marginal product (an amount of corn) is equal to the wage rate he finds ruling in the market.³⁴³

³⁴³ The notion of a net marginal product may be traced back in Marshall's *Principles*: "Every business man indeed, according to his energy and ability, is constantly endeavouring to obtain a notion of the relative efficiency of every agent of production that he employs; as well as of others that might possibly be substituted for some of them. He estimates as best he can how much *net* product (*i.e.* net addition to the value of his total product) will be caused by a certain extra use of
(*cont.*)

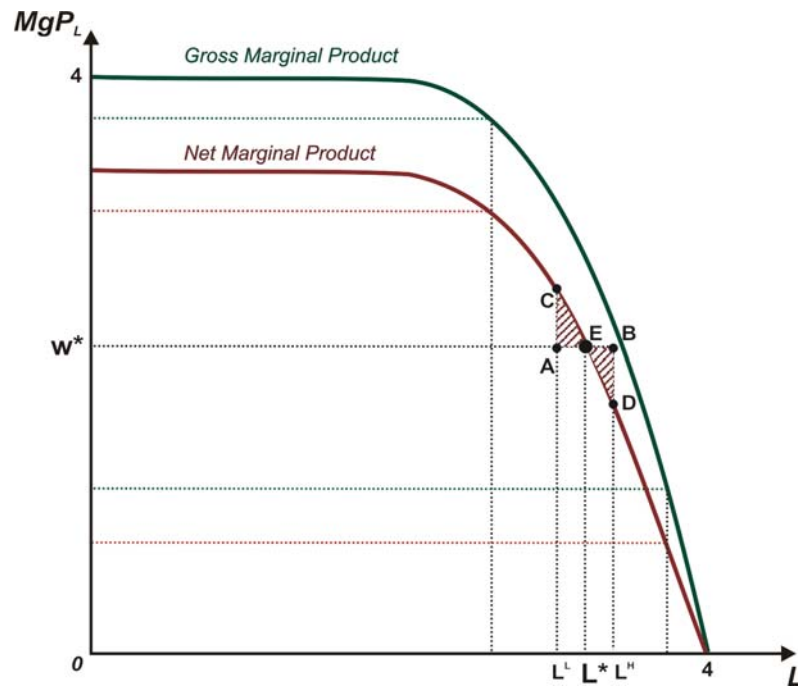


Figure A.1.

If the entrepreneur finds in the market $w=w^*$, then the optimal quantity of L demanded should be L^* . Were it otherwise, *e.g.* if the labour demanded were L^L which is lower than L^* , the entrepreneur would lose the shadowed area ACE .

any one agent; *net* that is after deducing for any extra expenses that may be indirectly caused by the change” (Marshall, 1959[1890]; pp. 336-7, emphasis in the original).

APPENDIX B

*Discussion of the ‘supply-side’ of capital in Wicksell’s general equilibrium system*³⁴⁴

B.1. This appendix attempts to illustrate, by using Wicksell’s general equilibrium system present in his *Lectures*, the arguments raised in ch.2 above concerning the supply-side or supply-aspect problem of capital.³⁴⁵ Moreover, it will be attempted to illustrate the *other* problem which arises with groups (ii) and (iii) of the data: in group (ii) the quantities of factors define the methods of production, whilst in group (iii) those quantities appear in the role of a ‘factor endowment’, that is the role on which the supply *functions* of each factor are based. In both of these roles the factors should be *measured* -and hence known- *before* knowing distribution, in order to obtain the quantities corresponding to *any* given method of production (group (ii)) and to any given physical output (group (iii)). The crucial point seems to be to measure the factors of production before determining distribution, in order to avoid circular reasoning.³⁴⁶

Let us not forget that the discussion of these problems meets those theorists who regarded capital in value terms as data (and not other theorists who considered capital, as part of the data, as the vector of the heterogeneous capital goods, like Walras and the “new” versions of the theory).

B.2 To begin with, we shall simplify the exposition by assuming the production of a single consumption good in yearly cycles.³⁴⁷

³⁴⁴ Appendix to ch.2, par. 2.8.

³⁴⁵ Cf. Wicksell (1904[1954]) pp. 144-206. These problems have already been dealt with by Garegnani (1958, 1960, 1990), and on his works shall I rely for presenting these two groups of difficulties of capital associated, on the one hand, to the contradiction between the role in group (ii) and group (iii) of the data, and on the other, to its supply-role.

³⁴⁶ Lastly, it will be useful to bear in mind how the problem of capital appears in Wicksell’s theory –which we illustrate in the present par.– in order to discuss, later, with Joan Robinson’s way to deal with capital, which has been associated to a discussion of “aggregate” production functions. (See ch.5).

³⁴⁷ Wicksell does so initially (*op. cit*) p. 181, and later in the book (pp. 196-206) he proceeds to the case of more than one consumption good.

The consumption good is produced by means of current labour, current land, and the saved-up (services of) land and labour: that is ‘capital’ in the form of past labour and land applied at different periods of time to the consumption good’s production before the latter emerges.³⁴⁸ It is here assumed, as Wicksell did, an economy where labour and land are saved up for no more than two years.³⁴⁹

Let us consider the general system set out by Wicksell. Therefore let:

Q : be the yearly output of the consumption good

K : be the value in terms of Q of the capital stock available in the economy

A : be the quantity of labourers available (they may be regarded as the absolute number)

B : be the acres of land available

A_0 : be the quantities of labour (labours-year) applied “0” years before Q emerges: that is current labour applied to Q .

A_1 : be the quantities of labour (labours-year) applied 1 year before Q emerges.

A_2 : be the quantities of labour (labours-year) applied 2 years before Q emerges.

B_0 : be the acres of land applied “0” years before Q emerges: that is current land applied.

B_1 : be the acres of land applied 1 year before Q emerges.

B_2 : be the acres of land applied 2 years before Q emerges.

w : be the yearly wage rate in terms of Q paid in arrears.

r : be the yearly rent rate in terms of Q .

i : be the yearly rate of interest in terms of Q .

Wicksell takes as given the quantities A , B , and K . He further supposes the quantities supplied for productive use to be independent of the rates of remuneration.

³⁴⁸ *E.g.* If in year t , corn, the consumption good Q , is produced by means of labour, land, and “ploughs”, then the latter are regarded as the (past) labour and (past) land used in year $t-1$ to produce those ploughs (suppose to be of one-year duration), which in turn are used to produce corn in t . In t , therefore, we reckon capital as the saved-up labour and saved-up land necessary to produce those ploughs.

³⁴⁹ *Cf.* Wicksell (*op. cit.*) p. 158. Or, $n=3$ if the reader is familiar with Garegnani’s (1990, p. 29) reconstruction of the system. Anyway, there is no loss of generality if the period of saved up labour and land is either one, or two, or n years. As Wicksell put it: “What we have to say in this connection can easily be extended to processes of production and capital investment over any period whatever.” (*Ibid.*)

The unknowns of the system are A_0, A_1, A_2 , (dated quantities of labour), B_0, B_1, B_2 , (dated quantities of land), Q, w, r , and i . For that determination we have the following relations:

$$Q = f(A_0, A_1, A_2; B_0, B_1, B_2) \quad (1.B)$$

Equation (1B) defines the production function of the year's consumption good, Q , –which is produced by means of current labour and land, and (saved and capitalised) resources during the two preceding years– and gives the combination which minimises the cost of production that the entrepreneur has to incur: wages w of the current labour (A_0), rents r of the current land (B_0), and interest i of the saved up labour (A_1, A_2 , that is the labours-years of the two preceding years) and of saved up land (B_1, B_2 , that is the utilised acres over the two preceding years).

A caveat here is in order. Firstly, to give, say A_1 , economic sense, we must assume that $w_1 > w$, so that the rate of interest will be $\frac{w_1 - w}{w} = i$ (w_1 is the wage rate which corresponds to the quantity of labour applied one year ago necessary to produce the capital goods to be reckoned to-day). For investment of “labour”-capital for two years, we must have $\frac{w_2 - w_1}{w_1} = i$. Furthermore, Wicksell assumed that, under competition, “all capital will receive approximately the same return” (p. 156), so: $\frac{r_1 - r}{r} = i$ and $\frac{r_2 - r_1}{r_1} = i$, (r_1 and r_2 are the rent rates which corresponds to the quantity of land necessary to produce the capital goods one and two years ago respectively, and to be reckoned to-day). Secondly, these equilibrium conditions (however not being used to determine equilibrium in the system)³⁵⁰ indicate us that, in equilibrium, uniformity of returns on the different capital goods is endogenously reached through equations (2.B) and (3.B)-(4.B)³⁵¹ below, that is through the relations expressed by, on the one hand, the equilibrium conditions supplied by the role that the factors play

³⁵⁰ In fact, once i and r are determined, one can determine r_1 by $r_1 = r(1+i)$, and r_2 by $r_2 = r(1+i)^2$. Similarly, once w is determined, both w_1 and w_2 will be determined too. It is worth pointing that those equilibrium equations entail a compound, and not simple interest.

³⁵¹ Discussion of equation (4.B) is postponed later in the present appendix.

in the definition of the methods of production, and on the other, by the supply and demand equilibrium functions for factors. Lastly, we would like to underlie that, equation (1.B) together with the following groups of equations do express “half” of the problem which arises between the role of productive factors in both its roles: group (ii) & (iii) of the data. In other words, the factors’ quantities defining the methods of production in the arguments of equations (1.B) and (2.B) are expressed in technical units.

Then we have the equations expressing the equilibrium relations between marginal productivity of each factor of production (measured in technical units) and their respective rates of remuneration. Then:

$$\begin{array}{ll}
 \frac{\partial Q}{\partial A_0} = w & \frac{\partial Q}{\partial B_0} = r \\
 \frac{\partial Q}{\partial A_1} = w(1+i) & \frac{\partial Q}{\partial B_1} = r(1+i) \\
 \frac{\partial Q}{\partial A_2} = w(1+i)^2 & \frac{\partial Q}{\partial B_2} = r(1+i)^2
 \end{array} \tag{2.B}$$

This group of equations provide the equilibrium condition of uniformity of rates of returns over costs of the capital goods.

Then there are two equations, which Wicksell derives by assuming “stationary conditions”³⁵²: year by year the total sum of the quantity of labour and of land annually consumed –saved up or current– must be equal to the supply of labour and of land annually available in the economy. Therefore:

$$\begin{array}{l}
 A_0 + A_1 + A_2 = A \\
 B_0 + B_1 + B_2 = B
 \end{array} \tag{3.B}^{353}$$

So far we have nine independent equations for determining our ten unknowns. The missing equation, which Wicksell however introduced verbally, is the one that it

³⁵² Wicksell (*op. cit.*) p. 204.

³⁵³ As Wicksell put it: “[I]f conditions are to remain stationary, two quantities of labour and land must be withdrawn from the production of the consumption goods during the current year and
(*cont.*)

makes the total sum of the values of “labour”-capital and “land”-capital equal to “a certain given quantity – the total exchange value of the capital employed.”³⁵⁴

Before putting forth the formal expression of that assertion, let us however remember that, under conditions of zero net accumulation, the expression K is obtained by bearing in mind that, say, to each A_j , $j= 1, 2$ there will correspond j elements of capital, each embodying A_j labour years. For instance, if A_2 were the quantity of labour for producing *coal* for producing *ploughs* (of one-year duration), used in turn to produce *corn* (the consumption good Q), then we would have, besides the quantity A_2 of labour embodied in ploughs, and applied one year before the moment of our reckoning of capital, a “second” quantity A_2 of labour paid for at the time of reckoning and embodied in the coal which is going to be worked upon by labourers producing ploughs. Hence:

$$K = wA_1 + wA_2[1 + (1 + i)] + rB_1 + rB_2[(1 + (1 + i))] \quad (4.B)^{355}$$

Through equations (3.B) and (4.B) we can see the other “half” of the problem we raised above, that is the roles these factors occupy in defining the endowments of the economy (group (iii) of the data). As to capital, it is evident that its measurement is not independent of distribution –contrary to what it is necessary in order to lie the grounds for a theory of distribution like the one we are considering.

B.3. Wicksell himself was aware of the difficulties associated to a value measurement of capital in order to provide the foundations of marginalist theory³⁵⁶:

devoted (1) to production of goods which will only be used in the following year, (2) to goods which will only be used in the year after that.” (*ibid.* p. 158).

³⁵⁴ *Id.*, pp. 204-5. The complete citation runs: “...the total exchange value of the capital employed in *the two industries together*, expressed in terms of the first commodity...” (*ibid.* emphasis in the original). The importance of the quotation in the main text, together with this complement, is the realisation that Wicksell assumes a given capital in value terms as the endowment of the economy, in order to fulfil the condition of a uniform rate of return on the supply prices of the capital goods; capital goods, that is, which are required by the productive conditions of each of the “two industries”. This passage also provides insight into the issue of why the composition of the capital endowment must be *endogenously* determined, as it has been argued in ch.2 section II, above.

³⁵⁵ The term wA_2 on the RHS of (4.B) is the “first” quantity of A_2 reckoned as capital (from our example: it is the labour employed in producing ploughs), while the term $wA_2(1+i)$ is the “second” quantity of A_2 , that is, in our example, the capital embodied in coal produced two years ago to produce ploughs. Cf. also n.98 (ch.3) above.

³⁵⁶ Cf. also Garegnani, (1960), p. 136 n, p. 143 n.

“Whereas labour and land are measured each in terms of its own *technical unit*, capital, on the other hand, is reckoned, in common parlance, as a sum of *exchange value* – whether in money or as an average of products. (...) However good the practical reasons for this may be, it is a theoretical anomaly which disturbs the correspondence which would otherwise exist between all the factors of production.”³⁵⁷

The “correspondence” between all factors of production is expressed in equations (1.B)-(2.B), where “capital” is measured in technical units. However, its existence is at stake, as Wicksell recognises, as “capital” is then introduced in value terms – equation (4.B).

Despite the value measurement of K in equation (4.B), Wicksell states that “capital is saved-up labour and saved-up land.”³⁵⁸ However, he does not take as given the capital endowment of the economy in terms of physical quantities $A_1, A_2; B_1, B_2$. Had he been consistent with such a definition, A_0 and B_0 would have had resulted, in accordance with equations (3.B), by subtracting from A and B the quantities needed to replace the saved up labour and land as they are being used up. Therefore, the number of unknowns would nevertheless have *fallen below* the number of independent equations, with the unknowns *decreasing by 4* (i.e., A_1, A_2, B_1, B_2) but only 2 equations (the first two in equation (2.B): that is $\frac{\partial Q}{\partial A_0} = w$ and $\frac{\partial Q}{\partial B_0} = r$) disappearing.

This over-determinacy would of course have engendered a conflict between the conception of capital in value terms, as Wicksell expressed it on some pages of his *Lectures* (and quoted above), and equations (2.B). Therefore, both the expression for capital (equation 4.B), and equations (2.B) establish the *uniformity* of the rate of return on wages (w_1, w_2) and rents (r_1, r_2) advanced for the several periods of time and therefore, on the supply price of the capital goods (the rate of interest i).

In fact, it is not that Wicksell would have been more satisfied with his results had he used a physical measurement of capital to express the endowment. In this regard it is worth bringing in the following quote:

³⁵⁷ Wicksell (*op. cit*) p. 149, emphasis in the original.

³⁵⁸ *Id.* p. 154.

“If capital also were to be measured in technical units, the defect [of measuring capital in value terms] would be remedied and the correspondence [between factor of production demanded and remuneration] would be complete. But, in that case, productive capital would have to be distributed into as many categories as there are kinds of tools, machinery, etc., and *a unified treatment of the rôle of capital in production would be impossible*. Even then we should only know the yield of the various objects, but nothing at all about the value of the goods themselves, which it is necessary to know in order to calculate the rate of interest, which *in equilibrium is the same on all capital*.”³⁵⁹

One can perceive, through the perusal of this long quotation, that Wicksell was aware that the only way to reach the uniformity of return on the supply prices of capital goods is by treating capital as a single magnitude in value terms, leaving the particular endowments of the stock to be endogenously determined. Further, he bestows the “unified treatment of the rôle of capital in production” the meaning that most of traditional economists had already supplied to economic theory: capital is conceived in value terms to determine the uniform equilibrium rate of interest over the costs of the capital goods. And this is the supply aspect of the problem of capital.³⁶⁰

³⁵⁹ *Id.* p. 149, emphasis added.

³⁶⁰ Wicksell, as noted by Garegnani (1960) “falls back” to a value measurement of capital. He “falls back” because he had already provided in *Value, Capital and Rent* (1954) a value measurement of “capital” based, as had Böhm Bawerk (1889), on the notion of an average period of production. Wicksell, as Garegnani (1960, part II) has argued, was very unsatisfied with the assumptions necessary for such a measurement, in particular: i) the rate of interest involved is a simple one, ii) the absence of fixed capital, iii) the existence of only one original factor to derive the average period of production. In the *Lectures* Wicksell drops the first and the third assumptions, but, however, for the reasons we have provided in the main text, he falls back, again, on a value measurement of capital. Some of the quotations provided in the text of this appendix somehow reflect how unsatisfied the Swedish economist felt with the definition of the capital endowment in value terms.

APPENDIX C

*Sraffa's landmark in the critique of the marginal theory*³⁶¹

C.1. Underlying the results of 'reswitching' and 'reverse capital deepening' is Sraffa's outstanding contribution *Production of Commodities by means of Commodities* (1960). In that work the famous Italian economist, who had established residence in Cambridge since the mid-20s, reached the results known in the debates as 'reswitching' and 'reverse capital deepening'. In that work (though we are told in the preface that the bulk of the ideas present in the book had been developed in the 20s and 30s) Sraffa shows that as the rate of profits³⁶² (or the real wage) varies, both the relative prices and the value of capital per labourer have to change.³⁶³ This is of course not new. In fact, the value of capital will change with changes in distribution when neither the techniques of production nor the output change. Sraffa goes further. He demonstrates that, given the technical production coefficients and outputs, when the rate of profits is increased, the movement of the price of a commodity relative to another need *not* be monotonic.

Sraffa shows that a commodity's price can be obtained by "reduction to dated quantities of labour", that is the sum of an infinite series of terms, representing the wages paid, to direct labour, to the direct labour necessary to produce the means of production, to the direct labour necessary to produce the means of production of the means of production, and so on. Each term of this sum must be multiplied by 1 , $(1+r)$, $(1+r)^2$, etc. (Wages of current labour are paid in arrears) For a generic commodity j its price can be represented as follows:

³⁶¹ Appendix to ch.3, par. 3.1.

³⁶² It has been preferred in this appendix to speak of rate of profits rather than the rate of interest as does traditional theory, because Sraffa is reviving the classical approach and the classical economists from Smith to Ricardo (and also Marx) spoke of a rate of profits. Profits are envisaged by these authors, and Sraffa and current the surplus approach's theorists, as a surplus after having deduced from the social product the necessaries for the reproduction of the economy (wages and replacements of capital goods). However, in this study we have used sometimes the term "rate of interest" as a synonym of rate of profits.

$$p_j = wL_{j(t)} + wL_{j(t-1)}(1+r) + wL_{j(t-2)}(1+r)^2 + wL_{j(t-3)}(1+r)^3 + \dots \quad (1.C)$$

where the terms $L_{j(t)}$, $L_{j(t-1)}$, ... represent the *dated quantity of labour* one obtains with the reduction. Sraffa then considers the case of two commodities whose reduction to dated quantities of labour generates identical dated labour terms except for three:

“One of them, ‘a’, has an excess of 20 units of labour applied 8 years before, whereas the excess of the other, ‘b’, consists of 19 units employed in the current year and 1 unit bestowed 25 years earlier.”³⁶⁴

In mathematical terms it means that the difference between both prices can be written in the following way:

$$p_a - p_b = 20w(1+r)^8 - [19w + w(1+r)^{25}] \quad (2.C)$$

The difference on both prices is zero when $r=0$, $r=17\%$ and $r=25\%$, which is the value of the maximum rate of profits (the wage rate is zero). The price of commodity ‘a’ rises relative to ‘b’ when $0 < r < 9\%$ and when $22\% < r < 25\%$. Instead, the price of commodity ‘a’ falls relative to ‘b’ when $9\% < r < 22\%$. Sraffa comments this result:

“The reversals in the direction of the movement of relative prices, in the face of unchanged method of production, cannot be reconciled with *any* notion of capital as a measurable quantity independent of distribution and prices.”³⁶⁵

C.2. To understand Sraffa’s conclusion, imagine that corn and cloth are produced by means of labour and *heterogeneous* capital goods. Then Sraffa’s result means that, as the wage rate rises (and the rate of profits decreases), the relative price of *cloth*

³⁶³ The only exception being the special case of “equal organic composition” noted by Marx (1961) in *Das Kapital*. In this case relative prices are proportional to labour embodied and independent of the distribution of income.

³⁶⁴ Sraffa, (1960) p. 37.

³⁶⁵ *Id.* p. 38. For a first echo of Sraffa’s discovery (in particular, of his striking conclusion quoted in the main text) cf. Harrod (1961) and Sraffa (1962).

(which is “more” labour intensive) relative to corn first *increases* and then *decreases*. Hence cloth is more labour-intensive for certain values of the rate of interest, but more “capital” intensive for other values. Hence there is no monotonic correlation between changes in distribution and changes in the changes in the prices of consumption goods (or costs of production). This explains why there does not exist any measurement independent of distribution of the “capital” intensity production method. Thus, ‘reverse capital deepening’ may arise even when nothing technically changes.

APPENDIX D

*'Reverse capital deepening' without 'reswitching'*³⁶⁶

D.1. Even in the absence of 'reswitching' there might be a "perverse" relationship between "capital" and the rate of interest. That will be the case when the value of capital per worker (k) *increases* when r increases too. For instance, consider a family of wage curves such that all of them are concave. While the "jumps" through the switch points from technique to technique goes in the neoclassical behaviour – as r increases k decreases *at those switch points* (Cf. in figure D1 below: k in B at $r=r_2$ is lower than k in A at $r=r_1$, and $r_1 < r_2$) – there are, on the other hand, increments in k as r increases i) from 0 to r_1 , ii) from r_1 to r_2 , iii) from r_2 to R^γ .

D.2. There is no general reason to presume that the "jumps" could more than offset k associated with values of r in between switch points. The figure below illustrates the case in which k more than offset the "jumps" *in between the switch points*.

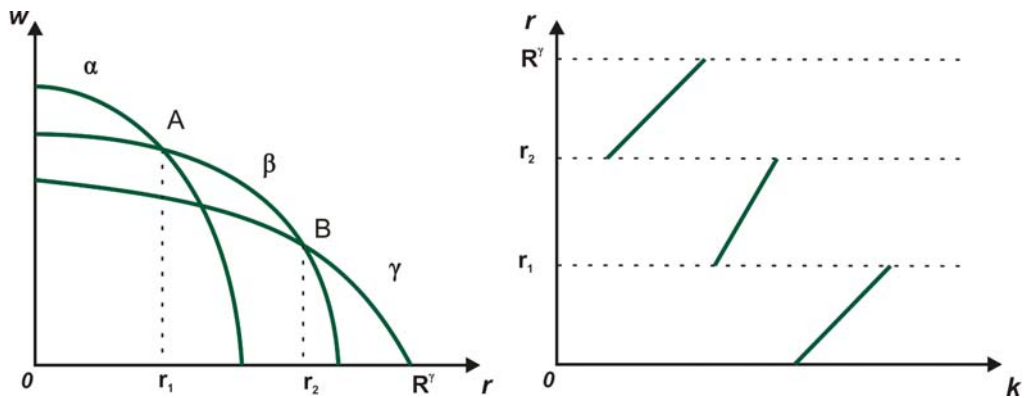


Figure D.1. Concave wage-curves and the employment function of "capital"

³⁶⁶ Appendix to ch.3, par. 3.4.

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AER: American Economic Review
Aust.EP: Australian Economic Papers
CJE: Cambridge Journal of Economics
CanJE: Canadian Journal of Economics
EJ: Economic Journal
ER: Economic Record
EJHET: European Journal of the History of
Economic Thought
HPE: History of Political Economy
JEL: Journal of Economic Literature
JEPersp: Journal of Economic Perspectives
JPE: Journal of Political Economy
JHETH: Journal of the History of Economic
Thought
NLR: New Left Review
OEP: Oxford Economic Papers
QJE: Quarterly Journal of Economics
RES: Review of Economic Studies
REStat: Review of Economics and Statistics
SEJ: Southern Economic Journal
ScanJE: The Scandinavian Journal of Economics
WEJ: Western Economic Journal

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