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Essays on Dynamic Stochastic General Equilibrium Models

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Introduction

This research theoretically analyzes some stylized facts that characterize postwar U.S. business cycles, and in particular: i) the explanation of the positive comovement of employment (and other variables) across sectors; ii) the source of the difference between the average real return rate on the stock market and the average riskless real interest rate (i.e. the equity premium).

Relating to the first theme, Christiano-Fitzgerald (1998) and Huffman-Wynne (1999) among others, estimate the correlation between sectoral output and employment. Their works show that output and employment across a broad range of sectors move together. With respect to the second topic, a large literature estimates that the equity market has granted, on average, substantially higher returns than Treasury bills. For example, Campbell (2002) analyzes U.S. data over the period 1947.2 to 1998.4 and asserts that the average real return has been 8.1% at annual rate, and that the average real return on 3-month Treasury bills has been 0.9% per year.

Both subjects have been largely studied in economic literature, but although the understanding of these arguments has increased, some issues still deserve to be clarified, justifying further research on these topics.

Then the purpose of this work is to suggest a new explanation of both these themes following the dynamic stochastic general equilibrium (from now on DSGE) approach. DSGE models rely on the hypothesis that important features of the aggregate economy can be analyzed by the formalization of the behavior of representative agents (as firms and households) that act rationally. These models generally assume the presence of exogenous shocks that generate fluctuations of the economic variables (such as output, employment, consumption and investment) around the long run equilibrium.

Due to the high flexibility of the DSGE approach, we choose to use this modelling scheme to perform our analysis for both topics of interest. Indeed, DSGE permits the formalization of a large range of hypotheses and the introduction of many elements.

Moreover, with thanks to the progress of the last decades, it allows to run numerical experiments. In this way, it is possible to calibrate the model economy so that it mimics the real economy along a carefully specified set of dimensions. Then, it provides a means of studying the quantitative effects of selected events.¹

Thesis Overview

The first chapter of this work analyzes the sources of sectoral comovement. The literature that studies this topic has failed to highlight the role of consumer's tastes as a source of positive correlation between employment in different sectors.

Indeed, this literature has mainly invoked aggregate shocks that directly affect the whole economy (i.e. productivity or monetary shocks) or has highlighted the role of interlinkage in production process by conveying sectoral fluctuations in the entire economy.

We discuss sectoral comovement by developing a two-sector model. We show that a shift in relative preferences between consumption goods is sufficient to explain positive comovement of output, consumption, investment and employment across sectors.

This result contrasts with the argument that in a standard business cycle model a relative preference shift between goods is followed by a negative comovement between sectoral employment.²

In order to justify how a change in the consumer's preferences determines a shift in aggregate variables, we assume that the consumers can change their tastes with respect to their consumption goods. Then, the consumer's satisfaction related to the entire consumption basket changes as well, determining a "perception effect" that, under specific conditions, can offset the "substitution effect" induced by the preference shift.

To explain how the mechanism works, it can be useful to consider the structure of the representative household's preferences. The household's utility is increasing in leisure and

¹See Kydland-Prescott(1996).

²See Christiano-Fitzgerald (1998) and Phelan-Trejos (2000).

consumption of two goods (denoted with c_1 and c_2). So the household optimally chooses both how to allocate its time between leisure and working hours in each sector and how to spend its income between the two goods.

Now suppose that the relative preferences move towards a type of consumption good that represents a small share in the consumption basket (suppose c_1), then the household associates to the actual standard of consumption a lower level of satisfaction. In order to return to the optimal resource allocation, the household is willing to modify the allocation of time and income to compensate for the decrease in consumption satisfaction. Then the household proceeds as follows. It allocates time and income within the sector that produces the type of good whose relative preference has increased (i.e. c_1). But, if the rise of c_1 is not enough to compensate the loss of satisfaction related to c_2 (because now c_2 has lost "appealing") then the allocation of time and income within the sector producing c_2 also increases. On the contrary, if the taste change is related to some goods that are largely available to the consumer, his satisfaction from consumption increases and the dynamics are reversed. The role of the composition of the consumption basket on the household's choice is what we identify as perception effect.

Technically speaking, a preference shift directly affects the marginal rate of substitution between consumption goods (in the standard way), whereas it affects the marginal rate of substitution between consumption goods and leisure according to the direction of the perception effect.

We show that if the shift is sufficiently persistent the described mechanism involves investment and labor supply in the same way, explaining a dynamic characterized by positive sectoral comovement of consumption, investment, output and employment.

It is noteworthy that the model proposed in Chapter 1 is formalized to clearly elicit the role of preference shifts. In fact, it does not consider aggregate shock, productivity shock and input-output interlinks.

The other two contributions presented in the second and the third chapters of this work develop a theoretical framework that investigates the sources of equity premium.

Most of the literature has modified the preference structure of the representative agent in order to reconcile the principle of consumption smoothing with high asset returns and low, risk free rates.³

These attempts have been quite successful in replicating selected elements of financial variables, but they have showed some difficulties in explaining the behavior of wages and employment along the business cycle. In fact (as we extensively explain in Chapter 2), the standard modelling of labor supply needs highly volatile and pro-cyclical real wages in order to generate consistent equity premium. Such implication, however, is strongly rejected by empirical evidence.⁴

Maintaining a Consumption Capital Asset Pricing framework, we follow Danthine-Donaldson (2002) and locate the core of the explanation of the equity premium in labor relations. Although our analysis is established within this work, we propose a different institutional setting, as we investigate the interaction between labor union and firm instead of the relation between worker and employer. Particularly, the introduction of labor unions allows disentangling labor supply from consumption path, and this hinders labor supply decisions from acting as an insurance device against fluctuations in consumption.

Moreover, from a theoretical point of view, the framework presented in our study allows us to incorporate insights of financial literature within DSGE macroeconomics models which include labor unions.⁵

Detailed in Chapter 2, it is assumed that the relative bargaining power of the labor union is an increasing function of the aggregate employment rate. Under this assumption the equilibrium wage depends positively on the aggregate employment rate and on firm's dividend (or liquidity). We show that these two elements move in the opposite directions

³See Campbell-Cochrane (1999) for a detailed review.

⁴For data analysis see Christiano-Eichenbaum (1992) among others. Recently, to overcome this problem Uhlig (2007) proposes exogenous habit formation in both leisure and consumption, and some unmodelled friction that prevents all labor supply from reaching the market.

⁵See Maffezzoli (2001) and Chiarini-Piselli (2005) regarding the study of the role of labor union in a DSGE model, and see Ramirez (2006) with regard to the study the presence of labor union on financial market performance.

along the business cycle, and this generates a-cyclical wages. The linkage between investment, dividend and wage induces high volatility on the asset returns, explaining the source of the equity premium.

Otherwise, in Chapter 3 we investigate another hypothesis concerning labor relations. We suppose that in each firm there is a standard monopoly union, but we assume that its preferences are characterized by a concern for firm's performance. In particular, the union's relative preference for wage, with respect to employment, are formalized as an increasing function of both firm's profits and dividends. Under this assumption, the dynamics following a productivity shock strongly depend on the chosen indicator of firm performance.

The model is capable of explaining equity premium only when the labor union links relative preferences for wage to distributed dividends of shareholders. In fact, similar to the model proposed in the previous chapter, a relationship between investment, dividend and wage emerges, generating high volatility of asset returns and explaining the equity premium.

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CHAPTER 1

Relative Preferences Shifts and Intersectoral Comovements

Abstract

This paper develops a two-sector general equilibrium model in which aggregate fluctuations are driven by shocks to relative preferences between consumption goods. These shocks can be represented as possible consequences of taste changes. When such a shift in preferences occurs the consumers associate different levels of satisfaction to the same basket of consumption depending on the state of preferences. We show that, under specific conditions concerning the initial composition of the consumption basket, a shift in relative preferences produces a "perception effect" that induces positive inter and intra sectoral comovements of selected macroeconomic variables (i.e. output, consumption, investment and employment). It is remarkable that the results are reached without introducing technology shocks, input-output linkages or direct changes in relative preferences between aggregate consumption and leisure.

Journal of Economic Literature Classification Numbers: F11, E320

Keywords: Demand Shocks, Two-sector Dynamic General Equilibrium Models

1.1. Introduction

The positive comovement of economic activity across different sectors is one of the most important regularities of all business cycles.¹ Burns-Mitchell (1946) included intersectoral comovement in the definition of business cycles, and many empirical studies prove procyclical behavior of cross-sector measures of employment, output and investment.² A first way to explain comovements along business cycles is to consider shocks that affect all the economy. Proceeding in this direction it is possible to explain two important stylized facts of business cycles: persistence of deviations and positive comovements between sectors. In fact, it is generally assumed that exogenous stochastic variables follow an autoregressive process (persistence), and these shocks concern the aggregate supply or demand side of the economy (positive comovement).

This approach is not fully satisfying because it is difficult to identify reasonable aggregate disturbances capable of explaining historical business cycles. So a vast literature emphasizes the transmission mechanisms from sectoral shocks to aggregate fluctuations. We follow this research field and propose a preference-based mechanism to explain sectoral linkages.

Most of the multi-sectoral literature considers the input-output structure of the economic system the most important transmission channel of sector-specific shocks. A seminal paper of Long-Plosser (1983) details a model in which the output of each sector can be used in the production process of all the other sectors, and consequently, an idiosyncratic technology shock modifies the possibilities of the production process in each sector. However, as emphasized by Murphy et al. (1989), Long-Plosser's model explains comovements only of output and not of employment. Afterwards, many contributions have tried to explain why employment should increase in sectors that experience a reduction in relative productivity. Hornstein-Praschnik (1997) distinguish between the production of durable goods and the production of nondurable goods and highlight the great use of the latter

¹Lucas (1977).

²See Christiano-Fitzgerald (1998) and Huffman-Wynne (1999).

as intermediate goods in the production of the former.³ By this way, a sector-specific shock in either sector affects the accumulation of capital (durable goods) and, therefore, the demand of intermediate goods (nondurable goods). Huffman-Wynne (1999) develop a two-sector model with only one sector producing capital goods to be employed in both sectors (the other type of capital goods is sector specific). They introduce intratemporal adjustment costs for switching production between the two types of capital. Consequently, it becomes costly to modify the composition of capital goods, and thus investment goods will be positively correlated. Horvath (1998, 2000) displays that in presence of a particular kind of not full input-use matrix, the law of large numbers does not work and then aggregate volatility could be induced by sector-specific disturbances.⁴ Horvath's analysis reveals that aggregate volatility can be generated by sectoral volatility when the input-use matrix is characterized by a few full rows and many sparse columns. Thus, two requirements are needed. First, the economy has to be characterized by the presence of sectors that produce intermediate inputs for "all" the sectors of the economy. Second, only a few rows have to be full and subsequently only a few sectors have to play the role of input-supplying sectors for most of economic system.

All quoted contributions rely on both productivity shocks and technological linkages between sectors. In fact, the input-output structure grants that after an idiosyncratic productivity shock fluctuations in each sector have the same direction. On the contrary, Cooper-Haltiwanger (1990) suggest that the normality of demands for consumption is the channel through which sectoral shocks spread over the economy; meanwhile, the presence of only a few sectors that hold inventories is the main intertemporal mechanism of transmission. For example, an increase in inventories immediately reduces the production in sectors holding inventories and then the income of workers employed in those sectors.

³The Authors support the significance of their classification of sectors noting that from U.S. input-output tables it emerges that nondurable goods are 26% of total payments to inputs of the durable goods producing sector, while durable goods reach only 4% of total payments to inputs of the nondurable goods producing sector.

⁴The Author points out that the traditional argument against multi-sectoral models is that whether sector-specific shocks are i.i.d. variables, then the law of large numbers implies that positive and negative shocks offset one another.

Therefore, these workers reduce the demand of the goods produced in the other sectors, thus emerging positive comovements of employment and output.

Unlike the quoted works, we develop a framework without changes to productivity. Fluctuations are induced by shocks to the structure of preferences; in particular, shocks concern consumers' relative preference between two consumption goods. We show that such kind of preference shock is able to explain both the persistence of fluctuations and the positive comovements of output, consumption, investment and employment between sectors. We remark that this kind of shock affects only the relative preference for different goods and does not directly modify the preference structure between the composite consumption good and leisure. That differs from Bencivenga (1992) and Wen (2005, 2006) who assume that shocks directly affect the marginal rate of substitution between consumption and leisure.

The stylized economy is characterized in the following way. Within each sector, a distinct output is produced using employment and sector-specific capital; this sector-specific output yields one type of consumption good and the investment good used to accumulate capital for the sector. This unusual assumption excludes that sectoral comovement is induced by complementarity in the production process. Utility is defined over leisure and over a consumption basket composed of the consumption goods from both sectors. In order to not consider a shock to the relative preference between total consumption and free time, we assume the Cobb-Douglas (homogeneous of degree 1) preferences between consumer goods. There are no other types of shocks, therefore inter and intra sectoral comovements of employment, consumption, output and investment are totally explained by shifts in relative preferences.

We interpret the dynamics focusing on different ways of perceiving the same consumption basket, depending on relative preferences. The paper is organized as follows. Section 1.2 details the benchmark economy. Section 1.3 presents the theoretical mechanism, Section 1.4 presents the selected numerical results and Section 1.5 concludes. Finally, the Appendix includes all proofs and derivations.

1.2. A Two-Sector Model with Relative-Preference Shifts

This section presents the baseline dynamic equilibrium model with relative-preference shocks. Since there are no restrictions to trade, we solve the dynamic planning problem of a benevolent planner.

The benchmark model is structured as a two-sector, two-good economy, with endogenous labor supply choice. There exists a continuum of identical households of total measure one. The relative demand for goods are driven by autonomous changes in preferences of the representative household. Aggregate uncertainty originates from the demand side, and it is modelled using a state dependent utility function. Consumption and capital goods are sector specific, while labor services can be reallocated across sectors, without bearing any adjustment cost.

1.2.1. Preferences

Define a Cobb-Douglas aggregate consumption index in the following way:

$$C_t = c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}} \quad (1.1)$$

where $c_{1,t}$ and $c_{2,t}$, respectively denote the consumption of good 1 and good 2 at time t ; $s_{1,t}$ and $s_{2,t}$ denote the preference weights, following stochastic processes (defined below).⁵ In this framework, a positive shock to $s_{1,t}$ (i.e. an increase in $s_{1,t}$) changes the instantaneous structure of preferences in favor of c_1 . In other words, such a shock would make the consumer perceive the commodity 1 as relatively more important with respect to the other good. The economic interpretation of this kind of shock is immediate. A preference shift is like a change in tastes. If we interpret c_1 as a specific kind of good, for example clothes, cars or food, and c_2 as the remaining goods composing the consumption basket, a positive shock to s_1 implies that the relative preference for clothes (or cars or food) increases with respect to the relative preference for the other goods. In the following section, it will be analyzed how the relative weight of c_1 in the consumption basket,

⁵Also Stockman-Tesar (1995) use the Cobb-Douglas aggregator for tradable consumption goods in a two country framework.

affects the way the consumer "feels" immediately after the shock. This feeling represents the key element to explain positive comovement in our framework. In order to highlight the aggregate effects of only relative shocks, we preserve the homotheticity of degree 1 of preferences and assume that $s_{1,t} + s_{2,t} = 1$, \forall all $t = 1, 2, \dots$

Preferences over aggregate consumption index C_t and leisure ℓ_t are described by a state dependent felicity function $u(C(\mathbf{c}_t), \ell_t; \mathbf{s}_t) : \mathbb{R}_+^2 \cdot \mathcal{S}^2 \cdot [0, 1]^2 \rightarrow \mathbb{R}$:

$$u(\mathbf{c}_t, \ell_t; \mathbf{s}_t) = \frac{(C_t)^{1-\gamma} - 1}{1-\gamma} + B\ell_t, \quad (1.2)$$

where γ is a parameter that measures the degree of risk aversion and is inversely proportional to the elasticity of intertemporal substitution; ℓ_t denotes leisure hours. In order to better understand the behavior of demands for consumption goods, we assume that the marginal utility of leisure is constant and equal to B .⁶ Leisure hours are defined as:

$$\ell_t = 1 - n_{1,t} - n_{2,t} \quad (1.3)$$

where $n_{1,t}$ and $n_{2,t}$ denote working hours in sector 1 and 2. It implies that available hours are normalized to 1 and labor services shift across sectors without adjustment costs.

1.2.2. Production Technologies

Each good is produced with physical capital and labor, using a sector-specific Cobb-Douglas technology:

$$y_{1,t} = \lambda_1 k_{1,t}^{\alpha_1} n_{1,t}^{1-\alpha_1} \text{ and } y_{2,t} = \lambda_2 k_{2,t}^{\alpha_2} n_{2,t}^{1-\alpha_2}, \quad (1.4)$$

where $k_{j,t}$ and λ_j denote, respectively, capital stock and technology level in sector j , for $j = 1, 2$. There is no exogenous technology process (i.e. λ_j parameters are constant over time) so the production is not subject to exogenous technology shocks. As remarked in the introduction, this strongly differentiates our model from the traditional approach that focuses on the effects of idiosyncratic productivity shocks.

⁶In a following section we show that linearity in leisure is not a necessary condition. Such assumption simplifies the explanation of the mechanism underlying relative-preference shifts in consumption goods.

In each sector, capital accumulation follows the standard formulation

$$k_{1,t+1} = (1 - \delta_1)k_{1,t} + i_{1,t} \quad \text{and} \quad k_{2,t+1} = (1 - \delta_2)k_{2,t} + i_{2,t}, \quad (1.5)$$

where the δ_j denotes depreciation rates of capital stocks and $i_{j,t}$ denotes investment flows at time t , for $j = 1, 2$. Eq.(1.4) and eq.(1.5) dictate that the investment good for the capital stock used in sector j is produced entirely in sector j . This hypothesis makes capital goods fixed across sectors and then rules out input-output transmission mechanisms. In this way, it is possible to isolate the way preferences drive intersectoral comovements with no influences from production processes. If we assume that the output of a sector can be used in the production of the other sector, we could observe a positive comovement with no clear understanding of the role of preferences.

The allocation constraint is specific for each sector and is given by

$$c_{1,t} + i_{1,t} = y_{1,t} \quad \text{and} \quad c_{2,t} + i_{2,t} = y_{2,t}, \quad (1.6)$$

1.2.3. Preference Shocks

As just explained, we preserve the homotheticity of degree 1 of preferences and assume that $s_{1,t} + s_{2,t} = 1$. Then, it is sufficient to specify the characteristics of s_1 . It follows an autoregressive process, $s_{1,t+1} = \rho s_{1,t} + (1 - \rho) s_1 + \varepsilon_t$, where $0 \leq \rho \leq 1$ and s_1 is the steady state value. ε_t is a random variable characterized by the following degenerated distribution $\{\varepsilon_{t+h} = \frac{s_1}{100} \text{ for } h = 0, \varepsilon_{t+h} = 0 \forall h > 0\}$. Consequently the relative-preference shock $\{\varepsilon_{1,t}\}_{t=1}^{\infty}$ is transitory, but because of the preference structure, it has persistent effects. Roughly speaking, we are analyzing the effects of a one-shot shock to relative preferences on the stylized economy.

1.2.4. Model's Solution and Equilibrium Characterization

Planner maximizes the expected present discounted value of the return function $\mathcal{V}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(\mathbf{c}_t, \ell_t; \mathbf{s}_t)$, where β ($0 < \beta < 1$) is a subjective discount factor, subject to the allocation constraints (eq.(1.6)), the capital accumulation constraints (eq.(1.5)), and

the total-hour constraint (eq.(1.3)). The state of the economy at time t is represented by a vector $\mathbf{x}_t = \langle k_{1,t}, k_{2,t}, s_{1,t}, s_{2,t} \rangle$. Controls for the problem are consumption flows \mathbf{c} , investment flows \mathbf{i} , and the labor services \mathbf{n} . Introducing dynamic multipliers $\phi_{1,t}$ and $\phi_{2,t}$, forming the Hamiltonian \mathcal{H} yields:

$$\begin{aligned} \max_{\{c_{j,t}, n_{j,t}, k_{j,t+1}\}_{j=1,2}^2} \mathcal{H} = & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}} \right)^{1-\gamma} - 1}{1-\gamma} + B(1 - n_{1,t} + n_{2,t}) + \right. \\ & + \phi_{1,t} \left[\lambda_1 k_{1,t}^{\alpha_1} n_{1,t}^{1-\alpha_1} - c_{1,t} + (1 - \delta_1) k_{1,t} - k_{1,t+1} \right] + \\ & \left. + \phi_{2,t} \left[\lambda_2 k_{2,t}^{\alpha_2} n_{2,t}^{1-\alpha_2} - c_{2,t} + (1 - \delta_2) k_{2,t} - k_{2,t+1} \right] \right\}, \quad (1.7) \end{aligned}$$

where \mathbb{E}_0 is the conditional expectation operator on time 0 information. First order conditions with respect to j -th consumption flow and working hours ($\text{FOC}(c_{j,t})$, $\text{FOC}(n_{j,t})$ for $j = 1, 2$ hereafter) read:

$$\begin{aligned} c_{1,t} & : \quad s_{1,t} c_{1,t}^{s_{1,t}-1} c_{2,t}^{s_{2,t}} \left(c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}} \right)^{-\gamma} = \phi_{1,t} \\ c_{2,t} & : \quad s_{2,t} c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}-1} \left(c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}} \right)^{-\gamma} = \phi_{2,t} \end{aligned} \quad (1.8)$$

$$\begin{aligned} n_{1,t} & : \quad B = \phi_{1,t} (1 - \alpha_1) \lambda_1 k_{1,t}^{\alpha_1} n_{1,t}^{-\alpha_1} \\ n_{2,t} & : \quad B = \phi_{2,t} (1 - \alpha_2) \lambda_2 k_{2,t}^{\alpha_2} n_{2,t}^{-\alpha_2} \end{aligned} \quad (1.9)$$

where $(1 - \alpha_j) \lambda_j k_{j,t}^{\alpha_j} n_{j,t}^{-\alpha_j} = w_{j,t}$ is the marginal productivity of labor in sector j .

Combining the previous equations, the FOCs for both consumption goods can be rewritten as:

$$\begin{aligned} s_{1,t} \frac{C_t^{1-\gamma}}{c_{1,t}} w_{1,t} & = B \\ s_{2,t} \frac{C_t^{1-\gamma}}{c_{2,t}} w_{2,t} & = B \end{aligned} \quad (1.10)$$

Optimality conditions (eq.(1.10)) indicate the standard equality between the weighted marginal utility of consumption $(s_{j,t} \frac{C_t^{1-\gamma}}{c_{1,t}})$ and the weighted marginal utility of leisure $(\frac{B}{w_{j,t}})$. Notice that if $\gamma > 1$, the sectoral consumption is negatively related to the aggregate consumption index.

After little simple algebra, investment dynamics are determined by the following two Euler Equations:

$$\begin{aligned} \beta \mathbb{E}_t \frac{s_{1,t+1} \frac{C_{t+1}^{1-\gamma}}{c_{1,t+1}} w_{1,t+1}}{s_{1,t} \frac{C_t^{1-\gamma}}{c_{1,t}} w_{1,t}} \left(\alpha_1 \lambda_1 k_{1,t+1}^{\alpha_1-1} n_{1,t+1}^{1-\alpha_1} + (1 - \delta_1) \right) &= 1 \\ \beta \mathbb{E}_t \frac{s_{2,t+1} \frac{C_{t+1}^{1-\gamma}}{c_{2,t+1}} w_{2,t+1}}{s_{2,t} \frac{C_t^{1-\gamma}}{c_{2,t}} w_{2,t}} \left(\alpha_2 \lambda_2 k_{2,t+1}^{\alpha_2-1} n_{2,t+1}^{1-\alpha_2} + (1 - \delta_2) \right) &= 1 \end{aligned} \quad (1.11)$$

where \mathbb{E}_t denotes the expectations operator, conditional on information available at time t . Notice that the pricing kernel $\Pi_{j,t} = \frac{C_{t+1}^{1-\gamma}}{C_t^{1-\gamma}} \frac{s_{j,t+1} c_{j,t}}{s_{j,t} c_{j,t+1}}$ is affected by the relative-preference parameters and it depends on both the level of consumption of the specific kind of good and the level of consumption index.

Combining optimal conditions and resource constraints we determine the deterministic steady state.⁷ Then we proceed to log-linearize the model around the steady state to study the dynamics.⁸ In the next section we illustrate the parameterization of the model and then we show the results.

1.3. Numerical simulations

1.3.1. Parameterization

The system of equations we use to compute the dynamic equilibrium of the model depends on a set of **twelve** parameters. **Six** pertain to technology (the capital share in both sectors α_j , the capital stock quarterly depreciation rate δ_j , the equilibrium value of technology λ_j), while the other **six** pertain to consumer's preferences (the subjective discount factor β , the relative risk aversion coefficient γ , the marginal utility of leisure B , the relative

⁷See the Appendix.

⁸The Appendix reports the dynamic equations of the model following Uhlig (1999).

preference for good 1, s_1 , and for good 2, s_2 , and the autoregressive coefficient of the relative preference process ρ).⁹

The two sectors are characterized by the same production process, so the eventual differences between equilibrium values derive from consumers' preferences. So sectoral technology parameters are set perfectly equal in both sectors. This assumption makes it easy to associate the parameterization of the relative preferences to the composition of the consumption index. In fact, under the symmetric hypothesis concerning the supply side, it emerges that $c_1 \gtrless c_2 \Leftrightarrow s_1 \gtrless s_2$. Assuming differences in the supply side, it would complicate the exposition of the mechanisms with no significant added value in the understanding of the role of preferences. The model is parameterized for the U.S. economy based on the post-war period, apart of relative preference parameters that are set to develop the theoretic investigation. The parameterization reads:

Technology parameters $(\delta_j, \alpha_j, \lambda_j)$ are set to commonly used values in RBC literature. We consider a symmetric economy by the supply side so, $\delta_1 = \delta_2 = 0.025$, $\alpha_1 = \alpha_2 = 0.36$ and $\lambda_1 = \lambda_2 = 1$.

Consumer's preference $(\beta, \gamma, B, s_j, \rho)$: the quarterly subjective discount factor β is set to correspond to an annual real interest rate of 4%; it yields $\beta = 0.99$. The relative risk aversion γ is equal to 5. The relative preference for good 1, s_1 , varies in the range $0 < s_1 < 1$ but in the baseline version is set equal to 0.1. The autoregressive coefficient of the preference process ρ is 0.99. The marginal utility of leisure, B is endogenously calibrated to generate $n_1 + n_2 = 0.3$.¹⁰

1.3.2. Structure of the simulation exercises

In this section we investigate how the stylized economy responses to an increase in the relative preference for good 1. We assume a temporary 1% positive deviation of s_1 from the steady state value. Here we anticipate that, in our framework, the key element that determines the rise of comovement, is the ratio $\frac{s_1}{s_2}$. Then, the first set of simulations reports impulse responses for different values of $\frac{s_1}{s_2}$. Precisely, the different values of this ratio are:

⁹Recall that the preference parameter for good 2, s_2 , is set equal to $1 - s_1$.

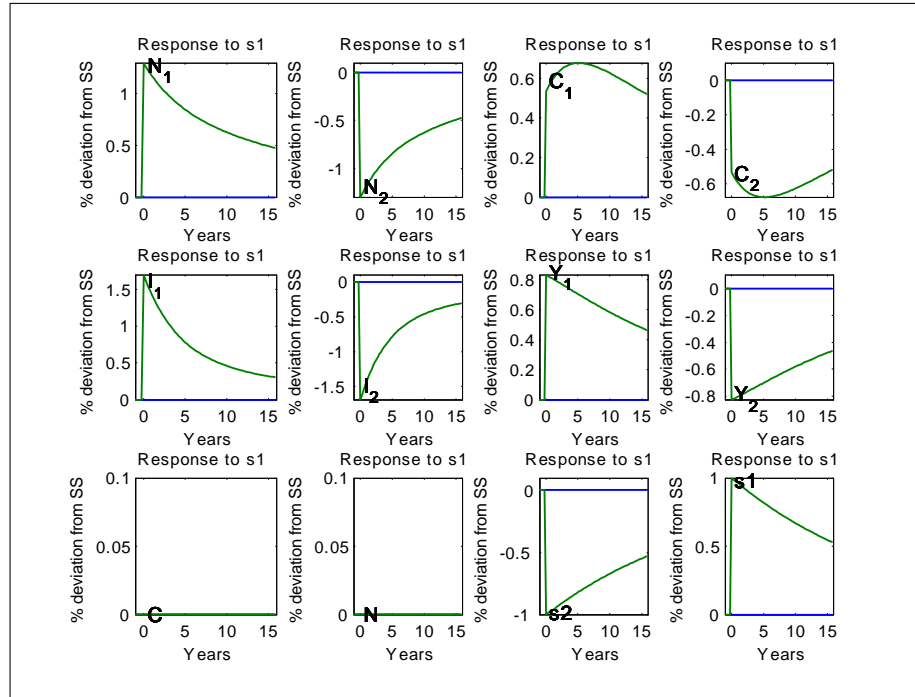
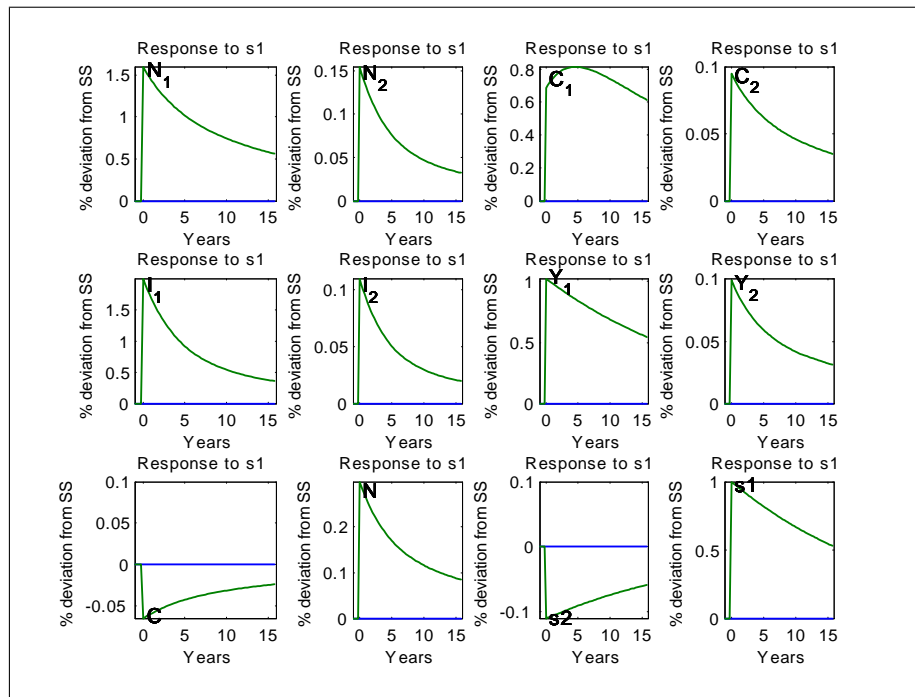
¹⁰See the Appendix.

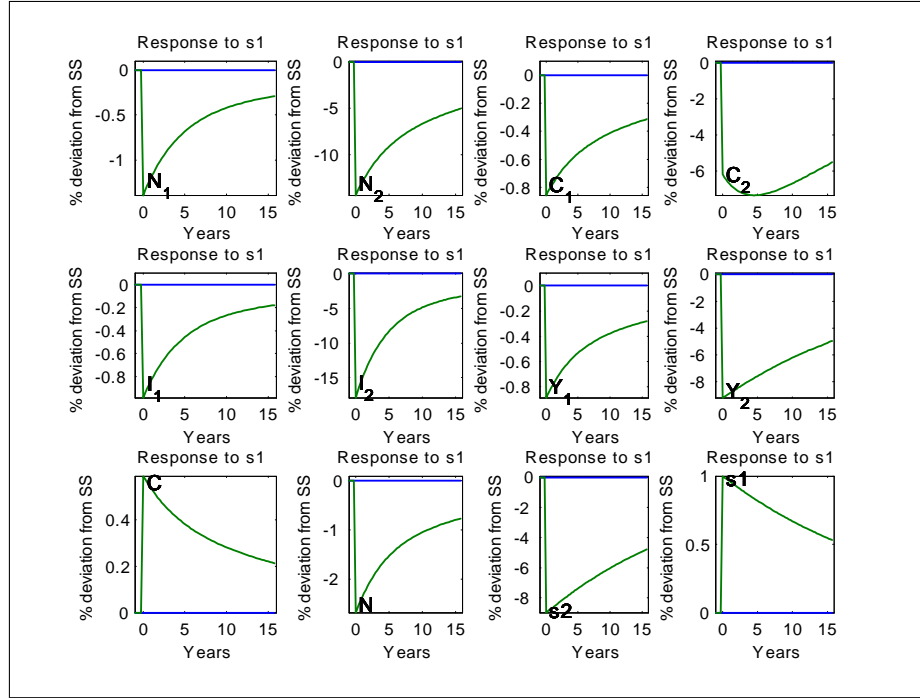
$\frac{s_1}{s_2} = 1$ ($s_1 = 0.5$ and $s_2 = 0.5$), $\frac{s_1}{s_2} < 1$ ($s_1 = 0.1$ and $s_2 = 0.9$) and $\frac{s_1}{s_2} > 1$ ($s_1 = 0.9$ and $s_2 = 0.1$). These simulations help to show how the effects of a preference shock change according to the initial structure of preferences. To explain the emerging results, we need to introduce what we call "*perception effect*" that we consider the main element inducing positive inter-sectoral comovement after a preference shock. As we will explain later in the paper, the perception effect is related to the different levels of satisfaction that the representative household can attribute to the same consumption basket according to the state of preferences. Then we conduct sensitivity analysis to study the role of the parameters that characterize the dynamics of the model. From the sensitivity analysis the following results will emerge. Firstly, the autoregressive coefficient of the preference shock principally affects the responses of investments, because it determines how long the immediate impact on consumptions extends to the future. Secondly, the coefficient of intertemporal substitution $\frac{1}{\gamma}$, together with the ratio $\frac{s_1}{s_2}$, determines if the perception effect is sufficiently high to outrun the substitution effect following a preference shock (i.e. we will show that the value of γ is very important to make emerge positive inter-sectoral comovement). Finally we will remove the linearity assumption concerning the preferences for leisure. As well, we will show that such assumption is not necessary and that the characterization of preferences for leisure contributes to strengthening or weakening inter-sectoral comovements without significantly modifying economic mechanisms.

1.3.3. Baseline simulations

Initially, we run the model three times maintaining the baseline calibration with the exception of the steady state value of s_1 .¹¹ We set s_1 equal to 0.5, to 0.1 (the benchmark value) and to 0.9 and report results respectively in Figure 1.1, Figure 1.2 and Figure 1.3.

¹¹Obviously, this implies different steady state values for all the endogenous variables of the model.

Figure 1.1. $s_1 = 0.5$ (Perfectly symmetric model)Figure 1.2. $s_1 = 0.1$

Figure 1.3. $s_1 = 0.9$

Detailed, within the figures are reported the impulse response functions of employment (N_1, N_2), consumption (C_1, C_2), investment (I_1, I_2) and output (Y_1, Y_2) of both sectors; C is the aggregate consumption index as defined in eq.(1.1), N is the total employment and the last two boxes refer to the preference weights for consumption goods¹².

Figure 1.1 (with $s_1 = s_2 = 0.5$) shows an economy where input factors (N_2 and I_2) are withdrawn from the production of good 2 (Y_2 and C_2 decrease) and are allocated to sector 1. Intra sectoral comovements between consumption, investment, employment and output are positive in both sectors, but inter-sectoral comovements are negative. In fact, the sector characterized by an increase in preference (sector 1) goes through an expansive phase while the other sector goes through a recessive phase.

Figure 1.2 (with $s_1 = 0.1, s_2 = 0.9$) shows an economy with both sectors in expansion. Both inter-sectoral and intra sectoral comovements are positive.

Figure 1.3 (with $s_1 = 0.9, s_2 = 0.1$), similar to Figure 1.2, shows an economy characterized by both positive inter-sectoral and positive intra-sectoral comovements, but the dynamics are completely reversed. The stylized economy experiences a recession in both

¹²We refer to N_j as employment and working hours. In this context the difference is not relevant.

sectors and in all variables (consumption, investment, employment and output). Reported figures raise some questions that need an explanation.

1.4. Economic Results

1.4.1. In which cases do preference shifts generate positive inter and intra sectoral comovement of consumption and employment?

The key element to understand the different dynamics reported above is the impulse response function of the aggregate consumption index C . It represents a first measure of the representative household's felicity. In fact, assuming a standard utility function (i.e. increasing and concave in consumption), the behavior of C affects both the level of satisfaction and the marginal utility associated with the consumption of each good.¹³ To better understand the role of the aggregate index, assume that a shock to preferences occurs and that c_1 and c_2 are fixed. Notwithstanding, the value of the consumption index changes because of preference parameters. Therefore, the total and marginal utilities related to each good and to the aggregate index also change. In this scenario, we will show that the marginal utility of C is the key element that could induce positive inter-sectoral comovements and aggregate fluctuations. First, let's try to make clear the economic intuition and then describe the mechanism more analytically.

Roughly speaking, we can argue that the influence of c_j on consumer's utility (i.e. the marginal utility of c_j) is given by the effect of c_j on C and the effect of C on the utility. If preferences shift, both of the effects vary but only the latter can induce positive inter-sectoral comovement. In fact, in the previous three examples (reported in Figures 1.1-1.3), after the positive shock to s_1 the *ceteris paribus* effect of c_1 on C increases while the effect of c_2 on C decreases.¹⁴ This is a sort of "*substitution effect*" that acts in the standard direction: after a positive shock to s_1 the representative consumer desires to substitute good 2 for good 1. On the contrary, it is not unique the way the influence of

¹³In our model C does not affect marginal utility of leisure because we assume that the utility function is separable in consumption and leisure.

¹⁴In the Appendix we prove that $C_{c_1 s_1} > 0$ and that $C_{c_2 s_1} < 0$ to show that only the direct effect of the shock on C can explain positive inter-sectoral comovements. This is confirmed also by the impulse responses of C_{c_1} reported in the next figures.

C on the utility changes, because it depends on the way C changes after the shock. If C decreases (increases) then the marginal impact of C on utility increases (decreases). We call this change "*perception effect*" that affects the marginal rate of substitution between consumption (consumption index and single good consumption) and any other argument of the utility function, as leisure in this case. In other words, C does not influence the marginal rate of substitution between c_1 and c_2 , but influences the marginal rate of substitution between c_i and leisure $(1 - n_1 - n_2)$.¹⁵

So, when C falls because of the preference shift (independently on what happens to c_1 and c_2), the marginal utility of consumption index increases and this positively influences the marginal utility of both consumption goods. This scenario is in accordance to Figure 1.2. The reverse occurs in Figure 1.3: the marginal utility of consumption index has reduced so highly that also sector 1 experiences a negative phase. In both cases positive sectoral comovements emerge driven by sectoral (and not aggregate) preference shock. The following table resumes possible scenarios after a positive shock to s_1 .

TABLE 1.1 POSSIBLE DYNAMICS OF CONSUMPTION AFTER A POSITIVE SHOCK TO s_1

	Perception	Substitution	Final Result
$c_1 < c_2$	$c_1 \uparrow$ $c_2 \uparrow$	$c_1 \uparrow$ $c_2 \downarrow$	$c_1 \uparrow$ $c_2 \updownarrow$
$c_1 = c_2$	$c_1 \leftrightarrow$ $c_2 \leftrightarrow$	$c_1 \uparrow$ $c_2 \downarrow$	$c_1 \uparrow$ $c_2 \downarrow$
$c_1 > c_2$	$c_1 \downarrow$ $c_2 \downarrow$	$c_1 \uparrow$ $c_2 \downarrow$	$c_1 \updownarrow$ $c_2 \downarrow$

¹⁵The preference shock generates a sort of "*real wealth effect*", if it is assumed that real wealth can be measured by the level of utility that consumer can reach. In fact, after the shock the level of satisfaction has changed because the consumer associates different satisfaction to the same goods. So, the level of satisfaction changes even if the consumption choices are unchanged. But it is necessary to note that even if we adopt the previous definition, the mechanism proposed in this paper is quite different from that emerging in standard microeconomic problems. This is because the first element to change is not the budget constraint but the indifference curve.

Now, let's try to develop more analytically the argument. Consider the optimal condition ruling the choice between consumption and leisure in each sector (with $j = 1, 2$),

$$U_{c_j} w_j = U_C C_{c_j} w_j = B \quad (1.12)$$

Equation (1.12) imposes that in equilibrium the marginal utility of consumption of good j , U_{c_j} , weighted with the marginal productivity of labor in sector j , w_j , has to be equal to the marginal utility of leisure, B .¹⁶ The marginal utility of c_j can be decomposed in the product between the first derivative of the utility function with respect to the aggregate consumption index, U_C (i.e. the marginal utility of C), and the first derivative of such index with respect to the single consumption good, C_{c_j} . The key question concerns what happens to U_C after an increase in s_1 (as in our simulations). To answer, it is necessary to focus on the signs of two derivatives. The first is the sign of the first derivative of the marginal utility U_C with respect to the consumption index (i.e. the second derivative of the utility function with respect to the consumption index). This sign is univocal negative (in fact, $U_{CC} = -\gamma C^{-\gamma-1} < 0$).

The second sign concerns the derivative of the consumption index with respect to the exogenous shock, C_{s_1} .¹⁷ The sign of this derivative depends on the ratio between the consumption goods composing the consumption index, in fact $C_{s_1} = c_1^{s_1} c_2^{1-s_1} \ln(\frac{c_1}{c_2})$. Recall that in our model the supply sides of the sectors are perfectly symmetric; it follows that the relative dimension of steady state values of sectoral variables depends only on consumer's preferences. Then $c_1 \gtrless c_2$ iff $s_1 \gtrless s_2$. In the benchmark version (reported in Figure 1.2) $s_1 = 0.1$ so $\frac{c_1}{c_2} < 1$, and then $C_{s_1} < 0$. If $C \downarrow$ then $U_C \uparrow$. In this case, the direct effect of a positive shock to s_1 reduces C and then increases U_C . So, according to the optimal conditions (eq.(1.12)) the product between $C_{c_j} w_j$ has to fall in both sectors, given the marginal utility of leisure, B . This can occur by an increase in c_j (the derivative

¹⁶ Assuming B constant, the dynamic equation of eq.(1.12) can be expressed in the following way: $\tilde{s}_{j,t} - \tilde{c}_{j,t} + (1 - \gamma) \tilde{C}_t + \alpha_j \tilde{k}_{j,t} - \alpha_j \tilde{n}_{j,t} = 0$. This equation is very helpful to follow the mechanism described in the present section.

¹⁷ We are interested in the direct effect of s_1 on C taking the rest as given. So we are not considering the indirect effect generated by variations in consumption composition.

of C_{c_j} with respect to c_j is negative, in fact $C_{c_1 c_1} = s_1 (s_1 - 1) c_1^{s_1-2} c_2^{1-s_1} < 0$, $C_{c_2 c_2} = -s_1 (1 - s_1) c_1^{s_1} c_2^{-s_1-1} < 0$ and also by an increase in employment n_j (to reduce w_j).¹⁸

This mechanism contributes in explaining the positive inter-sectoral comovements reported in Figure 1.2 and in Figure 1.3 and why the economic booms (dooms) occur after a preference shock when s_1 is set low (high).

Finally, Figure 1.1 represents the case of a perfectly symmetric economy: $s_1 = 0.5$, $\frac{c_1}{c_2} = 1$, $C_{s_1} = 0$. The direct effect of preferences on U_C is null; so the dynamics of the economy is simply driven by substitution effects between sectoral goods. In this case the marginal rate of substitution between leisure and consumption index does not change and neither the aggregate employment does.

As confirmation, consider these other figures that report the impulse responses of other variables (with the exception of the consumption index, C).¹⁹ " C_{c_1} " and " C_{c_2} " are the first derivative of C with respect to c_1 and c_2 .²⁰ " U_C " is the first derivative of the utility function with respect to the consumption index, and its dynamics describe the behavior of perception effect.²¹ Finally, " U_{c_1} " and " U_{c_2} " are the marginal utilities with respect to c_1 and c_2 .²²

Figure 1.4 confirms that in the symmetric case with no perception effect the marginal utility of each good depends only on the way the good affects the consumption index.²³

¹⁸It is noteworthy that it is also relevant the way C varies because of sectoral specific consumption changes (C_{c_j}). So C_{s_1} does not explain the entire variation of U_C .

¹⁹These tables are indicated as "bis" because they replicate the corresponding previous exercises.

²⁰ $C_{c_1} = s_1 c_1^{s_1-1} c_2^{1-s_1}$ and $C_{c_2} = (1 - s_1) c_1^{s_1} c_2^{-s_1}$ and the dynamic equations are respectively $0 = -\tilde{C}_{c_1} + [1 + s_1 \ln(\frac{c_1}{c_2})]\tilde{s}_1 + (s_1 - 1)\tilde{c}_1 + (1 - s_1)\tilde{c}_2$ and $0 = -\tilde{C}_{c_2} + [-\frac{s_1}{1-s_1} + s_1 \ln(\frac{c_1}{c_2})]\tilde{s}_1 + s_1\tilde{c}_1 - s_1\tilde{c}_2$ where the tilde indicates the variation rate of the variable. To simplify we have substituted $s_2 = 1 - s_1$.

²¹ $U_C = C^{-\gamma}$ and the dynamic equation is $0 = -\tilde{U}_C - \gamma\tilde{C}$;

²² $U_{c_1} = U_C C_{c_1}$ and $U_{c_2} = U_C C_{c_2}$ and the dynamic equations are $0 = -\tilde{U}_{c_1} + \tilde{U}_C + \tilde{C}_{c_1}$ and $0 = -\tilde{U}_{c_2} + \tilde{U}_C + \tilde{C}_{c_2}$.

²³Referring to note (18), notice that in this case C does not varies because $C_{s_1} = 0$ but also because $C_{c_1} + C_{c_2} = 0$.

Figure 1.5 and Figure 1.6 show two cases in which the perception effect is so high that it offsets the substitution effect. In the case with $s_1 = 0.1$ (Figure 1.5) the perception effect works in the same direction of the substitution effect for good 1 but in the opposite direction for good 2. Consequently, the response of " U_{c_1} " is stronger than the response of " C_{c_1} ", while the response of " U_{c_2} " is reversed with respect to the response of " C_{c_2} " (because the perception effect predominates). In the case with $s_1 = 0.9$ the reverse occurs; in fact, despite the increase in the relative preference for good 1, the marginal utility of c_1 decreases while the marginal utility of c_2 strongly falls.²⁴

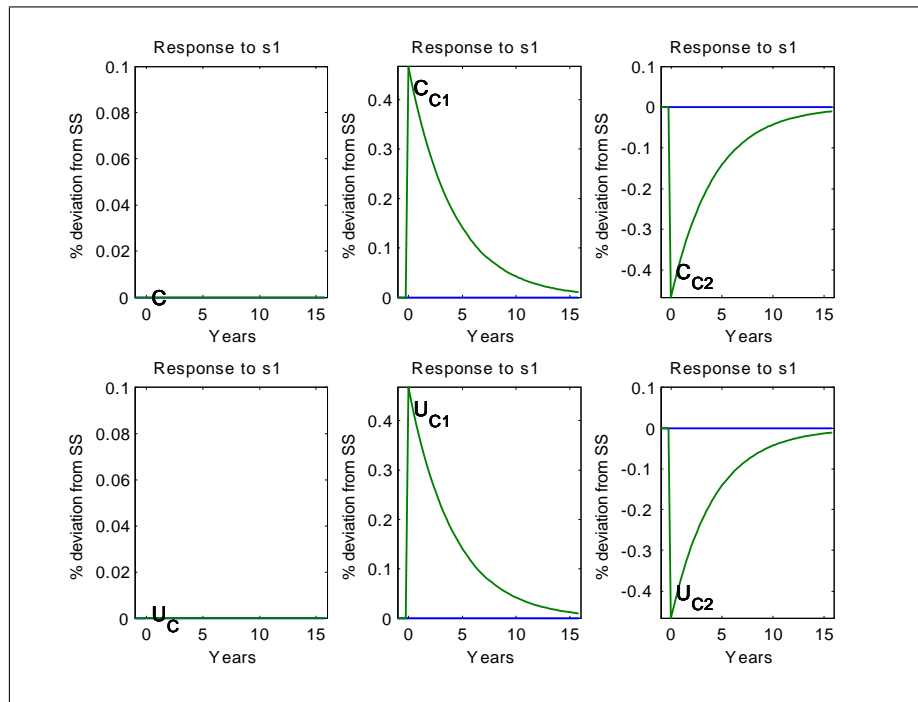
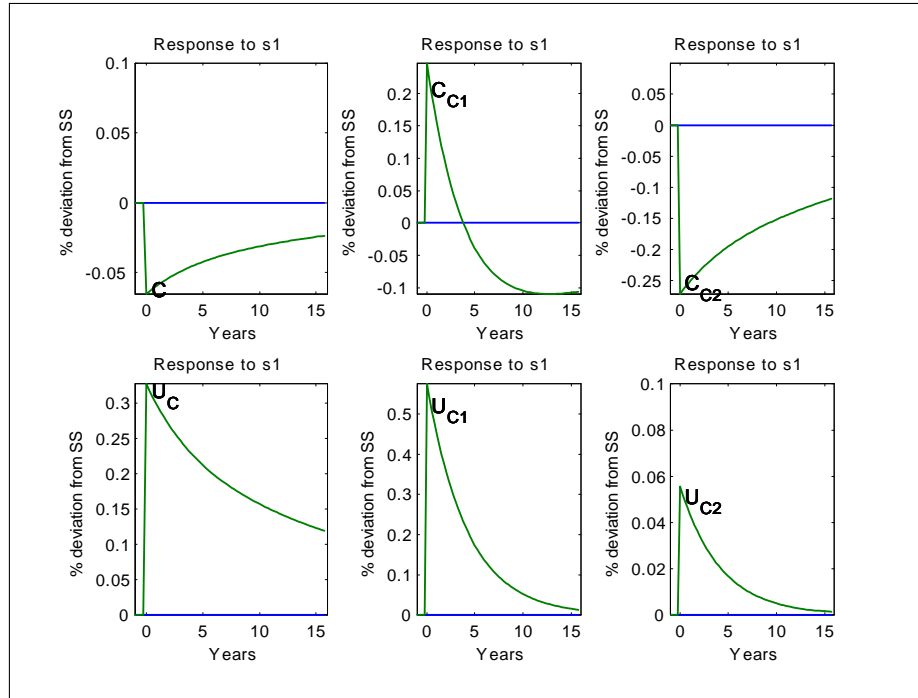
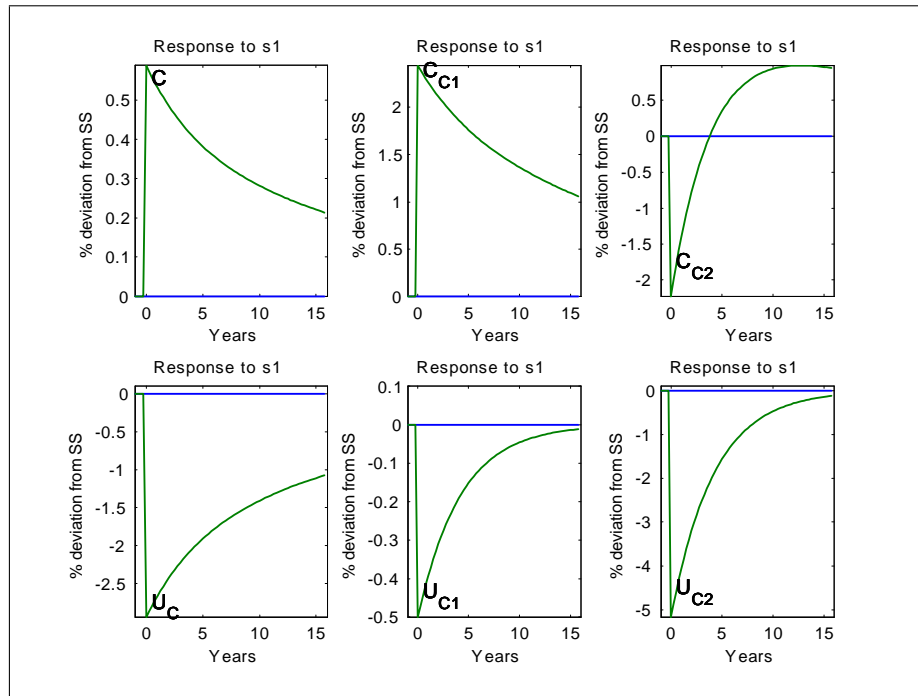


Figure 1.4. $s_1 = 0.5$ (Perfectly symmetric model)

²⁴Notice that Figure 1.5 and Figure 1.6 are not perfectly symmetric because the entity of the shock is different. Respectively 1% of 0.1 and 1% of 0.9. Notwithstanding, the comparison of the relative responses are very similar.

Figure 1.5. $s_1 = 0.1$ Figure 1.6. $s_1 = 0.9$

In this section we have not described all the forces determining the dynamics of this stylized economy, because we are principally interested in the mechanisms that induce positive inter-sectoral comovements. So, as evidenced in Table 1.1 (by the use of arrows), the ratio between preference parameters (or consumption goods) represents a sort of necessary, but not sufficient, condition to observe positive inter-sectoral comovements in response to relative-preference shifts.

1.4.2. And what about investment choice?

To answer we need two other sets of impulse response functions. Starting from the benchmark calibration (with $s_1 = 0.1$ and $\rho = 0.99$, reported in Figure 1.2), reduce the autoregressive coefficient of preference process to $\rho = 0.92$ and to $\rho = 0.80$. Results are represented respectively in the following two figures (Figure 1.7 and Figure 1.8).

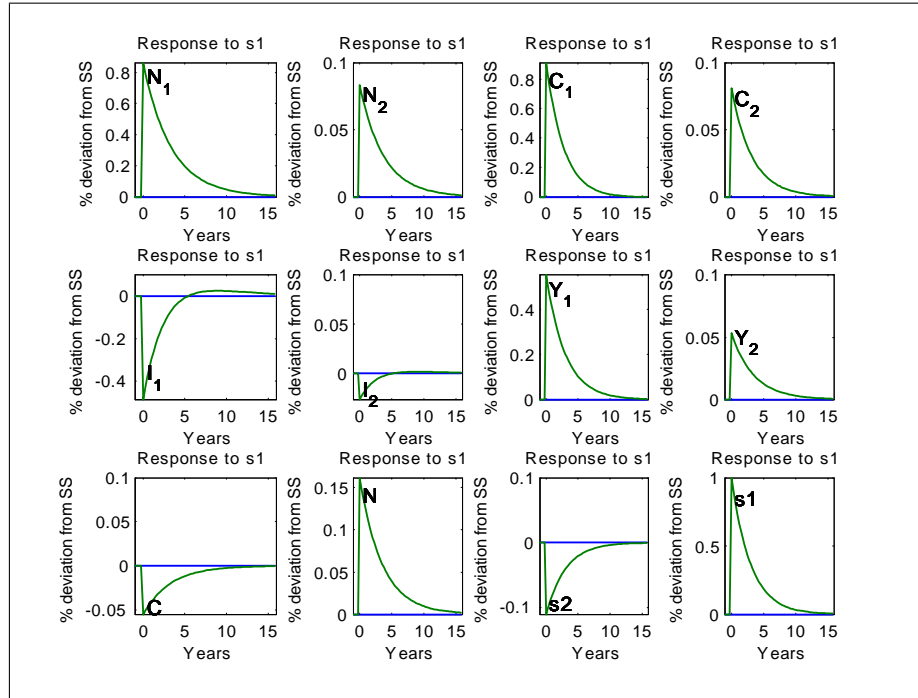
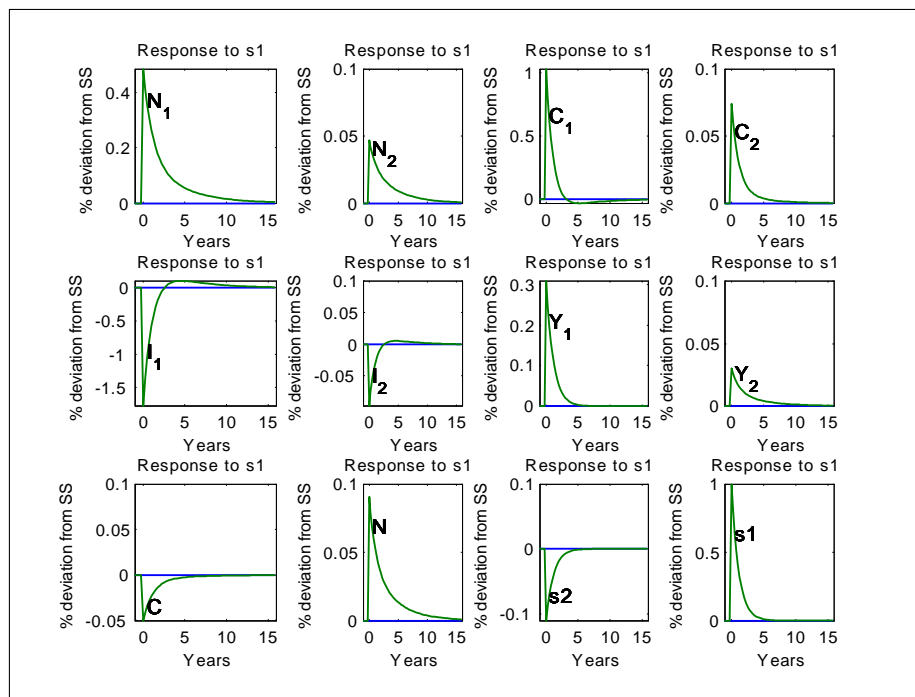


Figure 1.7. $\rho = 0.92$

It is immediately noted that the impulse responses of consumptions and outputs are less persistent (it would be strange if the opposite occurred). Another evidence is that the selected changes in the value of the persistence parameter from the preference process

Figure 1.8. $\rho = 0.80$

does not reverse the sign of inter-sectoral comovements. In fact these are still positive in both cases.

To our aim, the most important effect of a reduction in ρ concerns the change in the intra-sectoral comovements; in fact investments gradually become counter-cyclical with respect to the other variables. The negative correlation between consumption and investment is due to the quicker return to the original consumption index. In fact, after the preference shock the representative consumer "feels worse" and prefers reducing leisure to increase consumption. If the preference process is highly persistent, this feeling is long lasting and then it is optimal to plan a long lasting increase in consumption. Immediately this aim needs both higher consumption and higher investment. On the contrary, if the preference process is not highly persistent, the consumer needs to increase only actual consumption because the marginal rate of substitution between actual leisure and future consumption (to say actual investment) does not change significantly. Then, with low values of ρ , actual consumption increases in detriment to both actual leisure and future

consumption, while when $\rho = 0.99$ actual and future consumption increase in detriment to actual leisure.²⁵

Again, it is worth noticing that in this model the investment good for the capital stock used in sector j is produced entirely in sector j . This means that differing from labor services, capital is not mobile across sectors. Such assumption is fundamental to eliminate every kind of input-output linkage between sectors. In this way, the emerging inter-sectoral comovements are not generated by a combination of technological and preference reasons, but they are entirely due to preference shifts. In fact, relaxing this hypothesis the transfer of capital goods would ease the explication of inter-sectoral comovements of outputs and investments.

1.4.3. The role of the intertemporal elasticity of substitution in consumption

Regarding the role of γ , the impulse response functions continue to be a very clarifying instrument. So, we maintain the benchmark calibration and substitute $\gamma = 5$ with $\gamma = 1.5$.

The dynamics after a shock to s_1 are reported in Figure 1.9.

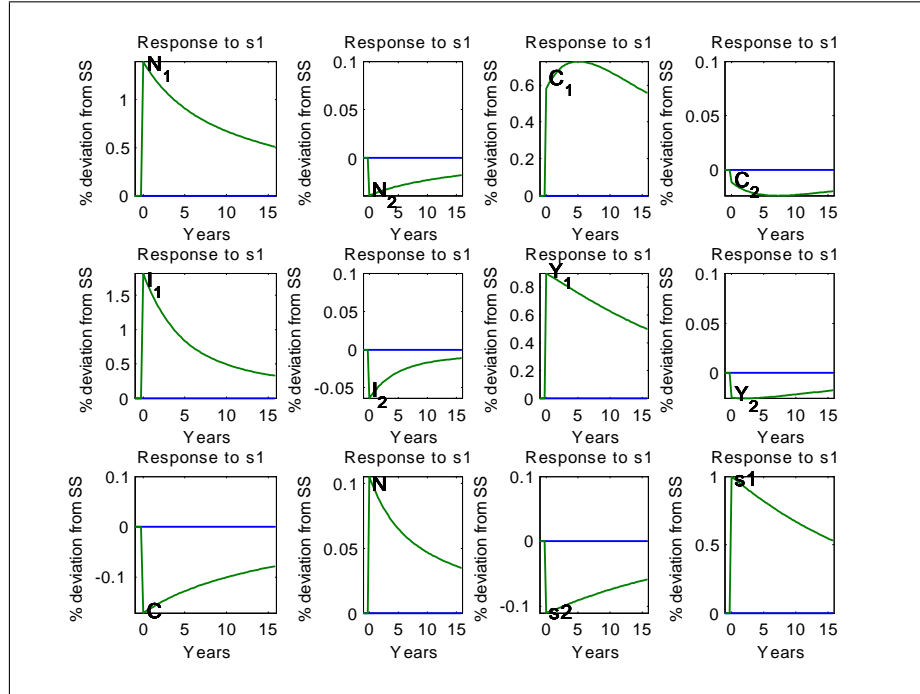


Figure 1.9. $\gamma = 1.5$

²⁵The impulse responses confirm the entire argument. For example, observe that the immediate deviation of c_1 with respect to that of n_1 and y_1 is relatively higher when ρ is low.

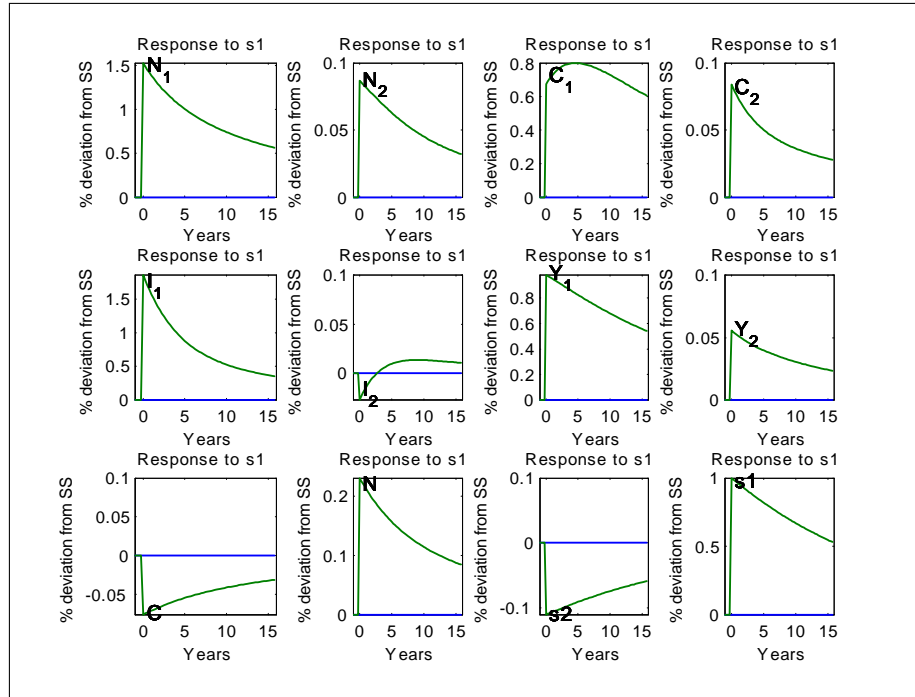
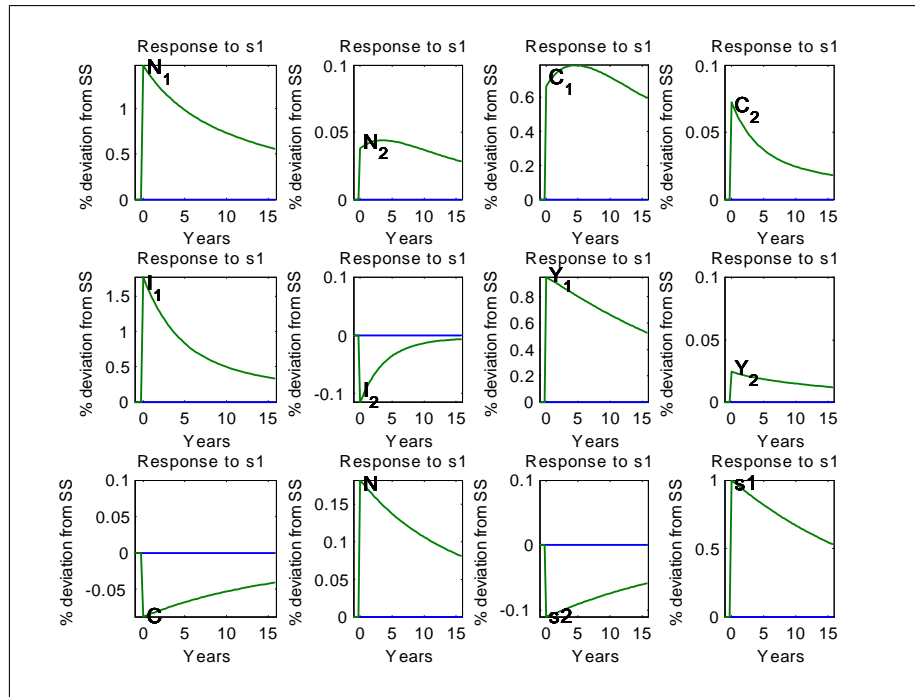
The figure shows that with low values of γ the positive comovements between sectors vanish. The reason is that the importance of the perception effect has decreased. Analyzing the dynamic equation of the first derivative of the utility function with respect to the consumption index ($\tilde{U}_C = -\gamma\tilde{C}$), or considering dynamic equation reported in note (16), it is clear that γ is a scale factor of the effect of C on the marginal utility of consumption. So if γ is low, variations in C poorly affect the relative preference between consumption goods and leisure and thus inter-sectoral comovements are infrequent.

1.4.4. The role of the intertemporal elasticity of substitution in leisure

The model has been developed assuming that the marginal utility of leisure is constant. The assumption serves a specific purpose. It permits to isolate the substitution and perception effects in consumption with no variation in marginal utility of leisure.

But this assumption is not necessary. In fact, we can solve the model assuming the following utility function: $u(\mathbf{c}_t, \ell_t; \mathbf{s}_t) = \frac{(C_t)^{1-\gamma}-1}{1-\gamma} + \frac{1}{v}B(1 - n_{1,t} - n_{2,t})^v$, where $(1-v)$ controls the degree of the risk aversion and is inversely proportional to the elasticity of intertemporal substitution in leisure. Obviously, with $v = 1$ the model collapses in eq.(1.2). To analyze the influence of v , we set $v = 0.2$ and then $v = -1$, and run the model in the usual way. Results are reported in the following figures.

Figure 1.10 and Figure 1.11 indicate that decreasing v (to say decreasing the intertemporal elasticity of leisure), it becomes less frequent to observe positive inter-sectoral comovements. Clearly, this parameter does not directly modify the relative weight of perception effect and substitution effect. The different dynamics emerge because of the behavior of the marginal utility of leisure that, with $v < 1$, is positively related to labor supply. In fact, even if the perception effect is sufficiently high to generate positive dynamics of the marginal utility of good 2, the positive inter-sectoral comovements are dampened by the increase in the marginal utility of leisure. As shown in the figures, the representative household is less inclined to reduce leisure time, so the increase in time employed in sector 2 reduces and the effects are particularly significant for investment in this sector.

Figure 1.10. $v = 0.2$ Figure 1.11. $v = -1$

1.5. Conclusions and economic intuition

In this model we have studied theoretical implications of shocks to relative preferences between consumption goods. We have shown that if preference structure is strongly asymmetric, positive inter and intra sectoral comovements could be generated by relative-preference shifts. Moreover, we have shown that results are consistent with respect to different kinds of sensitivity analysis. A strong implication of the model is that positive employment comovements can be explained by sectoral shocks without the introduction of input-output linkages.

The sketched mechanism is quite new in economic literature. In fact, positive comovements of economic sectors are generally explained by either a kind of input-output structure that transmits sectoral shocks over the entire economy or a kind of aggregate shocks. In the last field Bencivenga (1992) and Wen (2005, 2006) can also be inserted, who consider direct variations in relative preference between consumption and leisure. In fact, in the cited works, independently of the composition of consumption basket, the relative importance of consumption with respect to leisure is subject to change. This can be due to the fact that leisure time is employed in other activities (as homework production, see Benhabib et al., 1990), or it can be induced by alternative phases of the level of "consumerism" (or of "the urge to consume", see Wen, 2006) that modifies the importance of consumption. On the contrary, our model does not explicitly focus on consumerism but indicates two elements: the starting composition of consumption basket and the shifts in relative preference between consumption goods²⁶. If preferences shift to a (kind of) good that represents a highly minor or a highly major share of the actual consumption basket, then business cycles with positive sectoral comovements could emerge²⁷. The first case (minor share) can be represented in the following way. Suppose that the representative consumer increases the preference for a good that actually concerns a little share of her

²⁶Phelan-Trejos (2000) investigate preference shifts between consumption goods, but they find negative comovement in sectoral employment. It is very interesting that they observe changes in the sector not affected by the preference shock (their model considers three sectors), and they guess the relevance of the complementarity of consumption goods, but they do not see that the complementarity of consumption goods can generate positive comovement of sectoral employment.

²⁷In section(1.4.1) we have shown that aggregate booms and dooms are more frequent when the values of s_1 are respectively low and high.

consumption; she increases the demand for this good but she still gets an amount of it that is perceived as not enough to maintain the previous level of satisfaction. Consequently, to compensate such lack of satisfaction related to consumption, the consumer increases the demand for the other goods too. A particularly interesting implication of the model concerns the effects of advertising. In fact, this model suggests that the attempt to capture consumers' preferences can generate "not obvious effects". For example, supposing that s_1 is high (low), and interpreting positive shocks to s_1 as a consequence of advertising of sector 1, the model predicts that we could observe a reduction (increase) in the demand for good 1 (good 2). This opens other research fields that lie outside this model.

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Appendix

A. Steady State

From Euler equations (1.11):

$$\frac{k_1}{n_1} = \left(\frac{\alpha_1 \lambda_1}{\frac{1}{\beta} - 1 + \Omega_1} \right)^{\frac{1}{1-\alpha_1}}$$

$$\frac{k_2}{n_2} = \left(\frac{\alpha_2 \lambda_2}{\frac{1}{\beta} - 1 + \Omega_2} \right)^{\frac{1}{1-\alpha_2}}$$

Then combining the first order conditions of consumption and labor:

$$c_1 = \left(s_1 T T^{s_2(1-\gamma)} \frac{(1-\alpha_1) \lambda_1}{B} \left(\frac{k_1}{n_1} \right)^{\alpha_1} \right)^{\frac{1}{\gamma(s_1+s_2)+1-s_1-s_2}}$$

$$c_2 = T T c_1$$

$$C_t = c_1^{s_1} c_2^{s_2}$$

where $TT = \frac{s_2}{s_1} \frac{(1-\alpha_2) \lambda_2 k_2^{\alpha_2} n_2^{-\alpha_2}}{(1-\alpha_1) \lambda_1 k_1^{\alpha_1} n_1^{-\alpha_1}}$. By the feasible constraint it emerges:

$$k_{1,t} = \left(\lambda_1 \left(\frac{k_1}{n_1} \right)^{\alpha_1-1} - \delta_1 \right)^{-1} c_{1,t}$$

$$k_{2,t} = \left(\lambda_2 \left(\frac{k_2}{n_2} \right)^{\alpha_2-1} - \delta_2 \right)^{-1} c_{2,t}$$

Finally using the capital accumulation process:

$$I_1 = \delta_1 k_1$$

$$I_2 = \delta_2 k_2$$

$$n_1 = \left(\frac{k_1}{n_1} \right)^{-1} k_1$$

$$n_2 = \left(\frac{k_2}{n_2} \right)^{-1} k_2$$

B. Calibration of B

We assume that:

$$n_1 + n_2 = N = 0.3$$

Using eq.(1.8) and eq.(1.9) it is possible to express working time in the following way:

$$N = \left(\left(\frac{k_1}{n_1} \right)^{-1} \left(\lambda_1 \left(\frac{k_1}{n_1} \right)^{\alpha_1-1} - \delta_1 \right)^{-1} + \left(\frac{k_2}{n_2} \right)^{-1} \left(\lambda_2 \left(\frac{k_2}{n_2} \right)^{\alpha_2-1} - \delta_2 \right)^{-1} TT \right) c_1$$

Substituing the steady state value of c_1 and finding the value of B according to the parametrization of n_j :

$$B = \left(\left(\left(\frac{k_1}{n_1} \right)^{-1} \left(\lambda_1 \left(\frac{k_1}{n_1} \right)^{\alpha_1-1} - \delta_1 \right)^{-1} + \left(\frac{k_2}{n_2} \right)^{-1} \left(\lambda_2 \left(\frac{k_2}{n_2} \right)^{\alpha_2-1} - \delta_2 \right)^{-1} TT \right)^\gamma \times \right. \\ \left. \times \left(\tilde{s}_{1,t} T T^{\tilde{s}_{2,t}(1-\gamma)} (1 - \alpha_1) \lambda_1 \left(\frac{k_1}{n_1} \right)^{\alpha_1} \right) N^{-\gamma} \right)$$

C. Log-Linearization

- (1) $\tilde{s}_{1,t} - \tilde{c}_{1,t} + (1 - \gamma) \tilde{C}_t + \alpha_1 \tilde{k}_{1,t} - \alpha_1 \tilde{n}_{1,t} = 0$
- (2) $\tilde{s}_{2,t} - \tilde{c}_{2,t} + (1 - \gamma) \tilde{C}_t + \alpha_2 \tilde{k}_{2,t} - \alpha_2 \tilde{n}_{2,t} = 0$
- (3) $y_1 \tilde{y}_{1,t} = c_1 \tilde{c}_{1,t} + i_1 \tilde{i}_{1,t}$
- (4) $y_2 \tilde{y}_{2,t} = c_2 \tilde{c}_{2,t} + i_2 \tilde{i}_{2,t}$
- (5) $k_1 \tilde{k}_{1,t+1} = (1 - \delta_1) k_1 \tilde{k}_{1,t} + i_1 \tilde{i}_{1,t}$
- (6) $k_2 \tilde{k}_{2,t+1} = (1 - \delta_2) k_2 \tilde{k}_{2,t} + i_2 \tilde{i}_{2,t}$
- (7) $\tilde{y}_{1,t} = \lambda_1 \tilde{\lambda}_{1,t} + \alpha_1 \tilde{k}_{1,t} + (1 - \alpha_1) \tilde{n}_{1,t}$
- (8) $\tilde{y}_{2,t} = \lambda_2 \tilde{\lambda}_{2,t} + \alpha_2 \tilde{k}_{2,t} + (1 - \alpha_2) \tilde{n}_{2,t}$
- (9) $r_1 \tilde{r}_{1,t} = \alpha_1 (\alpha_1 - 1) k_1^{\alpha_1-1} n_1^{1-\alpha_1} \tilde{k}_{1,t} + \alpha_1 (1 - \alpha_1) k_1^{\alpha_1-1} n_1^{1-\alpha_1} \tilde{n}_{1,t}$
- (10) $r_2 \tilde{r}_{2,t} = \alpha_2 (\alpha_2 - 1) k_2^{\alpha_2-1} n_2^{1-\alpha_2} \tilde{k}_{2,t} + \alpha_2 (1 - \alpha_2) k_2^{\alpha_2-1} n_2^{1-\alpha_2} \tilde{n}_{2,t}$
- (11) $\tilde{C}_t = s_1 \tilde{c}_{1,t} + s_2 \tilde{c}_{2,t} + s_1 \ln(c_1) \tilde{s}_{1,t} + s_2 \ln(c_2) \tilde{s}_{2,t}$
- (12) $s_2 \tilde{s}_{2,t} = -s_1 \tilde{s}_{1,t}$
- (13) $\tilde{s}_{1,t} - \tilde{s}_{2,t} - \tilde{c}_{1,t} + \tilde{c}_{2,t} + \alpha_1 \tilde{k}_{1,t} + (1 - \alpha_1) \tilde{n}_{1,t} = 0$
- (14) $\tilde{s}_{2,t} - \tilde{s}_{1,t} - \tilde{c}_{2,t} + \tilde{c}_{1,t} + \alpha_2 \tilde{k}_{2,t} + (1 - \alpha_2) \tilde{n}_{2,t} = 0$

Forward equations:

$$15. \ 1 = \tilde{s}_{1,t+1} - \tilde{s}_{1,t} + \tilde{c}_{1,t} - \tilde{c}_{1,t+1} + (1 - \gamma) \tilde{C}_{t+1} - (1 - \gamma) \tilde{C}_t + \tilde{r}_{1,t+1}$$

$$16. \ 1 = \tilde{s}_{2,t+1} - \tilde{s}_{2,t} + \tilde{c}_{2,t} - \tilde{c}_{2,t+1} + (1 - \gamma) \tilde{C}_{t+1} - (1 - \gamma) \tilde{C}_t + \tilde{r}_{2,t+1}$$

D. Proof I

Proof. We now proof that $\frac{\partial^2 C}{\partial c_1 \partial s_1} > 0$. From eq.(1.1):

$$\frac{\partial C}{\partial c_1} = s_1 c_1^{s_1-1} c_2^{1-s_1}, 0 < s_1 < 1$$

and the corresponding steady state equation is:

$$\frac{c_1}{c_2} = \frac{s_1}{1 - s_1}$$

The derivative of $\frac{\partial C}{\partial c_1}$ with respect to s_1 is given by:

$$\frac{\partial^2 C}{\partial c_1 \partial s_1} = c_1^{s_1-1} c_2^{1-s_1} \left(1 + s_1 \ln \left(\frac{c_1}{c_2} \right) \right)$$

and then

$$1 + s_1 \ln \left(\frac{c_1}{c_2} \right) > 0$$

from which:

$$e > \left(\frac{c_2}{c_1} \right)^{s_1}$$

substituting the steady state values of c_1 and c_2 , leads to:

$$e^{\frac{1}{s_1}} s_1 + s_1 - 1 > 0$$

as $e^t > t$, for $t > 1$, the following always holds:

$$\frac{e^t}{t} + \frac{1}{t} - 1 > 0$$

□

E. Proof II

Proof. We now proof that $\frac{\partial^2 C}{\partial c_2 \partial s_1} < 0$. From (1.1)

$$\frac{\partial C}{\partial c_2} = (1 - s_1) c_1^{s_1} c_2^{-s_1}, 0 < s_1 < 1$$

and in steady state:

$$\frac{c_1}{c_2} = \frac{s_1}{1 - s_1}$$

The derivative of $\frac{\partial C}{\partial c_2}$ with respect to s_1 is:

$$\frac{\partial^2 C}{\partial c_2 \partial s_1} = -c_1^{s_1} c_2^{-s_1} \left(1 - (1 - s_1) \ln \left(\frac{c_1}{c_2} \right) \right)$$

and it proves that

$$1 - (1 - s_1) \ln \left(\frac{c_1}{c_2} \right) > 0$$

it follows that:

$$e > \left(\frac{c_1}{c_2} \right)^{1-s_1}$$

substituting the steady state values of c_1 and c_2 leads to:

$$e^{\frac{1}{1-s_1}} (1 - s_1) - s_1 > 0$$

Setting $t = \frac{1}{1-s_1}$, leads again to:

$$\frac{e^t}{t} + \frac{1}{t} - 1 > 0$$

CHAPTER 2

Bargaining Power, Labor Relations and Asset Returns

Abstract

This paper analyzes how the bargaining process between firms and labor unions affects the performance of the real economy and selected properties of asset returns in a general equilibrium model. We assume that firms set unilaterally employment, and that in the wage setting process the *relative bargaining power* of labor unions increases with the aggregate employment rate. The economy is numerically simulated under an aggregate supply shock identified as a productivity shock. The model suggests a possible explanation for both the difference between the equity return rate and the risk free rate and the low correlation between employment and wage. Following a positive and persistent impulse, firms experience a rise in the labor demand and a fall in the net cash flow, due to the increase in investment and to the reduction in their bargaining power. This mechanism generates sufficiently high volatility of firm's returns still predicting a-cyclical wages, which is an empirically robust stylized fact. The model is calibrated for the U.S. economy for the postwar period.

J.E.L. Classification Numbers: D81, E24, J23.

Keywords: Equity Premium, General Equilibrium, Union models.

2.1. Introduction

The interaction between the real side and the financial side of the economic system is a growing interest issue in current economic literature as stressed in recent contributions by Uhlig (2004) and Cochrane (2006).¹

In another recent paper, Sill (2005) empirically supports the relevance of such interaction. Starting from the evidence reported in Jagannathan et al. (2000), Sill investigates the linkage between aggregate risk and asset returns, suggesting that the recent reduction in the volatility of the technology process, and then of the aggregate GDP, explains a substantial fraction of the decline in the U.S. equity premium.²

Following this line of research, our paper focuses on the way labor relations influence macroeconomic dynamics and selected financial aspects. Particularly, we try to reconcile two empirical evidences that are not satisfactorily explained by the standard literature: the low correlation between wage and employment, or GDP (see among the many Uhlig, 2004, Boldrin-Horvath, 1995, and Stadler, 1994), and the difference between equity returns and returns of short term Treasury bonds (see Cochrane, 2006, or Jagannathan et al., 2000). Both themes have been extensively discussed in economic literature.

With regards to the **cyclical properties of wage**, many Authors have improved standard models introducing a further stochastic process, in addition to the standard productivity shock, sometimes specifying a second sector; among the others, Christiano-Eichenbaum (1992) introduce government consumption, Chiarini-Piselli (2005) analyze the effects of shocks to unemployment benefits while Benhabib et al. (1991) specify the home

¹Uhlig (2004): “..to properly conduct macroeconomic risk management policy, it is of paramount importance to understand the nature of the quantitative significance of macroeconomics risks.(...) These risks show their consequences in two important places in particular. First, on asset market, risks are priced. Second, the allocation in the economy results from risk-averse economic actors taking actions in the face of existing risks and their prices. Thus, asset prices and the allocation of economic resources in the economy as a whole are tightly intertwined. Observations on asset markets impose discipline on economic choices and models of the macroeconomic and vice versa. The quest therefore is on to provide models which can jointly explain the behavior of asset prices and of the economy as a whole”.

And Cochrane (2006) adds: “Understanding the marginal value of wealth that drives asset markets is most obviously important for macroeconomics. The centerpieces of dynamic macroeconomics are the equation of saving to investment, the equation of marginal rates of substitution to marginal rates of transformation, and of allocation of consumption and investment across time and states of nature. Asset market are the mechanism that does all this equating. If we can learn the marginal value of wealth from asset markets, we have a powerful measurement of the key ingredient of all modern, dynamic, inter-temporal macroeconomics”.

²This is important for stressing the role of a model that uses "small shocks".

production sector. From a technical perspective, in these works, the additional source of uncertainty is important to contrast the shift of labor demand induced by technology shocks. This paper, departing from the cited contributions, does not rely on the coexistence of multiple sources of uncertainty, but describes a dynamic system characterized by only productivity shocks, that endogenously decreases the correlation between wage and output.

The literature concerning the **difference between the returns of risky and risk-less assets**, relying on consumption-based models, is endless. Kocherlakota (1996) and Campbell (2002) offer illuminating surveys. Generally speaking, the equity premium is explained by a sufficiently high volatility of the discount factor and high volatility of equity returns. This paper explores the latter element, investigating how labor market equilibrium allocations affect the behavior of asset returns along business cycles. We show that the a-cyclical behavior of wage combined with a sufficient employment volatility (that emerge from data) are key elements to understand the source of the equity premium.³ In fact, the low business-cycle frequency of wages suggests that firms can adjust the level of employed labor services without sustaining strong variations in their costs. This generates, in a model characterized by the complementarity among input factors, high volatility in some firm indicators, such as output, capital returns and liquidity. In particular, the high volatility of firm returns helps to explain the excess return of equities with respect to risk-free assets. Moreover, the presence of labor unions implies that agents can not freely use the labor supply choice to smooth the consumption path. This permits to generate consistent volatility of the stochastic discount factor without adopting particular preference structures. Finally, it is worth to note that the volatilities of endogenous variables rely on "very small" exogenous shocks.⁴

³We refer indifferently to volatility of employment and to volatility of working hours because in our stylized economy there is no distinction between them.

⁴In macroeconomic and financial literature it is generally assumed that the standard deviation of the technology shock is about 0.7% (see Danthine-Donaldson, 2002a, and Hansen-Wright, 1992). In our baseline version it is equal to 0.525%.

To resume, this work investigates the dynamics underlying a general equilibrium business cycle model with labor unions and shocks to the technology process, focusing on the role of the relative power in the bargaining process between labor unions and firms.

We elicit the contribution of this paper discussing the effect of the (endogenous) relative bargaining power over real and financial variables. The most important result concerns the ability of the so called rent sharing hypothesis, with endogenous relative bargaining power, to conciliate selected evidence of financial market and labor market.

Particularly, the paper investigates how the responsiveness of the relative bargaining power to the employment rate affects the comovement between wage and GDP during the business cycle and it analyzes its effects on the risk premium. It is shown that under plausible parameterization, equilibrium wage is almost a-cyclical and employment volatility is sufficiently high; in this case the dynamics underlying our model generate high (absolute and relative) volatility of firm's cash flow and the equity premium near to empirical estimates.⁵

The economic intuition can be rationalized in the following way. Concerning the financial side of the economy, when a positive productivity shock occurs the firm increases investment and employment to take advantage of the higher productivity. As the cash flow is negatively affected by investment, and the bargaining power of unions is increasing in employment, the firm accepts a temporary yet strong liquidity reduction. Symmetrically, after a negative technology shock, the firm decreases investment and becomes stronger in the bargaining process because employment falls; consequently, liquidity temporarily increases. This mechanism generates high fluctuations of liquidity (dividends) giving a possible explanation for the equity premium.

With respect to the labor market dynamics, the determines the equilibrium wage as an increasing function of the relative bargaining power of labor unions and of the firm's liquidity per worker (the latter on account of the rent sharing hypothesis). After a positive productivity shock we observe a decrease in firm's liquidity and an increase in union's bargaining power (driven by the rise in employment). These changes influence wages

⁵In the paper, we will refer to the relative volatility of a variable as the ratio between the standard deviation of this variable and the standard deviation of GDP.

in opposite ways so that, in a range of calibrations, wages do not change significantly. This mechanism characterized by low response of wages to technology shocks contributes to explain the "puzzle" concerning the low correlation between employment and labor productivity.

The paper proceeds as follows. Section 2.2 briefly cites some references to the paper. Section 2.3 shows why a standard RBC model fails to explain the cyclical behavior of wages and why the resulting dynamics are not able to generate a volatility of firm returns consistent with the empirical equity premium. Section 2.4 details the model. Section 2.5 analyzes the results emerging from simulations, and Section 2.6 concludes. Proofs and derivations are sketched in the Appendix. The model is calibrated for the United States economy over the post-war sample period.

2.2. Related literature

There is a large resource of literature for studying the role of labor unions in the economic system.⁶ During the '80s many partial-equilibrium models have been developed focusing on the bargaining process between labor unions and firms over wage and employment.⁷ Such models represent labor union as a maximizing agent. They investigate how final allocation changes based on the bargaining framework and of labor union preferences.⁸

Nevertheless the relevance of the theme clearly emerged in theoretical and empirical literature, we are aware of only a few real-business-cycle models that include labor unions. Maffezzoli (2001) and Chiarini-Piselli (2005) develop monopoly-union models where labor unions substitute households in setting the labor supply. In these papers, labor unions have the full power to set wages, while firms choose the level of employment. This depicts an extreme case of the "right to manage" models. Our model maintains the hypothesis concerning the employment decision, but it assumes that the relative bargaining power in the wage setting is endogenous. Particularly, we assume that the relative bargaining power

⁶Farber (1986), Booth (1995) and Hirsch (1997) survey different ways to investigate the role of labor unions and their effects on the economic system.

⁷Between the many authors, see Oswald (1982).

⁸The debate between Dunlop (1944) and Ross (1948) concerns the possibility to clearly determine unions' preferences, in order to analyze the bargaining process rigorously (with mathematical expressions). Dunlop considers assigning a well-defined objective function to labor unions as fruitful; Ross considers such methodology completely misleading.

of labor unions depends positively on the employment rate. Neither the relevance of relative bargaining power, nor the linkage between employment rate and workers' bargaining power are new in economic literature. For example, in the search and matching literature, the relative bargaining power is often endogenous and related to the "tightness" of labor market.⁹ Moreover, the relevance of relative bargaining power in a DSGE model has been used by Danthine-Donaldson (2002a) to introduce a source of idiosyncratic uncertainty in a framework characterized by general uncertainty that has been generated by technology innovations. Unlike the cited Authors, we consider the bargaining power as an endogenous (and not exogenous) process.

Finally, in Danthine-Kurman (2005) it is possible to find many recent contributions supporting the rent sharing hypothesis and we limit to cite Christofides-Oswald (1992) and Blanchflower et al.(1996) as examples of empirical analysis supporting this hypothesis.¹⁰

2.3. Theoretical Background

This section discusses selected theoretical building blocks of equilibrium production economies by highlighting the connection between labor markets and asset returns. Consider first the labor market.

The standard neoclassical growth model supposes that a representative agent optimally chooses between consumption and leisure, so the so called "intra-temporal" optimality condition takes the following form:

$$\underbrace{u'_{c_t}(\cdot)}_{\text{marg. utility of cons.}} \times \underbrace{w_t}_{\text{wage rate}} = \underbrace{u'_{1-n_t}(\cdot)}_{\text{marg utility of leisure}} \quad (2.1)$$

This condition equates the marginal utility of consumption weighted by the wage rate to the marginal utility of leisure. Eq.(2.1) holds on a period by period basis, and drives the labor-supply choice, depending on the income/substitution effects. Generally, it is assumed that the substitution effect prevails on the income effect and, consequently, the *ceteris paribus* representation of the labor supply is a positively sloped curve in the

⁹See Pissarides (2000).

¹⁰Note that the rent sharing hypothesis does not imply the presence of labor unions.

employment-wage space (see LS curves in Figure 1). In the same space we can represent the firms' demand for labor services as a downward curve, because it is assumed that the marginal productivity of labor is decreasing (see LD curves in Figure 2.1).¹¹ What happens when a positive productivity shock hits the economy? The labor demand schedule shifts to the right (because of the rise in the productivity of labor services) pushing up the equilibrium wage. The new equilibrium allocation depends on the slope and the shift of the labor supply curve. The first element indicates an increase in both wage and employment. The shifts of the LS curve depend on the behavior of the marginal utility of consumption, consequently shifts to the left seem more likely. In fact, after a positive technology shock, especially in a general equilibrium framework, it is plausible that agents increase the level of consumption independently of the changes in labor income. This reduces the marginal utility of consumption and subsequently the labor supply shifts to the left.

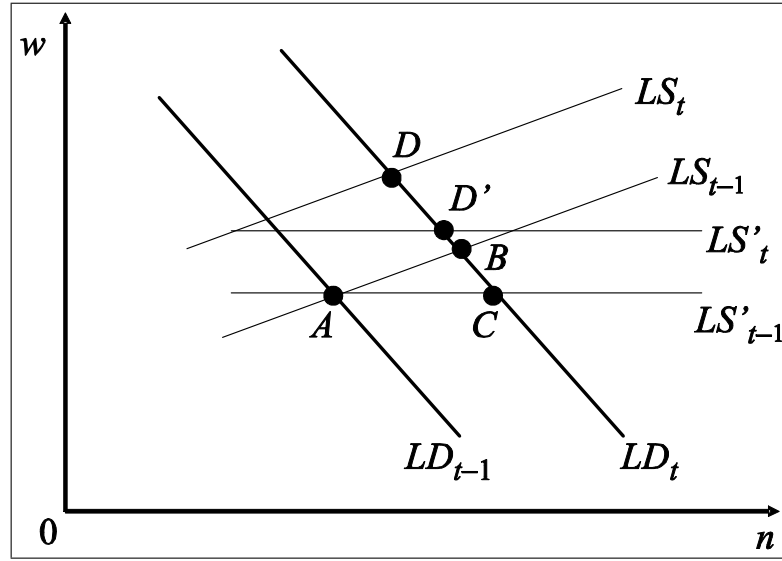


Figure 2.1. Consequences of a positive technology shock in a neoclassical growth model. The figure represents the labor market, in which n denotes the employment level and w the wage rate; LD and LS denote, respectively, the labor demand and the labor supply, while the subscript denotes the time index.

Such dynamics are represented in Figure 2.1. Referring to Figure 2.1, suppose to stay at time $t - 1$ at the equilibrium allocation corresponding to point A ; then, suppose that at time t a positive productivity shock occurs. The LD curve shifts to the right

¹¹In models with imperfect competition, the downward slope of firm labor demand can be explained by the elasticity of substitution between goods.

($LD_{t-1} \rightarrow LD_t$) because of the higher productivity. To identify the new allocation it is necessary to consider the behavior of labor supply. At first do not consider what happens to the marginal utility of consumption. If the Frisch elasticity of labor supply is positive, the new equilibrium is at point B characterized by higher employment and higher wage, while if Frisch elasticity of labor supply is zero the economy moves to point C characterized by higher employment and the same wage.¹² Now, consider the role of the marginal utility of consumption. It is reasonable to assume that it decreases because, during expansionary phases of the business cycle, consumption increases. Therefore, for each level of labor supply, the representative consumer-worker claims higher wage. So, apart from extreme cases, the equilibrium at time t is characterized by higher employment and by higher wage (for instance if Frisch elasticity is positive the LS_{t-1} shifts to LS_t and the new equilibrium is at point D , or if Frisch elasticity is null LS'_{t-1} shifts to LS'_t and the new equilibrium is at point D').¹³ Consequently, the consideration of the role of the marginal utility of consumption tends to reinforce the positive correlation between employment and wage.

Next, to relate leisure/consumption choice to asset returns, it is convenient to use the outline isoprofit curves in the employment-wage space (see Figure 2.2). Profits are measured as the difference between output (value) and labor costs. Again, suppose that at time t the economy experiences an increase in productivity, and that firms can optimize the employment of labor services; the labor demand moves to the right and the variation of profits increases with employment.¹⁴ This suggests that an economy characterized by a-cyclical wages and that has no restrictions on hiring choice of firms (i.e. allocation C) experiences high volatility of profits. Our aim is to show that this scenario is also consistent with a significant equity premium. To conclude the section, notice the difference between a standard RBC model with a positively sloped labor supply and a model with a-cyclical wage. The former contemplates highly pro-cyclical wage and moderate volatility of

¹²The Frisch elasticity of labor supply is identified as the elasticity of the desired labor supply with respect to a change to wage, maintaining the marginal utility of consumption constant.

¹³Some types of extreme cases are: *i*) negative values of Frisch elasticity that induces wage decrease; *ii*) decrease in consumption after a positive technology shock, that induces wage decrease; *iii*) very high variation in marginal utility of consumption, such that the new equilibrium would be characterized by lower employment.

¹⁴Roughly speaking profits increase moving from E to D , see Appendix. In this case, referring to higher employment or to lower wage does not matter because of the negative slope of the labor demand.

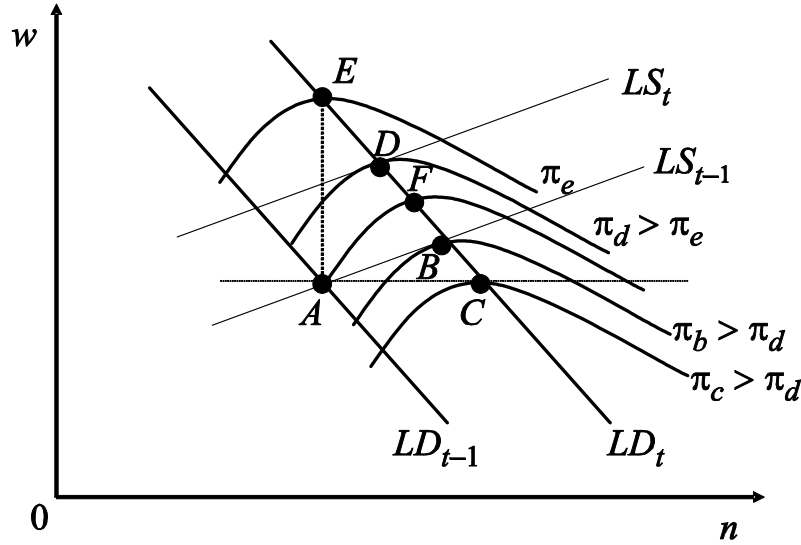


Figure 2.2. Firm's profit volatility after a positive technology shock in a neoclassical growth model. The figure represents the labor market, in which n denotes the employment level and w the wage rate; LD and LS denote, respectively, the labor demand and the labor supply, and π indicates an iso-profit curve. The subscript of π denotes the corresponding points along the LD curve.

employment and profits; consequently wage and employment are highly correlated and the volatility of equity returns does not explain the equity premium. The latter disentangles wage and employment and generates high volatility of equity returns. Intuitively, it is like comparing allocation D (standard RBC model) with allocation C (a-cyclical wage).

2.4. The Economy

2.4.1. Baseline economy

The stylized economy is populated by four kinds of agents: the household, the firm, the labor union, and the government. The firm owns capital and issues equities that the household purchases to transfer income period by period.¹⁵ The household obtains utility from consumption while the labor union is interested in employment and wage. All markets are in perfect competition excluding the labor market, in fact it is assumed that the labor union manages the labor supply. There is a labor union in each firm: the firm sets the employment and bargains with the union about wages. The relative bargaining power depends on the aggregate employment rate. The production function

¹⁵The selected decentralization scheme follows Danthine-Donaldson (2002b).

includes technology that follows a stochastic AR(1) process. No other exogenous stochastic processes is introduced.

2.4.1.1. Labor union. Suppose that in each firm there exists an agent that coordinates and manages the household's labor supply.¹⁶ We call this agent "Labor Union" or just "Union". The dimension of the firm is too small to influence the outcome of the market. The union cares about average labor income $w_t n_t + \bar{B}(1 - n_t)$, that formally is equivalent to consider $n_t(w_t - \bar{B})$, to say the product between employment (n_t) and the difference between wage (w_t) and unemployment benefit (\bar{B}).¹⁷ In this context a firm's payoff is represented by the dividends. We assume that the players might have different bargaining power when discussing wage and employment. To model this, suppose that the firm and the union simultaneously engage in two bargaining processes over equilibrium employment and wage. It is like saying that the bargaining processes happen simultaneously but in two different "tables".¹⁸

To technically accommodate this structure we formalize two simultaneous state dependent problems, differing only along the bargaining power of the players:

$$\max_{w_t} (n_t (w_t - \bar{B}))^{\phi_t} (\lambda_t k_t^\alpha n_t^{1-\alpha} - w_t n_t - i_t)^{1-\phi_t} \quad (2.2)$$

$$\max_{n_t} (n_t (w_t - \bar{B}))^{\eta_t} (\lambda_t k_t^\alpha n_t^{1-\alpha} - w_t n_t - i_t)^{1-\eta_t}, \quad (2.3)$$

where $\frac{\phi_t}{1-\phi_t}$ and $\frac{\eta_t}{1-\eta_t}$ represent the relative bargaining power of the union respectively in the wage setting and in the employment setting. Starting from the logarithmic transformation of the previous equations the first order conditions are:

¹⁶Formally, we can suppose, following Maffezzoli and Zanetti, that each firm is endowed with a pool of household from which it can hire and that the individual household is of measure zero. Since each household supplies its labor to only a clearly identified firm, workers organizes themselves into a firm specific labor union in order to extract some firm surplus.

¹⁷This formalization of union objective function is quite standard in literature but needs two clarifications. First, to maximize the average labor income is different from maximizing the average utility of the representative household, unless supposing that household utility is linear. Second, in our formalization of the union objective function there is no intertemporal dimension. As highlighted by Maffezzoli 2001 this is not relevant if we assume that the firm and the labor union solve a sequence of independent games.

¹⁸This assumption allows us to explicitly model the fact that the players have different bargaining power when discussing wage and employment. It differs from Manning (1987) that studies sequential bargainings.

$$w_t : \frac{\phi_t}{w_t - \bar{B}} - \frac{(1 - \phi_t) n_t}{d_t} = 0$$

$$n_t : \frac{\eta_t}{n_t} - \frac{(1 - \eta_t) ((1 - \alpha) \lambda_t k_t^\alpha n_t^{-\alpha} - w_t)}{d_t} = 0$$

To ensure consistency with the firm's problem, we exogenously set $\eta_t = 0$. This implies that the employment-wage combination is always along the firms' labor demand.¹⁹ As anticipated, we assume that problem (2.2) is state dependent, i.e. that the relative bargaining power depends on the status of the economy. In particular, we assume that in the wage setting process the relative bargaining power of the labor union is increasing in aggregate employment rate; roughly speaking this means that *"when employment is high then labor union is relatively stronger"*.

The relationship between relative bargaining power and aggregate employment is expressed in the following way (ψ_t indicates the relative bargaining power of union):

$$\frac{\phi_t}{(1 - \phi_t)} = \psi_t = \omega + (\mu N_t)^\theta \quad (2.4)$$

where ω , μ and θ are parameters. Combining the optimal conditions and the assumption about bargaining powers, employment and wage must respect:

$$n_t = ((1 - \alpha) \lambda_t)^{\frac{1}{\alpha}} k_t w_t^{-\frac{1}{\alpha}} \quad (2.5)$$

$$w_t = \bar{B} + \left(\omega + (\mu N_t)^\theta \right) \frac{d_t}{n_t} \quad (2.6)$$

Eq.(2.6) represents the wage setting function and denotes that the wage is increasing in the unemployment benefit, in the relative bargaining power of union and in firm's dividend per employee.²⁰

¹⁹Such assumption implies that the wage equal the marginal productivity of labor, as in the right to manage models, Dunlop-Solow.

²⁰Relative bargaining power depends on aggregate employment, so in the decentralized bargaining process the union and the firm do not directly internalize the effect of the employment choice on the relative bargaining power.

2.4.1.2. Household. The primary problem of the representative household concerns consumption and savings into the equity share (that represents the only way to save). It does not include the labor supply choice because it is managed by the labor union. It is assumed that the population distribution is reproduced in each household and that inside it, the same level of consumption is granted to each agent²¹ The utility function takes a constant relative risk aversion form such as $\frac{c_t^{1-\gamma}}{1-\gamma}$ where c_t is the individual consumption flow and γ measures the relative risk aversion coefficient.

The representative household's problem is the following:

$$v(z_t, \lambda_t) = \max_{z_{t+1}, c_t} \left[\frac{c_t^{1-\gamma}}{1-\gamma} + \beta E_t v(z_{t+1}, \lambda_{t+1}) \right] \quad (2.7)$$

$$s.t. \quad c_t + q_t^e z_{t+1} = (q_t^e + d_t)z_t + w_t n_t + \bar{B}(1 - n_t) - \tau_t \quad (2.8)$$

$$c_t, z_{t+1} \geq 0 \quad (2.9)$$

where β is the subjective discount factor, q_t^e denotes the price of the equity security and z_t is the share of the single equity that the household owns. Quantity λ_t , then, represents the technology that follows a stationary stochastic AR(1) process. Quantities n_t and w_t represent, respectively, the employment rate and the wage; then, $(1 - n_t)$ measures unemployment rate²². In each period the Government levies a lump sum tax τ_t and delivers unemployment benefits to unemployed workers equal to \bar{B} , under a balanced government budget constraint.

Under standard assumptions the necessary and sufficient first order condition for z_{t+1} reads

$$-q_t^e c_t^{-\gamma} + \beta v'_z(z_{t+1}, \lambda_{t+1}) = 0, \quad (2.10)$$

²¹Similarly to what happens in the Hansen (1985) model, it is assumed that the union provides unemployment insurance redistributing part of the income from employed to unemployed workers. Union does not makes profit, so that the insurance is actually fair. In this way the marginal utility of consumption is equalized across employed and unemployed workers, so that households can be aggregated in a representative household.

²²It is assumed that each working-age person actively participates in the labor market. Moreover, it is assumed that the labor force is constant and normalized to one, then the employment (unemployment) level is equal to the employment (unemployment) rate.

where $v'(z_t, \lambda_t) = (q_t^e + d_t)c_t^{-\gamma}$; leading the latter equation one period ahead $v'(z_{t+1}, \lambda_{t+1}) = E_t(q_{t+1}^e + d_{t+1})c_{t+1}^{-\gamma}$, substituting back into (2.10), and solving for the unique non-explosive solution leads to the following asset prices equation:

$$q_t^e = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left(\frac{c_t}{c_{t+j}} \right)^{\gamma} d_{t+j} \right\} \quad (2.11)$$

Eq.(2.11) shows how the household prices equities. The maximization process leads to the following result:

$$c_t = E_t \left\{ \left(\frac{q_t^e}{\beta(q_{t+1}^e + d_{t+1})} \right)^{\frac{1}{\gamma}} c_{t+1} \right\} \quad (2.12)$$

The eq.(2.12) describes the way the household keeps a consumption path in line with financial returns (more details to come in the subsequent sections).

2.4.1.3. Firm. The representative firm begins period t with the stock of capital k_t carried over from a previous period. After observing the realization of the technology process λ_t the firm sets the level of production. The proceeds of the output sale are used to pay the wage bill $w_t n_t$, to finance investments i_t under the knowledge of the equation of motion for capital stock $k_{t+1} = (1 - \delta) k_t + i_t$ and, residually, to pay dividends

$$d_t = y_t - w_t n_t - i_t, \quad (2.13)$$

where y_t and d_t represent respectively the output and the dividend. The production process follows a Cobb-Douglas production function that employs physical capital k_t and labor services n_t and it is subject to the technology λ_t , which evolves as $\lambda_t = \rho \lambda_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$ with $\rho \in (0, 1)$.

In this setting of effectively complete markets, the firm's objective function is clear: the maximization of the pre-dividend stock market value of the firm, $d_t + q_t^e$ period by period, optimally choosing the next period capital stock (k_{t+1}) and the demand for labor services (n_t).

More formally, the representative firm solves the following decision problem:

$$\mathcal{J}(k_t, \lambda_t) = \max_{n_t, k_{t+1}} [d_t + q_t^e] \quad (2.14)$$

$$s.t. \quad d_t + q_t^e = E_t \left(\sum_{\kappa=0}^{\infty} \beta^{\kappa} \left(\frac{c_t}{c_{t+\kappa}} \right)^{\kappa} d_{t+\kappa} \right); \quad d_t = y_t - w_t n_t - i_t \quad (2.15)$$

$$y_t = \lambda_t k_t^{\alpha} n_t^{1-\alpha}, \text{ with } 0 < \alpha < 1 \quad (2.16)$$

$$\lambda_t = \rho \lambda_{t-1} + \varepsilon_t, \text{ with } 0 < \rho < 1, \quad \varepsilon_t \sim N(0; \sigma_{\varepsilon}^2) \quad (2.17)$$

$$k_{t+1} = (1 - \delta)k_t + i_t, \quad (2.18)$$

where eq.(2.16) defines the production function and α measures the elasticity of substitution between capital and labor. The eq.(2.17) describes the technology process and the eq.(2.18) is the law of motion of capital where δ is the depreciation rate of capital.

The structure of the firm's optimization process requires that shareholders convey to the firm a complete listing of their future inter-temporal marginal rates of substitution. In the present complete markets setting and, *a fortiori*, in a homogenous agent environment, there would be perfect unanimity vis-à-vis. The information is to be provided.²³

Problem (2.14) admits an equivalent sequential formulation; it may be recursively expressed as:

$$\begin{aligned} \mathcal{J}(k_t, \lambda_t) &= \max_{n_t, i_t} (\lambda_t k_t^{\alpha} n_t^{1-\alpha} - w_t n_t - i_t) + \beta E_t \left(\frac{c_t}{c_{t+1}} \right)^{\gamma} \mathcal{J}(k_{t+1}, \lambda_{t+1}), \\ s.t. \quad k_{t+1} &= (1 - \delta)k_t + i_t; \quad \lambda_t = \rho \lambda_{t-1} + \varepsilon_t \text{ with } 0 < \rho < 1, \quad \varepsilon_t \sim N(0; \sigma_{\varepsilon}^2) \end{aligned}$$

The necessary and sufficient first order conditions are

$$w_t = (1 - \alpha) \lambda_t k_t^{\alpha} n_t^{-\alpha} \quad (2.19)$$

$$\beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^{\gamma} R_{t+1} \right] = 1, \quad (2.20)$$

²³Alternatively, the shareholders could appoint one of their own members to manage the firm, knowing that his preference for future consumption is an exact representation of their own.

where eq.(2.19) suggests that the firm chooses labor demand so that wage equals marginal productivity of labor; eq.(2.20) is the Euler equation that determines the optimal level of investment where R_{t+1} (equal to $\alpha\lambda_t k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + (1 - \delta)$) defines the net return in terms of the consumption good of investing in the capital stock. Eq.(2.20) holds on a period by period basis and indicates that the firm discounts the future returns ($R_{t+s} = \alpha\lambda_{t+s} k_{t+s}^{\alpha-1} n_{t+s}^{1-\alpha} + (1 - \delta)$, $s = 1, 2, \dots$) by consumer's endogenous discount factor, i.e. $\left(\frac{c_t}{c_{t+s}}\right)^\gamma$.

2.4.1.4. Government. Government simply collects lump sum tax revenue and returns it to the shareholder-workers as unemployment benefits. It follows that in each period government's balance is in equilibrium

$$\int_0^1 B_{i,t}(1 - n_{i,t})di = \int_0^1 \tau_{i,t}di \quad \forall t \quad (2.21)$$

where i identifies the single shareholder-worker.

2.4.1.5. Competitive Equilibrium Characterization. The competitive equilibrium consists of a set of decision rules $c_t(\lambda_t)$, $i_t(\lambda_t)$, $w_t(\lambda_t)$, $n_t(\lambda_t)$, $z_t(\lambda_t)$; a set of aggregate decision rules $C_t(\lambda_t)$, $I_t(\lambda_t)$, $W_t(\lambda_t)$, $N_t(\lambda_t)$, $Z_t(\lambda_t)$; a set of price function $D_t(\lambda_t)$, $R_t(\lambda_t)$; a value function $F(\lambda_t)$ satisfying:

- the firm's value maximization;
- the union's welfare maximization;
- the shareholder-worker's utility maximization;
- the aggregate resources constraint;
- the government constraint.

Assuming a continuum of each kind of agent (firm, shareholder-worker, labor union), uniformly distributed, market clearing conditions hold for each market. Specifically

- for employment, $\int n_i(M_t, k_t, K_t)di = \int n_j(M_t, k_t)di = n_i(M_t, k_t, K_t) \equiv N_t$
- for consumption, $\int c_i(M_t, k_t, K_t)di = \int c_j(M_t, k_t)di = c_i(M_t, k_t, K_t) \equiv C_t$
- for investment, $\int i_i(M_t, k_t, K_t)di = \int i_j(M_t, k_t)di = i_i(M_t, k_t, K_t) \equiv I_t$
- for capital, $\int k_i(M_t, k_t, K_t)di = \int k_j(M_t, k_t)di = k_i(M_t, k_t, K_t) \equiv K_t$

- for equity, $\int z_i(M_t, k_t, K_t) di = \int z_j(M_t, k_t) di = z_i(M_t, k_t, K_t) \equiv Z_t$

2.5. Quantitative analysis

2.5.1. Calibration

The model is parameterized for the United States Economy for the post-war period. The system of equations we use to compute the dynamic equilibria of the model depends on a set of **ten** parameters. **Two** pertain to household preferences, (the relative risk aversion coefficient γ , the subjective discount factor β), **one** to the structural-institutional context (the subsidy B), **three** to labor union's bargaining power (the constant parameter of the relative bargaining power function x , the linear and exponential coefficients s and θ linking the employment rate and the relative bargaining power) and the remaining **four** parameters to technology (the capital share α , the capital stock quarterly depreciation rate δ , the autoregressive coefficient of the technology process ρ and the standard deviation of technology shock σ_ε).

Shareholder-worker's preference (β, γ) : the subjective discount factor β is set to correspond to an annual real interest rate of 4% ($\beta = 0.99$). The relative risk aversion γ is set equal to 2.34. These are low and plausible values.

Technology $(\delta, \alpha, \rho, \sigma_\varepsilon)$ are set to commonly used values in this literature (e.g. Danthine Donaldson, 2002a, and Jerman, 1998). More precisely, we set, $\delta = 0.025$, $\alpha = 0.36$, $\rho = 0.95$. Concerning the standard deviation of exogenous productivity shock σ_ε , this is calibrated to match the GDP standard deviation, over the postwar period. Quantitatively speaking, the σ_ε ranges between 0.5% and 0.7%. Notice that these are lower than standard values used in this literature (about 0.712%). That is due to the fact that the model generates sufficiently high endogenous propagation.

Labor union' bargaining power (ω, μ, θ) : ω and μ are scale parameters that affect the deterministic steady state of the model. Numerical simulations confirm that these parameters do not affect the dynamic properties. In the baseline model we set ω equal to unity ($\omega = 1$), so that without considering the effect of employment, unions and firms have the same power (such assumption is not influent). Parameter μ is calibrated to

match the equilibrium employment of the model to the mean value of the post-war US employment rate (equal to 59.2% in the period January 1948 - October 2007, source B.L.S. monthly data seasonally adjusted). Starting from the equation determining the deterministic steady state value of K , we solve the equation for μ in the following way $\mu = \left(\frac{(1-\alpha)\lambda\left(\frac{K}{N}\right)^\alpha - B}{\left(\alpha\lambda\left(\frac{K}{N}\right)^\alpha - \delta\left(\frac{K}{N}\right)\right)} - \omega \right)^{\frac{1}{\theta}} \frac{1}{N}$. Notice that all elements are determined in the model independently of μ : ω , θ , α , δ and B are exogenously fixed, $\frac{K}{N}$ comes from the Euler's equation and W and N emerge from the bargaining problem. Finally, the parameter θ determines the way the dynamics of employment rate influences the dynamics of the relative bargaining power. This is a free parameter in the model. It represents a proxy for the elasticity of relative bargaining power with respect to employment level. Plausibly calibrated values range between $\theta = 5.9$ and $\theta = 7.0$. A low θ (i.e. $\theta = 5.9$) represents a case in which the employment has a low effect over relative bargaining power. Anticipating a result, a low θ , induces a pro-cyclical wage rate (more details in section 4.2). The benchmark calibration is $\theta = 6.45$, which offers the best fit of the model to the data.

Subsides (B) is set to a fifth of the steady state wage (more precisely to $B = 0.19 \cdot W$) as it emerges in OECD 1994.

TABLE 2.1: BENCHMARK PARAMETRIZATION

β	0.99	σ_ε	0.565
γ	2.34	ω	1
δ	0.025	μ	$\left(\frac{(1-\alpha)\lambda\left(\frac{K}{N}\right)^\alpha - B}{\left(\alpha\lambda\left(\frac{K}{N}\right)^\alpha - \delta\left(\frac{K}{N}\right)\right)} - \omega \right)^{\frac{1}{\theta}} \frac{1}{N}$
α	0.36	θ	6.5
ρ	0.95	B	$0.19 \cdot W$

2.5.2. The Real Economy

At first, we analyze how the real side of the stylized economy responds to a positive technology shock, considering three values of the parameter θ . Subsequently, we highlight how the dynamics of real variables influence the performance of selected financial variables. Technically, we analyze the impulse response functions and then we simulate the model (1.000 simulations of time series of 111 observations following Uhlig, 1999). The first set of simulations (results reported in Table 2.1 and Table 2.2) maintains the baseline calibration. In particular, the standard deviation of the technology shock is the same in each simulation. Then, we run a set of simulations removing the previous hypothesis and choosing the value of the standard deviation that, each time, permits to fit the empirical volatility of the standard deviation of GDP.²⁴

The first step is to analyze the macroeconomic dynamics with different values of θ . We report the graphical representation of impulse response functions in Figures 2.1, 2.2 and 2.3 and resume in Table 2.2 the results of the simulations. Table 2.2 suggests that the model is quite successful in replicating selected important aspects of the U.S. economy.

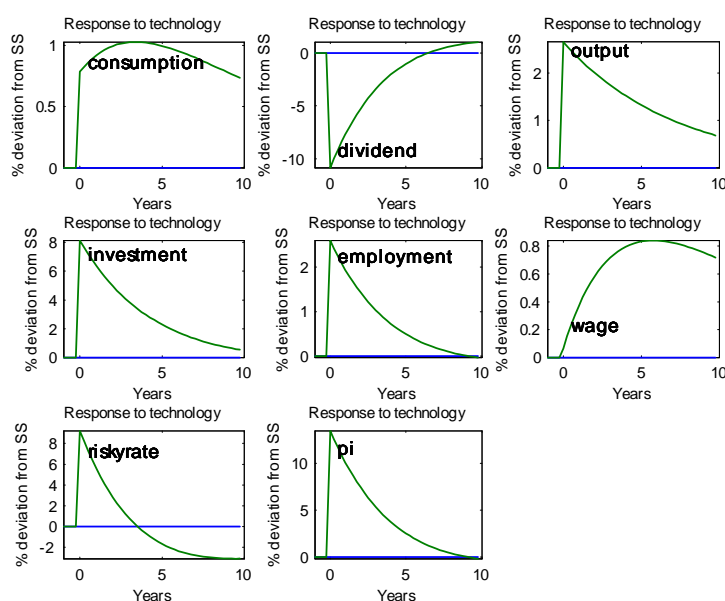


Figure 2.3. The figure shows the impulse response functions of selected variables after a 1% positive shock to s_1 , with $\theta = 6.45$.

²⁴This method of calibrating the volatility of technology shocks is largely used in literature. See Danthine-Donaldson (2002a).

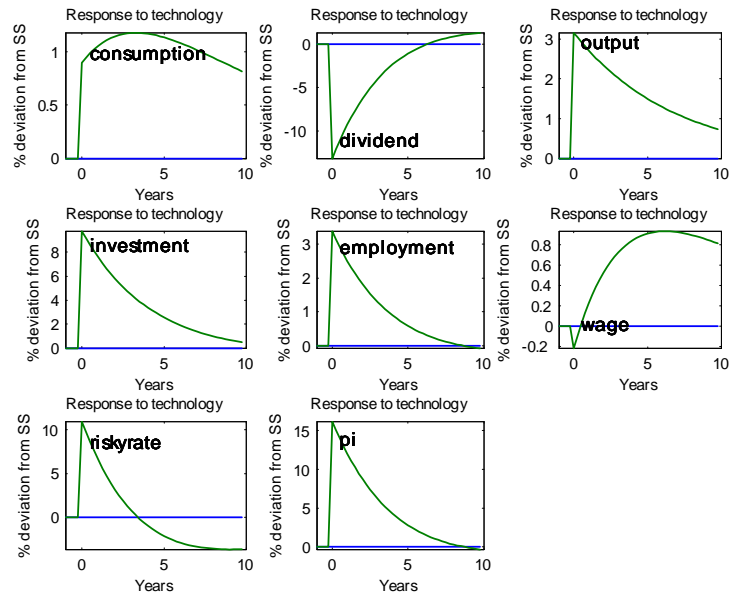


Figure 2.4. The figure shows the impulse response functions of selected variables after a 1% positive shock to s_1 , with $\theta = 5.9$.

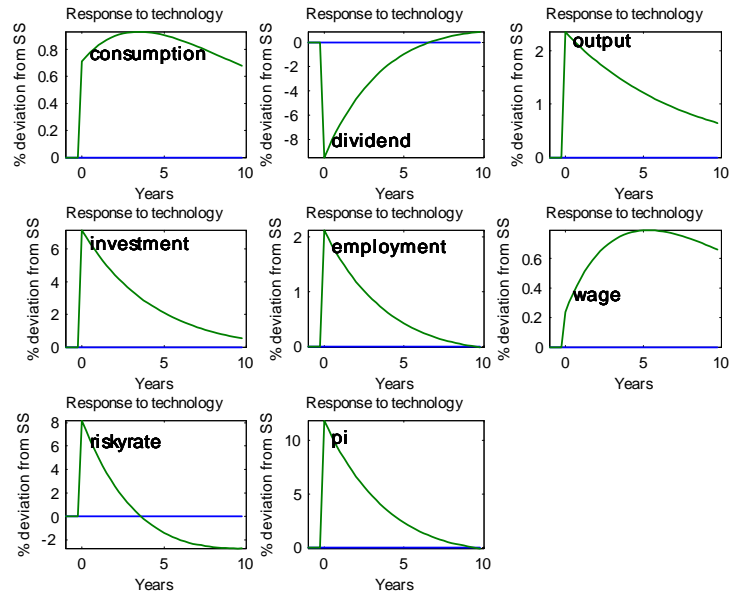


Figure 2.5. The figure shows the impulse response functions of selected variables after a 1% positive shock to s_1 , with $\theta = 7.0$.

TABLE 2.2 SIMULATIONS WITH DIFFERENT VALUES OF θ (REAL SIDE)

		US	$\theta = 5.9$ $\sigma_\varepsilon = 0.536$	$\theta = 6.45$ $\sigma_\varepsilon = 0.536$	$\theta = 7.0$ $\sigma_\varepsilon = 0.536$
y	s.d.	1.76*	2.09	1.76	1.55
c	s.d.(.)/s.d.(y)	0.38**	0.30	0.31	0.31
i	s.d.(.)/s.d.(y)	8.60*–1.96**	3.09	3.05	3.03
n	s.d.(.)/s.d.(y)	0.83**–0.99***	1.08	0.98	0.90
w	s.d.(.)/s.d.(y)	0.34**	0.14	0.12	0.15
	corr(.,y)	0.33**	-0.42	0.39	0.78
	corr(.,n)	0.29***	-0.51	0.28	0.70

(*) Data source: Danthine-Donaldson (2002a). (**) Data source: Boldrin-Horvath (1995). (***) Our estimation between US average hourly real earnings (private nonfarm industries) and US total private-sector employment with Hodrick-Prescott filter from 1972:Q1 to 1999:Q4, source US Department of Labor Bureau of Labor Statistics.

With respect to our objective, two important evidences emerge. **First**, the value of θ strongly affects the behavior of the wage rate along the business cycle. In fact, the wage rate can be pro-cyclical, a-cyclical and counter-cyclical according to the calibration of θ . **Second**, the volatility of the stylized economy is negatively related to the value of θ . From the previous considerations, it follows that the volatilities of investment and employment increase (decrease) when the wage rate is counter-cyclical (pro-cyclical), i.e. when the linkage between the relative bargaining power and the employment rate is high (low).

Both evidences have an intuitive explanation. In our model, eq.(2.6) indicates that the cyclical behavior of the wage rate depends on the firm's liquidity and on the relative bargaining power. Consequently, when the impact of the employment on the relative bargaining power is high (low), wages grow (fall) notwithstanding the reduction (increase) in the firm liquidity (see Figure 2.3).

Concerning the second result, when the economy experiences a positive shock, firms desire to increase the accumulation of input factors to take advantage of the increased productivity. This incentive is reduced by the way employment rate affects the relative

bargaining power. If little variations of the employment rate strongly affect the relative bargaining power (i.e. $\theta = 7.0$), firms limit the hiring of further labor services, notwithstanding the productivity improvement. A symmetric mechanism operates in downturn because the decrease in employment increases the relative bargaining power of firms and consequently labor input becomes relatively less costly. In both directions the effects on the hiring choice spread within the entire economy because of the complementarity among the input factors.

2.5.3 Asset Returns

In this section, predictions of the model are compared with selected financial stylized facts. The focus is on the ability to replicate the empirical equity premium with empirically consistent risk free rate and sharpe ratio. The consistency of the risk free rate is an important result because it shows that the equity premium, emerging from simulations, is due not only to volatility of firm returns. The results of different simulations are reported in the following tables. We propose two different kinds of simulation, choosing in each one, three different value of our key parameter, θ . The first set of simulations (results reported in Table 2.3) keeps constant the volatility of the technology shock. In this case, the value of σ_ε is calibrated so that the baseline version (with $\theta = 6.45$) fits historical GDP volatility. In the second set of simulations (reported in Table 2.4) the volatility of technology shock is adjusted to the historical GDP volatility in each simulation.

TABLE 2.3 SIMULATIONS WITH DIFFERENT VALUES OF θ (σ_ε CONSTANT)(**)

		US*	$\theta = 5.9$ $\sigma_\varepsilon = 0.536$	$\theta = 6.45$ $\sigma_\varepsilon = 0.536$	$\theta = 7.0$ $\sigma_\varepsilon = 0.536$
risk free	mean	1.91-0.80	-0.54	0.79	1.51
	s.d.	5.67	5.81	4.81	4.20
equity returns	mean	6.98	7.63	6.57	5.81
	s.d.	18.2-16.54	28.59	23.71	20.36
excess returns	mean	4.82-6.18	8.16	5.77	4.30
	s.d.	19.1-16.67	29.77	24.69	21.19
Sharpe Ratio		0.30***	0.29	0.24	0.20

(*) Data source: Danthine-Donaldson (2002a). (**) Annualized data. (***) Data source: Uhlig (2007)

TABLE 2.4 SIMULATIONS WITH DIFFERENT VALUES OF θ (σ_ε VARIABLE)(**)

		US*	$\theta = 5.9$ $\sigma_\varepsilon = 0.450$	$\theta = 6.45$ $\sigma_\varepsilon = 0.536$	$\theta = 7.0$ $\sigma_\varepsilon = 0.610$
risk free	mean	1.91-0.80	0.77	0.79	0.86
	s.d.	5.67	4.96	4.81	4.72
equity returns	mean	6.98	6.51	6.57	6.41
	s.d.	18.2-16.54	24.17	23.71	23.28
excess returns	mean	4.82-6.18	5.73	5.77	5.55
	s.d.	19.1-16.67	25.16	24.69	24.23
Sharpe Ratio		0.30***	0.24	0.24	0.23

(*)Data source: Danthine-Donaldson (2002a). (**) Annualized data. (***) Data source: Uhlig (2007)

Table 2.3 reports results consistent with the mechanism described in previous sections (see Section 2.1). When the elasticity of the relative bargaining power to employment is set to generate non-pro-cyclical wages (i.e. $\theta \leq 6.45$), the endogenous propagation mechanism is so strong that the equity premium emerges despite the low exogenous volatility of the

technology shock and the standard structure of preferences²⁵. To clearly understand the underlying mechanism consider the representation of risk premia (r^{rp}) used in Lettau (2003),

$$r^{rp} = \gamma \eta_{c\lambda} \eta_{r\lambda} \sigma_\varepsilon^2.$$

Risk premium is an increasing function of: the coefficient of risk aversion (γ), the variance of technology shock (σ_ε^2) and the elasticities of consumption ($\eta_{c\lambda}$) and asset return ($\eta_{r\lambda}$) with respect to the technology shock. Maintaining plausible parameterization of γ and σ_ε^2 , the aim is to generate and explain high values of $\eta_{c\lambda}$ and $\eta_{r\lambda}$. The last has been broadly explained in Section 2.1, where it has been argued that an economy, moving along firms' labor demand shows higher volatility of firm returns when it is characterized by non-pro-cyclical wages, than when it is characterized by pro-cyclical wages. So the explanation remains of which mechanism determines the elasticity of consumption to technology shock. In a recent paper, Uhlig argues that "*Endogenous labor supply decisions on a frictionless labor market provide agents with an insurance device against fluctuations in consumption. This insurance possibility then renders these models incapable of generating high Sharpe ratios or equity premia, unless additional frictions on labor markets such as separated labor markets or wage rigidities are introduced*"²⁶. The main point is that with endogenous labor supply, the labor supply choice represents an additional dimension to smooth consumption. Consequently, apart from strong assumptions on preferences, models with a representative agent can not explain the volatility of the stochastic discount factor. In this way, fixed labor supply becomes a comfortable hypothesis in reducing the ability of smoothing consumption but it is inconsistent with data²⁷. Considering this argument we refuse the fixed supply hypothesis because it reduces the role of the volatility of firms' returns and obstructs the study of employment behavior along the business cycle²⁸. So, maintaining a structure able to generate high volatility of firm returns, the model designs

²⁵In particular, the reference is to the large use of habit formation in the preference structure.

²⁶Uhlig (2007)

²⁷Employment volatility is similar to the volatility of GDP, see Table 2.1.

²⁸Refer to figures 2.2 in Section 2.1. If we assume a positive technology shock and fixed employment the variation of firm returns is negatively related to wage. If wage is fixed, firm profits are next to π_c (is an isoprofit curve next to that including point A) where the variation of profits is lower than in π_d .

a mechanism with endogenous employment where labor supply is not an instrument to smooth consumption because is not related to consumption choice²⁹. In fact, the labor union, not the household, bargains with the firm about wage-employment allocations. In the baseline version of the model, after a technology shock, consumption has an immediate strong response because it follows the income increase, and households cannot control consumption path with labor supply choice. The "insurance" mechanism of endogenous-employment models fades away. Considering Table 2.3, it appears that the risk free rate is sufficiently low (with non-pro-cyclical wage, the risk free rate ranges between 0.26 and 0.79) although it is assumed a standard CRRA utility function, while Table 2.2. reports a low relative volatility of consumption with respect to GDP. So, the representative household tries to smooth consumption (this explains the low volatility of consumption), but can not influence its own income. Consequently, after a technology shock, consumption jumps because of the high variation of output, and this explains why the model predicts low risk free rate³⁰.

At this point we have all the elements to understand the results of simulations reported in Table 2.3 and Table 2.4. First, to fit the volatility of GDP it is necessary to increase the volatility of the exogenous shock the higher is the value of θ (see Table 2.4). Second, the equity premium is negatively related to θ (see Table 2.3).

2.6. Conclusions

We have developed a SDGE model with labor union. We have assumed that the firm and the labor union set the wage with relative bargaining power depending on employment rate.

²⁹Instead, Uhlig maintains the linkage between consumption choice and leisure choice but inserts external habit in both the arguments and some unmodelled frictions.

³⁰Notice that in the actual model the eventual introduction of habit does not directly affect labor-market allocations. Including habit, it could be conveniently to select dynamics characterized by lower volatility of firm returns (higher values of θ) but there would be no significant variation in the way the model works, because labor supply does not depend on marginal utility of consumption. This is not true for standard models with endogenous labor supply, because the introduction of habit strongly affects labor-supply choice.

The model performs quite well from both the real and the financial point of view, in fact it is able to give an explanation to the productivity puzzle (concerning the labor market) and to the equity premium (concerning the financial market).

We support that the relation between bargaining power and employment rate is a key element to explain the a-cyclical behavior of wages and, consequently, the equity premium. If the influence of the employment rate on the relative bargaining power is not too strong, firm's revenue is highly volatile because the price of labor services does not immediately increase during expansive phases which are characterized by high investment costs and low firm liquidity.

Moreover, the assumptions concerning the bargaining process are capable of generating also high volatility of the standard stochastic discount factor (explaining by this way the low values of the risk free rate) because they disentangle the labor supply choice from the consumption choice.

It is remarkable that the framework adopted here is very simple, as only a few elements differ from the standard RBC models. Notwithstanding, it suggests a possible explanation for two important issues emerging from this literature, the equity premium and the productivity puzzle.

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Appendix

A. Volatility of Firm Revenue

Proof. Consider as a measure of the firm profits the difference between revenue and labor cost (see Blanchflower et al., 1996). Then:

$$\pi_t = y_t - w_t n_t = \alpha \lambda_t k_t^\alpha n_t^{1-\alpha}$$

The map of isoprofit of π in the space (n, w) is qualitatively the same of the capital revenue defined as

$$r_t k_t = \pi_t - \delta k_t$$

A maximizing firm is characterized by the following labor demand function:

$$n_t^d = ((1 - \alpha) \lambda_t)^{\frac{1}{\alpha}} k_t w_t^{-\alpha}$$

We can then consider the figure (2.3), in which A is the equilibrium at time $t - 1$. After a positive technology shock the labor demand schedule shift to the right (from LD_{t-1} to LD_t); if the labor supply is fixed (*i.e.* $n_t = n_{t-1} = n$) the economy moves to E and profits increase as:

$$\pi_{e,t} - \pi_{a,t-1} = (\lambda_t - \lambda_{t-1}) \alpha \bar{k}^\alpha \bar{n}^{1-\alpha} > 0$$

Otherwise, if the labor supply is endogenously provided and the wealth effect is positive (consumption increases independently of labor income) the economy moves towards the point D where profits are higher than in the point E , as

$$\pi_{d,t} - \pi_{e,t} = \alpha \lambda k^\alpha (n_d^{1-\alpha} - n_e^{1-\alpha}) > 0$$

Moreover, without the wealth effect the economy moves to B and

$$\pi_{b,t} - \pi_{d,t} = \alpha \lambda k^\alpha (n_b^{1-\alpha} - n_d^{1-\alpha}) > 0$$

The same mechanism is true if the wage rate does not change; in this case the economy moves to C where

$$\pi_{c,t} > \pi_{b,t} > \pi_{d,t} > \pi_{e,t} > \pi_{a,t-1}$$

□

B. Equilibrium Equations

- (1) $B_t(1 - N_t) = \bar{B}(1 - N_t) = T_t$
- (2) $C_t = D_t + W_t N_t + (1 - N_t) B_t - T_t$
- (3) $K_{t+1} = (1 - \delta)K_t + I_t$
- (4) $N_t = ((1 - \theta)\lambda_t)^{\frac{1}{\alpha}} K_t W_t^{-\frac{1}{\alpha}}$
- (5) $Y_t = \lambda_t K_t^\alpha N_t^{1-\alpha}$
- (6) $W_t = B_t + \psi_t \frac{D_t}{N_t}$
- (7) $\psi_t = \omega + (\mu N_t)^\theta$
- (8) $D_t = Y_t - W_t N_t - I_t$
- (9) $RR_t = \alpha \lambda_t K_t^{\alpha-1} N_t^{1-\alpha} - \delta$
- (10) $U = N_t(W_t - B_t)$
- (11) $\beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^\gamma (\theta \lambda_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + (1 - \delta)) \right] = 1$
- (12) $\ln \lambda_{t+1} = \rho \ln \lambda_t + \varepsilon_t$

C. Calibration

Proof. Imposing that the bargaining wage must stay along the firm's labor demand, it follows:

$$\begin{aligned}
 B + \psi \frac{D}{N} &= (1 - \alpha) \lambda \left(\frac{K}{N} \right)^\alpha \\
 B + \left(\omega + (\mu N)^\theta \right) \left(\alpha \lambda \left(\frac{K}{N} \right)^\alpha - \delta \left(\frac{K}{N} \right) \right) &= (1 - \alpha) \lambda \left(\frac{K}{N} \right)^\alpha \\
 \left(\omega + (\mu N)^\theta \right) &= \frac{(1 - \alpha) \lambda \left(\frac{K}{N} \right)^\alpha - B}{\left(\alpha \lambda \left(\frac{K}{N} \right)^\alpha - \delta \left(\frac{K}{N} \right) \right)} \\
 \mu &= \left(\frac{(1 - \alpha) \lambda \left(\frac{K}{N} \right)^\alpha - B}{\left(\alpha \lambda \left(\frac{K}{N} \right)^\alpha - \delta \left(\frac{K}{N} \right) \right)} - \omega \right)^{\frac{1}{\theta}} \frac{1}{N}
 \end{aligned}$$

□

D. Steady State

Starting from eq.(2.20) find capital-employment ratio:

$$\frac{k}{n} = \left(\frac{\alpha\lambda}{\frac{1}{\beta} - (1-\delta)} \right)^{\frac{1}{1-\alpha}},$$

and by this, the equilibrium wage rate is given by:

$$w = (1-\alpha)\lambda \left(\frac{k}{n} \right)^\alpha$$

Now, substituting eq.(2.4) and (2.13) into eq.(2.6) leads to:

$$K = \left(\frac{w - B - \omega \left(\frac{k}{n} \right) \left(\alpha\lambda \left(\frac{k}{n} \right)^{\alpha-1} - \delta \right)}{\mu \left(\frac{k}{n} \right)^{1-\theta} \left(\alpha\lambda \left(\frac{k}{n} \right)^{\alpha-1} - \delta \right)} \right)^{\frac{1}{\theta}}.$$

E. Dynamic Equations

- (1) $0 = -\tilde{C}_t + \frac{D}{C}\tilde{D}_t + \frac{WN}{C}\tilde{N}_t + \frac{WN}{C}\tilde{W}_t$
- (2) $0 = -\delta\tilde{I}_t + \tilde{K}_{t+1} - (1-\delta)\tilde{K}_t$
- (3) $0 = -\tilde{N}_t + \frac{1}{\alpha}\tilde{\lambda}_t - \frac{1}{\alpha}\tilde{W}_t + \tilde{K}_t$
- (4) $0 = -\tilde{Y}_t + \tilde{\lambda}_t + \alpha\tilde{K}_t + (1-\alpha)\tilde{N}_t$
- (5) $0 = -\tilde{D}_t - \tilde{\psi}_t + \frac{nw}{\pi d}\tilde{W}_t + \tilde{N}_t - \frac{nw}{\pi d}B_t$
- (6) $0 = -\psi\tilde{\psi}_t + \omega\tilde{\omega}_t + \theta\mu^\theta N^\theta\tilde{N}_t + \theta\mu^\theta N^\theta\tilde{\mu}_t + \theta\mu^\theta N^\theta \ln(\mu^\theta N^\theta)\tilde{\theta}_t$
- (7) $0 = -\tilde{D}_t + \alpha\frac{Y}{D}\tilde{Y}_t - \frac{I}{D}\tilde{I}_t$
- (8) $0 = -rr_t + \frac{1-(1-\delta)\beta}{1-\beta}[\tilde{Y}_t - \tilde{K}_t]$
- (9) $0 = -\gamma\tilde{C}_t + \gamma\tilde{C}_{t+1} - \beta\alpha\frac{Y}{K}(\tilde{Y}_{t+1} - \tilde{K}_{t+1})$

CHAPTER 3

Fair wages, labor relations and asset returns

Abstract

The paper investigates the nexus between labor and financial markets, focusing on the way the interaction between the labor union's attitude in the wage setting and the firm's investment strategy affect asset returns. We assume that the labor union's relative preferences between wage and employment depend on selected financial performances of the firm. We show that if the labor union links the preference for wage to the firm's dividends (or available liquidity) then the volatility of the firm's returns increases. Consequently, equities have to grant high expected returns in order to remunerate the strong volatility. This mechanism offers an explanation for the "equity premium" (that is the difference between the equity return rate and the risk free rate). It is a welcome result that the simulated excess return is about the empirical estimate and that it is obtained with a plausibly low parameterization of the shareholders' risk aversion.

J.E.L. Classification Numbers: Asset Pricing, Business Fluctuations, Labor Union models.

Keywords: G12, E32, J51.

3.1. Introduction

The paper investigates the nexus between the labor market and the pricing of financial assets, and it follows a recent literature concerning the influence of industrial relations on financial market performances. In particular, it analyzes labor relations in order to identify the origin of the so called "equity premium" concentrating on labor union's preferences and the complementarity between input factors.¹ We analyze these elements within a dynamic general equilibrium model composed of four representative agents (household, firm, labor union, government) and parameterized to replicate selected characteristics of the United States economy. The paper's distinctive element is the explicit introduction of a labor union into a general equilibrium model, where the union's relative preferences follows the firm's profitability.

Now we move towards the contribution of this model. It departs from the standard literature about firm-union bargaining processes and introduces a labor union that is characterized by a state dependent objective function. The arguments of the labor union's objective function are the usual ones: the actual number of employees and the difference between wages and unemployment benefits. The innovation is that the relative preference between these arguments depends on selected indicators of the firm's performance. The higher the value is of the selected indicator, the higher the relative preference is for wage in the employment-wage trade-off. The main point of the paper emerges when the labor union considers the firm's dividends (or available liquidity) as an indicator of the firm's performance. In this case, the bargaining process supports a capital-accumulation profile that better explains the excess return of equities over safe assets.

More generally, we discuss two possible schemes for the union's preference structure. The "fair wage" scheme assumes that the union considers the dividend (in the benchmark version) or the profit (in the alternative version) as an indicator of firm's performance. On the contrary, the "sticky wage" scheme assumes that the union's relative preference between wage and employment is constant, hence not depending on the firm.

¹The literature about this matter is very vast and in this place we limit to indicate Kocherlakota (1996) as an enlightening survey.

In particular, the paper focuses on the benchmark version of the first scheme, assuming that the labor unions' relative preference between wage and employment depends on firm liquidity, which in our framework is equal to the dividends distributed to the shareholders. Firm liquidity (i.e. its net cash flow) depends positively on the output and negatively on the labor cost and investment. Given the structure of labor union's preferences, it emerges that, after a positive technology shock, an increase in the investment has a negative influence on the dividends and, subsequently, on the wage. The last effect raises the demand for labor services and, because of the complementarity of input factors, it stimulates higher accumulation of capital. This mechanism implies that the optimal investment choice in response to a technology shock is higher (as compared to the standard context) when wages are influenced by the liquidity effect of investment. It follows that the volatility of the dividends increases, and consequently, equities must grant high returns. These dynamics explain why the model generates an equilibrium equity premium consistent with the historical data and noticeably higher than the equity premium emerging from a standard Real Business Cycle model. This is a welcome result, obtained with a standard specification (and a plausibly low risk aversion) of the shareholders' preferences.

The paper proceeds as follows. Section 3.2 discusses the relations with the existing literature and Section 3.3 details the model. Section 3.4 analyzes the results emerging from simulations, and Section 3.5 concludes. Proofs and derivations are sketched in the Appendix.

3.2. Related literature

The relationship between labor and financial markets is a growing-interest issue that, in the last years, has been studied with different approaches, including the financial and the macroeconomic ones. Moreover, the role of fairness and reciprocity in economic relations has been widely explored, especially in the labor market analysis.

Surprisingly, at our knowledge there are no theoretical contributions that analyze in a DSGE framework the effects of the fairness hypothesis on the labor union's attitude towards the firm's financial structure or profitability.

On the contrary, many empirical works demonstrate the existence of a relationship between the union's presence and the firm's financial structure.

In this literature, the labor union is considered an agent able to extract revenues from the firm; so, firms try to prevent union formation or to reduce union capacity of profit extraction.

Some papers estimate the effect of unionization on debt-equity ratio: Bronars-Deere (1991) argue that firms modify the capital structure whereby they limit the negative effect that the presence of a union has on shareholder's wealth. In their framework, debt is associated with higher probability of bankruptcy and such risk should limit the action of the labor unions.² In the same line, Cavanaugh-Garen (1997) find that the effect of the union bargaining power on the debt-equity ratio is positive but smaller when the firm's assets are less specific.³

Regarding the analysis of financial-market performance, Chen et al. (2007a) find that the expected returns are higher for firms operating in more unionized industries and that the influence of the labor union depends on the bargaining environment. They estimate that a one-standard-deviation increase in the unionization rate increases the implied cost of equity by 1.5% points per year.⁴

Moreover, De Angelo-De Angelo (1991) find that reported net income is significantly lower during union negotiations.⁵ This provided an empirical evidence to our "fairness hypothesis", as the labor union attitude is linked to the firm's performance.⁶

Another confirmation comes from Ramirez (2004). The Author argues that managers have an incentive to use dividend payments as a signal of future earnings. However, the main drawback of this policy is the effect on the labor union's wage claims. His

²The Authors find that evaluated at the sample mean, a 0.1 increase in the probability of unionization increases the ratio of debt to equity by 12.3%.

³The Authors find that, calculated at mean values of the sample, a 10% increase in the unionization of the firm increases the debt-equity ratio by a range between 7.2% and 10.5%.

⁴In a companion paper (Chen et al., 2007b) the Authors find that unionization reduces the moral hazard between the owners of the firm and the bondholders reducing agency costs of debt. This is consistent with the evidence concerning the debt-equity ratio because it cheapens the strategic use of debt.

⁵More precisely, the Authors investigate how seven steel producers used the managerial compensation, financial reporting, and dividend policy in negotiations with the labor unions during the 1980s.

⁶More doubtful results concerning the role of the ability to pay in wage determination are reported in Levine (1993).

empirical test supports the hypothesis that managers use dividends to convey information about future earnings to investors. Moreover, it emerges that the power of dividends as predictors of future earnings is higher for non-unionized firms than for highly unionized firms. Also the response of financial markets to firm indicators has a different behavior in presence of labor unions.

These stylized facts suggest firms characterized by the internal presence of a strong labor union show some "peculiarities". We suggest that these peculiarities can be due to the influence that firm performance has on labor union's preferences.

This paper analyzes this topic, and demonstrates that when the labor union links the relative preference for wages to firm performance and it adopts as reference an indicator that is counter-cyclical (pro-cyclical), the firm's investment response to productivity shocks is higher (lower). In this case firms' returns are more (less) volatile and the stylized economy is (not) capable of explaining equity premium.

In the next sections we firstly present a brief sketch of the literature on the "standard" approaches to the analysis of the relationship between firms and labor unions, then we discuss the most related contributions of the fairness hypothesis that suggest the relevance of including fairness concern in labor relations. This is necessary in order to better understand the underlying relations between the union's characterization and the final economic results.

3.2.1. Firms and labor union: standard approaches

As we have seen in the previous section, the financial literature concerning the interaction between labor union and firm has mainly explored how the existence of labor unions affects the investment and financing strategies of firms.⁷ For example, the firm's capital structure affects the size of the available resources that can be shared with the labor union, so the firm prefers to decrease the liquidity and eventually to run into debt.⁸ Moreover, a capital structure characterized by a high weight of debt increases the bankruptcy risk

⁷Recent references are Matsa (2006) and Chen et al. (2007a,b).

⁸See Dasgupta-Sengupta (1993).

and discourages wage claims.⁹ On the other side, sunk investment increases the loss that the firm holds in case of disagreement with unions, lessening the bargaining power of the firm.¹⁰

The other theoretical framework, related to our study, in which labor relations have been studied is the macroeconomic one. This approach is interested in labor relations in order to uncover which elements of the labor market are relevant to explaining economic fluctuations and how they influence the financial side. Labor unions are typically introduced into dynamic general equilibrium models as an additional agent; their role helps to understand how the labor market could stand in an equilibrium with involuntary unemployment and wages above the competitive level. Unions interact with firms along many dimensions, but the macroeconomic approach tends to focus on the effects over wage and employment. The formalization of the bargaining process between firms and labor unions dates back to Oswald's contributions and recently has been inserted into DSGE models.¹¹ These contributions follow the "right to manage" approach where unions set the wage internalizing that firms will choose employment moving along the own labor demand function.

In particular, Chiarini-Piselli (2005) propose unions and stochastic unemployment benefits as institutional elements in order to explain the low correlation between productivity and production. This paper presents some particularly interesting elements as it explicitly considers the possibility that the labor union has her own relative preferences between wage and employment.

However, in their framework, the only way to explain a change in the relative preferences between wages and employment is to allow the unemployment benefits to vary, as the weights of union preferences are exogenously given. The novelty of our model, with respect to the cited contribution is represented by the introduction of fairness elements in union's preferences. What do we mean with "fairness" is explained in the following section.

⁹See Bronars-Deere (1991).

¹⁰See Cavanaugh-Garen (1997).

¹¹See Oswald (1982), Maffezzoli (2001), Zanetti (2003) and Chiarini-Piselli (2005).

3.2.2. Fairness, Reciprocity and the labor market

Standard economic theory assumes agents as completely selfish; agents' objective function includes only their material self interest with no bearing on the other agents' welfare. In a framework characterized by perfect competition, each agent maximizes his objective function taking the other agents' behavior and the market conditions as given. Even removing some assumptions that are implicit in perfect competition (perfect information, little dimension of agents with respect to market dimension, etc.), a fully selfish and rational agent must consider the behavior and objective function of the other agents in order to maximize. But, also in this case, the other agents exist only to define the framework. For the fully selfish agents, the others affect how to reach the aim, but not the aim itself.

On the contrary, there is some evidence that, especially when interaction occurs between a small number of agents, people are not solely motivated by their material gains. There are laboratory experiments and motivational interviews that show some evidence unexplainable by this fully rational and self-interested hypothesis.¹² In particular, experiments such as the Gift Exchange Game and the Ultimatum Game prove that people are willing to support a cost in order to reward or to punish interacting people whom are judge as fair or unfair.¹³

A field of the economic literature has shown that the introduction of fairness and reciprocity is very fruitful in explaining some peculiarities of labor market. For example, the efficiency wage approach, following Akerlof (1982), explains the existence of flawed competition wages resulting from gifts exchanged between employer and employee. According to this approach, when workers estimate the wage offered by the firm as fair, they desire to provide a higher effort, even though this represents a costly activity. Indeed, this hypothesis is empirically supported. As well, many empirical investigations have been conducted to elicit the elements that are relevant to workers' perception of fairness. Studies such as

¹²A survey of laboratory experiments is provided by Fehr-Gächter (2000, 2001) and Fehr-Schmidt (2000), while Kahneman et al.(1986) employ motivational interviews to show which standards of fairness influence the behavior of firms and explain some market anomalies.

¹³A reference for Gift exchange Game is Fehr et al. (1993), and a reference for Ultimatum Game is Camerer-Thaler (1995).

Bewley (2002) and Campbell III-Kamlani (1997) show that workers' propensity to reciprocate is high and that firm-internal reference plays a prevalent role. Campbell III-Kamlani (1997) report some important evidence: *i*) 69.2% of their interviewed workers link a fall in effort to wage cuts due to a change in gratitude and loyalty; *ii*) most of the interviewed workers think that the issue regarding wage cuts can differ greatly depending on the firm's profitability.¹⁴ The last result is confirmed by Kahneman et al. (1986) which give a similar conclusion by investigating the notion of fairness, without reference to an efficiency-wage framework. The Authors show that workers' perception towards firm choices (and not only effort decision) strongly depends on the interaction and the distribution of benefits inside the firm.¹⁵ They conclude that in labor market "the employment relation is governed by an invisible handshake, rather than by the invisible hand".¹⁶

Our paper develops a model where labor relations are affected by fairness elements. In fact, it is to note that when the labor union shifts preferences toward employment, in effect, it is modifying its own preferences in favor of the firm. The last assertion holds because, in this context where final wage-employment allocations are always along firm's labor demand, the firm's revenue is increasing (decreasing) in employment (wage). So, the labor union shifts its own preferences toward employment (wage) when it perceives the firm going through a negative (positive) phase.

The way we introduce fairness elements has two important peculiarities. Firstly, we do not consider the choice about effort. Secondly, we refer to the relation between firm and labor union.

In efficiency wage models (as in Danthine-Kurman, 2005), the fairness indicator modifies the marginal rate of substitution between wage and effort, while in our model the fairness indicator modifies the marginal rate of substitution between wage and employment. But in both cases, preferences are state dependent. With regard to the second feature, this option is supported by the fact that the reference to labor unions seems more

¹⁴The last result is stronger between white-collar and blue-collar than between less skilled workers.

¹⁵To similar results compare Blinder-Choi (1990).

¹⁶The Authors openly refer to Okun (1981).

accurate when it is studying bargaining processes. Moreover, it is reasonable to assume that the labor union (better than the single worker) is able to know the firm's performance.

Finally, it is worth remarking that, with respect to the standard literature that investigates the rent sharing between union and firm (see authors as Blanchflower, Oswald), our model suggests another possible explanation of the empirical relation between wage and firm returns. In fact, in the cited literature agents' payoffs are related to each other, and the relative bargaining power determines the proportion between the labor union's payoff and the firm's payoff. On the contrary, in our model each agent has a monopolistic power in their own choice (the labor union sets wages and the firm sets employment), but the linkage between wages and firm's performance emerges because of labor union's preferences.¹⁷

3.3. The Economy

3.3.1. Baseline economy

The stylized economy is populated by four types of agents: the household, the firm, the labor union and the government. The firm owns capital and issues equities that households purchase to transfer income period by period. The households acquires utility from consumption. It is assumed that labor supply never limits the employment emerging from the bargaining process between labor union and firm, therefore the labor supply choice of the household is not analyzed. The labor union chooses the wage and the firm sets the employment level. The novelty with respect to the literature of the "right to manage models" is that labor union's relative preferences depend on firm's dividends. The production function includes capital, labor and technology. The latter follows a stochastic AR(1) process.

The economy is formalized following Danthine-Donaldson (2002b); in particular, we assume that the firm owns capital and maximizes the present discounted value of future cash flows (i.e. the dividends). This technique permits to evidence equities as a means of financing (for the firm) and of saving (for the household).

¹⁷In the versions with endogenous preferences, the model predicts a relationship between wages and dividends or profits similar to that estimated by Blanchflower et al.(1996)

3.3.1.1. Household. The decision problem of the representative household concerns consumption and investment into the equity shares, which represent the only way to save. The labor supply is managed by the labor union. It is assumed that the population distribution is reproduced in each household and that inside it, the same level of consumption is granted to each agent. Assume, next, that the utility function takes a CRRA form such as $\frac{c_t^{1-\gamma}}{1-\gamma}$ where c_t is the individual consumption flow and γ measures the consumer's relative risk aversion.

Representative household's problem is the following:

$$v(z_t, \lambda_t) = \max_{z_{t+1}, c_t} \left[\frac{c_t^{1-\gamma}}{1-\gamma} + \beta E_t v(z_{t+1}, \lambda_{t+1}) \right] \quad (3.1)$$

$$s.t. \quad c_t + q_t^e z_{t+1} = (q_t^e + d_t)z_t + w_t n_t + \bar{B}(1 - n_t) - \tau_t \quad (3.2)$$

$$c_t, z_{t+1} \geq 0 \quad (3.3)$$

where β is the subjective discount factor, q_t^e and d_t denote respectively the price of the equity security and the distributed dividend, and z_t is the share of the single equity that household owns. Quantity λ_t , next, represents the technology process and follows a stationary AR(1) process precised below. Quantities n_t and w_t represent, respectively, the employed labor and the wage rate; $(1 - n_t)$ measures unemployment.¹⁸ In each period the Government levies a lump sum tax τ_t and delivers unemployment benefits to unemployed workers equal to \bar{B} , under a balanced government budget constraint.

Under standard assumptions the necessary and sufficient first order condition for z_{t+1} reads

$$-q_t^e c_t^{-\gamma} + \beta v'_z(z_{t+1}, \lambda_{t+1}) = 0, \quad (3.4)$$

where $v'(z_t, \lambda_t) = (q_t^e + d_t)c_t^{-\gamma}$; leading the latter equation one period ahead $v'(z_{t+1}, \lambda_{t+1}) = E_t(q_{t+1}^e + d_{t+1})c_{t+1}^{-\gamma}$, substituting back into (3.4), and solving for the unique non-explosive solution leads to the following asset prices equation:

¹⁸Labor force is normalized to 1.

$$q_t^e = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left(\frac{c_t}{c_{t+j}} \right)^{\gamma} d_{t+j} \right\} \quad (3.5)$$

The eq.(3.5) shows how the price equities. The difference with the standard expression is that the employment and the capital accumulations differ from the Pareto-optimal choices. The maximization process leads to the following result:

$$c_t = E_t \left\{ \left(\frac{q_t^e}{\beta(q_{t+1}^e + d_{t+1})} \right)^{\frac{1}{\gamma}} c_{t+1} \right\} \quad (3.6)$$

The eq.(3.6) describes the way the representative household keeps consumption path in line with financial returns (more details to come in the subsequent sections).

3.3.1.2. Firm. The firm begins period t with the stock of capital k_t carried over from previous period, and the equity share outstanding $z_t = 1$. After observing the realization of the productivity process λ_t the proceeds of the output sale are used to pay the wage bill $w_t n_t$, to finance investment i_t under the knowledge of the equation of motion of capital stock $k_{t+1} = (1 - \delta) k_t + i_t$ and, residually, to pay dividends

$$d_t = y_t - w_t n_t - i_t, \quad (3.7)$$

where y_t represents the output. The production process follows a Cobb-Douglas production function that utilizes capital k_t , labor n_t and technology λ_t , which evolves as $\lambda_t = \rho \lambda_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$ with $\rho \in (0, 1)$.

In this setting of effectively complete markets, the firm's objective function is clear: maximize the pre-dividend stock market value of the firm, $d_t + q_t^e$ period by period.

More formally, the representative firm solves the following decision problem:

$$\mathcal{J}(k_t, \lambda_t) = \max_{n_t, k_{t+1}} [d_t + q_t^e] \quad (3.8)$$

$$s.t. \quad d_t + q_t^e = E_t \left(\sum_{\kappa=0}^{\infty} \beta^{\kappa} \left(\frac{c_t}{c_{t+\kappa}} \right)^{\kappa} d_{t+\kappa} \right); \quad (3.9)$$

$$y_t = \lambda_t k_t^{\alpha} n_t^{1-\alpha} \quad (3.10)$$

$$\lambda_t = \rho \lambda_{t-1} + \varepsilon_t \text{ with } 0 < \rho < 1, \varepsilon_t \sim N(0; \sigma_{\varepsilon}^2) \quad (3.11)$$

$$k_{t+1} = (1 - \delta)k_t + i_t, \quad (3.12)$$

where eq.(3.10) defines the production function where α and $(1 - \alpha)$ measure the elasticity of output with respect to capital and labor. The eq.(3.11) describes technology process and the eq.(3.12) is the law of motion of capital where δ is the depreciation rate of capital.

Formulation (3.8) requires that shareholders convey to the firm a complete listing of their future inter-temporal marginal rates of substitution. In the presence of complete markets setting and in a homogenous agent environment, there would be perfect unanimity vis-a-vis the information to be provided.¹⁹

Problem (3.8) admits an equivalent sequential formulation; it may be recursively expressed as:

$$\begin{aligned} \mathcal{J}(k_t, \lambda_t) &= \max_{n_t, i_t} (\lambda_t k_t^{\alpha} n_t^{1-\alpha} - w_t n_t - i_t) + \beta E_t \left(\frac{c_t}{c_{t+1}} \right)^{\gamma} \mathcal{J}(k_{t+1}, \lambda_{t+1}), \\ s.t. \quad k_{t+1} &= (1 - \delta)k_t + i_t; \lambda_t = \rho \lambda_{t-1} + \varepsilon_t \text{ with } 0 < \rho < 1, \varepsilon_t \sim N(0; \sigma_{\varepsilon}^2) \end{aligned}$$

The necessary and sufficient first order conditions are

$$w_t = (1 - \alpha) \lambda_t k_t^{\alpha} n_t^{-\alpha} \quad (3.13)$$

$$\beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^{\gamma} (\alpha \lambda_t k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + (1 - \delta)) \right] = 1. \quad (3.14)$$

¹⁹Alternatively, the shareholders could appoint one of their own members to manage the firm, knowing that his preference for future consumption is an exact representation of their own.

Eq.(3.13) suggests that the firm hires labor services so that wage equals marginal productivity of labor; eq.(3.14) is the Euler equation that determines the optimal level of investment. By construction, our firm discounts the future flow of dividends by the consumer discount factor.

3.3.1.3. Labor unions. Our economy is characterized by a monopolistic labor union, a variant of the “right to manage” models.²⁰ The bargaining process is modelled as a static Stackelberg game where the union is the leader and the representative firm is the follower. The union sets the wage knowing the firm’s labor demand schedule, and the firm reads off from the labor demand curve the number of workers to hire at the union wage.

As in Chiarini-Piselli (2005) we assume that labor union’s objective includes two components: employment and “wage rent”. The latter of the two denotes the difference between actual wage and union’s floor wage, B_t , that we can represent as the unemployment benefit. It is assumed that B_t is exogenous and constant over time. The novelty of our model is that labor union’s relative preference between employment and wage rent depends on the firm’s financial performance. In the following sections we propose different quantities which measures firm’s financial performance.²¹ For the moment we maintain an implicit form for the preference function.

Formally, the representative labor union’ maximization problem reads:

$$\mathcal{U} = \max_{w_t} (w_t - \bar{B})^{g(\cdot)} n_t^{1-g(\cdot)} \quad (3.15)$$

$$s.t. \quad n_t = ((1 - \alpha)\lambda_t)^{\frac{1}{\alpha}} k_t w_t^{-\frac{1}{\alpha}} \quad (3.16)$$

$$0 \leq g(\cdot) \leq 1, \quad g'(\cdot) > 0 \quad (3.17)$$

$$B_t > 0 \quad (3.18)$$

²⁰Oswald (1982).

²¹The results are not modified while considering an intertemporal objective function of unions, as expressed in in Maffezzoli (2001), if it is assumed that (i) the services of physical capital and labor are to be purchased in each period; (ii) pre-commitment is ruled out; (iii) the union takes the rental rate as given. Thereby unions do not internalize the effects of present wage on capital accumulation. The same argument explains why unions do not consider the effects on dividends.

Defining $h(\cdot) = \frac{g(\cdot)}{1-g(\cdot)}$ as the relative weight of wage rent in labor union's preference, it follows that it increases in dividends (in fact $\frac{\partial h(\cdot)}{\partial \cdot} = \frac{g'(\cdot)}{(1-g(\cdot))^2} > 0$). It can be showed that the labor union optimization problem is well behaved; given that, the necessary and sufficient first order condition with respect to wage rate w_t is:

$$w_t : g(\cdot) (w_t - B_t)^{g(\cdot)-1} w_t^{-\frac{1-g(\cdot)}{\alpha}} - \frac{1-g(\cdot)}{\alpha} (w_t - B_t)^{g(\cdot)} w_t^{-\frac{1-g(\cdot)}{\alpha}-1} = 0. \quad (3.19)$$

Proposition 1 below derives the optimal contract set by the labor union; the concept of optimality refers to the maximization of the labor union welfare function (3.15):

Proposition 1. *The optimal wage contract imposed by labor union is the following:*

$$w_t^* = \left(\frac{1-g(\cdot)}{1-(1+\alpha)g(\cdot)} \right) B_t. \quad (3.20)$$

Proof. See the Appendix. □

Eq.(3.20) suggests that the labor union chooses a wage that is a growing function of the unemployment benefits and the indicator of firm's performance. The first relationship has been largely highlighted in labor market literature; here, we limit to suggest that an increase in subsidies can be interpreted as a reduction in the cost of being unemployed, shifting preferences toward wages. Moreover, it clearly emerges how the indicator of firm's performance can affect wage claims. This is in accordance to the cited literature. To solve the model and run simulations, we specify the argument of function $g(\cdot)$. It will be done in Section (3.4.1).

3.3.1.4. Government. The government simply collects lump sum tax revenue and returns it to the households as unemployment benefits. It follows that in each period the government's balance is in equilibrium; formally:

$$\int B_{i,t}(1 - n_{i,t})di = \int \tau_{i,t}di \quad \forall t, \quad (3.21)$$

where i identifies the single shareholder-worker.

3.4. Calibration and quantitative analysis

3.4.1. Labor Union's preferences and firm's financial performance

This section investigates how the model responds to technology shocks, which represent the only source of uncertainty in this environment. To do this it is necessary to elicit the method in which fairness enters in labor relations. In particular, this section compares the outcomes of two different versions of the model. The possible schemes of the labor union preferences are:

- "Fair wages": under such scheme we include two possible specifications. In the first case (the benchmark model version), the hypothesis is that the union considers dividends as the indicator of the firm's performance. In this case function $g(\cdot)$ is increasing in d , and it can be expressed in the following way: $g(\cdot) = g(d_t) = (\eta d_t)^\theta$. Assuming that the union's relative preference for wage is positively related to the firm's dividends, implies that the firm is encouraged to invest because it reduces wage claim. This effect is reinforced by the fact that, in this model, the firm has no strategic choice about capital structure and dividend distribution. In fact, there is no possibility to run into debt and firm liquidity is entirely distributed as dividends. Consequently, the firm deals with a stringent trade-off between investment and dividend: an increase in investment surely reduces resources distributed as dividends. The second case (the alternative model version), assumes that the union considers profit as the indicator of firm's performance. We define profit as the difference between revenue and (labor and capital) costs, so $\pi_t = y_t - w_t n_t - \delta k_t$.²² The g function can be expressed in the following way: $g(\cdot) = g(\pi_t) = (\eta \pi_t)^\theta$.²³ In this case the investment choice does not produce immediate effects on labor union attitude.
- "Sticky wages": this scheme assumes that union's relative preference between wage and employment is a constant parameter, i.e. not depending on the state of

²²The interest rate on capital stock is not included in costs because the firm owns capital.

²³Notice that in steady state $d = \pi$. We set $\bar{g} = d = \pi$ so that the different schemes have the same steady state solutions.

the firm. Technically speaking the objective function ($\mathcal{U}_t = (w_t - \bar{B})^{g(\cdot)} n_t^{1-g(\cdot)}$) is no longer state dependent. In this case $g(\cdot) = \bar{g}$. This preference structure is common in labor union literature.²⁴

3.4.2. Calibration

The model is parameterized for the United States Economy for the postwar period. The system of equations we use to compute the dynamic equilibrium of the model depends on a set of **nine** parameters. **Two** pertain to household preferences, (the relative risk aversion coefficient γ , the subjective discount factor β), **one** to the structural-institutional context (the unemployment benefit B), **two** to labor union (the linear and exponential coefficients η and π linking the firm indicator to union's relative preference) and the remaining **four** parameters to technology (the capital share α , the capital stock quarterly depreciation rate δ , the autoregressive coefficient of the technology process ρ and the standard deviation of technology shock σ_ε).

Shareholder-worker's preference (β, γ): the subjective discount factor β is set to correspond to an annual real interest rate of 4% ($\beta = 0.99$). The relative risk aversion γ is set equal to 3. This is a low and plausible value.

Technology ($\delta, \alpha, \rho, \sigma_\varepsilon$): parameters are set to commonly used values in this literature (e.g. Lettau, 2003 and Sill, 2005). More precisely, we set, $\delta = 0.025$, $\alpha = 0.33$, $\rho = 0.9219$. The standard deviation of exogenous productivity shock σ_ε is calibrated to match the GDP standard deviation. Quantitatively speaking, σ_ε ranges between 0.2835 per cent and 0.7 per cent. Notice that these are lower than standard value used in this literature (0.72 per cent). That is due to the fact that the model generates sufficiently high endogenous propagation of exogenous shocks.

Labor union preferences (η, θ): η is a scale parameters that affect the deterministic steady state of the model. Numerical simulations confirm that this parameter does not modify the dynamic properties. It is calibrated to match the equilibrium employment of the model to the mean value of the post-war US employment rate (equal to 59.2% in

²⁴See Pencavel (1984) or Chiarini-Piselli (2005)

the period January 1948 - October 2007, source BLS monthly data seasonally adjusted). Starting from the equation determining the deterministic steady state value of N , we solve the equation for η in the following way $\eta = \frac{\left(\frac{w-B}{(w(1+\alpha)-B)}\right)^{\frac{1}{\theta}}}{\left(\alpha\lambda\left(\frac{k}{n}\right)^{\alpha}-\delta\left(\frac{k}{n}\right)\right)n}$.²⁵ Notice that all elements are determined in the model independently of η : λ , θ , α and δ are exogenously fixed, $\frac{K}{N}$ comes from the Euler's equation and W and N emerge from the bargaining problem and B is linked to W . Finally, the parameter θ determines the way the dynamics of firm indicator influences the dynamics of the union relative preferences. This is a free parameter in the model. Plausibly calibrated values ranges between $\theta = 0.0$ and $\theta = 0.0052$. A low θ (i.e. $\theta = 0.0012$) represents a case in which the firm indicator has a low effect over union relative preferences. The best fit is for $\theta = 0.005$.

Subsides (B) is set to a fifth of the steady state wage (more precisely to $B = 0.19 \cdot W^*$) as it emerges in OECD (1994).

All parameters with corresponding values are included in Table 3.1.

TABLE 3.1: PARAMETRIZATION

β	0.99
γ	3
δ	0.025
α	0.33
ρ	0.9219
σ_{ε}	0.2835
η	$\frac{\left(\frac{w-B}{(w(1+\alpha)-B)}\right)^{\frac{1}{\theta}}}{\left(\alpha\lambda\left(\frac{k}{n}\right)^{\alpha}-\delta\left(\frac{k}{n}\right)\right)n}$
θ	0.0050
B	$0.19 \cdot W$

²⁵Later on, the capital letters indicate the aggregate variables.

3.4.3. Results

Technically, we simulate the different model versions (1.000 simulations of time series of 111 observations following Uhlig, 1999). In the final part of this section we report the impulse response functions of dividends under the different model versions in order to elicit the dynamics of the stylized economy.

Table 3.2 compares selected moments of real and financial variables, which refer to the U.S. postwar economy with the results of simulations. The table shows that the benchmark scheme fits empirical data quite well. Under this scheme the model endogenously generates sufficient GDP volatility and consistent equity premium. The other schemes are characterized by a weaker propagation mechanism and they predict inconsistent results on the financial side. To explain the differences between the simulations, it is firstly opportune to identify the characteristics of the benchmark scheme that generate high excess return. Subsequently, it will be simpler to explain why the other schemes do not perform well.

TABLE 3.2: SELECTED EMPIRICAL AND SIMULATED DATA

\		US economy(*)	benchmark	sticky wage	alternative
risk free	mean	0.80	1.61	3.42	3.65
	std. dev	5.67	5.62	3.11	2.20
equity returns	mean	6.98	6.23	4.82	4.43
	std. dev	16.54	22.32	12.52	8.79
equity premium	mean	6.18	4.16	1.41	0.78
	std. dev	16.67	24.98	13.17	9.25
Sharpe ratio		0.25	0.21	0.11	0.09
y	std. dev	1.76	1.78	1.06	0.70
c	std. dev	1.29	0.97	0.65	0.40

In order to understand where the equity premium comes from, and how it is related to the labor union attitude during business cycles, it is useful to recall the equilibrium wage equation, (i.e. eq.(3.20)) which is reported below.

$$w_t^* = \left(\frac{1 - g(\cdot)}{1 - (1 + \alpha)g(\cdot)} \right) B_t$$

The labor union's preference structure and the bargaining process generate a positive relationship between the wage and the chosen indicator of the firm's performance. The benchmark scheme assumes that the relative preferences of the labor union depend on the firm dividends. This means that the union links wage claims to the resources distributed to shareholders (i.e. the resources not utilized in firm production). So what happens after a positive technology shock? Both output and investment increase; dividends fall because the increase in investment more than offsets the increase in revenue (recall the time lag between the purchase and the actual use of capital). Consequently, the counter-cyclical behavior of dividends, modifying the relative preferences of the labor union, moderates the wage claim. We are drawing near the key point. In this case, the stylized economy experiences, at the same time, the increase in input-factor productivity and the moderation in wage claims. The combination of these elements amplifies the reaction of the firm to the productivity shock.²⁶ The resulting volatility of dividends imposes that in equilibrium equities must pay high premium to remunerate the riskiness of the asset. This mechanism explains why the benchmark scheme is capable of generating consistent output volatility and consistent equity premium despite the use of low exogenous volatility. The dynamics are completely reversed in the alternative version of the fair wage scheme. Here, the wage claim is linked to a pro-cyclical variable, i.e. profits. This implies that after a positive technology shock the relative preferences of labor union change, thus favoring wage. In this scenario, the firm has to afford higher wage claims when productivity is high. Sustaining pro-cyclical costs, the reaction of the firm to a productivity shock would be dampened. The resulting dividends are slightly volatile and the equity has to remunerate a low degree of riskiness. Finally, the sticky wage scheme represents an intermediate case characterized by constant wage during the business cycle. Thus, it follows that the simulated real and financial variables assume intermediate values with respect to the other two model

²⁶Wage moderation induces to employ more labor services and this increases the productivity of capital (with Cobb-Douglas production function the marginal productivity of a factor depends positively on the relative presence of the other production factors). This stimulates investment, and so a further reduction in firm liquidity. It follows a further reduction in wage and so on. Under our parameterization the mechanism converges to the equilibrium allocation; endogenous growth dynamics are excluded.

versions. Notice that previous mechanisms work in the same way in the event of a negative technology shock.

The mechanism is confirmed by the following impulse response functions of dividends under the three model versions, as reported in Figure 3.1. The highest response emerges under the benchmark version of the fair wage scheme.

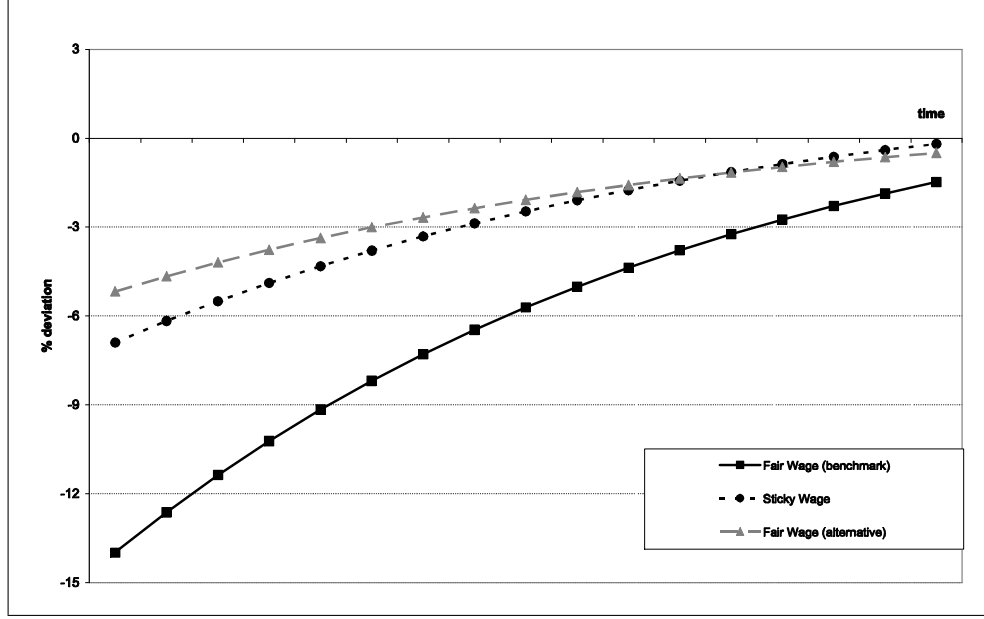


Figure 3.1. Impulse response functions of investment under different values of θ .

3.4.4. Sensitivity Analysis

We have previously analyzed the relevant choice of the indicator of firm's performance, concluding that when the wage claim is linked to dividends the model predicts a strong propagation mechanism and consistent equity premium. Now, we investigate how the dynamics of the benchmark model version change according to the value of θ . This parameter measures the strength of the linkage between the wage claim and the selected indicator of firm's performance. Maintaining the previous calibration for the other parameters, we simulate the benchmark version of the model with three different values of θ , 0.0052, 0.0032 and 0.0012. Results are reported in Table 3.3.

TABLE 3.3: SENSITIVITY ANALYSIS WITH RESPECT TO θ

benchmark scheme		$\theta = 0.0052$	$\theta = 0.0032$	$\theta = 0.0012$
risk free	mean	1.56	2.62	3.20
	std. dev	5.61	4.33	3.47
equity returns	mean	6.33	5.42	5.01
	std. dev	22.39	17.43	14.00
equity premium	mean	4.77	2.79	1.82
	std. dev	23.54	18.34	14.73
Sharpe ratio		0.21	0.16	0.13
y	std. dev	1.77	1.40	1.16
c	std. dev	0.99	0.80	0.69

The table shows that the propagation mechanism is increasing in θ . This indicates that the stronger is the relevance of the firm in the union preferences, the higher is the response of the firm to technology shock. Such a result is consistent with the mechanism explained in the previous section. In the benchmark scheme, the counter-cyclical behavior of firm liquidity generates counter-cyclical pressure on wage claims. This enhances the firm to strongly modify the employment of both input factors. This effect is reasonably stronger when the influence of firm liquidity on union preferences is higher. This explains the positive linkage between the equity premium and θ .

3.5. Conclusions

Our research is related to Danthine and Donaldson's contributions (2002a, 2002b, 2005). They analyze how the contracting structure between workers, delegated managers and firm owners influences the volatility of residual claimants' revenue. Workers desire a sort of insurance against income fluctuations, meanwhile delegated managers try to hold high dividends when dividends represent the majority of their income. The first mechanism tends to increase dividends volatility and, by this way, gives a possible explanation of equity premium. The second one could generate under-investment, and a sort of smoothness in dividends' path. Our model suggests a completely different mechanism to explain the excess return rate of the equities with respect to the risk free rate. Union's preferences and the complementarity between input factors are the most important elements driving

the dynamics of the model. When a firm decides to invest, it suffers a reduction in cash flow that induces unions to freeze wage claims. The reduction of the labor cost pushes to hire more workers, and because of the complementary between inputs, generates a further increase in investment. The described dynamics amplify the volatility of dividends. It follows that in equilibrium the excess return has to be sufficiently high to remunerate the augmented volatility of firms' revenues.

This paper formulates and calibrates an equilibrium model for a non-Walrasian Economy. Labor market is influenced by monopolistic labor unions that set wages while considering firms' dividends (or available liquidity). In this framework, investment strongly responds to technology shocks and generates high volatility to firm's returns. Such dynamics explain high equity premium similar to that emerging from empirical studies.

The model follows a research area that studies how the features of the labor market affect the performances of real and financial markets. Under specific assumptions regarding the labor union's preference, the model replicates selected financial features quite well. A peculiarity of our contribution is that the volatility of firm returns is not generated by some type of stickiness or adjustment costs that generally produce "leverage" effects. Instead, the volatility derives from optimizing choices of firms operating with labor costs linked to their financial performances. In particular, we have showed that when the attitude of the labor union changes according to the quantity of resources distributed to shareholders the responses to technology shocks are amplified. Therefore, the introduction of fairness-reciprocity elements allows enlarging the analysis of labor relations. This enlargement includes contributions coming from sociological and institutional analyses, and it offers reasonable explanations to quantitative issues of the financial market.

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Appendix

A. Concavity of Labour Union's Objective Function

Proof of Proposition 1. The first order condition the Union's problem is:

$$g(\cdot) (w_t - \bar{B})^{g(\cdot)-1} w_t^{-\frac{1-g(\cdot)}{\alpha}} - \frac{1-g(\cdot)}{\alpha} (w_t - \bar{B})^{g(\cdot)} w_t^{-\frac{1-g(\cdot)}{\alpha}-1} = 0,$$

that can be reduced to:

$$g(\cdot) w_t - \frac{1-g(\cdot)}{\alpha} (w_t - \bar{B}) = 0.$$

Then, the equilibrium wage is

$$w_t^* = \left(\frac{1-g(\cdot)}{1-(1+\alpha)g(\cdot)} \right) \bar{B},$$

that we impose to hold with:

$$1 - (1 + \alpha) g(\cdot) > 0. \quad (3.22)$$

The second order condition is given by:

$$g(\cdot) - \frac{1-g(\cdot)}{\alpha} < 0,$$

that is always true if eq.(3.22) holds. □

B. Equilibrium Equations

- (1) $B_t(1 - N_t) = \bar{B}(1 - N_t) = T_t$
- (2) $C_t = D_t + W_t N_t + (1 - N_t) B_t - T_t$
- (3) $K_{t+1} = (1 - \delta)K_t + I_t$
- (4) $N_t = ((1 - \alpha)\lambda_t)^{\frac{1}{\alpha}} K_t N_t^{-\frac{1}{\alpha}}$
- (5) $Y_t = \lambda_t K_t^\alpha N_t^{1-\alpha}$
- (6) $W_t = \left(\frac{1-(\eta D_t)^\theta}{1-(1+\alpha)(\eta D_t)^\theta} \right) \bar{B}$
- (7) $D_t = Y_t - W_t N_t - I_t$
- (8) $R_t = \alpha \lambda_t K_t^{\alpha-1} N_t^{1-\alpha} - \delta$
- (9) $\beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^\gamma (\alpha \lambda_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + (1 - \delta)) \right] = 1$

$$(10) \ln \lambda_{t+1} = \rho \ln \lambda_t + \varepsilon_t$$

C. Steady State

Proof. Starting from the Euler equation

$$\frac{K}{N} = \left(\frac{\alpha \lambda}{\frac{1}{\beta} - (1 - \delta)} \right)^{\frac{1}{1-\alpha}}$$

The wage is determined along the firm's labor demand

$$W = (1 - \alpha) \lambda \left(\frac{K}{N} \right)^\alpha$$

From the bargained wage we determine dividend

$$D = \left(\frac{W - B}{(W(1 + \alpha) - B)} \right)^{\frac{1}{\theta}} \frac{1}{\eta}$$

and from the definition of dividend we find employment and then capital and output

$$N = \frac{D}{\left(\alpha \lambda \left(\frac{K}{N} \right)^\alpha - \delta \left(\frac{K}{N} \right) \right)}$$

$$K = \left(\frac{K}{N} \right) N$$

$$y = \lambda K^\alpha N^{1-\alpha}$$

□

D. Calibration

Proof. Starting from the definition of dividend we elicit the following relation between employment and dividend

$$N = \frac{D}{\left(\alpha \lambda \left(\frac{K}{N} \right)^\alpha - \delta \left(\frac{K}{N} \right) \right)}$$

Substituting the steady state equation of dividend and solving for η , it follows

$$\eta = \frac{\left(\frac{W-B}{(W(1+\alpha)-B)} \right)^{\frac{1}{\theta}}}{\left(\alpha \lambda \left(\frac{K}{N} \right)^\alpha - \delta \left(\frac{K}{N} \right) \right) N}$$

□

E. Dynamics Equations

- (1) $0 = -\tilde{C}_t + \frac{D}{C}\tilde{D}_t + \frac{WN}{C}\tilde{N}_t + \frac{WN}{C}\tilde{W}_t$
- (2) $0 = -\tilde{I}_t + \frac{1}{\delta}\tilde{K}_{t+1} - \frac{1-\delta}{\delta}\tilde{K}_t$
- (3) $0 = -\tilde{N}_t + \frac{1}{\alpha}\tilde{\lambda}_t - \frac{1}{\alpha}\tilde{W}_t + \tilde{K}_t$
- (4) $0 = -\tilde{Y}_t + \tilde{\lambda}_t + \alpha\tilde{K}_t + (1-\alpha)\tilde{N}_t$
- (5) $0 = -\tilde{D}_t + \frac{1}{\theta} \left(\frac{\alpha\eta BW}{(W(1+\alpha)-B)\eta(W-B)} \right) \tilde{W}_t$
- (6) $0 = -\tilde{D}_t + \alpha\frac{Y}{D}\tilde{Y}_t - \frac{I}{D}\tilde{I}_t$
- (7) $0 = -\widetilde{RR}_t + \frac{1-(1-\delta)\beta}{1-\beta}[\tilde{Y}_t - \tilde{K}_t]$
- (8) $0 = -\gamma\tilde{C}_t + \gamma\tilde{C}_{t+1} - \beta\alpha\frac{Y}{K} \left(\tilde{Y}_{t+1} - \tilde{K}_{t+1} \right)$